IL NUOVO CIMENTO Vol. 34 C, N. 4 Luglio-Agosto 2011
DOI 10.1393/ncc/i2011-10951-5

Colloquia: Channeling 2010

Self-amplified Cherenkov radiation from relativistic particles in layered dielectric-filled waveguide

L. Sh. Grigoryan(*), H. F. Khachatryan and S. R. Arzumanyan

Institute of Applied Problems in Physics - 25 Hr. Nersessian Str., 0014 Yerevan, Armenia

(ricevuto il 22 Dicembre 2010; pubblicato online il 29 Agosto 2011)

Summary. — The radiation from a charged particle uniformly moving along the axis of cylindrical waveguide filled with semi-infinite layered dielectric material that weakly absorbs the radiation is investigated. Expressions for calculation of the spectral distribution of the total energy of radiation passing through the transverse section of a waveguide at large distances from the boundary of a layered medium are derived with no limitations on the amplitude and variation profile of the layered medium permittivity. The results of numerical calculations for emission of Cherenkov radiation (CR) in the layered material consisting of dielectric plates alternated with vacuum gaps are given. It is shown that in some special cases CR from a relativistic particle-in-flight from the layered medium to the vacuum at a single waveguide mode may prove to be many times as strong as CR in the waveguide filled with semi-infinite solid dielectric without vacuum gaps. The visual explanation of this effect is given and a possible application is discussed.

PACS 61.80.Fe – Electron and positron radiation effects.
PACS 41.60.Bq – Cherenkov radiation.
PACS 07.57.Hm – Infrared, submillimeter wave, microwave, and radiowave sources.

1. – Introduction

Modern electron accelerators show great potential as tools for production radiation in the terahertz spectral region that is of considerable interest for applications in physics, chemistry and biology (see, *e.g.*, [1]). A typical scenario involves the passage of an electron beam through a perturbing element such as a bend magnet, undulator, stack of plates, or another structure that causes it to radiate. The structure under our study is a cylindrical waveguide filled with a dielectric material that weakly absorbs the radiation. At the flight of a relativistic charged particle (*e.g.*, an electron) along the axis of the waveguide, it may generate Cherenkov radiation (CR) inside the dielectric material.

(*) E-mail: levonshg@mail.ru

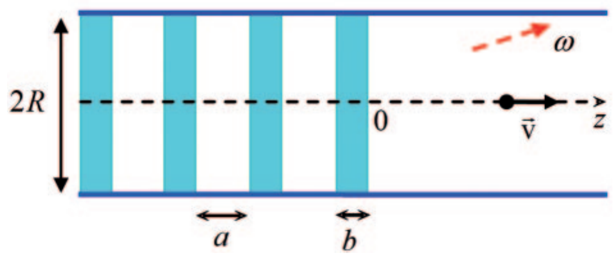

Fig. 1. – The radiation from a relativistic charged particle uniformly moving along the axis of a cylindrical waveguide filled with a semi-infinite stack of dielectric plates. The particle emerges from the stack of plates to the vacuum. The period of the layered medium is $l = a + b$.

In 2009 Rosenzweig with co-authors [2] reported the fist direct observation of narrow-band THz coherent CR driven by a subpicosecond electron bunch traveling along the axis of a hollow cylindrical dielectric-lined waveguide. Now the authors of [2] "consider structure variations that would give improved results". The purpose of this paper is to develop appropriate theoretical grounds that would permit formulating in the future the recommendations to modify the experimental set-up presented in [2]. According to our preliminary estimates this kind of modernization may considerably increase the value of peak power of THz coherent CR.

In the present work the motion of a relativistic particle along the axis of a cylindrical waveguide filled with a semi-infinite layered dielectric material (*e.g.*, a semi-infinite stack of plates alternated with vacuum gaps) is considered. We try to show how in this system the CR generated by relativistic particle may be self-enhanced at the waveguide presence. In its absence the radiation from a relativistic particle generated in a semi-infinite medium is not self-enhanced [3, 4]. In 1996 and 1997 Wiedemann with co-authors [5, 6] observed a stimulated transition radiation from a chain of relativistic electron bunches of sub-picosecond duration. Later on a similar effect of self-stimulation (self-amplification) for CR from a relativistic particle rotating about a dielectric ball was theoretically investigated in [7] as well as for synchrotron radiation from a relativistic particle rotating in a spherical cavity inside a homogeneous and transparent dielectric material in [8]. Now we report the self-amplification of CR from a relativistic particle inside a layered dielectric-filled waveguide.

2. – Formulation of the problem

Let us consider the uniform motion of q charge travelling with v velocity along the axis of R radius in an ideal cylindrical waveguide loaded with semi-infinite layered material (see fig. 1 where the case of a waveguide filled with a semi-infinite stack of plates is shown). Let us suppose, the Z axis of the cylindrical system of coordinates r, φ, z is directed along the waveguide axis and the charge travels from a semi-infinite layered medium that weakly absorbs the radiation ($z < 0$, range 1) to the vacuum ($z > 0$, range 2) so that $v_z = v > 0$. The plane $z = 0$ is selected as an interface between the layered medium and the vacuum.

We shall assume that the permittivity ε_1 and permeability μ_1 of the layered medium are independent of the transverse coordinates r, φ, but alternate along the direction of the particle motion according to the arbitrary law

$$\varepsilon_1(z-l)=\varepsilon_1(z), \qquad \mu_1(z-l)=\mu_1(z) \tag{1}$$

(l—the period of layered medium). The permittivity ε and permeability μ of all the system may be written in the form

$$\begin{aligned} &\varepsilon(z)=\varepsilon_1(z), && \mu(z)=\mu_1(z) && \text{when} && z<0 && \text{(range 1)}\\ &\varepsilon(z)=1, && \mu(z)=1 && \text{when} && z>0 && \text{(range 2).} \end{aligned} \tag{2}$$

Consider the energy of radiation

$$W=\sum_{n=1}^{\infty}\int_{\omega_n}^{\infty} I_n^{(2)}(\omega)\,\mathrm{d}\omega \tag{3}$$

traversing the waveguide cross-section in the $z=z_0$ point, that is distant from $z=0$ and located in range 2 during all the time of particle motion. In (3) the allowance was made of the fact that the radiation is bounded by bandwidths (8) due to the presence of conducting walls of the waveguide, ω_n is the lower boundary of the n-th bandwidth of the waveguide, and $I_n^{(2)}(\omega)$ is the spectral distribution of the radiation energy of particle in this bandwidth, in the vacuum, that propagates along the direction of particle motion.

The aim of the present work is to identify, analyze, and clear explain the cases when both waveguide and periodical structure of the layered medium jointly influence the spectral distribution $I_n^{(2)}(\omega)$ of CR of the particle.

3. – The stages of analytical calculations and final expressions

Taking into account the azimuthal symmetry of the problem and making the Fourier transform

$$f_\omega=\frac{1}{2\pi}\int f(t)\exp[i\omega t]\mathrm{d}t, \tag{4}$$

one may reduce the set of Maxwell equations to the solution of one equation [9]

$$\left[\varepsilon\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\varepsilon}\frac{\mathrm{d}}{\mathrm{d}z}\right)+\frac{\omega^2}{c^2}\varepsilon\mu-\frac{\alpha_n^2}{R^2}\right]A_n=\frac{\varepsilon}{v_z}\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\varepsilon}\exp[i\omega\, z/v_z]\right)-i\frac{\omega}{c^2}\varepsilon\mu\,\exp[i\omega\, z/v_z] \tag{5}$$

that determines the longitudinal component $E_{z\omega}$ of the electric field strength by the following

$$\varepsilon E_{z\omega}(r,z)=\sum_{n=1}^{\infty}\frac{2q}{\pi R^2 J_1^2(\alpha_n)}J_0\left(\frac{\alpha_n}{R}r\right)A_n(z). \tag{6}$$

Here α_n is the n-th root of the 0th-order Bessel function: $J_0(\alpha_n)=0$, and the coefficient $2q/\pi R^2 J_1^2(\alpha_n)$ is introduced to simplify the solution ($J_1(x)$ is the 1st-order Bessel function). In these expressions $\varepsilon\equiv\varepsilon_\omega$ and $\mu\equiv\mu_\omega$ are the Fourier transforms of the permittivity and permeability (the contribution of spatial dispersion is assumed to be negligibly small). The functions $\varepsilon_{1\omega}$ and $\mu_{1\omega}$ are complex (to take into account the absorption of radiation by the material of layered medium).

In the range 2 (vacuum) eq. (5) has the following solution:

$$A_n(z) = A_n^q(z) + \frac{i}{\omega} a_n^{(2)} \exp\left[ik_n^{(2)} z\right], \qquad \text{where} \qquad k_n^{(2)} = \sqrt{\omega^2/c^2 - \alpha_n^2/R^2}, \tag{7}$$

and $a_n^{(2)}$ is a dimensionless quantity; the first term describes the known field of charge inside the hollow waveguide, and the second term describes the free field (the radiation field), if the following condition is observed:

$$\omega > \omega_n = \alpha_n c/R. \tag{8}$$

Far from the interface $z = 0$ of the layered medium with vacuum (range 2) the field of produced radiation is "separated" from the field of charged particle and freely propagates inside the waveguide along the positive sense of the axis Z. The spectral distribution of the energy of this radiation over the whole waveguide cross-section on its n-th bandwidth is determined by the expression

$$I_n^{(2)}(\omega) = \frac{4q^2}{\pi\omega} \frac{k_n^{(2)} \left|a_n^{(2)}\right|^2}{\alpha_n^2 J_1^2(\alpha_n)}. \tag{9}$$

For calculation of $a_n^{(2)}$ amplitude it is sufficient to compare (7) with the complete solution of eq. (5) that is valid for all $-\infty < z < \infty$. The complete solution can be obtained by means of the Green's functions method. Indeed, it is rather convenient as it is based on the solution of the simpler homogeneous equation

$$\left[\varepsilon \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{\varepsilon}\frac{\mathrm{d}}{\mathrm{d}z}\right) + \frac{\omega^2}{c^2}\varepsilon\mu - \frac{\alpha_n^2}{R^2}\right] L_n = 0. \tag{10}$$

Therewith, according to the Bloch theorem the general solution of eq. (10) inside the layered medium may be represented as a superposition

$$L_n(z) = c_1 L_{n+}(z) + c_2 L_{n-}(z) \tag{11}$$

of Bloch waves

$$L_{n\pm}(z) = w_{n\pm}(z) \exp\left[\pm ik_n^{(1)} z\right], \qquad \text{where} \qquad w_{n\pm}(z-l) = w_{n\pm}(z) \tag{12}$$

(travelling waves $\exp[\pm ik_n^{(1)} z]$ modulated with the period of the layered medium, see, *e.g.*, [10]). Here the quasiwave number $k_n^{(1)}(\omega)$ is determined with an accuracy to the sign. To fix the idea we shall assume that

$$k_n^{(1)} = {k'}_n^{(1)} + i{k''}_n^{(1)}, \qquad \text{where} \qquad \omega\, {k''}_n^{(1)}(\omega) \geq 0 \tag{13}$$

(the imaginary part ${k''}_n^{(1)} \neq 0$, because the allowance for radiation absorption by the material of the layered medium is made).

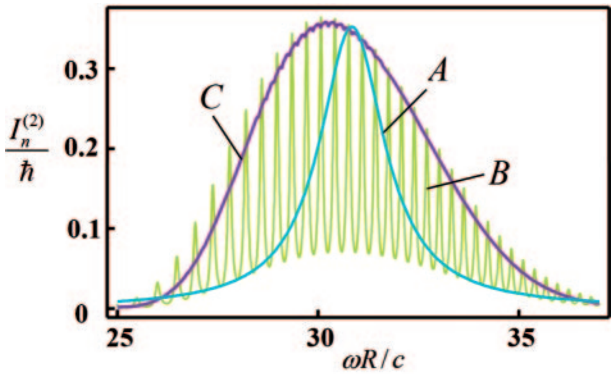

Fig. 2. – The density of spectral distribution $I_n^{(2)}(\omega)$ of the CR energy from an electron on the 3rd mode of waveguide, half-filled either with a continuous dielectric (curve A), or a stack of plates of the same dielectric (curves B and C).

Omitting the intermediate calculations we now obtain the final expression

$$a_n^{(2)} = \frac{\omega}{2k_n^{(2)} v_z} \left\{ \frac{1 - v_z^2/c^2}{1 - k_n^{(2)} v_z/\omega} - \frac{1 - v_z^2/c^2}{1 + k_n^{(2)} v_z/\omega} \cdot \frac{k_n - \varepsilon_1 k_n^{(2)}}{k_n + \varepsilon_1 k_n^{(2)}} - \right. \\ \left. - \left[\varepsilon_1 - 1 + \frac{\sigma_n}{1 - \exp\left[i \left(k_n^{(1)} - \omega/v_z \right) l \right]} \right] \frac{2k_n^{(2)}}{k_n + \varepsilon_1 k_n^{(2)}} \right\}_{z=0}, \tag{14}$$

where $k_n = i\dot{L}_{n-}(0)/L_{n-}(0)$ and

$$\sigma_n = i \frac{\omega \varepsilon_1(0)}{v_z L_{n-}(0)} \int_{-l}^{0} \left(1 - \varepsilon_1 \mu_1 \frac{v_z^2}{c^2} + \frac{i v_z \dot{\varepsilon}_1}{\omega \varepsilon_1} \right) \frac{L_{n-}}{\varepsilon_1} \exp\left[i \frac{\omega}{v_z} z \right] \mathrm{d}z \tag{15}$$

(the point over the function implies the differentiation by its argument). In (14) there are no limitations on the amplitude and variation profile of the layered medium parameters. In the particular case of $\varepsilon_1(z)$, $\mu_1(z) = \mathrm{const}$, eq. (14) describes the radiation field of particle leaving out the uniform material for the hollow part of the waveguide.

4. – Results of numerical calculations

In fig. 2 three curves of the spectral distribution $I_n^{(2)}(\omega)$ of the CR energy from an electron on the 3rd mode of waveguide are shown.

The energy of electron is 1.2 MeV,

$$\varepsilon_1 = \varepsilon_1' + i\varepsilon_1'' = 1.3 + 0.005i \tag{16}$$

is the dielectric permittivity of the material filling the waveguide, $\mu_1 = 1$.

In the case of curve A the waveguide is filled with a semi-infinite continuous dielectric. The position of maximum in the spectrum of CR is determined from the well-known equality

$$\omega_n^{Ch} = \frac{\alpha_n v}{R\sqrt{\varepsilon_1' v^2/c^2 - 1}} \tag{17}$$

TABLE I. – *The parameters of the semi-infinite medium.*

Curve	a/R	b/R	Semi-infinite medium
A	0	-	Solid dielectric
B	308.6	13.79	Stack of plates
C	15.43	13.79	Stack of plates
D	15.05	13.79	Stack of plates

for $n = 3$ (the number of waveguide mode). Equality (17) determines the frequencies of CR from the relativistic electron traveling along the axis of the waveguide completely filled with dielectric. The finite height and finite width of the peak are conditioned by the fact of the allowance of radiation absorption in the matter (ref. to (16)).

The case B differs from case A in that some part of the dielectric is removed from the waveguide to leave there a semi-infinite stack of plates with vacuum gaps (layered medium, see table I). At its travel along the waveguide axis the particle generates CR in each plate and the pulses formed in different plates superimpose and interfere. The oscillations in curve B reflect this fact. The case C differs from case B by the thickness of vacuum gap (see table I).

It is natural that the integral (total) energy of radiation is practically the same for curves A and B, whereas for curve C it is twice as large:

$$W_3^A \approx W_3^B \approx 0.95c\hbar/R, \qquad W_3^C = 1.94c\hbar/R. \tag{18}$$

What is the explanation?

In fig. 3 the radiation spectra on the first five modes of the waveguide are shown. The numbers of modes are seen beside the curves. The curves on the left and right (C and D) correspond to slightly differing thicknesses of the vacuum gap between the plates (about several percents, see table I). Curve A is shown here for comparison (the waveguide filled with a continuous semi-infinite dielectric). From fig. 3 it follows that i) at the transition $A \to C$ the total energy of radiation increases on the 2, 3, 4 and 5th modes and does not increase on the 1st mode of waveguide and ii) at $A \to D$ transition the total energy of radiation increases by 3.6 times on the 1st mode and does not increase on

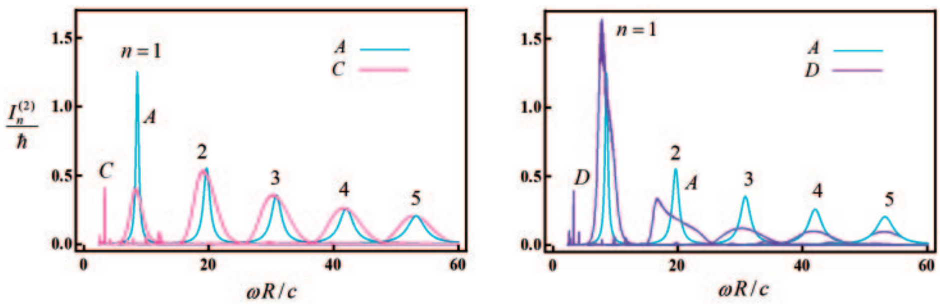

Fig. 3. – CR spectra from an electron at the first five modes of waveguide either half-filled with continuous dielectric (curve A on the left and right), or with stack of plates (curve C on the left and curve D on the right).

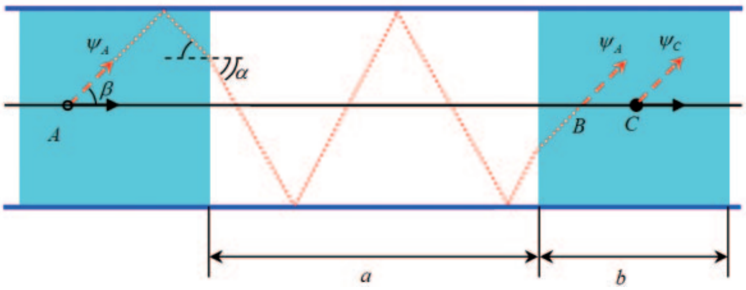

Fig. 4. – In case of $C \equiv B$ the pulse Ψ_C of CR generated by the particle in the vicinity of point C is "superimposed" on the pulse Ψ_A of CR generated by the particle in the vicinity of point A prior to that.

successive modes of waveguide (2,3,4,5). A slight (several per cent) variation in thickness of vacuum gaps in the stack of plates changes the waveguide modes, at which the energy of CR increases. How can this amplification be explained?

5. – Visual explanation

In fig. 4 two neighboring plates inside the waveguide are shown. Now let us consider the instant when the charged particle was in the left plate and observe Ψ_A *pulse of CR*, generated by a relativistic particle in the immediate vicinity of point A, that is *emitted at a specific angle* β *(angle of CR) with respect to the waveguide axis* (dashed line). At propagation this pulse is found in the right plate and at some instant of time traverses the particle trajectory (the waveguide axis) in a certain point B at the same angle β. By this time the relativistic particle emits Ψ_C pulse of CR in the immediate vicinity of some (generally) other point C (see fig. 4). However, the following equalities should be satisfied:

$$\frac{a+b}{v} = \frac{a}{c\cos\alpha} + \frac{b\sqrt{\varepsilon_1\mu_1}}{\cos\beta}, \qquad \frac{a\mathrm{tg}\alpha + b\mathrm{tg}\beta}{2R} = 4s, \quad s = 1, 2, 3... \tag{19}$$

(α is an angle giving the direction of CR with respect to the waveguide axis in the vacuum between the plates), then $C \equiv B$ and for this reason the particle will traverse the vicinity of point B concurrently with pulse Ψ_A. As a result, Ψ_C pulse emitted at the same angle β in the vicinity of point C will "superimpose" on Ψ_A pulse emitted earlier in the 1st plate. As a result of superposition (interference) the pulses may be either suppressed or amplified depending on the specific value of phase difference. The factor 4 in the right-hand side of the second equation of (19) ensures the fulfillment of the condition of pulse constructive superposition. Thus, if conditions (19) are met, the Cherenkov waves are generated inside the plate and at the same time constructively interfere with those emitted by the particle in the preceding plates.

For curve C in figs. 2 and 3 the values of a/R and b/R (see table I) have been determined from eq. (19) in case of $s = 1$. This accounts for the increase of total energy of CR on the 2,3,4 and 5th waveguide modes at the transition from curve A (waveguide filled with a semi-infinite continuous dielectric) to curve C (waveguide filled with a semi-infinite stack of plates).

Equalities (19) are valid in the framework of ray optics when the wavelength of the emitted wave is much less than R, a and b. For this reason eq. (19) requires more accurate definition on the first waveguide mode. It is not surprising hence that in fig. 3 the increase in the energy of CR on the 1st waveguide mode at the transition from curve A to curve D occurs for a/R somewhat different from its value determined from (19) (compare the values of a/R in rows C and D in table I).

Thus, the increase in total energy of CR at the replacement of the continuous dielectric in the waveguide by a stack of plates is due to the fact that *in each plate of stack in the CR formation zone the processes of generation and of constructive interference of Cherenkov waves emitted by the particle in the preceding plates occur simultaneously.*

The synchronous superposition of waves is followed by the increase in the resultant field in the radiation formation zone and, hence, also by the increase of the force retarding the particle motion along the waveguide axis. In this situation *additional work of external forces sustaining the uniform motion of the particle shall be spent on the generation of more intense CR.*

6. – Conclusions

In this paper the radiation from a charged particle uniformly moving along the axis of a cylindrical waveguide filled with a semi-infinite layered dielectric material that weakly absorbs the radiation is investigated.

a) Expressions for calculation of the spectral distribution of total energy of radiation passing through the transverse section of waveguide in the vacuum (at large distances from the boundary of layered medium) are derived with no limitations on the amplitude and variation profile of the layered medium permittivity.

b) The results of numerical calculations for emission of Cherenkov radiation (CR) in the layered material consisting of dielectric plates alternated with vacuum gaps are given. The values of the problem parameters for which CR is self-amplified at a separate waveguide mode have been determined. This radiation may prove to be many times as strong as CR in the waveguide filled with a semi-infinite solid dielectric without vacuum gaps. The visual explanation of this self-amplification effect of CR is given.

c) It is proposed to use this effect for amplification of coherent CR (CCR) from subpicosecond electron bunches observed by Rosenzweig *et al.* in [2].

We propose to improve the part of experimental setup of [2], in which CCR from a subpicosecond electron bunch is generated, by cutting L long hollow cylindrical dielectric tube inside the waveguide into N identical parts (each of b length, $b = L/N \gg \lambda$, where λ is the wavelength of CCR) and then construct a periodic structure consisting of these parts alternated with vacuum gaps. *If the length a of* each *vacuum gap* in the stack of shorter tubes inside the waveguide *is selected correctly* (see, *e.g.*, (19)), then (according to our preliminary estimates) *we expect* nearly *N-fold increase in the peak power of CCR* from subpicosecond electron bunch as compared with $a = 0$ case (the stack is one continuous tube [2] with total length $Nb = L$).

REFERENCES

[1] Williams G. P., *Rep. Prog. Phys.*, **69** (2006) 301.
[2] Cook A. M., Tikhoplav R., Tochitsky S. Y., Travish G., Williams O. B. and Rosenzweig J. B., *Phys. Rev. Lett.*, **103** (2009) 095003.
[3] Arzumanyan S. R., Grigoryan L. Sh., Khachatryan H. F. and Wagner W., *J. Phys: Conf. Ser.*, **236** (2010) 012012.
[4] Grigoryan L. Sh., Mkrtchyan A. R., Khachatryan H. F., Arzumanyan S. R. and Wagner W., *Proceedings of 35th International Conference on Infrared Millimeter and Terahertz Waves* (IRMMW-THz 2010) 2010, Mo-P.13.
[5] Settakorn C., Hernandez M. and Wiedemann H., *Stimulated Transition Radiation in the Far-Infrared*, SLAC-PUB-7587, August 1997.
[6] Lihn H. C., Bocek D., Hernandez M., Kung P., Settakorn C. and Wiedemann H., *Phys. Rev. Lett.*, **76** (1996) 4163.
[7] Grigoryan L. Sh., Khachatryan H. F., Arzumanyan S. R. and Grigoryan M. L., *Nucl. Instrum. Methods Phys. Res. B*, **252** (2006) 50.
[8] Grigoryan L. Sh., Khachatryan H. F., Arzumanyan S. R. and Grigoryan M. L., *Nucl. Instrum. Methods Phys. Res. B*, **266** (2008) 3715.
[9] Fainberg Y. B. and Khizhniak N. A., *Zh. Eksp. Teor. Fiz.*, **32** (1957) 883.
[10] Flugge S., *Practical Quantum Mechanics 1* (Springer-Verlag, Berlin) 1971.