A two-stage stochastic inventory management model for an intermodal trucking company

by

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Abstract

Intermodal transportation faces several challenges due to uncertainty in rail schedules and customer demand. However, this uncertainty is rarely considered for determining asset management at the Intermodal rail yards. Typically, each Intermodal rail yard requires certain inventory of chassis to serve the demand for either empty containers or loaded containers. It is crucial for any transportation firm to optimally allocate and move chassis between rail ramps to overcome random demand.

This thesis develops a two stage stochastic optimization model to determine the optimal allocation and repositioning decisions for chassis and empty boxes across the rail yards to minimize costs and meet service levels. The first stage formulation contains the initial chassis allocation decisions which are independent from random parameters in the following time periods. The second stage formulation determines the empty boxes and chassis repositining decisions for subsequent time periods when the random demand is realized. This thesis applies the L-Shaped Method to efficiently solve this problem.

Using numerical experiments, this thesis analyzes the impact of system parameters on the run time performance. The thesis also analyzes the impact of initial chassis inventory and demand patterns on the optimal decisions. We observe that the higher initial inventory or demand at one location than the other results in an increase in the required repositioning moves and expected cost. Conversely, the model is fairly robust to how inventory and demand values are distributed between resource types.

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Finally, I would like to thank and dedicate this thesis to my family. Their constant love and support have encouraged me every step of the way to this point. Without them, I likely would not have considered pursuing a Masters degree in the first place.

Dedication

To my family

Chapter 1

Introduction

With the advent of globalization and distributed customer base, transportation services account for the majority of the product life cycle. Depending on the type of the products and the length of the haul, transportation services can be classified into the following categories: Truckload, Intermodal, and Bulk transportation. Truckload transportation serves short distance hauls and covers multiple trips in a day. However, the Intermodal transportation requires multiple modes of travel (truck services within a hub and rail services across hubs) to cover long distances hauls. Bulk transportation is completely different and involves transportation of bulk products (like aerosols, oils, chemicals, etc.) and requires specialized operations. This thesis focuses on the inventory management issues in the Intermodal transportation industry.

The Intermodal transportation within a hub experiences much shorter and more frequent trips. Many of the efficiencies gained in improving truck routing is in the reduction of empty miles. Empty miles are typically created when the repositioning of a driver and/or chassis is required for the next fulfillment task. Minimizing these non-value-added repositioning moves is often the main mechanism through which a transportation management system increases operational efficiency. The main opportunities in the system examined in this research for reducing the number of empty miles is through the ordering of a larger chassis inventory and the better repositioning of assets at the end of the day. This thesis examines such issues with repositioning chassis and containers under uncertainty in demand and costs and develops a stochastic optimization model to efficiently solve across the possible realizations of these parameters.

The specific problem setting modeled in this research is that of a trucking company operating in a location with two transportation yards, such as two railyards or a railyard and a shipping dock. The primary role of the company in this setting is to facilitate the movement of both full and empty intermodal containers between these two hubs. In order to perform the movement of a container from one location to the other, an unloaded chassis must be present at that location. While initial inventory and additional inventory ordering costs are known at the beginning, future movement costs and demands are uncertain and could take on many different values with estimated probabilities. During each day, the company has the opportunity to move empty containers and chassis to better prepare for the next day's demand.

The stochastic chassis management model developed in this thesis would prove uniquely valuable to the intermodal trucking company for three main reasons. First, this model provides the framework for both ordering additional resources for each location in addition to the transfer of chassis between locations. Secondly, the proposed model is able to consider any random distribution for demands and costs. This provides value to the company by recommending management decisions that are robust to a variety of possible demands rather than optimal for a single value, which may not be the value that is actually seen in practice. Finally, the decomposition of the proposed model provides quicker solutions for the large problem sizes necessary for accurate chassis management recommendations.

Most research and industry applications in the transportation optimization field model problems using known and set parameters. The solutions generated by these models work very well when the values used as inputs to the model are the same as those realized in the real world. However, parameters such as travel times, demand, and costs very often take different values from those that were predicted at the beginning of a day or week. These different realizations of parameters can have out-sized effects on the performance of models that only considered their predicted values.

In contrast, this research aims to incorporate many different realizations of parameters into the model using stochastic optimization. In a sense, this stochastic model adds a layer of sensitivity analysis and risk management to a deterministic model. Having a reputation of reliability is very valuable in the transportation industry. Not being able to fulfill a client's order due to a slight increase in demand from what was expected can have large tangible and intangible costs to a company. Having a transportation management system that can account for variations in input data allows a company to position its fleet in a way that minimizes its average cost over a range of scenarios instead of one that minimizes the individual cost of the most likely scenario.

The solutions generated by this model will be more robust to variations in parameters at the expense of increased model complexity. The L-Shaped Method is applied in the proposed model to combat this increase in complexity and run-time. The goals of this research are to assess the feasibility of such a modeling technique, analyze the benefit of solution robustness, and examine how the distribution of stochastic parameters can impact decision-making.

1.1 Truckload Transportation

In the United States, the trucking industry employs about 6% of the total population and moves 71% of all freight (John, 2019). However, the way in which it moves varies within the trucking industry. Truckload transportation is typically used to carry goods short to medium distances and is categorized as either Full Truckload (FTL) or Less Than Truckload (LTL) shipping.

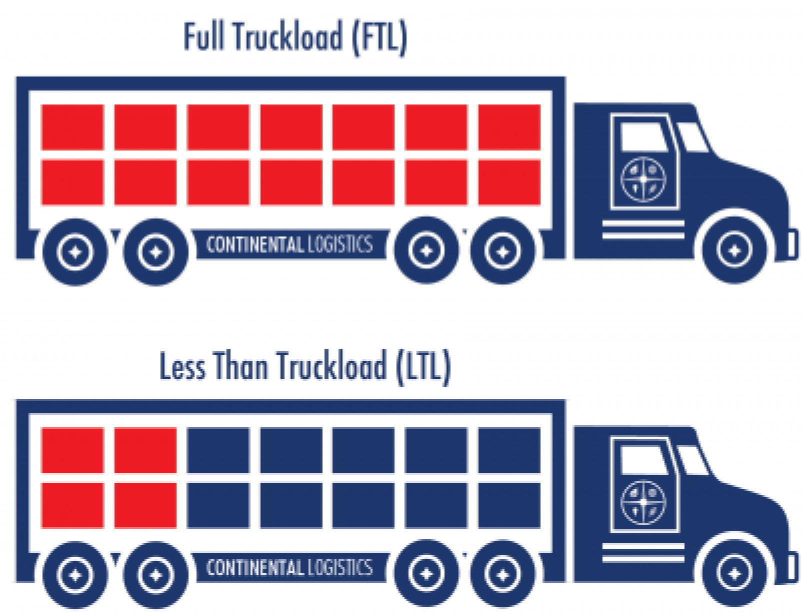

Figure 1.1: LTL vs. FTL (*FTL vs. LTL Trucking*, 2018)

FTL shipping is used when a single business has the volume to completely fill trucks with the products they are shipping between an origin and destination (LaGore, 2018). FTL could be used by a company that has the size to justify having its own dedicated supply chain, but wishes to outsource the ownership and/or management of the trucking fleet. Because the company is essentially renting entire trucks, it gets a much higher degree of control over when and how deliveries are made ("LTL vs.

FTL", 2019). This control can be especially valuable to companies that are shipping high-value or time-sensitive products ("LTL vs. FTL", 2019). This industry has fairly simple routes as each truck completely fills its trailer and then delivers it directly to the destination.

Comparatively, LTL shipping collects multiple smaller loads from different customers and delivers them to multiple destinations. Companies with shipment sizes generally less than 6 pallets will find LTL to be more cost-effective than FTL shipping ("LTL vs. FTL", 2019). While this option can provide less flexibility and lower service levels than FTL trucking, being able to combine shipments from multiple companies allows for a much higher truck utilization and lower costs than shipping multiple half-full trucks. Routing decisions in this industry are more complex as the company must decide on the order of both picking up the loads and delivering them, while accounting for the capacity of the truck.

1.2 Intermodal Transportation

Intermodal shipping refers to all modes of transportation that use intermodal containers. This often includes some combination of truck, rail, and waterway transportation. The usage of rail and cargo ships is the most economical option for shippers making deliveries to a distant destination. Shipping lanes over 700 miles typically use intermodal and generally see a 10% savings compared to using truckload shipping for that lane (LaGore, 2018).

Routing decisions in intermodal shipping have similarities to both FTL and LTL trucking. Similar to FTL trucking, intermodal containers are full, discrete loads that have one origin and one final destination. However, any intermodal container will be handed off between different entities and have multiple intermediate destinations

Figure 1.2: LTL vs. FTL (*Intermodal Container Loading*, n.d.)

before reaching its final destination. Intermodal trucking is most often used in these interactions between longer haul shipping entities, such as rail or cargo ships, and as a final mile delivery service. The specific problem setting examined in this research is that of an intermodal trucking company operating between intermodal transportation hubs. This company serves as an intermediary between these two hubs by transporting intermodal containers between the two locations.

The resources available to this company are loaded containers, empty containers, and chassis. Loaded containers contain product to be delivered from one location to the other. There is also demand for empty containers to be shipped from each transportation hub. Unlike loaded containers, empty containers are considered to be homogeneous and can be stored in inventory at each location. Finally, the company has access to an inventory of chassis which the containers are loaded onto and are required to transport resources from one location to the other.

1.3 Inventory Management

At its core, the problem that this research is modeling is one of inventory management. The key decision variables are the inventory levels of chassis and empty containers at each of the transportation hubs. These inventory levels determine the capacity of loaded container movement, the ability to fulfill empty demand, and the costs of potentially necessary chassis repositioning movements. However, this problem deviates from traditional inventory control models in a few important ways.

The first difference is that multiple locations share a total inventory supply and must allocate this supply between them. Most inventory control models examine how much inventory to order at a certain location given a distribution of demand realizations. This problem instead introduces the ability to reposition resources by incurring a travel cost. This allows for overall lower inventory levels as the resources required by one facility may be acquired from another facility.

The second distinction is that, after the first time period, no additional resources can be ordered. The inventory levels determined in the first time period decide how much of each resource is available to the system in all future time periods. This initial inventory ordering must take into account the distribution of demands for each location not only in the next time period but all following time periods in the problem.

The main decisions made before parameters are realized are how much of each resource is needed in the system and where to allocate those resources. These decisions will influence all other routing decisions after demands and costs are realized. In terms of chassis inventory, it will determine the number of repositioning moves required to satisfy demands at each location. For empty inventory, it will determine the number of repositioning moves as well as whether or not the realized demands are able to be satisfied.

1.4 Complexity Due to Uncertainty

The base deterministic model for many stochastic optimization applications can often be quite large and require a significant amount of computation time to solve. Adding uncertainty to a model roughly multiplies the original problem size by the number of random scenarios that are being considered. For example, if a problem has 10 variables present in 10 constraints that contain a parameter that will now be modeled as a random variable with 100 realizations, the new problem will contain 1,000 variables and constraints.

The solution time for general linear programming problems can be solved in polynomial time (Borgwardt, 1987). This non-linear relationship between problem size and solution-time can quickly become prohibitive for practically sized applications of stochastic optimization.

Many stochastic optimization techniques have been developed to combat this model complexity problem. In particular, the L-Shaped Method takes advantage of the independence of random variable realizations to separate each realization scenario into smaller models that can be solved faster individually than the aggregation of all scenarios (Birge & François, 2011). The L-Shaped Method's algorithm is based upon the logic in Benders' Decomposition (Benders, 2005).

Benders' Decomposition was developed to solve mixed-integer programming problems. It accomplishes this by separating the continuous and integer variables into different model classes (Benders, 2005). It then iterates between these model classes until an optimal integer-feasible solution is found. The L-Shaped Method adapts this by separating the decisions that happen before the realizations of random variables occur into a first-stage model and those decisions made afterward into a second-stage model (Birge & François, 2011).

The benefit of using this decomposition in stochastic optimization is that instead of having one model with 10,000 variables, the L-Shaped Method can have 100 second-stage models each with 100 variables. This allows the solution time of the overall problem to increase linearly with the number of scenarios considered instead of exponentially. Additionally, these models are completely independent within each iteration. This allows for separate cores within a computer, or separate computers within a server, to solve these models simultaneously and significantly improve overall computation time.

1.5 Research Tasks

The main objective of this research is to examine the optimal inventory and resource repositioning decisions made by an intermodal trucking company that operates in moving containers between two major transportation hubs with uncertain demands and costs. The company has three main resources: loaded containers that contain product to be shipped, empty containers that can be stored as inventory or shipped to meet external or internal demand, and chassis that are stored at each location and are used to carry the containers between hubs. In reality, not all intermodal containers are the same. They are typically either 20 or 40 feet long and can have other variations, such as refrigeration units or tanks for liquids. In the interest of keeping the base modeling of this problem simple in order to instead examine the complexity of adding uncertainty, these differences are neglected in this research. The goal of the company is to satisfy its clients' demands at the least cost to itself.

Figure 1.3 shows how resources are able to flow between the two rail ramps. In each time period, the intermodal company must determine how to reposition its resources between the two locations in order to best prepare for the next time period's

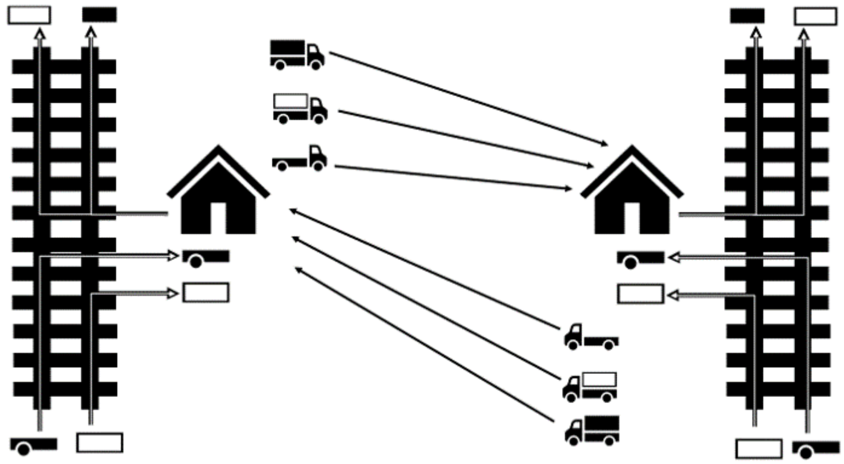

Figure 1.3: Intermodal Transportation Between Hubs

uncertain demand. In the initial time period, the company must also determine how many additional chassis and empty containers should be ordered to each location. Optimally positioning its resources allows the company to save costs and improve service rates, even when key parameters are uncertain.

To solve this problem, this thesis first proposes an extensive form model that captures all decisions and costs in all considered realizations of stochastic parameters. This extensive form model is then decomposed into a stochastic optimization model using the L-Shaped Method. This model is then solved and analyzed to investigate the following research questions:

RQ1: What are the optimal container and chassis inventory management decisions?

RQ2: How does the stochastic solution compare to the solution of a deterministic adaptation?

RQ3: How do system parameters such as demand distribution and initial inventory impact the optimal inventory decisions?

Answering these research questions will provide an in-depth understanding of the solutions generated by the model and how changes in parameter values/distributions affect the optimal resource allocation strategies.

1.6 Thesis Outline

This thesis is organized into five chapters. Chapter 2 provides a literature review of the material examined in this thesis. This includes reviews of intermodal trucking, parameter estimation, and optimization modeling techniques.

In Chapter 3, the problem is described in detail and an extensive form model is proposed to solve the problem. This extensive form model is then decomposed using the L-Shaped Method and an equivalent stochastic optimization model is proposed. The chapter also provides insights to research question **RQ1**.

This stochastic optimization model is solved and analyzed using numerical experiments in Chapter 4. Each section in this chapter examines experiments designed to investigate a different research question outlined in the previous section. The chapter also provides insights to research questions **RQ2** and **RQ3**.

Finally, the thesis is concluded in Chapter 5 with a summary of the model, the results of the numerical experiments, and a discussion of possible future work.

Chapter 2

Literature Review

2.1 Intermodal Trucking Industry

Intermodal transportation refers to the transportation of goods using two or more types of carriers. Any product that travels part of its journey on one type of carrier, such as a cargo ship, and then must switch to a different carrier, such as a railway, is considered to have traveled via intermodal transportation. The intermodal industry has a few standard shipping containers, typically either 20 or 40 feet long, that allow for easy transitions between rail, water, road and even air carriers (Mohit, 2019).

This thesis focuses specifically on the intermodal trucking industry. While the other modes of transportation typically haul containers over the greatest distance, trucking usually carries containers at the beginning or end of their journey or serves as an intermediary link between carriers ("What is Intermodal Trucking", 2018). Transferring containers from and to other transportation hubs makes up a large portion of a product's total transportation costs with estimates between 25-80% (Daham et al., 2017). Intermodal trucking companies often see a large volume of short distance trips when compared to other modes of transportation. Because of this, there is a large opportunity for efficiency gains through smart fleet positioning and routing.

Much of the research into how to realize these efficiency gains is focused on how to reduce the number of empty miles driven by a fleet's trucks. Empty miles occur when a truck reaches its delivery location and then must drive some distance to its next pickup location. This distance traveled is not value-added to the trucking company or their clients. It is estimated that 20-30% of the trucking industry's total miles are empty miles (Berman, 2019; Schulz, 2016). The largest portion of the cost of empty miles is in the drivers' time, but other costs range from fuel and truck wear to increased carbon emissions (Berman, 2019).

Strategies for mitigating the number of empty miles traveled include combining pickup and delivery trips (Daham et al., 2017), smarter routing algorithms, and even posting empty return lanes online to attract potential shippers (Kerr, 2010). Most studies that look at more efficient vehicle routing strategies use mixed integer programming models (Daham et al., 2017) to minimize total travel costs to the fleet. Examples of these models will be discussed in Section 2.3.2.

2.2 Data Estimation and Modeling

2.2.1 Travel Time Estimation

Optimization models depend heavily on the parameters input to them in order to determine the best solution. For transportation applications, this mostly includes the travel time and demand estimates. Travel time estimates heavily influence when and where a model decides to route products (Yang et al., 2010). Most transportation systems only consider the expected travel times, so research into better understanding travel times focuses heavily on their mean values rather than their associated variance.

Methods for estimating travel times vary widely in complexity. Perhaps the simplest is to use the historical average for a given starting point and destination (Yang et al., 2010). However, increasing the estimation complexity a small amount to include a departure time allows the expected travel time to account for fluctuations through the time of the day or day of the week (Rice & van Zwet, 2004; Wedin & Norinder, 2015). Relatively simple models that include this information can achieve fairly high accuracy. Daniel Wedin (2015) showed that using historical GPS data to estimate travel times given an expected departure time could give results with a mean absolute percentage error of 16.5%, outperforming Google Maps on the same data set.

It is expected that more complex models can provide more accurate and reliable results. Ghiani et al. (2008) look at a prediction model that accounts for two sets of parameters: deterministic and stochastic. Deterministic parameters include values that do not change for a given route. Examples of deterministic parameters include the length of the route and the number of lanes on the roads in the route. Stochastic parameters include values that are expected to change depending on when the route is traveled. Examples of stochastic parameters include weather and traffic conditions. Using these value, the authors developed two neural network models to predict travel times. Both models found the traffic conditions to be useful predictors, while one model found the weather conditions to be useful predictors (Ghiani et al., 2008).

Wei et al. (2018) further expand on the idea of using traffic conditions to predict travel times by predicting how congestion on downstream sections of a road propagate toward upstream sections. The authors also use neural networks to predict travel times, however they incorporate a feature that captures the time-shifted delays caused by downstream traffic congestion. This feature was calculated using historical data for the major roads in their study.

2.2.2 Demand Estimation

Demand estimations are traditionally used to plan inventory levels in production planning. In settings with variable demand, the estimate plus a safety stock level is often used to accommodate for fluctuations in the realized demand. While this research focuses on the shipment, rather than production, of goods, having accurate predictions of what resources are going to be needed and when is important to the efficiency of the company as well as the accuracy of a decision-making model using these predictions.

Estimating a future demand level from historic demand data is typically modeled as a time series forecasting problem. One of the most well-known time series forecasting methods is Autoregressive Integrated Moving Average (ARIMA) (Box & Jenkins, 1976). This method is a good starting point as it is very general and accounts for trends over time as well as seasonality. However, many applications have unique qualities that require more complex methods to model. Flores, Graff, and Rodriquez (2012) propose an algorithm for automatically setting parameters and performing feature selection for ARIMA and artificial neural network models using a genetic algorithm. The goal of this research was to create a general application that could accurately forecast the availability of fuel for renewable energy plants, such as wind speed or solar radiation levels, without the expertise of someone skilled in model tuning. They found that the artificial neural networks generally outperformed the ARIMA models and that the genetic algorithm was able to improve both methods through model tuning (Flores et al., 2012). This research shows that while simple methods can provide good and quick predictions, performing further tuning and testing on forecasting models can provide valuable gains in accuracy.

Gilbert (2005) researches how ARIMA time series models apply to multistage supply chains and proposes a model that demonstrates how variations in demand propagate through a supply chain. In this research, the author models a supply chain as multiple entities preparing for demand according to their own ARIMA time series equation based on historical values with a lead-time delay between each subsequent entity receiving new demand values. This delay in information sharing causes variations in demand values to increase in magnitude as they propagate through a supply chain. This phenomenon is commonly known as the bullwhip effect. By modeling a multistage supply chain in this way, it was shown that the main determinant in the magnitude of the bullwhip effect is the total supply chain cycle time and not the number of entities in the supply chain (Gilbert, 2005). This research also demonstrates how the bullwhip effect can occur through normal variation in demand values as well as actual shifts in the underlying demand distribution (Gilbert, 2005).

Most of the research into time series forecasting is focused on predicting a single value for each future time period. However, one of the main motivations for the model proposed in this thesis is that it is able to optimize inventory repositioning decisions for a range of random demand values. Simple statistics, such as mean and variance, can be collected from historical demand data and used to provide demand estimates and their respective probabilities. Demand data could also be used estimate a statistical distribution for the population using methods such as the Chi-Squared test. However, neither of these methods accounts for the fact that historical demand data are from a time series and could exhibit trends or seasonality. Not modeling these factors could significantly overestimate the variance in demand and negatively impact the accuracy of the stochastic optimization model. While there is not much research into this niche use of time series data, a good solution would be to simply adapt the methods discussed previously to forecast multiple parameters of a distribution rather than just the expected value. For example, if historical demand data are best represented by a Normal distribution, then ARIMA could be used to predict both the mean and variance of the next time period.

2.3 Optimization Techniques

2.3.1 Deterministic Models

Transportation routing problems are generally modeled using mixed integer programming (MIP) models. The general formulation for these problems fall into the vehicle routing problem (VRP) category (Daham et al., 2017; García et al., 2013; Rais et al., 2014). Even for medium-sized problems with deterministic parameters, this class of problems is computationally hard to solve (Daham et al., 2017). The general problem formulation must consider all routes that a fleet could take to serve its customers in certain time windows.

Rais et al. (2014) consider an extension of the general vehicle routing problem that allows for the transshipment of products along their route. Transshipment in this case is defined as the ability of a product to be handed off from one vehicle to another, even vehicles of different types (Rais et al., 2014). The authors build upon the general MIP model with additional variables and constraints to account for which products can be transshipped to specific customers. This model can be seen as incorporating the intermodal routing decisions into the larger supply chain routing model. While this thesis only looks at a particular slice of the supply chain industry, it is important to consider how decisions in the intermodal trucking industry can impact decisions in the rail and water transportation industries to which it is connected.

Because the general vehicle routing problem is hard to solve, many researchers attempt to model specific applications using different techniques. Similar to the strategy this paper uses in splitting one big problem into smaller and more manageable pieces, García et al. (2013) break up a large intermodal shipping problem into several assignment models that are solved using linear programming. An artificial intelligence then looks at the set of truck and route assignment solutions and selects the best combinations it can find for the fleet (García et al., 2013). The results of this study show that breaking a problem into several smaller models and then optimizing over their solutions can be effective strategy in reducing the solving times of large problems.

Another strategy for reducing the complexity of the VRP is to instead model it as a project assignment problem (Daham et al., 2017; Elimam & Dodin, 2013). The basic premise is that instead of considering routes for vehicles to take, the project assignment models consider how to pair vehicles with pickup and delivery tasks. These models are very effective in applications where the solution space of the VRP is able to be reduced to these simpler pairings of vehicles and tasks. Elimam and Dodin, (2013) show that this technique can handle complex scenarios in a supply chain model that encompasses everything from production to customer delivery of a product.

2.3.2 Stochastic Models

Stochastic optimization models consider similar problems to those of general optimization models, except they consider uncertainty in their parameters in the form of probability distributions or scenarios. There are many reasons to incorporate uncertainty into optimization models. Transportation companies may be motivated by reducing the likelihood of dissatisfied customers and being prepared for demands outside of their expected averages. However, many studies in stochastic optimization are centered around disaster relief logistics. Disaster relief planners must not only consider highly variable demand sizes and locations but also the possibility of supplies and even roads being destroyed in the disaster (Bozorgi-Amiri et al., 2013; Chang et al., 2007). Chang et al. (2007) and Borzogi-Amiri et al. (2013) examine two techniques for solving very similar disaster relief planning problems. Most stochastic optimization problems consider a first-stage set of decisions that are optimized prior to the realization of the stochastic parameters and a second-stage set of decisions that are optimized after the realization of the stochastic parameters, with the decisions and outcomes of one influencing the other. In both studies, the first stage decision is where to position supplies/distribution centers to most effectively service their surrounding communities. After making these decisions, a disaster occurs, which potentially destroys supplies and distribution centers. Now a fulfillment strategy must be enacted. This strategy is determined by a second-stage set of decisions. Chang et al. (2007) use two extensive form models (containing all scenarios in each model) and prioritize minimizing the variance in their solutions. Borzogi-Amiri et al. (2013) use a two-stage stochastic model that examines each second-stage scenario individually before aggregating the solutions to provide a first-stage solution.

While some problems can be solved in their extensive form (Chang et al., 2007; Lima et al., 2018), the solution time and memory space required for solutions becomes prohibitive with larger problems. One method for addressing this issue is through Monte-Carlo simulation. Instead of looking at all scenarios at once, these models solve scenarios one at a time. The results of these simulations can then be analyzed to identify trends common throughout and extract a decision policy (Pironet, 2015; Wang & Yang, 2013). Wang and Yang (2015) demonstrate how this idea can be used by applying a simple probability-based constraint in a simulation for optimizing sea cargo movements. Pironet (2013) shows that this can also be useful in much larger transportation problems that consider stochasticity in many parameters and constraints.

Another method for solving larger stochastic optimization problems is the L-Shaped Method. This method is based off of Benders Decomposition and separates the problem into first-stage and second-stage models that can be solved iteratively (Bozorgi-Amiri et al., 2013; Chu & You, 2013; Dentcheva & Martinez, 2012). Chu and You (2013) use this technique in modeling how to schedule batched sequential production processes while considering the uncertainty of individual task processing times. While expanding upon the base algorithm, the authors were able to solve a case study with more than 3 million variables and equations across 100 scenarios. Dentcheva and Martinex (2012) expand upon the general formulation by analyzing a risk-averse model. Their formulation adds a first-stage constraint that requires all second-stage scenario costs must below a set value. The authors go on to develop solution techniques for solving problems of this form.

Chapter 3

Stochastic Models for Chassis Management

3.1 Introduction

In this chapter, two-stage stochastic models are developed to manage chassis inventory in intermodal transportation. For an intermodal transportation company, the chassis are shared across multiple rail ramps in the same hub. For instance, Chicago has multiple rail ramps where one ramp serves demand for eastern hubs and the other serves demand for western hubs. Typically, the demands at each rail ramp are for either empty containers or loaded containers. Both of these containers share the same chassis. It is crucial for any transportation firm to optimally allocate and move chassis between rail ramps to overcome random demand. Using stochastic models, we aim to answer the following questions: (1) When and how many chassis to move between each rail ramp to leverage against random demand and (2) How do such decisions vary with system parameters and problem size.

3.2 Intermodal Chassis Management Model

This section describes the Intermodal Chassis Management Model and the possible decisions available to the intermodal trucking company in all time periods and scenarios. The resource flow for the Intermodal Chassis Management Model is shown in Figure 3.1.

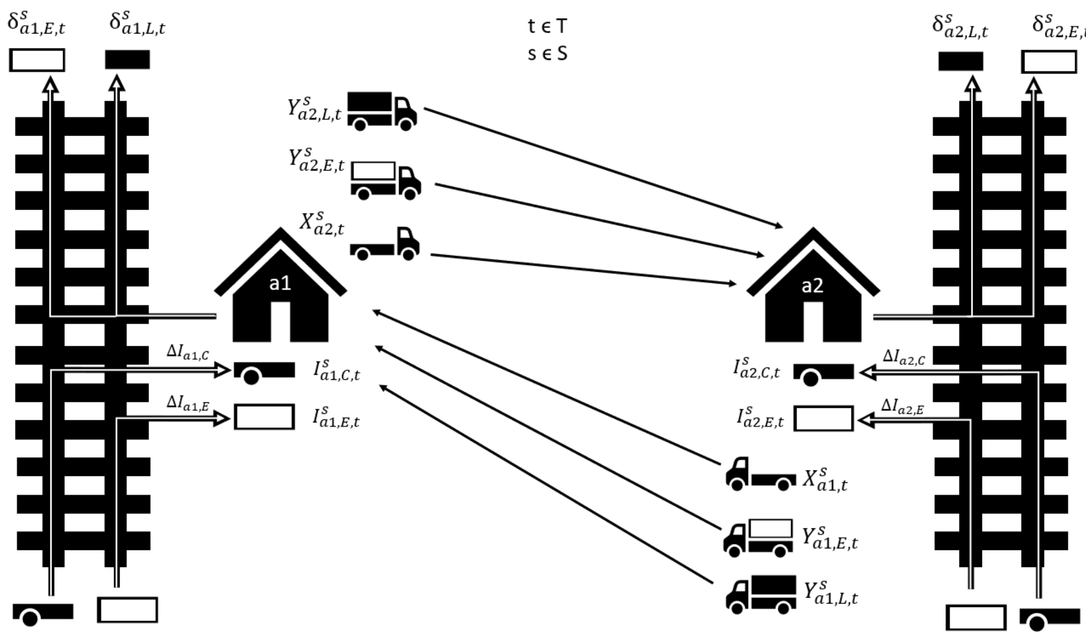

Figure 3.1: Intermodal Chassis Management Model Layout

There are two locations i ($i \in A$) in the problem representing the ramps that serve both railroads. The trucking company is able to store resources at each location $I_{i,j,k}$ ($i \in A, j \in \Gamma, j \neq L, k \in T$). Loaded containers are not kept in inventory because they must be shipped to their destination location in the same time period in which they arrive. After the first time period, a random demand for empty and loaded containers $\delta^s_{i,j,k}$ ($s \in S, i \in A, j \in \Gamma, j \neq C, k \in T$) arrives at each location. Because empty containers are homogeneous, their demand may be filled from existing inventory at that location or by repositioning existing inventory from the other location. Loaded demand must be filled by moving that loaded container $Y^s_{i,L,k}$ $(s \in S, i \in A, k \in T)$ from the other location to location i. Additional chassis and empty containers can be ordered $\Delta I_{i,j}$ $(i \in A, j \in \Gamma, j \neq L)$ in the first time period. Inventory repositioning moves can be made for both chassis $X^s_{i,k}$ $(s \in S, i \in A, k \in T)$ and empty container resources $Y^s_{i,E,k}$ $(s \in S, i \in A, k \in T)$ to location i from the other location. Loaded and empty container repositioning moves require a chassis to be present at that location or transferred to that location in the same time period.

3.3 Extensive Form Model for Intermodal Chassis Management

This section proposes an extensive form model to solve problems of the type described in Section 3.2. This formulation contains all time periods t $(t \in T)$ and random scenarios s $(s \in S)$ in one model.

Table 3.1: Sets for the Itermodal Chassis Management Model

Sets		
S	Set of scenarios	S = {1,2,3,...,$\|S\|$}
A	Set of hub locations	$A = \{a1, a2\}$
T	Set of time periods	T = {1,2,3,...,$\|T\|$}
Γ	Types of loads that a trailer can carry, denoted as E for empty containers, L for loaded containers, and C for chassis	$\Gamma = \{E,\ L,\ C\}$

Table 3.2: Variables for the Intermodal Chassis Management Model

Variables		
$I^s_{i,j,k}$	Inventory in scenario s at location i for load type j in time period k	$s \in S, i \in A, j \in \Gamma, j \neq$L, $k \in T$
$X^0_{i,j,0}$	Beginning repositioning moves before the first time period at location i for load type j	$i \in A, j \in \Gamma, j \neq L$
$\Delta I_{i,j}$	Additional inventory resources ordered before the first time period at location i for load type j	$i \in A, j \in \Gamma, j \neq L$
$X^s_{i,k}$	Chassis moves to location i in time period k in scenario s	$s \in S, i \in A, k \in T$
$Y^s_{i,j,k}$	Container moves in scenario s to location i of load type j in time period k	$s \in S, i \in A, j \in \Gamma, j \neq$C, $k \in T$

Table 3.3: Parameters for the Intermodal Chassis Management Model

Parameters

$\delta^s_{i,j,k}$	Demand in scenario s at location i for load type j in time period k	$s \in S, i \in A, j \in \Gamma, j \neq \text{C}, k \in T$
$I1_{i,j}$	Initial inventory at location i of load type j	$i \in A, j \in \Gamma$
$C^s_{i,j,k}$	Cost of move in scenarios s to location i of load type j in time period k	$s \in S, i \in A, j \in \Gamma, k \in T$
P^s	Probability of scenarios s	$s \in S$
$\Delta IC_{i,j}$	Cost of adding resources before the first time period of load type j at location i	$i \in A, j \in \Gamma, j \neq L$

Extensive Form model

$$min \sum_{i \in A} \sum_{j \in \Gamma, j \neq L} \Delta IC_{i,j} * \Delta I_{i,j} + \sum_{i \in A} \sum_{j \in \Gamma, j \neq L} C^0_{i,j,0} * X^0_{i,j,0} + \sum_{s \in S} [\sum_{i \in A} \sum_{j \in \Gamma, j \neq C} \sum_{k \in T} P^s * C^s_{i,j,k} * Y^s_{i,j,k} + \sum_{i \in A} \sum_{k \in T} P^s * C^s_{i,j,k} * X^s_{i,k}] \tag{3.1}$$

Subject to:

$$\delta^s_{i,L,k} \leq Y^s_{i,L,k} \ \ \forall\, s \in S, i \in A, k \in T \tag{3.2}$$

$$\delta^s_{i,E,k} \leq Y^s_{i,E,k} + I^s_{i,E,k} \ \ \forall\, s \in S, i \in A, k \in T \tag{3.3}$$

$$\begin{aligned} I^s_{a1,C,k+1} &= I^s_{a1,C,k} + X^s_{a1,k} - X^s_{a2,k} + Y^s_{a1,L,k} - Y^s_{a2,L,k} \\ &\quad + Y^s_{a1,E,k} - Y^s_{a2,E,k} \ \ \forall\, s \in S, k \in T, k \neq |T| \end{aligned} \tag{3.4}$$

$$\begin{aligned} I^s_{a2,C,k+1} &= I^s_{a2,C,k} + X^s_{a2,k} - X^s_{a1,k} + Y^s_{a2,L,k} - Y^s_{a1,L,k} \\ &\quad + Y^s_{a2,E,k} - Y^s_{a1,E,k} \ \ \forall\, s \in S, k \in T, k \neq |T| \end{aligned} \tag{3.5}$$

$$\begin{aligned} I^s_{a1,E,k+1} &= I^s_{a1,E,k} + Y^s_{a1,E,k} - Y^s_{a2,E,k} \\ &\quad - \delta^s_{a1,E,k} \ \ \forall\, s \in S, k \in T, k \neq |T| \end{aligned} \tag{3.6}$$

$$\begin{aligned} I^s_{a2,E,k+1} &= I^s_{a2,E,k} + Y^s_{a2,E,k} - Y^s_{a1,E,k} \\ &\quad - \delta^s_{a2,E,k} \ \ \forall\, s \in S, k \in T, k \neq |T| \end{aligned} \tag{3.7}$$

$$I^s_{a2,C,k} + X^s_{a2,k} \geq \sum_{j \in \Gamma, j \neq C} (Y^s_{a1,j,k} - Y^s_{a2,j,k}) \ \ \forall\, s \in S, k \in T \tag{3.8}$$

$$I^s_{a1,C,k} + X^s_{a1,k} \geq \sum_{j \in \Gamma, j \neq C} (Y^s_{a2,j,k} - Y^s_{a1,j,k}) \ \ \forall\, s \in S, k \in T \tag{3.9}$$

$$I^s_{i,C,k} \geq \sum_{i2 \in A, i2 \neq i} X^s_{i,k} \ \ \forall\, s \in S, i \in A, k \in T \tag{3.10}$$

$$\begin{aligned} I^s_{i,C,1} &= \sum_{j2 \in \Gamma, j2 \neq L} X^0_{i,j2,0} - \sum_{i2 \in A, i2 \neq i} \sum_{j2 \in \Gamma, j2 \neq L} X^0_{i2,j2,0} + I1_{i,C} \\ &\quad + \Delta I_{i,C} \ \ \forall\, s \in S, i \in A, j \in \Gamma \end{aligned} \tag{3.11}$$

$$\begin{aligned} I^s_{i,E,1} &= X^0_{i,E,0} - \sum_{i2 \in A, i2 \neq i} X^0_{i2,E,0} + I1_{i,E} \\ &\quad + \Delta I_{i,E} \ \ \forall\, s \in S, i \in A, j \in \Gamma \end{aligned} \tag{3.12}$$

$$X^0_{i,C,0} + X^0_{i,E,0} \leq \sum_{i2 \in A, i2 \neq i} (I1_{i2,E} + \Delta I_{i2,E}) \ \ \forall\, i \in A, j \in \Gamma \tag{3.13}$$

$$X^0_{i,E,0} \leq \sum_{i2 \in A, i2 \neq i} (I1_{i2,E} + \Delta I_{i2,E}) \ \ \forall\, i \in A, j \in \Gamma \tag{3.14}$$

The extensive form objective function (3.1) has two main components: the cost of first stage decisions and the cost of second stage decisions. The first stage cost has two terms: the cost of ordering additional empty containers and chassis and the cost of repositioning moves before the first time period. The second stage costs are expected costs over scenarios. The second stage cost are also comprised of two terms: the expected cost of empty and loaded repositioning moves and the expected cost of chassis repositioning moves.

Constraint (3.2) forces all the loaded demand to be sent to its destination in the same time period and scenario as it arrived. Constraint (3.3) ensures that there is enough empty inventory available at or arriving to the location in the time period and scenario that it is needed. Constraints (3.4) and (3.5) are the inventory balance equations for chassis at each location. The next period's inventory includes all chassis that were there previously, plus any chassis that were sent there, minus any chassis that were sent to the other location. Constraints (3.7) and (**??**) are the inventory balance equation for empty containers at each location. The next time period's empty container inventory is equal to the previous period's inventory, plus any empty containers sent to that location, minus those sent to the other location and those used to fulfill empty container demand. Constraints (3.8) and (3.9) state that we cannot move more empty or loaded containers to a location than were already available at that location or sent there in that time period. Constraint (3.10) states that we cannot move more chassis to a location than were present at the origin location at the beginning of the time period. Constraints (3.11) and (3.12) determine the chassis and empty container inventory for the first time period after initial adjustments through repositioning or additional chassis inventory ordering. Constraints (3.13) and (3.14) ensure that we do not reposition more chassis or empty containers from a location than were already available or added there before the first time period.

3.4 Two-Stage Stochastic Model

This section describes the two-stage stochastic optimization formulation of the problem described in Section 3.2. This formulation decomposes the extensive form model using the L-Shaped Method (Birge & François, 2011; Dantzig & Wolfe, 1960). The motivation for this decomposition is to improve the solution time of the model. The extensive form model's solution time increases exponentially with the size of the problem. One of the largest factors in determining the extensive form model's size is the number of scenarios. The two-stage stochastic model presented below improves this exponential relationship by separating each scenario into an independent model. These second-stage independent models generate optimality and feasibility constraints that are added to the first-stage model. Both model classes are solved iteratively until an optimal solution is achieved.

3.4.1 Master Problem Model

$$min \sum_{i \in A} \sum_{j \in \Gamma, j \neq L} \Delta IC_{i,j} * \Delta I_{i,j} + \sum_{i \in A} \sum_{j \in \Gamma, j \neq L} C_{i,j,0} * X_{i,j} + \theta \tag{3.15}$$

Subject to:

$$
\begin{aligned}
I_{i,C,1} &= \sum_{j2\in\Gamma,j2\neq L} X_{i,j2} - \sum_{i2\in A,i2\neq i}\sum_{j2\in\Gamma,j2\neq L} X_{i2,j2} \\
&\quad + I1_{i,C} + \Delta I_{i,C} \;\; \forall\, i \in A, j \in \Gamma && (3.16)\\
I_{i,E,1} &= X_{i,E} - \sum_{i2\in A,i2\neq i} X_{i2,E} + I1_{i,E} \\
&\quad + \Delta I_{i,E} \;\; \forall\, i \in A, j \in \Gamma && (3.17)\\
X_{i,C} + X_{i,E} &\leq \sum_{i2\in A,i2\neq i} (I1_{i2,E} + \Delta I_{i2,E}) \;\; \forall\, i \in A, j \in \Gamma && (3.18)\\
X_{i,E} &\leq \sum_{i2\in A,i2\neq i} (I1_{i2,E} + \Delta I_{i2,E}) \;\; \forall\, i \in A, j \in \Gamma && (3.19)\\
\sum_{i\in A}\sum_{j\in\Gamma,j\neq L} D_{I_{i,j,1},r} * I_{i,j,1} &\geq d_r \;\; \forall\, r \in \Omega && (3.20)\\
\sum_{i\in A}\sum_{j\in\Gamma,j\neq L} E_{I_{i,j,1},p} * I_{i,j,1} + \theta &\geq e_p \;\; \forall\, p \in \Phi && (3.21)
\end{aligned}
$$

There are three terms in the master problem's objective function. The first captures the cost of adding additional inventory of a certain type to a location. The second term captures the cost of repositioning resources in the first time period. The final term θ is the average cost of the second stage models.

Constraints (3.16) - (3.19) are described in Section 3.3. Constraints (3.20) and (3.21) are the cutting constraints generated by the L-Shaped Method algorithm. (3.20) is the feasibility cut constraint. Its parameters, $D_{I_{i,j,1},r}$ and d_r, are computed by the feasibility model formulation described in the next section. The set Ω represents the infeasible scenarios r in the current iteration of the L-Shaped Method. This thesis uses the single-cut method for feasibility cuts, so each infeasible second stage model will generate a constraint of this form. Constraint (3.21) is the optimality cut constraint. Its parameters, $E_{I_{i,j,1},p}$ and e_p, are generated by the optimality model described in Section 3.4.3. The set Φ represents the iterations completed p by the L-Shaped Method. The θ term represents the average cost of the optimality models. This research uses the multi-cut method for optimality cuts, so each L-Shaped Method iteration that contains no infeasible scenarios will generate a constraint of this form.

3.4.2 Feasibility Model

The feasibility model contains an additional set κ, which represent the set of second stage constraints. This set is used to define two additional variables: v_κ^+ and v_κ^-. v_κ^+ is a positive slack variable for each second stage constraint, while v_κ^- is a negative slack variable. Each constraint in this formulation contains one or both of these terms.

$$min \sum_{\kappa \in K} v_\kappa^+ + v_\kappa^- \tag{3.22}$$

Subject to:

$$\delta_{i,L,k}^s - v_\kappa^- \leq Y_{i,L,k}^s \;\; \forall\, i \in A, k \in T \tag{3.23}$$

$$\delta_{i,E,k}^s - v_\kappa^- \leq Y_{i,E,k}^s + I_{i,E,k}^s \;\; \forall\, i \in A, k \in T \tag{3.24}$$

$$\begin{aligned} I_{a1,C,k+1}^s + v_\kappa^+ - v_\kappa^- &= I_{a1,C,k}^s + X_{a1,k}^s - X_{a2,k}^s + Y_{a1,L,k}^s - Y_{a2,L,k}^s \\ &\quad + Y_{a1,E,k}^s - Y_{a2,E,k}^s \;\; \forall\, k \in T, k \neq |T| \end{aligned} \tag{3.25}$$

$$\begin{aligned} I_{a2,C,k+1}^s + v_\kappa^+ - v_\kappa^- &= I_{a2,C,k}^s + X_{a2,k}^s - X_{a1,k}^s + Y_{a2,L,k}^s - Y_{a1,L,k}^s \\ &\quad + Y_{a2,E,k}^s - Y_{a1,E,k}^s \;\; \forall\, s \in S, k \in T, k \neq |T| \end{aligned} \tag{3.26}$$

$$\begin{aligned} I_{a1,E,k+1}^s + v_\kappa^+ - v_\kappa^- &= I_{a1,E,k}^s + Y_{a1,E,k}^s - Y_{a2,E,k}^s \\ &\quad - \delta_{a1,E,k}^s \;\; \forall\, k \in T, k \neq |T| \end{aligned} \tag{3.27}$$

$$I^s_{a2,E,k+1} + v^+_\kappa - v^-_\kappa = I^s_{a2,E,k} + Y^s_{a2,E,k} - Y^s_{a1,E,k} - \delta^s_{a2,E,k} \;\; \forall\, k \in T, k \neq |T| \tag{3.28}$$

$$I^s_{a2,C,k} + X^s_{a2,k} + v^+_\kappa \geq \sum_{j\in\Gamma, j\neq C} Y^s_{a1,j,k} - Y^s_{a2,j,k} \;\; \forall\, k \in T \tag{3.29}$$

$$I^s_{a1,C,k} + X^s_{a1,k} + v^+_\kappa \geq \sum_{j\in\Gamma, j\neq C} Y^s_{a2,j,k} - Y^s_{a1,j,k} \;\; \forall\, k \in T \tag{3.30}$$

$$I_{i,C,k} + v^+_\kappa \geq \sum_{i2\in A, i2\neq i} X_{i,k} \;\; \forall\, i \in A, k \in T \tag{3.31}$$

The feasibility model is solved after the master problem for each scenario in the list of possible scenarios. The objective function to determine the feasibility cut is given by (3.22) and minimizes the positive and negative slack variables summed across all constraints in the model. The second stage scenario is feasible if and only if the objective function is zero.

Each constraint in this formulation has a v^+_κ and/or a v^-_κ slack variable term. The κ in these variables represents the specific constraint that the variable is found in. The constraints in this model are similar to the constraints described in the extensive form model in Section 3.3 with the addition of slack variable terms. Constraints with strict equality have both the slack variable terms added. The negative slack variable v^-_κ is added only to constraints with less-than-or-equal relations. The positive slack variable v^+_κ is added only to constraints with greater-than-or-equal relations.

Feasibility Cut Generation

After solving each feasibility model r, the L-Shaped Method computes the associated feasibility cut parameter values $D_{I_{i,j,1},r}$ and d_r, which will be used in constraint (3.20) of the master problem model. For each first stage variable $I_{i,j,1}$ and infeasible scenario r, the cut coefficient $D_{I_{i,j,1},r}$ is defined as the dual value of each constraint $\sigma_{\kappa,r}$ multiplied by the coefficient of the first stage variable in that constraint $\tau_{\kappa,I_{i,j,1}}$, summed over all constraints $\kappa \in K$. Note that this $\tau_{\kappa,I_{i,j,1}}$ term signifies the coefficient value if the variable were to be moved to the left-hand side of the constraint. Equation (3.32) defines the parameter $D_{I_{i,j,1},r}$.

$$D_{I_{i,j,1},r} = \sum_{\kappa \in K} \sigma_{\kappa,r} * \tau_{\kappa,I_{i,j,1}} \ \ \forall\, i \in A, j \in \Gamma, j \neq L, r \in \Omega \tag{3.32}$$

The calculation for d_r is similar to that of $D_{I_{i,j,1},r}$, except it is computed as a single value for each infeasible model. The cut coefficient d_r is defined as the dual value of each constraint $\sigma_{\kappa,r}$ multiplied by the constant in that constraint $h_{\kappa,r}$, summed over all constraints $\kappa \in K$. Note that this $h_{\kappa,r}$ term signifies the value of the constant if all parameters were moved to the right-hand side of the constraint and summed. Equation (3.33) defines the parameter d_r.

$$d_r = \sum_{\kappa \in K} \sigma_{\kappa,r} * h_{\kappa,r} \ \ \forall\, r \in \Omega \tag{3.33}$$

3.4.3 Optimality Model Formulation

The optimality models are solved for each scenario in each iteration of the L-Shaped Method. These models are used to generate optimality cut constraints to be added the master problem. These models contain all second stage decision variables and parameter realizations for a random scenario and use the previous first stage decision variables as parameters in determining recourse actions.

$$min \sum_{i \in A} \sum_{j \in \Gamma \neq C} \sum_{k \in T} C^s_{i,j,k} * Y^s_{i,j,k} + \sum_{i \in A} \sum_{j \in \Gamma \neq L} \sum_{k \in T} C^s_{i,j,k} * X^s_{i,k} \tag{3.34}$$

Subject to:

$$\delta^s_{i,L,k} \leq Y^s_{i,L,k} \;\; \forall\, i \in A, k \in T \tag{3.35}$$

$$\delta^s_{i,E,k} \leq Y^s_{i,E,k} + I^s_{i,E,k} \;\; \forall\, i \in A, k \in T \tag{3.36}$$

$$\begin{aligned} I^s_{a1,C,k+1} &= I^s_{a1,C,k} + X^s_{a1,k} - X^s_{a2,k} + Y^s_{a1,L,k} - Y^s_{a2,L,k} \\ &\quad + Y^s_{a1,E,k} - Y^s_{a2,E,k} \;\; \forall\, k \in T, k \neq |T| \end{aligned} \tag{3.37}$$

$$\begin{aligned} I^s_{a2,C,k+1} &= I^s_{a2,C,k} + X^s_{a2,k} - X^s_{a1,k} + Y^s_{a2,L,k} - Y^s_{a1,L,k} \\ &\quad + Y^s_{a2,E,k} - Y^s_{a1,E,k} \;\; \forall\, k \in T, k \neq |T| \end{aligned} \tag{3.38}$$

$$I^s_{a1,E,k+1} = I^s_{a1,E,k} + Y^s_{a1,E,k} - Y^s_{a2,E,k} - \delta^s_{a1,E,k} \;\; \forall\, k \in T, k \neq |T| \tag{3.39}$$

$$I^s_{a2,E,k+1} = I^s_{a2,E,k} + Y^s_{a2,E,k} - Y^s_{a1,E,k} - \delta^s_{a2,E,k} \;\; \forall\, k \in T, k \neq |T| \tag{3.40}$$

$$I^s_{a2,C,k} + X^s_{a2,k} \geq \sum_{j \in \Gamma, j \neq C} Y^s_{a1,j,k} - Y^s_{a2,j,k} \;\; \forall\, k \in T \tag{3.41}$$

$$I^s_{a1,C,k} + X^s_{a1,k} \geq \sum_{j \in \Gamma, j \neq C} Y^s_{a2,j,k} - Y^s_{a1,j,k} \;\; \forall\, k \in T \tag{3.42}$$

$$I_{i,C,k} \geq \sum_{i2 \in A, i2 \neq i} X_{i,k} \;\; \forall\, i \in A, k \in T \tag{3.43}$$

The objective function (3.34) minimizes the costs of all loaded and empty container moves plus the costs of all chassis repositioning moves. All the constraints in this model are the same as ones found in the extensive form model described in Section 3.3, except that a model is created for each scenario instead of all constraints being rewritten for each scenario.

Optimality Cut Generation

After solving all optimality models in an iteration (p), the L-Shaped Method computes the associated optimality cut parameter values, $E_{I_{i,j,1},p}$ and e_p, which will be used in Constraint (3.21) in the next iteration of the master problem model. For each first stage variable $I_{i,j,1}$, the cut coefficient $E_{I_{i,j,1},p}$ is defined as the dual values of each constraint $\pi_{\kappa,p,s}$ multiplied by the coefficient of the first stage variable in that constraint $\tau_{\kappa,I_{i,j,1}}$ and probability of that scenario p_s, summed over all constraints $\kappa \in K$ and all scenarios $s \in S$. Note that this $\tau_{\kappa,I_{i,j,1}}$ term signifies the coefficient value if the variable were to be moved to the left-hand side of the constraint. Equation (3.44) defines the parameter $E_{I_{i,j,1},p}$.

$$E_{I_{i,j,1},p} = \sum_{s \in S} \sum_{\kappa \in K} p_s * \pi_{\kappa,p,s} * \tau_{\kappa,I_{i,j,1}} \quad \forall\, i \in A, j \in \Gamma, j \neq L \tag{3.44}$$

The calculation for e is similar to that of E, except it is computed as a single value in each iteration. The cut coefficient e is defined as the dual value of each constraint $\pi_{\kappa,p,s}$ multiplied by the constant in that constraint $h_{\kappa,s}$ and the probability of the scenario p_s, summed over all constraints $\kappa \in K$ and all scenarios s ($s \in S$). Note that this $h_{\kappa,s}$ term signifies the value of the constant if all parameters were moved to the right-hand side of the constraint and summed. Equation (3.45) defines the parameter e_p.

$$e_p = \sum_{s \in S} \sum_{\kappa \in K} p_s * \pi_{\kappa,p,s} * h_{\kappa,s} \tag{3.45}$$

The master problem model, feasibility model, and optimality model and their respective cut equations define the two-stage stochastic decomposition of the extensive form model. This two-stage stochastic optimization model can be used to determine optimal intermodal transportation inventory management under uncertain demands and costs. The next chapter will solve and analyze this model to answer the research questions of this thesis.

Chapter 4

Numerical Experiments

In this chapter, we conduct multiple experiments and examine their impact on the Intermodal Chassis Management Model described in Chapter 3. In Experiment 4.1, we analyze the run-time performance of the L-Shaped Method on the stochastic Intermodal Chassis Management Model and investigate the impact of problem size on run-time. In Experiment 4.2, we analyze how the chassis management decisions change under uncertainty when compared with a deterministic model. Next, we analyze the impact of demand distribution on the optimal decisions in Experiment 4.3. Finally, we conduct sensitivity analysis with respect to initial inventory and additional ordering cost values in Experiment 4.4. Python 3.0 and CPLEX 12.9 were used to develop the code for all experiments. This code was executed on a computer with a 3.00 GHz processor and 24 GB of RAM.

4.1 Experiment 1: Impact of System Size on Run-Time Performance

The primary motivation for implementing the L-Shaped Method is to reduce the time complexity of the model by decomposing the problem into two stages. Note that the time complexity of the extensive form increases exponentially with the problem size. The two main factors affecting the size of the problem are the range of the time horizon and the number of random scenarios. By further decomposing each second-stage scenario into independent models, the L-Shaped Method is theoretically able to create a linear relationship between the number of scenarios and the solution time. This section analyzes experiments performed to demonstrate both of these relationships.

Experiments in this section used the parameter values described in Table 4.1. For each scenario in an experiment, random values were sampled from the parameter distributions given in the table. In this experiment, we set the initial inventories to zero, which forces ordering of all of the resources the model determines necessary in the first time period. Setting the additional inventory ordering cost significantly higher than the average movement cost provides an incentive to the model to find a balance between higher inventory costs and higher resource repositioning costs. These trade-offs result in non-trivial model decisions, such as ordering a chassis for each move or ordering the minimum feasible number of chassis.

Figure 4.1 shows that the run-time for the two-stage stochastic models increases exponentially in relation to the time range. In the extensive form model, the number of scenarios have a similar effect as the time range on the overall problem size. It is expected that an extensive-form model would demonstrate a similar relationship between the number of scenarios and the solve-time. However, the L-Shaped Method

Table 4.1: Solution Time Experiment Parameters

Parameter	Value/Distribution
$\delta^s_{i,E,k}$	Normal(20,4)
$\delta^s_{i,L,k}$	Normal(50,10)
$\Delta IC_{i,j}$	1000
$C^s_{i,j,k}$	Normal(100,10)
$I1_{i,j}$	0

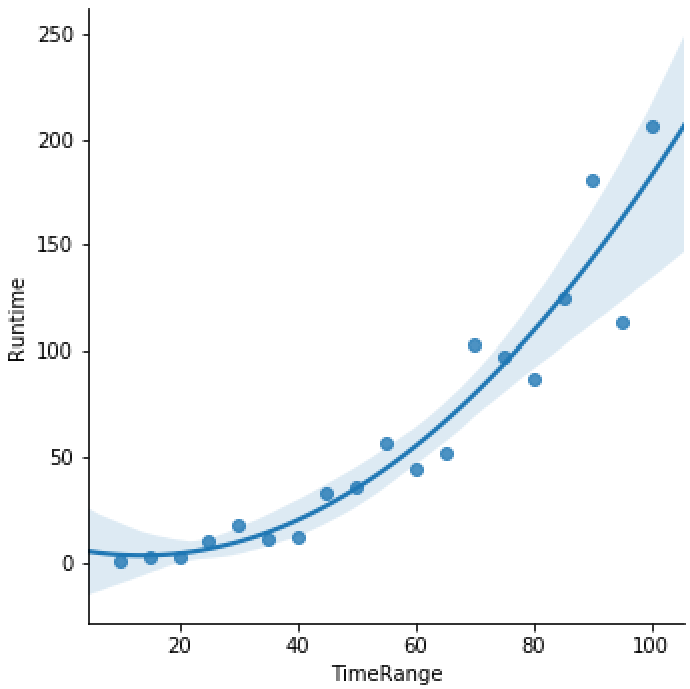

Figure 4.1: Time Range vs. Run-time (sec.)

should improve this relationship by decomposing the model.

Figure 4.2 shows a linear relationship between the number of scenarios and the solution time of the model. The number of scenarios in each of the 20 tests was varied between 50 and 1000. These two experiments validate the assumption that the model

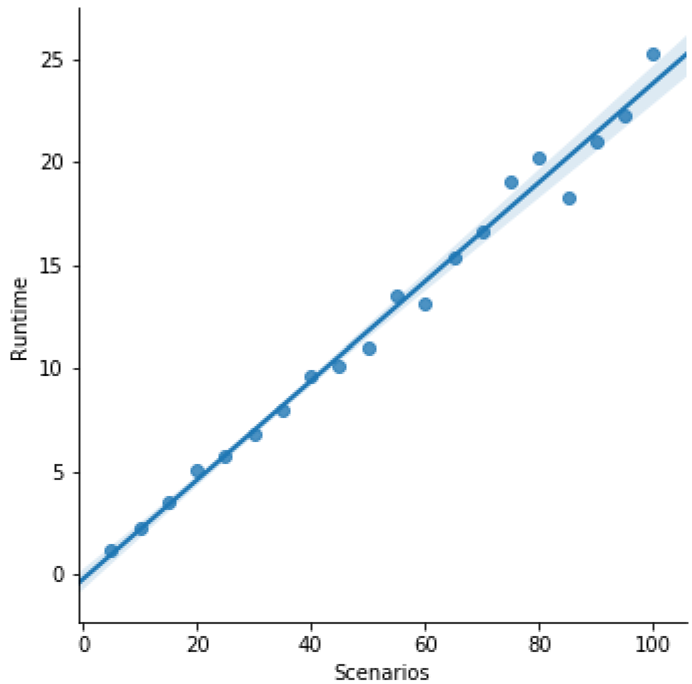

Figure 4.2: Scenarios vs. Run-time (sec.)

this thesis proposes exhibits an exponential relationship between problem size and solution time within each second stage model, but a linear relationship between the number of second stage models and solution time.

4.2 Experiment 2: Stochastic and Deterministic Cases

This section compares the optimal chassis management decisions obtained using the stochastic formulation to ones with a deterministic adaptation of the problem. The deterministic version is the same as the stochastic version except it ignores all randomness in parameters. Table 4.2 describes the parameters used in this experiment for the stochastic version. The deterministic adaptation uses the same values, except it uses the mean value for parameters modeled with distributions and a scenario size of one. The parameters used are similar to those used in the previous experiment. The average loaded container demand is higher than the average empty container demand, although they have the same coefficient of variance. The additional ordering cost is ten times the average movement cost. This ratio is important as it determines the break-even point between the average cost of repositioning moves a resource is expected to make versus the value of ordering another unit of that resource. Table 4.3 shows the results of this experiment.

Table 4.2: Deterministic Comparison Experiment Parameters

Parameter	Value/Distribution
$\Delta IC_{i,j}$	1000
$C^s_{i,j,k}$	Normal(100,10)
$\delta^s_{i,E,k}$	Normal(20,4)
$\delta^s_{i,L,k}$	Normal(50,10)
$I1_{i,E}$	10
$I1_{i,C}$	10
$\|S\|$	500
$\|T\|$	10

These results show that the deterministic version ordered less additional empty containers and had a lower expected cost. The variance in the stochastic model caused ordering of additional resources to meet demand in scenarios with higher demand values. The decision to order additional resources causes the stochastic version to have a higher expected cost, but this value is a better estimation of the actual costs realized in practice than the optimal cost calculated when only considering average

Table 4.3: Deterministic Comparison Experiment Results

	Total Expected Cost	Additional Empty Container Orders	Additional Chassis Orders
Stochastic	495061	390	0
Deterministic	444000	340	0

values. The empty container inventory value generated by the deterministic adaptation is insufficient to fulfill empty demand in 47.2% of the 500 scenarios modeled in the stochastic version. There is an average of 13.5 missed empty container demand in these under-supplied scenarios. Depending on the opportunity cost of lost demand and the desired service rate of the company, ignoring the distribution of parameters in the problem could have large negative operational impacts. The stochastic version provides optimal chassis decisions that are much more robust to variability in parameters.

4.3 Experiment 3: Impact of Demand Distribution

This section analyzes the relationship between the demand distribution and optimal inventory management decisions. This relationship is examined by conducting experiments that vary the demand by location, type, average value, and coefficient of variance.

The first study examines how changing the location of the demand affects the Intermodal Chassis Inventory Management model's optimal inventory allocations. Table 4.4 outlines the parameter settings for this experiment. The chassis repositioning and ordering costs are lower than than the empty to encourage chassis movements in this experiment and better demonstrate how adjusting the demand parameters affect decisions affecting chassis resources. The initial inventory levels are 10 for both empty containers and chassis at each location. While the average demand is varied, the variance of the demand distribution is kept at a constant value of 5.

Table 4.4: Demand Distribution Experiment Parameters

Parameter	Value/Distribution
$\Delta IC_{i,C}$	20
$\Delta IC_{i,E}$	100
$C^s_{i,j,E}$	Normal(20,2)
$C^s_{i,j,L}$	Normal(20,2)
$C^s_{i,j,C}$	Normal(2,0.2)
$I1_{i,E}$	10
$I1_{i,C}$	10
$\|S\|$	100
$\|T\|$	5

Table 4.5 shows how the total costs and inventories required in this experiment are impacted by how the demand is split between locations. The location of the inventories varies with where the demand is expected to be, but the total empty inventory level does not change much. A small number more are ordered in the cases where the demand is more unbalanced. The cost of ordering additional empty containers to accommodate unbalanced demand is far more than the cost of just repositioning the empty containers as needed. The opposite is true for the total chassis inventory as it increases dramatically with the imbalance in demand. The cases with more demand

Table 4.5: Demand Location Experiment Results

A1 Avg. Demand	A2 Avg. Demand	Expected Cost	Total Empty Inventory	Total Chassis Inventory	Expected Chassis Repositions
0	100	57410	441	143	80.6
25	75	53900.7	429	61	41.28
50	50	52619.3	432	20	1.74
75	25	54026.6	430	130	40.7
100	0	56998.6	435	147	80.69

at one location than the other require more resource repositioning, and thus more chassis, to meet those demands. These additional repositioning moves are reflected in the cost with the imbalanced cases having higher costs. In the fully unbalanced cases, nearly every chassis that fulfills demand to a location must travel with no cargo back to its original location to be of use again. In the more balanced cases, more chassis are able to travel to where they will next be required by simultaneously meeting a demand at that location. This is reflected in the number of expected chassis repositions varying from 1.74 to 80.69.

Next, we examine the impact of varying the demand type on the the optimal inventory allocations. The same parameters described in Table 4.4 were used in this experiment.

Table 4.6 shows the results of varying the average demand of both empty and loaded containers between 0 and 100. As would be expected, the total empty inventory increases with the empty demand. Because all cases in this study have demand that are balanced between locations, the same chassis inventory as the balanced case in the last study are required for all cases in this study. These results also show that the total expected cost increases as the ratio of empty to loaded demand increases.

Table 4.6: Demand Type Experiment Results

Empty Demand	Loaded Demand	Expected Cost	Total Empty Inventory	Total Chassis Inventory	Expected Chassis Repositions
0	100	22540.1	45	20	1.36
25	75	36359.9	229	20	2.11
50	50	52659	433	20	1.94
75	25	68255.4	628	20	1.84
100	0	84953.7	831	20	1.4

This is largely because the empty containers have both ordering and movement costs associated with them, instead of just the movement costs associated with loaded containers. The number of expected chassis repositions are low due to the demand being balanced between location. Overall, the distribution of demand between container types had a large impact on cost and empty container inventory, but a small impact on decisions concerning chassis.

Figures 4.3 and 4.4 show how the the demand distribution impacts the total inventory requirements of the inventory management model. The same parameters described in Table 4.4 were used in these analyses. Figure 4.3 shows the relationship between total inventory and total average demand as the demand is varied between 20 and 200 in increments of 20 for loaded container demand and varied between 10 and 100 in increments of 10 for empty container demand. Figure 4.4 shows the relationship between total inventory and the demand distribution's coefficient of variance. The average demand for this analysis was kept at a constant 100 for loaded containers and 50 for empty containers. The coefficient of variance for both of these distributions was varied between 2% and 66% in increments of 2%.

Figures 4.3 and 4.4 show a linear relationship between the total inventory require-

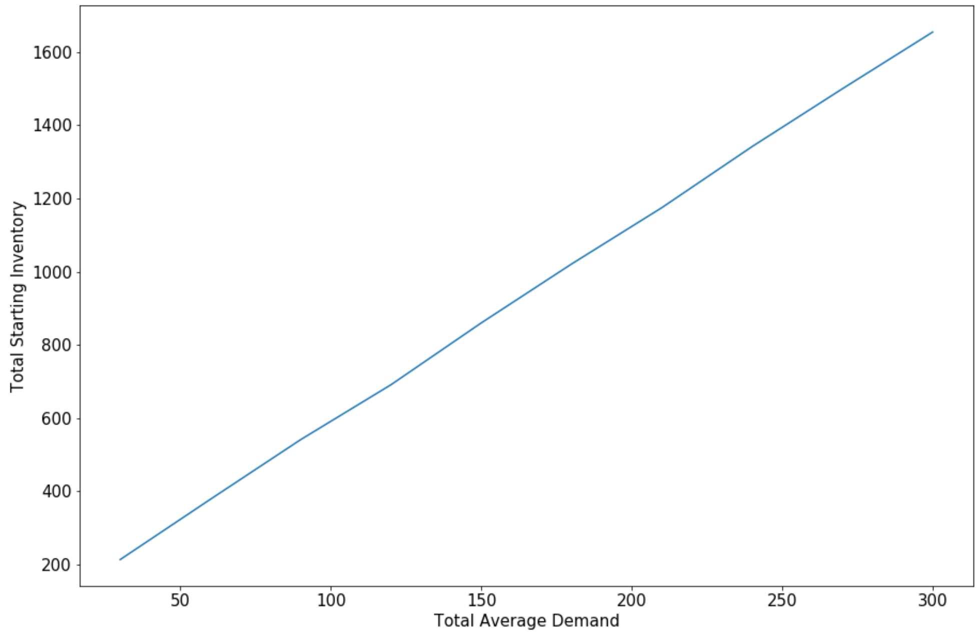

Figure 4.3: Inventory vs. Average Demand

ments and their respective demand distribution variable. For the average demand, this relationship is fairly intuitive. An increase in empty container demand has a direct relationship to the number of empty containers required to fulfill that demand. Increasing loaded or empty container demand also increases the number of chassis moves required and increases the value of ordering additional chassis inventory. The same basic logic is true for the coefficient of variance. The scenarios with higher demands that are unable to be satisfied have a larger affect on the initial inventory decisions than the scenarios of low demands with too much inventory. The linear relationship between initial inventory and the variance of demand exists because increasing the variance in demand increases the upper extremities of the demand across scenarios and these higher demand scenarios have a proportionally higher impact on the initial inventory.

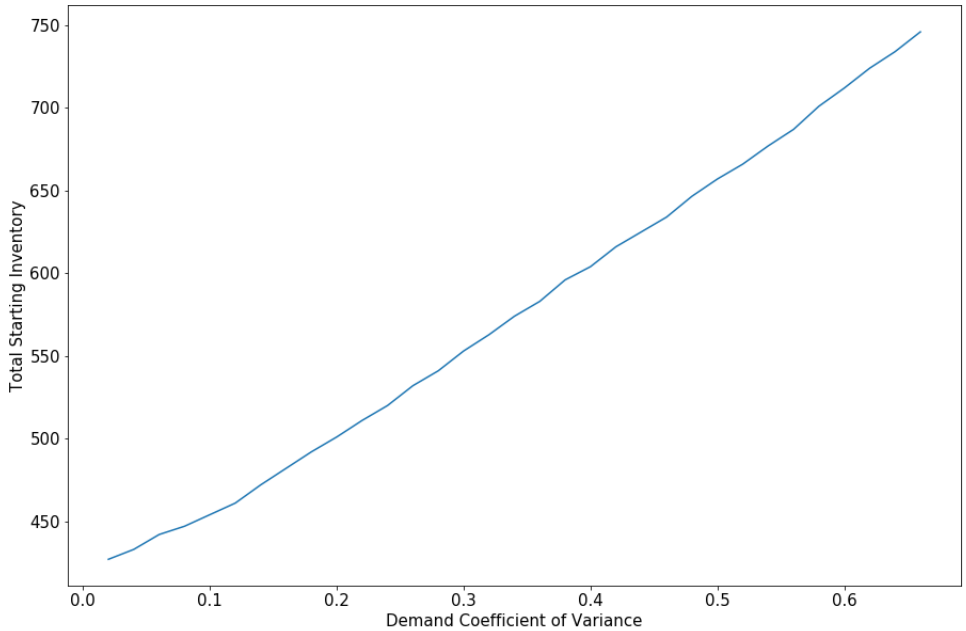

Figure 4.4: Inventory vs. Demand Coefficient of Variance

4.4 Experiment 4: Impact of Initial Chassis

This section of experiments analyzes the relationship between the beginning inventory values and the optimal inventory allocation of the problem. These experiments vary the initial inventory type, location, and ordering cost to demonstrate how these factors impact costs and decisions in the model. The parameters used in the inventory location and type experiments are described in Table 4.7 and are similar to those used in the last experiment with the average empty and loaded container demand set to 20 and 50 respectively.

The first experiment varies the initial inventories between 0 and 1000 for each location. These values are used for both the empty and chassis starting inventories. Table 4.8 shows the results of this experiment. Because there was a fairly large

Table 4.7: Inventory Experiment Parameters

Parameter	Value/Distribution
$\delta^s_{i,E,k}$	Normal(20,4)
$\delta^s_{i,L,k}$	Normal(50,10)
$\Delta IC_{i,C}$	20
$\Delta IC_{i,E}$	100
$C^s_{i,j,E}$	Normal(20,2)
$C^s_{i,j,L}$	Normal(20,2)
$C^s_{i,j,C}$	Normal(2,0.2)
$\lvert S\rvert$	100
$\lvert T\rvert$	20

starting chassis inventory and the chassis and empty containers started in the same location, no chassis repositioning moves are required in any of the cases. In the first case, no unloaded chassis repositioning moves were required. However, 287 empty containers were repositioned and this allows chassis to be available at both locations at the start of the next time period. The number of empty container repositions were highly sensitive to the balance of initial inventories between locations. This experiment shows that the Intermodal Inventory Management model is affected by how initial inventories are distributed between locations. In particular, matching empty and chassis inventory levels at locations helps decrease the number of unloaded repositioning moves.

The next experiment analyzes how varying the initial inventory type between 0 and 200 affects the optimal inventory allocations. Table 4.9 shows the results of this experiment. These results show that the initial empty inventory value is much more

Table 4.8: Inventory Location Experiment Results

A1 Initial Inventory	A2 Initial Inventory	Expected Cost	Empty Repositions	Expected Chassis Repositions
0	1000	4297.67	287	0
250	750	4040.72	36	0
500	500	3953.59	0	0
750	250	4064.77	32	0
1000	0	4335.21	287	0

important to determining the total expected cost than the initial chassis inventory. The parameters in this experiment assign a high cost to ordering additional empty containers, so more empty container resources provided at the beginning significantly reduce the cost of the model. While the cost of additional chassis resources is the same as empty containers, the initial chassis inventory does not have as large an impact on expected cost because the overall number required by the model is lower and these resources can be reused across time periods. There are not very many first stage repositioning moves in any scenario because the inventories and demands are balanced between locations in this experiment.

Table 4.9: Inventory Type Experiment Results

Initial Empty Inventory	Initial Chassis Inventory	Total Expected Cost	Additional Empty Orders	Additional Chassis Orders
200	0	31730.9	266	7
150	50	36636.2	318	0
100	100	41603.6	368	0
50	150	46469.2	417	0
0	200	52640.5	480	0

The final analysis examines how the ratio of additional inventory ordering costs to movement costs affects optimal inventory management decisions. The parameters used for this experiment are described in Table 4.10. Let α represent this ratio for this experiment, thus $\alpha = \Delta IC_{i,j}/C^s_{i,j,k}$. To generate cases in this experiment, $C^s_{i,j,k}$ was then set equal 100, $\Delta IC_{i,j}$ set equal to $100 * \alpha$, and α was varied between .01 and 1 in increments of .05. Figure 4.5 shows the amount of total additional inventory ordered, empty containers and chassis, compared to this ratio.

This graph shows that there is an inverse relationship between α and the total additional inventory ordered.The model effectively computes a break-even cost for each additional resource that compares the cost of ordering that resource to the cost of the expected number of moves that resource will be required to make. When the cost of ordering resources goes up compared to the movement cost, the model prioritizes using fewer resources to perform the required number of movements. There looks to be a minimum of around 300 additional resources required to meet demands in all scenarios. This is the number of empty resources required to fulfill demands. As α decreases, the number of additional inventory resources ordered increases dramati-

Table 4.10: Inventory Costs Experiment Parameters

Parameter	Value/Distribution
$\delta^s_{i,E,k}$	Normal(20,4)
$\delta^s_{i,L,k}$	Normal(50,10)
$I1_{i,E}$	10
$I1_{i,E}$	10
$\lvert S \rvert$	100
$\lvert T \rvert$	10
$C^s_{i,j,k}$	100
$\Delta IC_{i,j}$	$100 * \alpha$

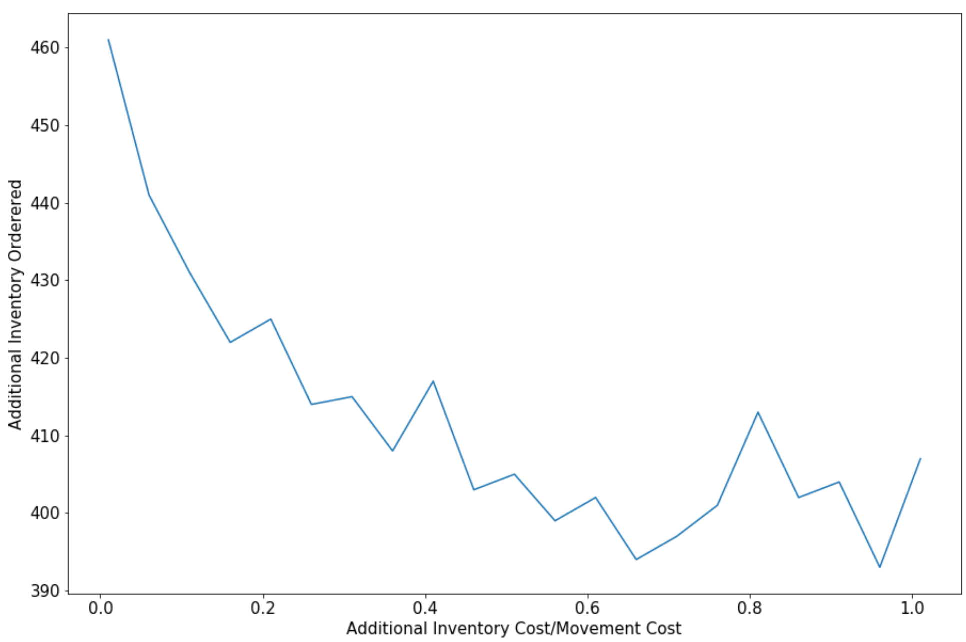

Figure 4.5: Additional Inventory vs. Additional Inventory Cost/Movement Cost

cally. This shows that the model prioritizes ordering more resources to each location instead of repositioning them as needed. The upper bound for this value would be the maximum total empty demand and chassis moves required across all scenarios. In this case, all demands and chassis moves would be satisfied from the existing inventory at that location.

Chapter 5

Conclusion

In this thesis, we present a two-stage stochastic optimization model to determine optimal inventory management strategies in the intermodal trucking industry. The motivation for this model is to reduce empty miles driven and improve service levels, with respect to uncertainty in demands and costs. By considering the randomness in certain parameters, a more robust inventory management policy is produced. The transportation industry is highly competitive and operates on low margins, so reducing operational costs while meeting customer demands is a high priority for companies.

This research differentiates itself through a combination of its specific application and its decomposition method. The model is able to optimize decisions made by an intermodal trucking company by considering their pooled chassis inventory, their vehicle routing requirements, and the presence of uncertainty in demand parameters. The decomposition methods takes advantage of the structure of these inventory management decisions to improve solution time and problem complexity.

First, an extensive form model was developed to optimize the inventory management of an intermodal trucking company operating between two transportation hubs. Due to problem size and solution time issues inherent in stochastic optimization, this extensive form was decomposed into the Intermodal Chassis Inventory Management model that this thesis proposes. This model uses the L-Shaped Method to separate independent realizations of random variables into scenarios that can be solved individually. Included in the proposed Intermodal Chassis Inventory Management model are formulations for the master problem, feasibility problem, and optimality problem, with equations for how each of these problems generates cuts in each iteration of the L-Shaped Method.

The Intermodal Chassis Inventory Management model was then analyzed to determine 1. How the problem size affects solution time, 2. How the stochastic solution differs from a deterministic adaptation, and 3. How inventory and demand values affect optimal inventory management.

The analysis of problem size shows that the solution time of this model increases exponentially with the number of time periods, but that the decomposition had the desired effect of creating a linear relationship between solution time and number of random variable realizations.

The experiment comparing the stochastic solution to a deterministic model using only mean values highlights the value of considering the variance in parameters. Using the deterministic solution would cause the company to be under-prepared for realized demand in roughly half of the scenarios possible. While the stochastic optimal inventory management solution reports a higher expected cost, this cost is a better estimate of the cost that would be realized in practice as it incorporates a range of different demands rather than a single predicted demand for each location and time period.

Finally, the analysis on system parameters shows that the model is highly sensitive to balance of values between the two transportation hubs. Having a significantly higher initial inventory or demand at one location than the other results in an increase in the required repositioning moves and expected cost. Conversely, the model is fairly robust to how inventory and demand values are distributed between resource types.

This thesis provides insights into the dynamics of intermodal trucking inventory management. Considering uncertainty in system parameters adds value to inventory management models of this type by providing more robust optimal strategies and more realistic cost estimations. This thesis also proposes a viable decomposition of the stochastic problem to improve the solution times of the larger problem sizes seen in the intermodal trucking industry.

Future work for this research may include expanding upon the base problem by considering additional hubs. Repositioning moves in the current model are implied to have come from the other location. Adding another location would significantly increase the complexity of the problem, because routing decisions would have to consider origin options as well as destination options. The model could also be reformulated as a multistage stochastic optimization problem. In this adaptation, each time period must make inventory management decisions prior to the realization of random variables.

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