IL NUOVO CIMENTO Vol. 34 C, N. 4 Luglio-Agosto 2011
DOI 10.1393/ncc/i2011-10969-7

Colloquia: Channeling 2010

Radiation of a relativistic electron with non-equilibrium own Coulomb field

A. S. Fomin, S. P. Fomin(*) and N. F. Shul'ga

Akhiezer Institute for Theoretical Physics, NSC KIPT
1, Akademicheskaya Str., Kharkov 61108, Ukraine

(ricevuto il 22 Dicembre 2010; pubblicato online il 21 Settembre 2011)

Summary. — The condition and specific features of the non-dipole regime of radiation is discussed in the context of the results of the recent CERN experiment NA63 on measurement of the radiation power spectrum of 149 GeV electrons in thin tantalum targets. The first observation of a logarithmic dependence of radiation yield on the target thickness that was done there is the conclusive evidence of the effect of radiation suppression in a thin layer of matter, which was predicted many years ago, and which is the direct manifestation of the radiation of a relativistic electron with non-equilibrium own Coulomb field. The special features of the angular distribution of the radiation and its polarization in a thin target at non-dipole regime are proposed for a new experimental study.

PACS `41.60.-m` – Radiation by moving charges.
PACS `41.75.Ht` – Relativistic electron and positron beams.

1. – Introduction

During last two years the results of recent experimental investigations on the special features of a relativistic electron bremsstrahlung in a thin target were published [1, 2]. These measurements were done by the international Collaboration NA63 in CERN using SPS secondary electron beam with energy around 200 GeV. One of the main motives to carry out this experiment was to make clear the unexpected result of the SLAC experiment E-146 [3-5] that showed a strange behavior of the radiation spectrum of 25 GeV electrons in a relatively small-thickness target, especially for the gold target with a thickness of 0.7% of the radiation length [3, 4].

The SLAC experiment E-146 was generally devoted to the verification of the Migdal quantitative theory of the Landau-Pomeranchuk-Migdal (LPM) effect [6, 7], which describes the suppression of radiation of relativistic electrons in an amorphous matter due

(*) E-mail: `sfomin@kipt.kharkov.ua`

to the multiple scattering on atoms in comparison with the predictions of the Bethe-Heitler theory [8]. The analysis of the data obtained in SLAC experiment E-146 showed good agreement between the calculations using the Migdal formula (LPM effect) and the experimental data for relatively thick targets and not very low photon energies. However, for the case of the gold target with a thickness of 0.7% of the radiation length there was a significant disagreement between theory and experiment [4]. Such "unexpected" behavior of the radiation spectrum at low frequencies was named in [4] as "edge effect" and firstly they tried to exclude it by subtraction procedure, because "no satisfactory theoretical treatment of this phenomenon" was found for that moment. Actually, they found out the Ternovskii article [9], in which the Migdal theory of the LPM effect that developed for the boundless amorphous medium was improved for the finite target thickness case. However, when they tried to use the Ternovskii formula to describe the "edge effect", they obtained the excess of the Bethe-Heitler result [8] instead of the expected suppression, and they wrote in [3] that this formula gives "unphysical result".

The discrepancy observed in SLAC experiment stimulated a new wave of theoretical investigations of the multiple scattering effect on radiation (see [10-15]). In [10] it was shown that the deviation from predictions of the Migdal theory observed in [3-5] takes place when the target thickness t is small in comparison with the coherence length (or formation zone) of the radiation process $l_\mathrm{c} = 2\varepsilon'\varepsilon/m^2\omega$ [16] (here m and ε are the mass and initial energy of an electron, ω is the emitted photon energy, $\varepsilon' = \varepsilon - \omega$, we use the system of units: $h = c = 1$). Exactly this case $t \ll l_\mathrm{c}$ was theoretically considered earlier in [17, 18], where the specific effect of the suppression of radiation in a thin layer of matter was described and discussed in details including its essential distinction from the LPM and BH regimes of radiation. As was shown in [10], the "unphysical result" obtained by the Ternovskii formula in [3] is connected with the usage of the asymptotic formula for a mean-square angle of multiple scattering, which is not applicable for the SLAC experiment E-146 conditions.

The quantitative theory of the radiation suppression effect in a thin layer of matter was developed later in [10-15] using different approaches. The results obtained in these works are in good agreement with the SLAC experimental data for the thin golden target (see, for example, reviews [4, 19]). However, it was the only one explicit manifestation of this effect during the SLAC experiment E-146 and it took place in a relatively narrow photon energy region for 25 GeV electrons. That is why it was necessary to carry out a special experimental investigation of this effect at higher electron energy that gives a wider photon energy region for observation of this effect and, that is even more important, to study the thickness dependence of radiation intensity in a thin-target case, which is fundamentally different from the BH and LPM regime of radiation (see [17-19]).

A new experimental study of the LPM and analogous effects at essentially higher electron energies (up to $\varepsilon = 287\,\mathrm{GeV}$) was carried out recently at CERN by the NA63 Collaboration (see [20, 21] and also [1, 2]). The results of measurements for Ir, Ta and Cu targets with thicknesses about 4% of the radiation length showed good agreement with the Migdal theory of the LPM effect [20]. The effect of suppression of radiation in a thin target, named in these papers as the Ternovskii-Shul'ga-Fomin (TSF) effect, was also considered, however, the photon energy region, in which the TSF effect could be observed for chosen target thicknesses, was below the energy threshold of measured photons $\omega_\mathrm{min} = 2\,\mathrm{GeV}$ for both experiments [20, 21]. The condition for the successful observation of the TSF effect in radiation spectrum was realized later in CERN for 206 and 234 GeV electrons radiation in Ta targets of 5–10 μm thickness [1].

Finally, probably the most complicated measurements for realization, but the most important for demonstration of the TSF effect essence, namely the logarithmic thickness dependence of radiation intensity in a thin target, were successfully carried out recently by the CERN NA63 Collaboration [2]. This is the first direct demonstration of the suppression of radiation effect for a relativistic electron with non-equilibrium own Coulomb field [18, 22]. Note that this effect should have its analog also in QCD at quark-gluon interaction.

In this paper we present the theoretical analysis and treatment of the recent CERN experimental results [1, 2]. We also propose to carry out a new experiment to study the special features of the angular distribution of radiation at the TSF effect conditions, which were theoretically described in [23]. These features can give a new opportunity for obtaining a high degree of linear polarization of gamma-quanta that was proposed in [24].

2. – General conditions and features of LPM and TSF effects

According to the standard Bethe-Hietler theory of bresstrahlung in amorphous matter the radiation power spectrum $\mathrm{d}E/\mathrm{d}\omega$ defined by scattering of the relativistic electron on target atoms is proportional to the target thickness t [8]:

$$\frac{\mathrm{d}E_{\mathrm{BH}}}{\mathrm{d}\omega} = \frac{2t}{3X_0}\left[\left(1+\frac{\varepsilon'^2}{\varepsilon^2}\right)+\frac{\omega^2}{2\varepsilon^2}\right], \tag{1}$$

where X_0 is the radiation length of the target material.

Landau and Pomeranchuk showed [6] that if the root-mean-square angle of electron multiple scattering θ_{ms} at the distance of the coherence length l_{c} exceeds the characteristic angle of relativistic particle radiation $\theta \sim \gamma^{-1}$, where $\gamma = \varepsilon/m$ is the Lorentz factor of an electron, then the radiation power spectrum will be suppressed in comparison with the Bethe-Hietler result given by formula (1).

The root-mean-square angle of electron multiple scattering on atoms in an amorphous medium at the depth t is inversely proportional to the electron energy ε [8, 19]

$$\theta_{ms}(t) = (\varepsilon_s/\varepsilon)\sqrt{t/X_0}\,[1+0.038\ln(t/X_0)]\,, \quad \varepsilon_s^2 = 4\pi\cdot \mathrm{m}^2/\mathrm{e}^2, \tag{2}$$

thus, the target thickness l_γ, at which $\theta_{\mathrm{ms}}(l_\gamma) = \gamma^{-1}$, does not depend on the electron energy ε and is determined by the target material only $l_\gamma \approx 0.15\% X_0$.

Thus, the condition of the suppression of radiation due to the multiple scattering effect $\theta_{ms}(l_{\mathrm{c}}) > \gamma^{-1}$ (the so-called non-dipole regime of radiation) can be written in the following form:

$$l_{\mathrm{c}} > l_\gamma. \tag{3}$$

If $t < l_\gamma$, *i.e.* the target thickness t is less than $0.15\% X_0$, the spectral density of radiation for all possible emitted photon energies is defined by the Bethe-Heitler formula (1).

If $t > l_\gamma$, there are three possible regimes of radiation in this case depending on the energy region of the emitted photon.

For the relatively hard part of the emitted spectrum, when $l_{\mathrm{c}} < l_\gamma$, we have a dipole regime of radiation described by the Bethe-Hietler formula (1) too.

For the non-dipole radiation, $l_c > l_\gamma$, there are two regions, defined by the ratio between the coherence length l_c and the target thickness t, with quite a different behavior of the radiation spectrum. If the target is thick enough, $t \gg l_c > l_\gamma$, the Migdal theory [7] of the LPM effect, which describes the suppression of radiation in a boundless amorphous medium, is applicable. For a relatively thin target, $l_c \gg t > l_\gamma$ (intermediate case), the TSF mechanism of radiation [9,17] is realized.

Condition (3) determines the photon energy region, where the LPM effect is essential:

$$\omega < \omega_{\text{LPM}} = \frac{\varepsilon}{1+\varepsilon_{\text{LPM}}/\varepsilon}, \quad \varepsilon_{\text{LPM}} = \frac{e^2m^2}{4\pi}X_0 \approx 7.7\,\text{TeV}\cdot X_0\,(\text{cm}). \tag{4}$$

It means that for ultrahigh electron energy ($\varepsilon \gg \varepsilon_{\text{LPM}}$) the whole radiation spectrum is suppressed due to the LPM effect: $\omega_{\text{LPM}} \approx \varepsilon$.

If $\varepsilon \ll \varepsilon_{\text{LPM}}$, then $\omega_{\text{LPM}} \approx \varepsilon^2/\varepsilon_{\text{LPM}} \approx 1600\gamma^2/X_0$.

The Migdal function Φ_{M} [7] describes the deviation of the radiation spectrum for $\omega < \omega_{\text{LPM}}$ from the Beth-Hietler formula (1) in a relatively soft part of the spectrum ($\omega \ll \varepsilon$):

$$\frac{\mathrm{d}E_{\text{LPM}}}{\mathrm{d}\omega} \approx \frac{\mathrm{d}E_{\text{BH}}}{\mathrm{d}\omega}\hat{O}_M(s),\ \hat{O}_M(s) = 24s^2\left(\int_0^\infty \mathrm{d}x\text{ctg}(x)e^{-2sx}\frac{\pi}{4}\right),\ s = \frac{1}{2}\sqrt{\frac{\omega}{2\omega_{\text{LPM}}}}. \tag{5}$$

The upper limit for the emitted photon energy for the TSF regime of radiation ω_{TSF} follows from the TSF effect condition

$$l_c \gg t > l_\gamma, \tag{6}$$

It is defined by the equality $t = l_c$ and can be written in the following form:

$$\omega_{\text{TFS}} = \frac{\varepsilon}{1+\varepsilon_{\text{TSF}}/\varepsilon}, \quad \varepsilon_{\text{TSF}} = m^2t/2 \approx 6.6\,\text{PeV}\cdot t\,(\text{cm}). \tag{7}$$

If $\varepsilon \ll \varepsilon_{\text{TSF}}$, one can use a simpler expression for the TSF effect threshold, $\omega_{\text{TSF}} \approx 2\gamma^2/t$.

The quantitative theory of the multiple scattering effect on a radiation of the relativistic electron in a thin layer of matter (the TSF effect) was developed in [10] using classical formulas for spectral density of radiation and the results of the Bethe-Moliere theory of multiple scattering [25]. This approach is valid if $\omega \ll \varepsilon$. Namely such a condition was realized for both experimental investigations at SLAC E-146 [3,4] and at CERN NA63 [1,2].

The radiation power spectrum in this case ($l_c \gg t$) is determined by the formula [11]

$$\frac{\mathrm{d}E_{\text{TSF}}}{\mathrm{d}\omega} = \frac{2e^2}{\pi}\int \mathrm{d}\boldsymbol{\theta}_s f_{\text{BM}}(\boldsymbol{\theta}_s)\left[\frac{2\xi^2+1}{\xi\sqrt{\xi^2+1}}\ln(\xi+\sqrt{\xi^2+1})-1\right], \quad \xi = \gamma\theta_s/2, \tag{8}$$

in which the averaging over the electron multiple scattering in a target is carried out with the Bethe-Moliere distribution function f_{BM} [25] (for details see [10,19]). At $t \ll l_\gamma$ (that means ($\xi \ll 1$) formula (8) gives the Bethe-Hietler result with a linear dependence on the target thickness. In the opposite case, *i.e.* at $t \gg l_\gamma$, formula (8) gives only a

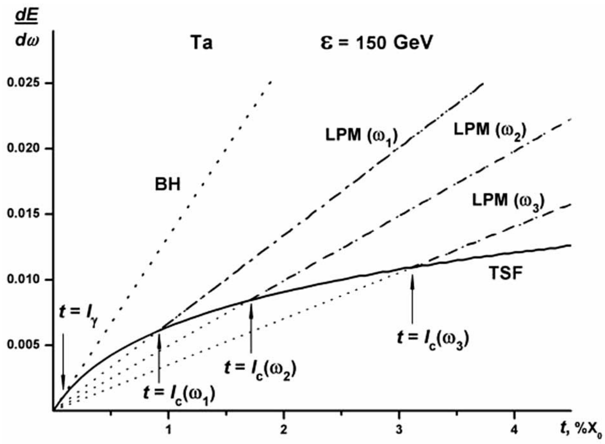

Fig. 1. – The radiation power spectrum of 150 GeV electrons in tantalum target via target thickness $t(\% X_0)$. The detailed description of curves is given in text.

logarithmic increase of the radiation power spectral density with increasing the target thickness.

Such a strange behavior of the radiation power (the scattering angle still increases linearly with thickness, but radiation does not) can be explained by the relativistic delay effect during regeneration of the own Coulomb field of the relativistic electron after its scattering on a large angle $\theta_s > \gamma^{-1}$, and it can be treated as a radiation of the "half-bare" electron, *i.e.* the electron with non-equilibrium own Coulomb field (see [18,19,22] for detailed discussion). This logarithmic behavior will be changed to the linear one again when the target thickness reaches the value of coherence length for the given photon energy ω.

The quantum treatment of the TSF effects was done in [11-15] using different approaches and it became important for ultrahigh electron energy ($\varepsilon \gg \varepsilon_{\mathrm{TSF}}$), when $\omega_{\mathrm{TSF}} \approx \varepsilon$ and the whole radiation spectrum is suppressed due to the TSF effect.

There are two additional factors that have an essential influence on the radiation process in matter, namely, the dielectric suppression (or the Ter-Mikaelyan effect [16]) and the transition radiation from the target bounds [5, 16]. Both these effects could be neglected, if we consider photons energies higher than $\omega_0 = \gamma\omega_p$, where ω_p is the plasma frequency [16]. For tantalum target and the electron beam energy $\varepsilon = 150\,\mathrm{GeV}$ this threshold is about $\omega_0 \approx 25\,\mathrm{MeV}$.

The qualitative difference between the different regimes of radiation in amorphous matter, namely the BH, LPM and TSF regimes and their consequent changing clearly demonstrates the thickness dependence of the radiation power spectrum $\mathrm{d}E/\mathrm{d}\omega$. The results of theoretical calculations of such dependence are presented in fig. 1.

For $t < l_\gamma$, *i.e.* when the target thickness t is so small that the multiple scattering of relativistic electrons in target is not enough to fulfill condition (3), the radiation process has a dipole character and the radiation power spectrum is described by the Bethe-Hietler formula (1). The soft part of the Bethe-Hietler spectrum ($\omega \ll \varepsilon$) does not depend on ω and is described by a very simple formula, $\mathrm{d}E_{\mathrm{BH}}/\mathrm{d}\omega = 4t/3X_0$. The corresponding curve is presented in fig. 1 by the dashed straight line "BH".

With increasing the target thickness the condition $t = l_\gamma$ could be fulfilled, and at this point the dipole regime of radiation is changed to the non-dipole one that leads

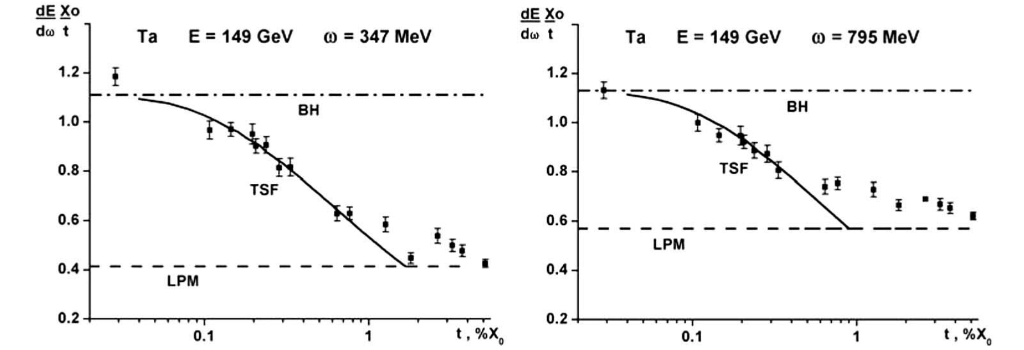

Fig. 2. – The radiation power per unit length for 149 GeV electron radiation in tantalum target via target thickness t ($\%X_0$). A detailed description of the curves is given in the text.

to suppression of radiation comparing with the Bethe-Hietler formula predictions. For relatively soft photons, for which $l_\mathrm{c} \gg t$, the radiation power for this part of radiation spectrum is determined by formula (8) that means the TSF regime of radiation with a logarithmic dependence on the target thickness t (solid line "TSF" in fig. 1). As follows from eq. (8) $\mathrm{d}E_\mathrm{TSF}/\mathrm{d}\omega$ does not depend on the emitted photon energy ω, however, the validity condition of the TSF regime (6) does. It means that for different ω_n the transition from the TSF to the LPM regime of radiation takes place at different values of target thickness $t_\mathrm{n} = l_\mathrm{c}(\omega_\mathrm{n})$. In fig. 1 there are three such points marked by arrows for different photon energies ω_n, namely $\omega_1 = 150$, $\omega_2 = 350$ and $\omega_3 = 800\,\mathrm{MeV}$. There are also three different dot-dashed lines "LPM", which are calculated using the Migdal formula (5) for these values of photon energy respectively. Thus, changing the target thickness one can consequently observe three different mechanisms of radiation of relativistic electron in the amorphous target such as the BH, TSF and LPM.

The first experimental investigation of the thickness dependence transformation from the linear regime (BH) via the logarithmic one (TSF) to the linear (LPM) one again was recently done in CERN by the NA63 Collaboration [2]. In spite of all difficulties connected with a very complicated experimental installation and by operating with a set of very thin targets of several micrometers thickness, this experiment gave a conclusive proof of the suppression effect of relativistic electron radiation in a thin layer of matter predicted many years ago [9,17] and *per se* it gave the unique demonstration of the space-time evolution of the radiation process in matter as an example of relativistic electron with non-equilibrium own Coulomb field [18, 19, 22].

The comparison of experimental data with the results of calculations using different approaches represented in [1, 2] shows a not only qualitative, but also quantitative good agreement. In this short paper we present the comparison of the results of our calculation with the experimental data [2] only for two values of the emitted photon energy, $\omega = 347$ and $795\,\mathrm{MeV}$ (see fig. 2). Following [2] we present here the radiation power spectrum per unit length, *i.e.* $\mathrm{d}E/\mathrm{d}\omega$ multiplied by X_0/t. In these units the linear dependence of the radiation power spectrum for the BH (dot-dashed line) and LPM (dashed line) regimes of radiation are the constants (see fig. 2). The curve TSF shows the logarithmic behavior of the radiation power spectrum in the intermediate region $l_\mathrm{c} > t > l_\gamma$ for a given ω.

For numerical calculations we used the original Fortran code based on the same formulas as the calculations of the SF curves presented in figures in [1,2]. Following [2] we took into account the multiphoton effect by corresponding normalization on the BH radiation spectrum. The results of our calculations give a little excess (about 10%) over the results presented in [1, 2] by SF curves in all figures, thereby they show good agreement with experimental data (see, for example, fig. 2). The essential discrepancy observed around the point $t = l_\mathrm{c}$ is easily explainable by the fact that the Migdal theory of the LPM effect is applicable at $t \gg l_\mathrm{c}$, whereas eq. (8) for the TSF regime of radiation is derived for $t \ll l_\mathrm{c}$. In the intermediate region ($2l_\mathrm{c} > t > l_\mathrm{c}/2$) we have a smooth transition between these two regimes.

3. – Angular distribution and polarization of radiation in the non-dipole regime in a crystal

As was shown in [23], the non-dipole regime of radiation changes essentially not only the spectrum of emitted gamma quanta, but also their angular distribution. In [24] it was proposed to use special features of angular characteristics of non-dipole coherent radiation in a thin crystal for production of intensive photon beams with high degree of linear polarization.

This idea is based on the fact that the non-dipole regime of radiation, when the scattering angle becomes larger than the characteristic angle of radiation of a relativistic electron γ^{-1}, gives the possibility to avoid a mixture of the radiation emitted under different (greater than γ^{-1}) angles. Using photon collimators with angular width about γ^{-1} one can organize the space-angular separation of photons emitted by electrons that were scattered in essentially different directions, for example, perpendicular ones. To realize the non-dipole regime of radiation a high energy of the electron beam is necessary. However, it means that very narrow photon collimators should be used for this purpose: for the electron energy $\varepsilon = 150\,\mathrm{GeV}$ the angle width of the collimator should be about $3\,\mu\mathrm{rad}$. So, it is necessary to find the compromise condition for realization of this idea.

To decrease the minimal electron energy for the non-dipole regime radiation is possible by using the coherent effect at relativistic electron scattering on atomic chains along the crystallographic axis (the so-called "doughnut scattering effect", see *i.e.* [19]). The mean-square angle of multiple scattering in this case can exceed essentially the analogous parameter for amorphous matter [26]. This effect is stronger if nuclear charge of the crystal material is higher, so, the best candidate for the crystal converter would be a tungsten monocrystal.

On the basis of the theoretical approach explained in details in [24] we have carried out the calculations of the angular distributions and polarization of radiation by 3.5 GeV electrons incident on a tungsten crystal at the angle $\psi = \psi_\mathrm{L}$ to the axis $\langle 111 \rangle$ (where ψ_L is the Lindhard angle [27]). In this case $\theta_\mathrm{ms} \approx \psi_\mathrm{L} = 0.6\,\mathrm{mrad}$ and the non-dipole parameter is $\gamma\theta_\mathrm{ms} \approx 4$. The multiple scattering of electrons on crystal atoms was simulated using the binary collision model that makes it possible to take into account both coherent and incoherent scattering on lattice atoms [24]. The angular distribution of radiation and polarization were calculated as the sum of the value from each scattered electron using the scheme described in details in [24]. The results of these calculations are presented in fig. 3.

The left part of fig. 3 presents the angular distribution of corresponding spectral-angular radiation density $\mathrm{d}^2E/\mathrm{d}\omega\mathrm{d}o$ of emitted photons. The right part of fig. 3 presents the angular distribution of the linear polarization degree of emitted photons from the 100% vertically polarized photons ($P = -1$) to the 100% horizontal polarization ($P = 1$). All angles in fig. 3 are measured in units γ^{-1}.

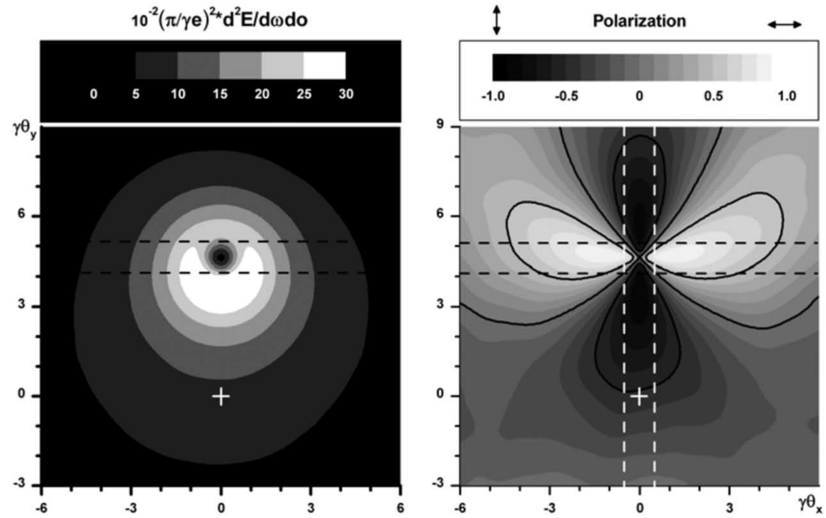

Fig. 3. – The angular distributions (in units γ^{-1}) of the radiation power spectrum emitted by the 3.5 GeV electron beam incident on the tungsten monocrystal of $10\,\mu$m thickness at the angle ψ_L to the axis $\langle 111\rangle$ (left) and the degree of linear polarization of this radiation (right).

The integral (over all angles) degree of linear polarization of radiation is close to zero. However, using the slit-type horizontal (or vertical) photon collimator with the angular width $\Delta\theta_\gamma = \gamma^{-1}$ and putting it as shown in fig. 3 by dashed lines it is possible to obtain a linearly polarized (along the collimator plane) photon beam with polarization degree of about 60%. Note that the radiation intensity in the case of axially oriented crystal is much higher than in the planar orientation case, which is applied normally for production of polarized photon beams.

* * *

We are grateful to all participants of CERN NA63 Collaboration for a brilliant performance of complicated measurements of radiation in a set of ultrathin targets. It allowed the first observation of the logarithmic thickness dependence of the radiation power of a relativistic electron at a non-dipole regime that was predicted many years ago. Special thanks to U. Uggerhoj for fruitful discussions on the subject of this investigation.

REFERENCES

[1] Thomsen H. D. *et al.*, *Phys. Lett. B*, **672** (2009) 323.
[2] Thomsen H. D. *et al.*, *Phys. Rev. D*, **81** (2010) 052003.
[3] Klein S. R. *et al.*, Preprint SLAC-PUB-6378 (1993).
[4] Anthony R. L. *et al.*, *Phys. Rev. Lett.*, **75** (1995) 1949.
[5] Klein S., *Rev. Mod. Phys.*, **71** (1999) 1501.
[6] Landau L. D. and Pomeranchuk I. Yu., *Dokl. Akad. Nauk SSSR*, **92** (1953) 535, 735.
[7] Migdal A. B., *Dokl. Akad. Nauk SSSR*, **96** (1954) 49; *Phys. Rev.*, **103** (1956) 1811.
[8] Bethe H. and Heitler W., *Proc. R. Soc. London, Ser. A*, **146** (1934) 83.
[9] Ternovskii F. F., *Sov. Phys. JETP*, **12** (1961) 123.
[10] Shul'ga N. F., Fomin S. P., *JETP Lett.*, **63** (1996) 873; *Sov. Phys. JETP*, **86** (1998) 32.

[11] Shul'ga N. F. and Fomin S. P., *Nucl. Instrum. Methods B*, **145** (1998) 180.
[12] Blankenbacler R. and Drell S. D., *Phys. Rev. D*, **53** (1996) 6265.
[13] Zhakharov B. G., *JETP Lett.*, **64** (1996) 781.
[14] Baier R., Dokshitser Yu. L., Mueller A. H. *et al.*, *Nucl. Phys. B*, **478** (1996) 577.
[15] Baier V. N. and Katkov V. M., *Phys. Rev. D*, **60** (1999) 076001.
[16] Ter-Mikaelian M. L., *High-energy electromagnetic processes in condensed matter* (Wiley Interscience, New York) 1972.
[17] Shul'ga N. F. and Fomin S. P., *JETP Lett.*, **27** (1978) 117.
[18] Fomin S. P. and Shul'ga N. F., *Phys. Lett. A*, **114** (1986) 148.
[19] Akhiezer A. I., Shul'ga N. F. and Fomin S. P., *Landau-Pomeranchuk-Migdal Effect* (Cambridge Scientific Publishers, UK) 2005.
[20] Hansen H. D., Uggerhoj U. I., Biino C. *et al.*, *Phys. Rev. D*, **69** (2004) 032001.
[21] Uggerhoj U. I., Knudsen H., Ballestrero S. *et al.*, *Phys. Rev. D*, **53** (2006) 6265.
[22] Feinberg E. L., *Sov. Phys. JETP*, **23** (1966) 132.
[23] Fomin S. P., Shul'ga N. F. and Shul'ga S. N., *Phys. At. Nuclei*, **66** (2003) 396.
[24] Fomin A. S., Fomin S. P. and Shul'ga N. F., *Proc. SPIE*, **5974** (2005) 177.
[25] Bethe H., *Phys. Rev.*, **89** (1953) 1256.
[26] Shul'ga N. F., Truten' V. I. and Fomin S. P., *Sov. Phys.-Techn. Phys.*, **27** (1982) 1399.
[27] Lindhard J., *K. Dan. Vidensk. Selsk. Mat.-Fys. Medd.*, **34**, no. 14 (1965).