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Frascati resummation schemes for IR radiation in QED and QCD

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Summary. — As part of the 70th birthday celebration of our dear friend Mario Greco, a brief history of the development of radiative effects at Frascati in QED and QCD covering the period between late sixties to the late eighties is presented. During these two decades, two parallel resummation schemes for QED were formulated and radiative corrections were made for the J/Ψ production and later for Z^0 production. The schemes were then extended to QCD to obtain realistic estimates of the e^+e^- hadronic cross-sections at all energies. It led to the introduction of several new concepts: frozen α_s and later to singular α_s and to the notion of the maximum value of transverse energy allowed in a radiative process. Some other fall outs from this research such as transverse momentum distributions and hadronic total cross-sections which are valid even now would be briefly touched upon.

PACS 13.85.Lg – Total cross sections.
PACS 13.60.Hb – Total and inclusive cross sections (including deep-inelastic processes.

1. – Resummed QED radiation

In 1960, Bruno Touschek had the seminal idea of making an electron positron machine. It led first to the construction at Frascati of ADA (in 1961, with an energy of 250 MeV) and later to ADONE (with an energy of 3 GeV). With the advent of ADONE, a sizeable theory group developed at Frascati, whose members in 1966, were Giovanni De Franceschi, Paolo Di Vecchia, Francesco Drago, Etim Etim, Giancarlo Rossi, Mario Greco and GP, one of the authors of this note (see fig. 1). Touschek understood that a proper quantitative analysis of the experimental results from ADONE (or any other e^+e^- colliding beam) necessitated precise computations of resummed finite radiative corrections. His philosophy being "We must do the administration of the radiative corrections to electron positron experiments". In his words, "We must earn our bread and butter" [1].

Fig. 1. – Mario Greco and YS in Frascati in 1972.

The first scheme towards infra-red (IR) radiative correction to the cross-section of a process through resummation was initiated by Touschek [2] through a relativistic formulation of the Bloch-Nordseick Theorem, Poisson statistics and energy-momentum conservation. The resummed probability distribution of a 4-momentum loss K^μ carried off by soft photons in a process was obtained as

$$\mathrm{d}^4P(K) = \left[\frac{\mathrm{d}^4K}{(2\pi)^4}\right] \int \mathrm{d}^4x \exp[-h(x) + iK \cdot x], \tag{1}$$

where h is defined in terms of the mean number of photons $\bar{n}(k)$ as

$$h(x) = \int \mathrm{d}^3\bar{n}(\mathbf{k})\big(1 - \exp[-ik \cdot x]\big). \tag{2}$$

If only the distribution in ω, the total energy lost through IR radiation is required, the closed form expression is

$$\mathrm{d}P(\omega) = \left[\frac{\mathrm{d}\omega}{2\pi}\right] \int_{\infty}^{\infty} \mathrm{d}t \exp[i\omega t - h(t)] = \mathcal{N}(\beta)\beta\frac{\mathrm{d}\omega}{\omega}\left(\frac{\omega}{E}\right)^\beta, \tag{3}$$

Fig. 2. – Mario Greco with the author (YS) and Guido Altarelli at Accademia dei Lincei, circa 1974.

where $\mathcal{N}(\beta)$ is a calculable normalization factor and β for electrons and positrons is given by

$$\beta = \frac{4\alpha}{\pi}\left(\log\frac{2E}{m_e} - \frac{1}{2}\right). \tag{4}$$

Unlike perturbation theory to any finite order, the resummed distribution in eq. (3) is integrable. If ΔE is the maximum allowed energy loss, the IR radiative correction factor to the cross-section is given by

$$\mathrm{d}\sigma = \mathcal{N}(\beta)\left(\frac{\Delta E}{E}\right)^{\beta}\mathrm{d}\sigma_0. \tag{5}$$

The method was further extended for resonant processes [3]. The width of the resonance provides an intrinsic cut-off and Γ replaces ΔE, provided $\Gamma \ll \Delta E$ (as is the case for J/Ψ) and for this case, a radiative tail also develops.

An equivalent very successful second scheme, also suggested by Touschek, —focused on correcting the scattering amplitude itself rather than the measurable cross-section— was developed by Greco and Rossi [4]. Both schemes were employed to do extensive and precise radiative corrections for the J/Ψ resonance and then for the Z^0 resonance [5].

2. – QCD radiation

During the period that QCD with its quarks and gluons was becoming the accepted theory behind hadrons largely thanks to the theoretical notion of asymptotic freedom, and the experimental observation of jets at high energies in e^+e^- and $pp/p\bar{p}$ reactions, theoretical need arose for saying something reasonable about the unknown non-perturbative IR "slavery" region of QCD. An immediate problem at hand was a description of the experimentally measured $e^+e^- \to$ *hadrons* at the initial energy $\sqrt{s}$. With point like quarks and hadrons, the famous ratio $R(s)$ was written down as

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\left[\sum_i Q_i^2\right][1 + \alpha_s(s)/\pi]\,, \tag{6}$$

where Q_i denotes the charge (2/3 for an up quark and $-1/3$ for a down quark), and $\alpha_s(s)$ is the QCD running coupling constant, whose AF expression is given by

$$\alpha_{AF}(s) = \frac{12\pi}{(33-2n_f)\ln(s/\Lambda^2)}\,. \tag{7}$$

As the threshold of a given quark channel is crossed, its contribution is included in the sum leading thereby to steps in the cross-section for charm and later bottom quark thresholds. What about the lower energy region which should give an average value of 2, in the free quark model due to the almost massless u, d, s quarks? Would there be an enhancement due to some average value of α_s term as in eq. (6)? Since theoretically eq. (7) is not expected to be valid at low energies, and experimentally the low energy cross-section has much variation due to low energy resonances $(\rho, \omega, \phi, \ldots)$, some device has to be found to theoretically estimate the QCD correction in this region and another to compare it to the experimental value. The technique adopted in [6] was to assume that α_s is "frozen" at low energies:

$$\alpha_{frozen}(s) = \frac{12\pi}{(33-2n_f)\ln(a + s/\Lambda^2)}\,, \tag{8}$$

where a is a constant (see fig. 2). To smooth out the ups and downs from resonances, a zeroth moment of the experimental $R(s)$ was constucted

$$M(\bar{s}) = \int_0^{\bar{s}} (\mathrm{d}s) R_{expt}(s). \tag{9}$$

From fig. 3, an average value of $R_{light} \approx 2.4$ was deduced. Also, the thresholds of various flavours being sharply delineated, allowed the authors to obtain an effective charm quark mass $m_c \approx (1.45 \pm 0.5)\,\mathrm{GeV}/c^2$. By the way, it also provided a neat direct check of semi-local quark-hadron duality for light and heavy quarks.

Over the years, IR behavior of α_s would become a crucial widely discussed topic and singular but integrable versions would be proposed [7]

$$\alpha_{IR}(s) \to_{s \ll s_0} (s_0/s)^p, \tag{10}$$

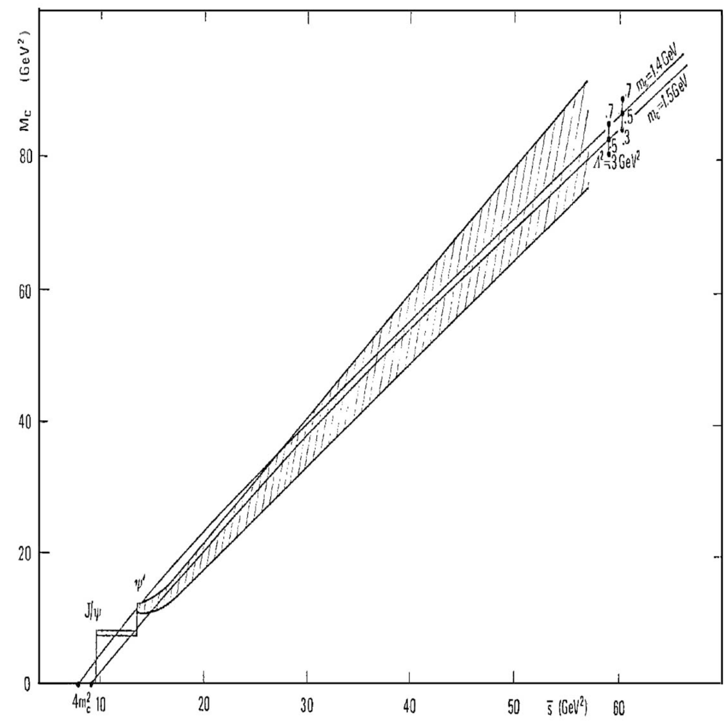

Fig. 3. – Zeroth moment of $R(s)$.

with $1/2 \leq p < 1$: the upper limit $p = 1$ being the Wilson value whereas the lower limit $p = 1/2$ corresponding to confinement but just. (α_{frozen} corresponds to $p \to 0$, which of course does not lead to confinement.)

The coherent state formalism for IR resummation in QED was successfully extended to QCD [8]. Very soon compact expressions for various jet processes in QCD were obtained using the coherent state formalism [9,10], see fig. 5.

3. – Two applications of the QCD radiation

When transverse momentum distributions were analyzed through α_{frozen}, an abitrary "intrinsic" transverse momentum had to be introduced. On the other hand, the singular version in eq. (10), was shown to generate spontaneously such a term with quite satisfactory results [7] as can be seen in fig. 4.

For the p_t- distribution for Drell-Yan pairs and other proceeesses, a very useful concept of the maximum transverse momentum Q_{max} was intoduced by Chiapetta and Greco [11]. The quantity $Qmax$ along with an IR singular α_s have been used by us extensively in our later papers on soft-gluon resummation in total and inelastic hadronic cross-sections [12]. Recent data on inelastic cross-sections at $\sqrt{s} = 7\,\text{TeV}$ from LHC have been successfully analyzed using this formalism.

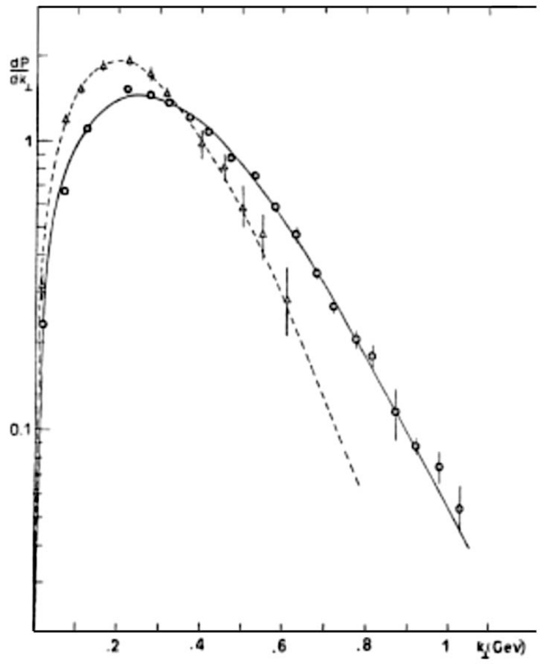

Fig. 4. – "High" transverse momentum distribution circa 1979 [7].

Fig. 5. – Mario Greco with GP.

4. – Coda

May we all meet at our 90th birthday.

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