ECONOMIC VIABILITY OF A WATER SUPPLY GRAVITY MAIN

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Abstract: The water supply gravity main does not require power input, thus it is preferred in comparison to water supply pumping main. Moreover, gravity main has greater reliability as it does not have moving parts, e.g. pumps and motors, and is independent of power requirement. Availability of regular power supply at required current and voltage is a problem in many parts of the globe. For a gently sloping topography the gravity main involves large pipe diameters. Thus, in comparison to a pumping main a gravity main may be uneconomical due to large size and associated overall cost. A review of literature indicated that there is no guideline available for the adoption of a gravity main for a gently sloping terrain. In this investigation, a criterion has been obtained to ascertain if gravity main or pumping main will be economic for a given gentle terrain for pipe laying.

Key words: Cost, gravity main, pumping main, terrain.

Introduction

A water supply pumping main, as seen in Fig. 1a, can be adopted in any type of topographic configuration for the supply of water. On the other hand, according to Fig. 1b, a water supply gravity main is feasible only if the input point is at a higher elevation than the exit point. In a pump driven water network, the designer has some degree of control over the location and amount of energy required in the network to maintain desired flow and pressure, while such luxury does not exist in gravity-driven systems (Jones 2011). In the gravity-driven systems, the elevation difference provides the potential energy to overcome the headloss due to frictional

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resistance to the flow in pipes. The pipe diameter required to flow desired quantity will depend upon the elevation difference. There is an inverse functional relationship between elevation difference and pipe diameter (Swamee et al. 2018). If the elevation difference between the input point and the exit point is very small, the required pipe diameters for gravity main will be large, which may not be economical in comparison with the corresponding pumping main. Thus, there exists a slope at which both gravity and pumping main will have the same life cycle cost. This slope may be called equal-cost slope. If the terrain slope is greater than the equal-cost slope, the gravity main will have an edge over the pumping alternative. In case of steep slopes, corrugated pipes can be used in which the friction factor is relatively large to avoid maximum velocity constraints (Calomino et al. 2015 and Calomino et al. 2018).

The research is focused on developing a method for selecting a gravity or pumping main for a gently sloping terrain based on cost considerations. Presented herein is an equation for equal-cost slope for gravity and pumping mains. The application of this criterion has been demonstrated by an example.

Analytical Considerations

Pumping main

The cost of a pumping main per unit length of pipe, F_p, is given by (Chapter 4, Swamee and Sharma 2008)

$$F_p = k_m D_p^m + k_T \rho g Q S_f - k_T \rho g Q S_{oa} \tag{1}$$

where D_p = pumping main diameter; k_m = pipe cost coefficient; and m = exponent; k_T = pumping cost coefficient (incorporating operation and maintenance cost of pumps); ρ = mass density of water; g = gravitational acceleration; and Q = discharge; and S_f = friction slope; and S_{oa} = available ground slope. The last term of Eq. (1) represents cost involved in lifting of the water. As there is no adverse slope in the present case, there cannot be a reduction in cost due to positive slope. Therefore, the last term in Eq. (1) should be zero. That is,

$$F_p = k_m D_p^m + k_T \rho g Q S_f \tag{2}$$

First term in Eq. (2), represents the capitalized cost of the pipes and second term the capitalized cost associated with operation and maintenance of pumping system (Swamee and Sharma 2008).

The pressure head h_o due to the water column in the balancing tank can vary significantly due to water inflow and outflow from the tank. Moreover, such water columns in balancing tanks even when completely full are just few meters and thus can be neglected. On the other hand, being small, the terminal head, the entrance and the exit losses are also neglected. With these assumptions the resistance equation is written as

$$S_f = \frac{8 f_p Q^2}{\pi^2 g D_p^5} \tag{3}$$

where f_p = friction factor for pumping main. Eliminating S_f between Eqs. (2) and (3)

$$F_p = k_m D_p^m + \frac{8 k_T \rho f_p Q^3}{\pi^2 D_p^5} \tag{4}$$

For minimum, differentiating F_p with respect to D_p and equating it to zero and simplifying optimum diameter D_p^* is obtained as

$$D_p^* = \left(\frac{40 k_T \rho f_p Q^3}{\pi^2 m k_m} \right)^{\frac{1}{m+5}} \tag{5}$$

Using Eqs. (4) and (5) the optimum cost per unit length F_p^* of a pumping main is

$$F_p^* = k_m \left(1 + \frac{m}{5}\right) \left(\frac{40 k_T \rho f_p Q^3}{\pi^2 m k_m} \right)^{\frac{m}{m+5}} \tag{6}$$

The friction factor for pipe is given by Swamee and Jain (1976) equation

$$f_p = 1.325 \left[\ln\left(\frac{\varepsilon}{3.7 D_p} + \frac{5.74}{\mathbf{R}_p^{0.9}} \right) \right]^{-2} \tag{7}$$

where ε = roughness height of pipe surface; and $\mathbf{R}_p$ = pumping main Reynolds number given by

$$\mathbf{R}_p = 4Q/(\pi \nu D_p) \tag{8}$$

Gravity main

The cost of a gravity main per unit length F_g given by

$$F_g = k_m D_g^m \tag{9}$$

where D_g = diameter of gravity main. The friction slope as given by the Darcy-Weisbach equation is equal to the bottom slope S_o given by

$$S_o = \frac{8 f_g Q^2}{\pi^2 g D_g^5} \tag{10}$$

where f_g = friction factor for gravity main. Eq. (10) gives the diameter D_g of the gravity main as

$$D_g = \left(\frac{8 f_g Q^2}{\pi^2 g S_o} \right)^{\frac{1}{5}} \tag{11}$$

Eqs. (9) and (11) yield

$$F_g = k_m \left(\frac{8 f_g Q^2}{\pi^2 g S_o} \right)^{\frac{m}{5}} \tag{12}$$

Equal-cost slope

It can be seen that whereas the pumping main cost F_p^* is independent of ground slope, the gravity main cost F_g depends on the ground slope. For a slope S_{oe} the gravity main cost will be equal to the pumping main cost. For ground slope less than S_{oe} pumping will be more economical. The cost associated with S_{oe} is given by putting $S_o = S_{oe}$ in Eq. (12). That is,

$$F_g = k_m \left(\frac{8 f_g Q^2}{\pi^2 g S_{oe}} \right)^{\frac{m}{5}} \tag{13}$$

Using Eqs. (6) and (13) the equal-cost slope S_{oe} at which both gravity and pumping modes of flow have equal preference is given by,

$$S_{oe} = \frac{8 f_g Q^2}{\pi^2 g} \left(\frac{5}{m+5} \right)^{\frac{5}{m}} \left(\frac{\pi^2 m k_m}{40 k_T \rho f_p Q^3} \right)^{\frac{5}{m+5}} \tag{14}$$

Using Eqs. (14), Eq. (11) is written for the equal cost slope S_{oe} as

$$D_g = \left(\frac{m+5}{5} \right)^{\frac{1}{m}} \left(\frac{40 k_T \rho f_p Q^3}{\pi^2 m k_m} \right)^{\frac{1}{m+5}} \tag{15}$$

Using Eqs. (5) and (15)

$$D_g = \left(1+\frac{m}{5}\right)^{\frac{1}{m}} D_p^* \tag{16}$$

Using Eqs. (11) and (16) for equal cost slope

$$S_{oe} = \left(\frac{5}{m+5}\right)^{\frac{5}{m}} \frac{8 f_g Q^2}{\pi^2 g D_p^{*5}} \tag{17}$$

in which f_g is given by Swamee and Jain (1976) equation

$$f_g = 1.325\left[\ln\left(\frac{\varepsilon}{3.7D_g} + \frac{5.74}{\mathbf{R}_g^{0.9}}\right)\right]^{-2} \tag{18}$$

where $\mathbf{R}_g$ = gravity main Reynolds number given by

$$\mathbf{R}_g = 4Q/(\pi \nu D_g) \tag{19}$$

Example

Find economic feasibility of a gravity main as compared to a cast iron pipeline to carry a discharge of 0.25 m^3/s on a longitudinal slope of 0.00075. For the design g = 9.79 m/s^2; and ν = 1.007×10^{-6} m^2/s (water at 20°C) have been adopted. Adopt k_T/k_m = 0.0131 SI units (for further details on k_T/k_m refer to Swamee and Sharma 2008), m = 1.6, roughness height of pipe surface ε = 0.25 mm (considering similar pipe material for both pumping and gravity mains).

Solution: Assuming f_p = 0.01 initially and using Eq. (5), one obtains D_p^* = 0.4506 m. Using Eq, (8) $\mathbf{R}_p$ = 701,574. Using Eq. (7) f_p is revised as 0.01785. Again using Eq. (5) with revised f_p, $D_p^* = 0.4919$m. In the next iteration the process converges to f_p= 0.01762l and $D_p^* = 0.4909$ m. The process of f_p and D_P^* computation is repeated till two consecutive values are very close. Using Eq. (6) F_p^* = 0.4242k_m. Using Eq. (16) D_g = 0.5840 m. Eq. (19) gives $\mathbf{R}_g$ = 541,302. Further, using Eq. (18) f_g = 0 .01723. Using these values in Eq. (17) gives S_{oe} = 0.00130, which is greater than the available slope S_{oa} = 0.00075. Thus, pumping option is more economical.

It can be seen by using Eq. (12) one gets F_g = 0.4242k_m, which is same as F_p^* for calculated S_{oe}.

On contrary, taking S_{oa} = 0.00075 and assuming f_g = 0.01 Eq. (11) gives D_g = 0.5858 m; further using Eq. (19) $\mathbf{R}_g$ = 539,508. For these values Eq. (18) gives f_g = 0.01722. In a subsequent iteration process the solution converges to D_g = 0.6516 m. Adopt D_g = 65 cm. Using Eq. (12) that gives $F_g = 0.5020k_m$, which is more expensive than pumping main option.

Conclusion

Pumping and gravity mains are the two options for a water supply based on the topography of the mains alignment. If the elevation difference between the supply and delivery points is small, although both mains can be functionally feasible, however only pumping or gravity main will be economical. A criterion has been developed to estimate equal-cost slope at which the cost of gravity and pumping mains will be the same. It is based on pipe cost exponent *m*, friction factor in pipe *f*, pipe diameter *D* and gravitational constant *g*. If the ground slope is less than the equal-cost slope, pumping main option will be economical and vice versa. The developed criterion will help water professional/ designers to decide if a pumping or gravity main will be more economical for a given terrain.

Notation

D^* = optimal pumping main diameter (m);

f = Darcy-Weisbach friction factor (nondimemsional);

F_g = Gravity pipeline cost for unit length ($/m);

F_p = pumping main cost per unit length ($/m);

F_p^* = optimum pumping main cost per unit length ($/m);

g = gravitational acceleration (m/s^2);

k_m = pipe cost coefficient($/m^{m+1});

k_T = pumping cost coefficient [$s^3/(mkg)];

m = pipe cost coefficient exponent (nondimemsional);

Q = Discharge (m^3/s);

$\mathbf{R}$ = Reynolds number (nondimemsional);

S_f = friction losses in pipe (nondimemsional);

S_{oa} = available topographic slope (nondimemsional);

S_{oe} = equal-cost slope (nondimemsional);

ε = the average roughness height of the pipe surface (m);

ν = kinematic viscosity of water (m^2/s); and

ρ = mass density of water (kg/m^3).

Data Availability Statement

No data, models, or code were generated or used during the study (e.g., opinion or data-less paper).

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