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How to make coherent bremsstrahlung circularly polarized

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Summary. — It is shown that if high energy positrons are moving in planar channeling conditions in a bent crystal at a small angle with respect to atomic strings which form the bent channeling planes, the hard part of the spectrum of gamma radiation emitted by the positrons will be highly circularly polarized. This Circularly Polarized String-of-Strings Radiation of Positrons Channeled in Bent Crystal (CPSOSRPCHBC) can be considered as a circularly polarized coherent bremsstrahlung manifesting itself efficiently from sub-GeV up to TeV and higher positron energies. The possibility to observe the CPSOSRPCHBC at the 500 MeV positron beam of Frascati National Laboratory is considered.

PACS 61.85.+p – Channeling phenomena (blocking, energy loss, etc.).
PACS 41.75.Ht – Relativistic electron and positron beams.
PACS 13.60.Fz – Elastic and Compton scattering.
PACS 41.60.-m – Radiation by moving charges.

1. – Introduction

Circular polarization plays a significant role in the study of fundamental interactions. Hard circularly polarized γ-quanta can be produced by backward Compton scattering of intensive circularly polarized laser beams at unpolarized high energy electrons. At the same time the electron (positron) coherent bremsstrahlung in crystals (see [1,2] and references therein) is in many respects analogous to the back Compton scattering of intensive electromagnetic waves. A natural question arises, can the coherent bremsstrahlung of unpolarized electrons or positrons, in that number of secondary beams of the largest proton accelerators, be circularly polarized? The negative answer has been commonly accepted for half a century while the linear coherent bremsstrahlung polarization is widely used in the study of nucleon structure. Nevertheless, we will show below that special conditions of positron interaction with crystals, namely a positron channeling motion at small angles

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with respect to atomic strings constituting bent channeling planes, allow to generate circularly polarized γ-quanta in the widest interval of both positron and gamma-quantum energies.

2. – Effective circular polarization of the crystal field

Radiative processes accompanying high energy particle motion at small angle ψ with respect to a major crystal axis are described by the averaged axial potential $V(\mathbf{r}) = \sum_{\mathbf{q}} V(\mathbf{q})e^{i\mathbf{qr}}$, where $\mathbf{q}$ are reciprocal lattice vectors normal to the axis. If the angle ψ is not very small, the γ-quantum radiation by electrons (positrons) is described by the dipole approximation [2, 1]. The latter allows to consider the radiation process as a backscattering of virtual photons with energies $\Omega_{\mathbf{q}} = \gamma\mathbf{qv} = \gamma q_{\|}v$ measured in the comoving reference frame of particles with velocity $\mathbf{v}$ and Lorenz-factor γ [1]. The linear polarization of the virtual photons representing a Fourier harmonic of the crystal electric field $\mathbf{E} = -\vec{\nabla}V(\mathbf{q})$ is determined by the transverse component $\mathbf{q}_{\perp} = \mathbf{q} - \mathbf{v}(\mathbf{qv})/v^2 \simeq \mathbf{q}$ of the reciprocal lattice vector $\mathbf{q}$. Since usually the virtual photons have different $q_{\|}$'s and $\Omega_{\mathbf{q}}$'s, the scattered γ-quanta do not interfere and preserve the linear polarization of virtual photons.

A qualitatively different situation is realized in the case of "string-of-strings" (SOS) alignment [3-5] when particles move parallel to the planes of atomic strings. The point is that the normal to the crystal planes component $q_x\mathbf{e}_x$ of the reciprocal lattice vector $\mathbf{q} = q_x\mathbf{e}_x + q_y\mathbf{e}_y$ does not contribute to the photon frequency $\Omega_{\mathbf{q}} = \gamma q_{\|}v$ in the SOS case. As a consequence all the harmonics with $q_x = q_{1x}n_x$, $n_x = 0, \pm1, \ldots$, have equal frequencies

$$\Omega_n = \Omega_{\mathbf{q}} = \gamma q_{\|}v = \gamma n q_{1y}\psi v \tag{1}$$

and are scattered coherently at each $n = n_y = \pm1, \pm2, \ldots$. For an illustration let us consider a simple crystal orthorhombic lattice with primitive reciprocal lattice vectors $q_{1x} = 2\pi/d$ and $q_{1y} = 2\pi n/d_{iax}$, where d and d_{iax} are the interplanar gap width and interstring distance in the plain, respectively—see fig. 1. Both the diverse directions of polarization $\mathbf{e}_{\mathbf{q}} = \mathbf{q}_{\perp}/|\mathbf{q}_{\perp}| \simeq \mathbf{q}/|\mathbf{q}|$ and different x-dependence of phases $\varphi_{\mathbf{q}} = q_x x$ of the coherent Fourier harmonics with different q_x and same q_y allow to envisage an appearance of a circular component of polarization of γ-quanta scattered in some local x regions.

For a quantitative treatment, the coherent action of all the harmonics with the same frequency (1) can be represented [6, 7] by the x-dependent electric field "q_y-harmonics" with Cartesian components

$$\begin{aligned} E_{x,q_y}(x) &= E_{xn}(x) = 2\sum_{n_x=0,1,\ldots} q_x V(q_x, q_y)\sin(q_x x), \\ E_{y,q_y}(x) &= E_{yn}(x) = 2q_y\sum_{n_x=0,1,\ldots} \frac{1}{1+\delta_{n_x,0}} V(q_x, q_y)\cos(q_x x), \end{aligned} \tag{2}$$

where $\delta_{n_x,0} = 1$ at $n_x = 0$ and $\delta_{n_x,0} = 0$ at $n_x = 1, 2, \ldots$. Jointly with the averaged field $-V'(x) = 2\sum_{n_x=0,1,\ldots} q_x V(q_x, 0)\sin(q_x x)$ of crystal plains these q_y-harmonics constitute

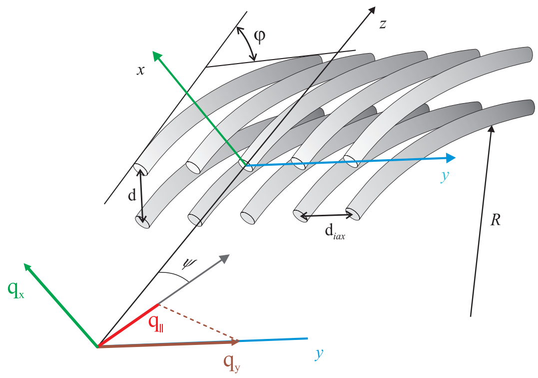

Fig. 1. – Positron beam orientation with respect to both axes and planes of the bent crystal at the "string-of-string" alignment, when positrons hit the crystal at an angle ψ with respect to its axes nearly parallel to the planes formed by the latter.

the averaged electric field

$$\mathbf{E} = -\vec{\nabla} V(x,y) = -\mathbf{n}_x V'(x) + \sum_{n=\pm1,\pm2,\dots} \left[\mathbf{n}_x E_{xn}(x) - i\mathbf{n}_y E_{yn}(x)\right] e^{iq_y y} \tag{3}$$

of crystal axes. A $\pi/2$ phase shift between the real components (2) directly indicates the presence of effective circular polarization of the q_y-harmonics. Indeed, the amplitudes (2) allow us to introduce the combinations

$$\xi_{2n} = 2E_{xn}E_{yn}/E_n^2, \quad \xi_{3n} = (E_{xn}^2 - E_{yn}^2)/E_n^2, \tag{4}$$

where $E_n^2 = E_{xn}^2 + E_{yn}^2$, which can be treated as the Stokes parameters of the virtual photons encountered by a particle in its comoving reference frame. Figure 2 demonstrates both a high circular polarization of the strongest first q_y-harmonic and an inversion of its sign on the opposite halves of the interplanar gap $[0,d]$ following from the symmetry properties

$$E_{xn}(d-x) = -E_{xn}(x), \qquad E_{yn}(d-x) = E_{yn}(x). \tag{5}$$

The inversion of the virtual photon circular polarization prevents any manifestation of the coherent bremsstrahlung circular polarization in usual conditions, though a couple of methods are known for a considerable time allowing to obtain circularly polarized γ-quanta in very thin crystals [6,8]. The main point of this paper is to apply the channeling process in bent crystals at the SOS conditions (see fig. 1) to restrict the positron motion by the nonsymmetric transverse regions (see figs. 2, 3) with a substantially dominating sign of the circular polarization of the q_y-harmonics (see fig. 2), a backward scattering of which allows to generate circularly polarized γ-radiation in *thick* crystals [7].

Since the scattering probability is proportional to the squared q_y-harmonic amplitudes, rapidly decreasing far from the planes, as fig. 2 demonstrates, a practically complete polarization asymmetry is reached at a relatively modest crystal bending. Remind that the minimal bending radius $R_{\min}$ admitting channeling in bent crystals equals

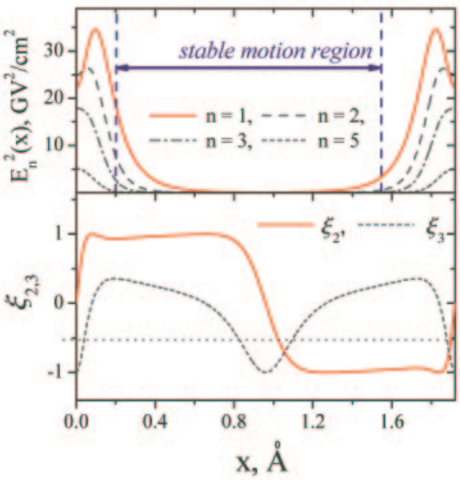

Fig. 2. – Dependence of the squared amplitudes of the 1st, 2nd, 3rd and 5th q_y-harmonics on the distance to the atomic plane (above) and the same dependence of the circular (ξ_2) and linear (ξ_3) polarizations of the first of them.

$\varepsilon/eE(x_c)$, where $E(x_c) = -V'(x_c)/e$ is the planar electric field strength at the effective dechanneling distance x_c from the plain, $x_c \simeq 0.2$ Å and $E(x_c) \simeq 5 \cdot 10^9$ V/cm for (110) Si plane at $T = 293$ K. A region of stable transverse e^+ motion at bending radius $R = 5R_{\rm min}$ which provides a sufficient dominance of one of the circular polarization directions is indicated in fig. 2, while the 500 MeV e^+ asymmetric distribution between the planes is illustrated by fig. 3.

3. – SOS radiation of channeled positrons

It should be reminded that the electron SOS radiation is, in fact, well known [3-5]. A considerable increase of probabilities of both the soft γ-quantum emission, steaming from the relatively slow planar channeling motion, and the hard one, representing itself

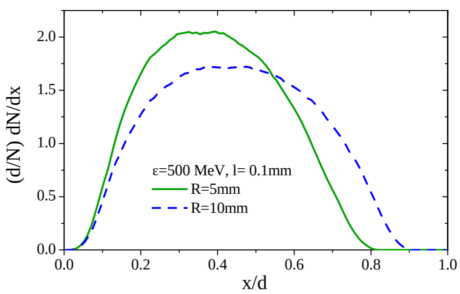

Fig. 3. – 500 MeV channeled positron distribution density in the interplanar region in the crystal bent with radii 5 (solid line) and 10 mm (dashed line). Crystal bending allows to depopulate the $0.8 \leq x/d \leq 1.0$ region, preventing the compensation of the polarization of gamma-quanta emitted most intensively in the region $0.10 \leq x/d \leq 0.20$.

a coherent bremsstrahlung in the field of crystal axes, have been predicted [3] and both successfully observed [4] and interpreted [5]. However possible influence of the interference of the crystal field harmonics on either intensity or polarization of the SOS radiation has never been mentioned.

A considerable difference of frequencies of the planar channeling oscillations and spiral vibrations in the axial field allows to represent the radiation spectrum at positron energies $\varepsilon \leq 200\,\mathrm{GeV}$ accessible at present as a sum of nearly independent soft contribution of either dipole or synchrotron-like planar channeling radiation, both of which can be found in [1, 2], and of the hard contribution of the coherent bremsstrahlung-like SOS radiation in the field of axes. The latter can be treated as a dipole-like radiation in the field (2) characterized by the interference of the Fourier harmonics with different q'_x s and equal q'_y s. Because of both the short formation length of the hard SOS radiation and the small angles formed by the velocity of channeled positrons with channeling planes the x coordinate and, consequently, the harmonics (2) can be treated as constants in the process of SOS photon formation. Such a really drastic simplification allows to introduce an "instant" polarization matrix

$$\frac{\mathrm{d}w_{ij}(x(t))}{\mathrm{d}\omega\mathrm{d}t} = \frac{1}{2}(A + \mathbf{B}\cdot\boldsymbol{\sigma})_{ij}, \tag{6}$$

where

$$\begin{gathered}(A, \mathbf{B}) = \frac{\alpha}{\gamma^2}\sum_{n=1,2,\ldots}\left(\frac{E_n}{mq_\parallel}\right)^2 (a, \mathbf{b})\,\theta(1-\nu), \\ a = \frac{\varphi(\varepsilon)}{2} + 2\nu(1-\nu), \quad \nu = \frac{\omega m^2}{2\varepsilon\varepsilon' q_\parallel}, \quad q_\parallel = nq_{1y}\psi v, \\ b_1 = 0, \quad b_2 = \xi_{2n}\varphi\left(\nu - 1/2\right), \quad b_3 = \xi_{3n}\nu^2,\end{gathered} \tag{7}$$

of photons emitted over a unit of the time t of positron motion at a constant distance $x(t)$ to the crystal plain. Equation (6) contains the Pauli matrices $\theta(1-\nu)$, a step function $\theta(1-\nu)$, the amplitudes and the two Stokes parameters ξ_{2n} and ξ_{3n} of the q_y-harmonics (2). However the third of them ξ_{1n} is absent in complete agreement with symmetry considerations and in contrast with [5].

The theta-function in (7) reflects the well-known kinematics of inverse Compton scattering of a photon with 4-momentum $q_\mu = (0, q_\parallel)$ at a positron with 4-momentum $p_\mu = (\varepsilon, p)$. The condition $\nu = u/s = 1$ corresponds to the pure backscattering and determines the edge energies

$$\omega_n = \frac{s_n}{1+s_n}\varepsilon, \qquad s_n = \frac{2q_\mu p^\mu}{m^2} = \frac{2\varepsilon|q_\parallel|v}{m^2}$$

of the SOS spectrum peaks, the first of which is usually the highest.

Remarkable that, as eqs. (6) and (7) show, the circular polarization of the virtual photons is nearly completely transferred to the emitted γ-quanta in the peak region $0 < 1-\nu \ll 1$, where $b_2/a \simeq 1$, in agreement with the pioneering paper [9] on the polarized inverse Compton scattering. That is why a high circular polarization of the virtual photons (see fig. 2) allows highly circularly polarized SOS radiation to be obtained—see fig. 4.

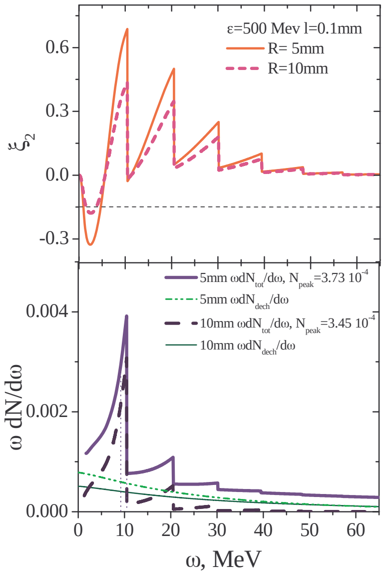

Fig. 4. – Polarization (top) and radiated photon number spectral distributions (bottom) *vs.* the energy of γ-quantum emitted by 500 MeV positrons moving at the angle $\psi = 1$ mrad with respect to $\langle 111 \rangle$ axis in the channeling regime between (110) planes of 0.1 mm Si crystal bent with radii $R = 5$ and 10 mm. Two lower smooth dashed lines represent the unpolarized contribution of dechanneled positrons.

4. – Suggested moderate energy experiment

The suggested method of Circularly Polarized String-of Strings Radiation of Positrons Channeled in Bent Crystal (CPSOSRPCHBC) will be most suitable to polarize gamma-quanta emitted by secondary (in fact, tertiary) positron beams generated unpolarized and in same quantity as the electrons at most of the high energy proton accelerators [7]. Many other effects (see [10, 11] and references therein) have been predicted to polarize secondary electron-gamma beams of proton accelerators, however only the CPSOSR-PCHBC effect can be observed both in TeV and GeV energy regions. In particular, the CPSOSRPCHBC can be studied at the 500 MeV positron beam of Frascati National Laboratory. In addition to wider beam availability, traditional gamma-polarimetry methods [12, 13] can be used at the energies of a few tens of MeV of the CPSOSRPCHBC gamma-quanta.

Because of relatively fast dechanneling length of 500 MeV positrons in bent crystals, we chose the crystal length l equal to 100 μm. Positron distributions in the inter-planar channel averaged along this length were simulated for bending radii of $R = 5$ and 10 mm (see fig. 3). The former evidently provides considerably better, close to a maximally possible asymmetry of positron distribution between the strong field regions $x/d = 0.1-0.2$ and $x/d = 0.8-0.9$ with opposite directions of the virtual photon circular polarization (see fig. 2). Spectral distributions of both the polarization and emitted photon number are present in fig. 4 allowing to compare 5 and 10 mm bending radii. The angle of positron incidence w.r.t. atomic strings in the channeling plane was taken equal to 1 mrad.

A novel crystal assembly allowing to pass a wide positron beam through very short crystals bent with very small radii is present in fig. 5. The crystal bending angle $\varphi = l/R$

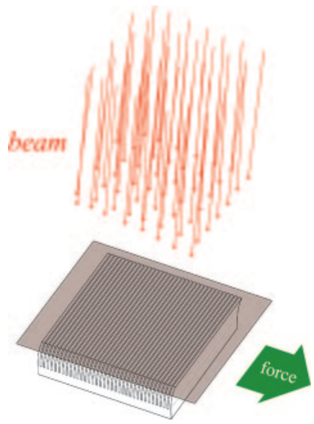

Fig. 5. – A crystal assembly allowing to pass a wide positron beam through very short crystals bent with very small radii.

will reach the value of 10–20 mrad allowing to separate polarized gamma-quanta from that emitted by nonchanneled positrons of a considerably diverging beam. Note that such a method of relatively low-energy wide beam deflection can also be applied for manipulation of ion beams used for the nuclear cancer therapy.

Since the channeled positrons experience a weaker incoherent scattering on both crystal electrons and nuclei than the nonchanneled ones, the CPSOSRPCHBC will manifest itself both clearer and under lower energies than the SOS radiation of nonchanneled positrons or electrons. The CB-like structure of the CPSOSRPCHBC spectrum will, however, be partially smeared out by an unpolarized radiation of dechanneled positrons, leaving the CPSOSRPCHBC regime along all the crystal length and giving depolarizing contribution to the CPSOSRPCHBC of the well deflected positrons. Random string approximation [2] was used to describe this effect numerically—see fig. 4.

5. – Conclusions

Thus, we have demonstrated that channeled positrons moving in bent crystals at small angles with respect to atomic strings emit hard circularly polarized string-of-strings radiation. The latter represents itself a specific type of the well-known coherent bremsstrahlung, the possibility of circular polarization of which we, thus, predict here for the first time.

Though the predicted Circularly Polarized String-of Strings Radiation of Positrons Channeled in Bent Crystal (CPSOSRPCHBC) will be most productive for polarizing secondary gamma-quanta beams of proton accelerators at multi-hundred-GeV energies considered in [7], it can also fortunately be observed in the GeV-energy region, in particular, at the positron beam of Frascati Nation Laboratory. A novel method of short crystal bending with small radii is suggested for this purpose.

The revealed circular polarization of the crystal field virtual photons which channeled positrons encounter moving in bent crystals opens up other possibilities to observe polarization effects involving both longitudinally polarized positrons and circularly polarized γ-quanta. Namely, the dependence of γ-quantum emission probability on the longitudinal polarization of channeled positrons allows both to obtain longitudinally polarized positrons and measure their longitudinal polarization.

The probability of production of an electron-positron pair, the positron of which finds itself in the channeled state, will depend on both gamma-quantum circular polarization and positron longitudinal one at the SOS crystal orientation. The dependence of pair production probability on the gamma-quantum circular polarization opens up a new way of its measurement at multi-GeV γ-quantum energies under which the traditional methods based on Compton scattering in magnetized iron become inefficient. In turn, the pair production probability dependence on the positron longitudinal polarization naturally allows to produce longitudinally polarized channeled positrons by unpolarized γ-quanta. The electrons produced in pairs with the channeled positrons will also be longitudinally polarized.

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