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Angular distributions of Cherenkov radiation from relativistic heavy ions: Stopping and isotopic effects

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Summary. — We studied numerically the structure of angular distributions of Cherenkov radiation (ChR) from moderately relativistic heavy ions (RHI) taking into account the decrease of the ion velocity due to stopping in the radiator. The calculations clearly show that both the width and fine structure of the ChR angular distribution in the vicinity of the Cherenkov cone are remarkably different for isotopes with different masses, at equal initial relativistic factor (velocity) of isotopes. This stopping and isotopic effects in ChR can be observed using RICH detectors of RHI.

PACS 61.85.+p – Channeling phenomena (blocking, energy loss, etc.).
PACS 78.70.-g – Interactions of particles and radiation with matter.
PACS 41.60.Bq – Cherenkov radiation.

1. – Introduction

Tamm-Frank's theory [1] describes ChR of a particle that moves rectilinearly with a constant velocity v in a medium of refractive index n larger than v/c (c is the speed of light in vacuum). This theory neglects both the bending of the particle trajectory (due to multiple scattering) and the decrease of its velocity (due to ionization energy loss). Dedrick [2] was the first who has taken into account the influence of the particle-radiator interaction on the angular density of the particle's ChR. He examined the influence of multiple scattering ignoring the slowing-down effect.

Kuzmin and Tarasov [3] considered another limiting case where the multiple scattering effect is insignificant, that is the slowing-down effect playing a key role in forming the angular distribution of ChR. It occurs when relativistic heavy ions ($Z, A \gg 1$) cross thin radiators. They ignored the fluctuations of ionization energy loss and the trajectory bending effect due to multiple scattering and studied the angular distribution of ChR emitted by an ion moving rectilinearly but at a decreasing velocity. This assumption

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allowed them to obtain the formula for the angular density of ChR when taking into account the slowing down in the thin radiator. The thin radiator approximation was used in [3] to obtain a simple expression for the velocity decrease with depth penetration x due to ionization energy loss.

Thus, the width of ChR radiation angular distribution in the vicinity of the Cherenkov cone $\Delta\vartheta$ becomes proportional both to the stopping power in the radiator and to the radiator thickness L. Within $\Delta\vartheta$, the authors of ref. [3] predict the diffraction-like structure in the angular distribution, with a diffraction parameter (the distance between maxima) depending on the emission wavelength, the initial velocity and the stopping power $\mathrm{d}E/\mathrm{d}x$ of the radiator.

In order to avoid the approximations made in [3] for the thin radiator, recently a new approach was developed [4], which uses the popular computer code SRIM'06 [5] to calculate the ion velocity depending on its penetration depth in the radiator. We subsequently substituted it into the formula of classical electrodynamics for the spectral-angular distribution of radiation from a particle moving in a medium along a trajectory $r(t)$ taking into account the slowing-down due to ionization energy loss in the radiator. New suggested method allowed investigating in more detail dependences of the ChR angular distributions on the square of the ion charge, on the radiator thickness, on the emission wavelength and on the refractive index of the radiator material.

In the present work, we pay attention to another peculiarity of ChR from RHI, which we call "isotopic effect", *i.e.* ChR from RHI with the same charges but different masses, which is connected with a slight difference in stopping in a radiator.

2. – RHI stopping: influence on ChR

The general expression for the intensity of the radiation from RHI with a charge Ze penetrating through a non-magnetic dielectric medium characterized by the dielectric function $\varepsilon = \varepsilon(\omega)$ may be written in the form (see, *e.g.*, [6])

$$\frac{\mathrm{d}I}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Z^2e^2\omega^2}{4\pi^2c^3}\sqrt{\varepsilon}\left|\int_{-\infty}^{+\infty} n\times[n\times v]\exp\left[i\omega\left[t-\frac{\sqrt{\varepsilon}nr\left(t\right)}{c}\right]\right]\mathrm{d}t\right|. \tag{1}$$

In the case of rectilinear motion, $r(t) = vt$, therefore,

$$\frac{\mathrm{d}I}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Z^2e^2}{c^3}\sqrt{\varepsilon}\,[n\times v]^2\left|\frac{\omega}{2\pi}\int_{-\infty}^{+\infty}\exp\left[i\omega t\left[1-\frac{\sqrt{\varepsilon}}{c}nv\right]\right]\mathrm{d}t\right|. \tag{2}$$

The integral in the above expression is the Dirac δ-function that means

$$\frac{\mathrm{d}I}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Z^2e^2}{c}\sqrt{\varepsilon}\beta^2\sin^2\vartheta\left|\delta\left(1-\sqrt{\varepsilon}\beta\cos\vartheta\right)\right|^2. \tag{3}$$

Here, ϑ is the angle between the emission direction n and the velocity vector v, and $\beta = v/c$. The appearance of δ-function in expression (3) is due to infinite penetration time with a constant velocity and leads to infinite spectral density of the radiation emitted at the Cherenkov angle. If one takes into account the finite time $2T$ of RHI penetration through the radiator, then the divergence disappears:

$$\frac{\omega}{2\pi}\int_{-T}^{+T}\exp\left[i\omega t\left[1-\frac{\sqrt{\varepsilon}}{c}nv\right]\right]\mathrm{d}t = \frac{\omega T}{\pi}\frac{\sin\left[\omega T\left(1-\sqrt{\varepsilon}\beta\cos\vartheta\right)\right]}{\omega T\left(1-\sqrt{\varepsilon}\beta\cos\vartheta\right)}. \tag{4}$$

Thus, the spectral-angular distribution of ChR (Tamm-Frank distribution) from RHI in the finite-size radiator is defined by the following equations:

$$\frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}\Omega} = \omega L \left(\frac{Ze \sin\vartheta}{c}\right)^2 f_{\mathrm{TF}}(\theta, \omega); \tag{5}$$

$$f_{\mathrm{TF}}(\theta,\omega) = \frac{1}{\Delta\vartheta_{\mathrm{TF}}}\left(\frac{\sin x}{x}\right)^2; \quad x = \frac{\pi}{\Delta\vartheta_{\mathrm{TF}}}\left(\cos\vartheta - \frac{1}{\beta\sqrt{\varepsilon}}\right); \tag{6}$$
$$\Delta\vartheta_{\mathrm{TF}} = \frac{\lambda}{L\sqrt{\varepsilon}}; \quad \lambda = \frac{2\pi c}{\omega}$$

The width $\Delta\vartheta_{\mathrm{TF}} = \lambda/L\sqrt{\varepsilon}$ of the Tamm-Frank distribution (concentrated in the vicinity of the Cherenkov cone) is inversely proportional to the radiator thickness L. Therefore, at large L there appears a very sharp maximum at the Cherenkov angle $\vartheta = \vartheta_C$ defined by $\cos\vartheta_C = 1/\beta\sqrt{\varepsilon}$.

In the case of RHI penetrating through a radiator, we may assume that the multiple scattering effect is negligible, *i.e.* the direction of RHI velocity $\mathbf{v}/v = \mathrm{const}$ but its magnitude decreases due to ionization energy loss, *i.e.* depends on penetration depth x, $v = v(x)$. As in [3], we can replace the integration over time in eq. (2) by the integration over the trajectory, by making use of the substitution

$$t(x) = \int_0^x \frac{\mathrm{d}x'}{v(x')}. \tag{7}$$

After some algebra we derive the following expression for the spectral-angular distribution of ChR from RHI, taking into account the stopping in the radiator:

$$\frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}\Omega} = \omega L \left(\frac{Ze \sin\vartheta}{c}\right)^2 f(\theta, \omega)$$
$$f(\theta,\omega) = \frac{1}{L^2}\left|\int_0^L \exp\left[i\omega\left[t(x) - \frac{\sqrt{\varepsilon}x\cos\theta}{c}\right]\right]\mathrm{d}x\right|^2; \quad t(x) = \int_0^x \frac{\mathrm{d}x'}{v(x')}. \tag{8}$$

According to eq. (8), the angular distribution of ChR in the vicinity of the Cherenkov angle is characterized by the new distribution function $f(\theta, \omega)$. The analytical form of this function was obtained in ref. [3] using the thin radiator approximation, which allows to write up a simple expression for the inverse velocity entering eq. (8):

$$\frac{1}{v(x)} \approx \frac{1}{v_0} - \frac{1}{v_0^2}\left(\frac{\mathrm{d}v(x)}{\mathrm{d}x}\right)_{x=0} x, \tag{9}$$

where v_0 is the initial velocity of an ion. In eq. (9), the velocity gradient is expressed through the stopping power $\mathrm{d}E/\mathrm{d}x$ of the radiator:

$$\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\mathrm{d}E}{\mathrm{d}x}\frac{\mathrm{d}v}{\mathrm{d}E} = \frac{1}{p\gamma^2}\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{1}{p\gamma^2}S; \quad p = vE; \quad S = \frac{\mathrm{d}E}{\mathrm{d}x}. \tag{10}$$

Here, p, E, γ are the momentum, energy and relativistic factor of the ion.

The substitution of (10) in eq. (8) leads to new spectral-angular distribution of ChR (Kuzmin-Tarasov) from RHI in the finite-size radiator:

$$f_{KT}\left(\vartheta,\omega\right)=\frac{1}{2\Delta\vartheta\sin\vartheta_0}\left\{\left[C\left(u_1\right)-C\left(u_0\right)\right]^2+\left[S\left(u_1\right)-S\left(u_0\right)\right]^2\right\}.$$

Here, $C(u), S(u)$ are the Fresnel integrals and u_1, u_0 are the functions of initial velocity, refractive index, radiation wavelength, radiator thickness and stopping power of the radiator [3].

Thus, the width of ChR angular distribution $\Delta\vartheta$ now becomes proportional to the stopping power S of the radiator and to the radiator thickness L. Since the aperture of the Cherenkov emission cone is fixed by the particle velocity, $\cos\vartheta_C = c/v\sqrt{\varepsilon} = 1/\beta\sqrt{\varepsilon}$, it is clear that one may expect an additional broadening $\Delta\vartheta$ of the angular distribution, the value of which is determined approximately by the initial and final velocities of the particle. Within $\Delta\vartheta$, the authors of ref. [3] predicted the diffraction-like structure in the angular distribution, with a diffraction parameter (the distance between maxima) depending on the emission wavelength, the initial velocity and the stopping power S of the radiator.

3. – ChR from isotopes taking account of stopping in radiator

Let us start with the Bethe-Bloch formula for stopping power (see, *e.g.*, [7]):

$$-\frac{\mathrm{d}E}{\mathrm{d}x}=4\pi N_A r_e^2 m_e c^2 Z^2\frac{Z_t}{A}\frac{1}{\beta^2}\left[\ln\frac{T_{\max}}{I}-\beta^2-\frac{\delta}{2}\right]. \quad (11)$$

The usual meaning is that the stopping of relativistic heavy particles is defined only by the square of projectile charge and its kinetic energy. Thus, the isotopes with initially equal velocities (relativistic factors) should have equal energy loss in the radiator. This is indeed true, but in the problem we are concerned with, *i.e.* ChR, the key role plays the change in velocity but not the change in the energy.

Let us consider, for example, penetration of two isotopes with equal charge Ze and different masses M_1, M_2 through a thin layer of radiator Δx. Let the isotopes before the radiator have equal velocities and relativistic factors v_0, γ_0. After the penetration through Δx both isotopes loose an equal amount of energy

$$\Delta E_1=\Delta E_2=-\mathrm{d}E/\mathrm{d}x=S\left(\gamma_0,v_0\right), \quad (12)$$

Here, the stopping $S(\gamma_0, v_0)$ depends on the relativistic factor and velocity *before* they pass Δx. Thus, *after* penetration through Δx the two isotopes will enter the next piece of radiator with energies

$$\begin{aligned}E_1(\Delta x)&=E_1(0)-\Delta E_1=E_1(0)-S\left(\gamma_0,v_0\right)\Delta x,\\ E_2(\Delta x)&=E_2(0)-\Delta E_2=E_2(0)-S\left(\gamma_0,v_0\right)\Delta x.\end{aligned} \quad (13)$$

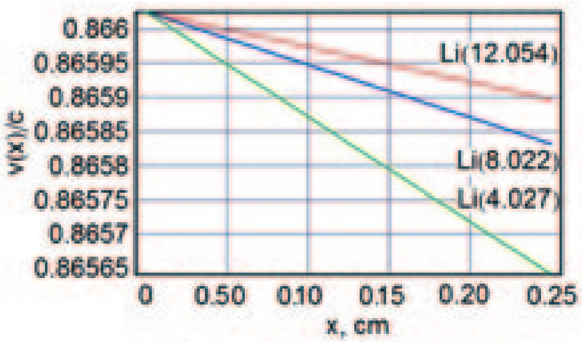

Fig. 1. – Dependence of Li isotopes velocities on the penetration depth in a thick LiF radiator. Initial ion beam energy equals 1000 MeV/u.

As a sequence, new values of their relativistic factors become *different*:

$$\begin{aligned} \gamma_1(\Delta x) &= \frac{E_1(\Delta x)}{M_1c^2} = \frac{E_1(0) - S(\gamma_0, v_0)\,\Delta x}{M_1c^2} = \gamma_0 - \frac{S(\gamma_0, v_0)\,\Delta x}{M_1c^2} = \gamma_0 - \Delta\gamma_1, \\ \gamma_2(\Delta x) &= \frac{E_2(\Delta x)}{M_2c^2} = \frac{E_2(0) - S(\gamma_0, v_0)\,\Delta x}{M_2c^2} = \gamma_0 - \frac{S(\gamma_0, v_0)\,\Delta x}{M_2c^2} = \gamma_0 - \Delta\gamma_2. \end{aligned} \tag{14}$$

Because of $\gamma = 1/\sqrt{1-\beta^2}$, one can easily connect the change in relativistic factor with the corresponding change in velocity, that is $\Delta\beta = \Delta\gamma/\beta\gamma^3$. Therefore, two isotopes with initial equal velocities before the piece of radiator, after penetration through this piece acquire different changes in velocities:

$$\begin{aligned} \Delta\beta_1 &= \left(\frac{1}{\beta_1\gamma_1^3}\right)_{x=0} \Delta\gamma_1 = \left(\frac{1}{\beta_1\gamma_1^3}\right)_{x=0} \frac{S(\gamma_0, v_0)\,\Delta x}{M_1c^2}, \\ \Delta\beta_2 &= \left(\frac{1}{\beta_2\gamma_2^3}\right)_{x=0} \Delta\gamma_2 = \left(\frac{1}{\beta_1\gamma_2^3}\right)_{x=0} \frac{S(\gamma_0, v_0)\,\Delta x}{M_2c^2}. \end{aligned} \tag{15}$$

As a sequence, comparing the change in velocity for two isotopes after penetration through a thin piece of radiator, we conclude:

$$\frac{\Delta\beta_1}{\Delta\beta_2} = \frac{M_2}{M_1}. \tag{16}$$

This means the change in velocity becomes dependent on isotope mass. This change is more remarkable for light isotopes, *e.g.*, hydrogen, lithium and berillium, when the masses may differ by a factor of two-three.

So, the two isotopes enter next piece of the radiator with different velocities and will loose the energy according to their new different stopping powers $S(\gamma_1(\Delta x), v_1(\Delta x))$ and $S(\gamma_2(\Delta x), v_2(\Delta x))$. The further calculation of velocity dependent on penetration depth in the case of the thick radiator is obvious.

The results of exact calculation of lithium isotopes velocities, as a function of the penetration depth, are presented in fig. 1. In the calculations, the SRIM'06 code [6] was used to obtain the isotopes stopping powers [6].

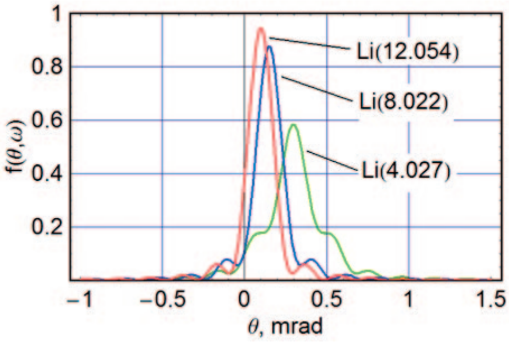

Fig. 2. – The change in the ChR angular distribution from Li isotopes ($Z = 3$) in the vicinity of the Cherenkov cone with the increase of the mass number. The emission wavelength $\lambda = 390\,\text{nm}$, the radiator material is LiF; the radiator thickness $L = 0.25\,\text{cm}$; the index of refraction $n = \sqrt{\varepsilon}$ depends on λ according to [8]; initial ion beam energy $1000\,\text{MeV}/u$. Here, $\vartheta = 0$ corresponds to the Cherenkov angle $\vartheta = \vartheta_C$, $\cos\vartheta_C = 1/\beta n$ and positive ϑ values in fact denote emission angles smaller than ϑ_C.

To conclude with velocity change due to stopping, one should remark the very weak dependence on the projectile mass in the original Bethe-Bloch formula for stopping. Indeed, the maximal energy transferred to the atomic electron of the target by a charged heavy relativistic particle (RHI in our case) is defined by the equation

$$T_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma\,(m_e/M) + (m_e/M)^2}. \tag{17}$$

Here, γ and βc are relativistic factor and velocity of the projectile particle, M and m_e are the RHI and electron mass, respectively. As a sequence, if γ and βc of two isotopes (masses M_1 and M_2) are equal, their stopping powers are very slightly different due to different $T_{\max}$. However, this difference is too small and cannot lead to additional change of angular distributions of ChR from isotopes.

Thus, our next goal is to study how a small difference, eqs. (15), (16) in the velocity change may influence the angular distributions of the ChR from isotopes. Since the velocity $v(x)$ enters the phase of the exponential under the integral sign in eq. (8), even a small variation of $v(x)$ may lead to serious changes of the angular distribution of ChR.

4. – Isotopic effect in angular distributions of ChR: numerical calculations

Based on eq. (8) and SRIM'06 calculations of $v(x)$ as in [4] (taking account of stopping), we studied numerically the ChR angular distributions from RHI with the same charge Ze, but different masses, *i.e.* isotopes, at the fixed radiator thickness L and emission wavelength λ.

We calculated the new distribution function $f(\vartheta, \omega)$ for three Li isotopes: Li-12 ($A = 12$), Li-8 ($A = 8$) and Li-4 ($A = 4$). The results of calculations of new distribution function for ChR in the case of lithium isotopes are presented in fig. 2 and clearly show the remarkable sensitivity of the position of the maximum and its width to the isotope mass, if the initial velocities before the radiator were equal.

Indeed, for an isotope with a smaller mass the change of velocity after penetration through the every next thin piece of the radiator is little bit greater compared to a much heavier isotope, and as a result, the final velocity at the exit from the radiator is smaller. As a result, the Cherenkov angle

$$\cos\vartheta = \frac{c}{vn(\lambda)}$$

is shifted to smaller values. That means, the center of gravity of new distribution function is shifted more remarkably in the case of light isotopes, in accordance with numerical calculations presented in fig. 2.

5. – Conclusion

The calculations of ChR angular distribution from moderately relativistic heavy ions (isotopes) taking into account their stopping in the radiator have been performed. The results of calculations show that the angular distribution of ChR from different isotopes (in the vicinity of the Cherenkov cone) has a complicated fine structure and width which depends remarkably on the isotope mass, if the initial velocities before entering a radiator are equal. It is astonishing, but it occurs unless the variations of the isotopes velocities are very small (see fig. 1).

The experimental studies of predicted stopping + isotopic effect are possible at existing (GSI, CERN) and future (FAIR) relativistic heavy ion accelerators, especially using RICH detectors. The measurements of the width of the ChR angular distribution in the vicinity of the Cherenkov cone may be even considered as a new method to measure the isotope masses.

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REFERENCES

[1] Tamm I. E. and Frank I. M., *Dok. Akad. Nauk SSSR*, **14** (1937) 107.
[2] Dedrick K. G., *Phys. Rev.*, **8** (1952) 891.
[3] Kuzmin E. S. and Tarasov A. V., *JINR Rapid Commun.*, **4** (1993) 64.
[4] Altapova V. R., Bogdanov O. V. and Pivovarov Yu. L., *Nucl. Instrum. Methods B*, **256** (2009) 109.
[5] *SRIM-06*: `http://www.srim.org`.
[6] Jackson J. D., *Classical Electrodynamics*, 2nd edition (John & Wiley Sons, New York) 1975.
[7] *Review of Particle Physics Journal of Physics G. Nucl. Part. Phys.* **37**, 2010, p. 287.
[8] *Springer Handbook of Condensed Matter and Materials Data* (Springer, Berlin-Heidelberg) 2005.