Analytical Description of a Series Fault on a dc Bus

Angelo L. Gattozzi, *Sr. Member, IEEE*, John D. Herbst, Fabian M. Uriarte, and Robert E. Hebner, *Fellow, IEEE*

***Abstract*–The solution of the equations of a dc circuit containing an arc is given and compared with experimental data. The arc is modeled according to its classical equivalent circuit and the adequacy of this model is discussed. The analytical solution for the circuit with an opening gap is given for the case of a constant gap and the results are extended to the cases of a gap opening with uniform velocity and a gap opening with constant acceleration, under the assumption of a quasi-static approximation for which the limits of applicability are estimated. Voltage and current evolutions in time are derived, including an estimate of the arc duration and quenching time. The results are compared to experimental data. Also provided is a generalized view of the transient behavior of an arc in a circuit that extends the description commonly used, in terms of only a voltage-current relationship, by also including inductive effects.**

***Index Terms*–arc model, dc bus fault, electric arc extinction, fault interruption, series fault**

I. INTRODUCTION

Power systems where substantial sections are dependent on a local dc bus are becoming ever more ubiquitous as the penetration of power electronics makes them more practical and attractive. In addition, dc loads in areas like computing and telecommunications continue to be added in greater numbers to power systems. Some complete systems based on dc bus architecture have already been commissioned and several more are being planned for both large and small installations, like microgrids, ship power systems, etc. As these applications grow, it becomes increasingly important to address the issue of proper circuit protection against faults, which has historically been more difficult for dc systems than for ac systems. One such potential fault is caused by the accidental opening of the dc bus due to conductor rupture or to the breaking of the connection between bus sections. This failure results in the injection of a gap in series with the main current flow and usually gives rise to an arc between the separated sections of the circuit (series fault). Therefore, the current may actually be maintained near its normal level and the fault may go undetected for some time, while considerable energy is dissipated in the arc with potential destructive results. An in-depth understanding of this fault, therefore, is very important for planning of proper circuit protective features, for estimating the potential damage resulting from it, and for comparing different circuit architectures in regard to their ability to survive such faults.

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In this paper, the analytical solution of the circuit equations applicable to a dc bus under a series fault is presented, making use of the classical representation of an arc in terms of an equivalent circuit. The results are then compared with experimental data obtained at the University of Texas Center for Electromechanics (UT-CEM) and a discussion of the limits of the arc model is provided. A generalization of the classical description of the arc is also included, obtained by adding an inductive voltage axis to the traditional two-dimensional steady state voltage-current diagram.

II. EQUIVALENT CIRCUIT

The equivalent circuit of the system is shown in Figure 1: a gap opens in the line conductor in series with a load resistor *R* and a line inductance *L* supplied by a dc voltage source *V*. The gap separation is *x* between one gap electrode assumed stationary and the other electrode that moves in a straight line with velocity *u* which may be a function of time. An arc is assumed to be formed at the gap upon initial separation, with an arc voltage *e* and an arc current *i*.

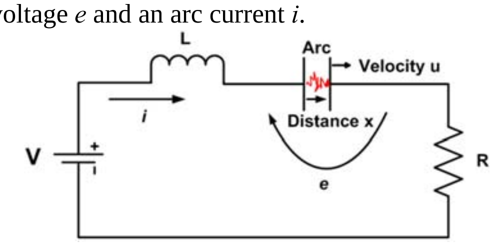

Figure 1: Equivalent circuit of the system.

The equations governing the system are:

$$L\frac{di}{dt} + Ri + e = V \qquad \text{Kirchhoff Law} \qquad (1)$$

$$e = e(i, x) \qquad \text{arc characteristic} \qquad (2)$$

$$x = x(t) \qquad \text{gap dynamic characteristic} \qquad (3)$$

The arc characteristic equation (2) describes the voltage at the arc terminals as a function of arc current and arc separation. This characteristic equation aims to synthesize, in terms of macroscopic parameters, the complex phenomena taking place in the gap and at the electrodes, reducing them to an equivalent circuit model for the arc and making possible the solution of the circuit using analytical procedures or simulations.

The electric arc between a pair of electrodes has been studied systematically for over a century [1-4] and several attempts have been made to condense the complex physical phenomena taking place at the arc gap into a representation amenable for use in electric circuit calculations. All circuit models proposed for an electric arc can be summarized in the following general relationship:

$$e = a + bx + \frac{c + dx}{i^{\alpha}} \tag{4}$$

where a, b, c, d, and α are all parameters to be found in the literature or experimentally. This relationship is empirical in nature.

It is important to clarify that one set of values for the a, b, c, d, and α parameters may cover only the range of arc currents of immediate interest and that, therefore, these parameters are not constant but themselves, in general, variable. However, to minimize the analytical complexity of the problem, it is customary to treat these parameters as true constants with the understanding that the solution may then be limited to a given current range.

For example, if $\alpha = 1$, we obtain the equation first proposed by Ayrton [1-4] and still the most commonly used. It must be pointed out, however, that in general α may not be an integer and that it could even be negative, thus allowing equation (4) to describe in its form the whole range of possible arcs, from low current "silent" arcs to high current "hissing" arcs.

The set of basic equations (1)-(3) has an interesting geometrical interpretation that highlights some properties of its solution in an intuitive way. If we assume the following as reference axes in a system variable space:

variable i for the x axis

variable di/dt for the y axis

variable e for the z axis

then it can be noticed that (1), written as $f(i, di/dt, e) = 0$, represents a plane in this reference system, whereas (2), written as $g(i, e, x) = 0$ in the same variable space, with x as a parameter, represents a cylindrical surface with its axis parallel to the di/dt axis. The arc will be sustainable only on the intersection of the plane surface and the cylindrical surface, that is to say on the solution curve for (1)-(3). However, there are only two steady state points of equilibrium for the arc, and they are located where the solution curve meets the plane $di/dt = 0$ (x-z plane). In fact, all points on the solution curve lying in the region $di/dt > 0$ will tend to migrate toward points of larger current, by definition, and likewise, points for which $di/dt < 0$ will move toward lower currents. Based on this fact, it can be quickly realized also that the only limit point of stable equilibrium is $i = i_Q$ (Figure 2) which becomes an attractor point for all others on the solution curve, whereas $i = i_P$ is an isolated point of unstable equilibrium. In fact, if the point representing the system on the solution curve is displaced slightly from i_P, it will tend to move farther from it.

Notice that the intersection of the surfaces shown in Figure 2 with the e-i plane results in the familiar two-dimensional arc stability diagram based only on resistive elements reported in the literature [1-4]. Therefore, the representation shown in Figure 2 is a generalization of this diagram to the case where an inductive component is present in the circuit.

As the contacts separate, the cylindrical surface describing the gap rises vertically according to (2) and it is clear from Figure 2 that the equilibrium points move until they eventually disappear when the solution curve has no more points in common with the e-i plane: when this happens, the arc quenches. Figure 2 also shows the potentially large inductive voltage that can be generated by a sudden change in arc gap length. This change in gap length may not necessarily be the result of physical motion of one electrode with respect to the other (as indicated in the figure), but can also be the result of the random fluctuations in the current path within the arc resulting in an unpredictable and rapid change in effective gap length. Correspondingly, the operating point of the arc then jumps rapidly from one solution curve to another with the current lagging behind and resulting in Ldi/dt voltage spikes.

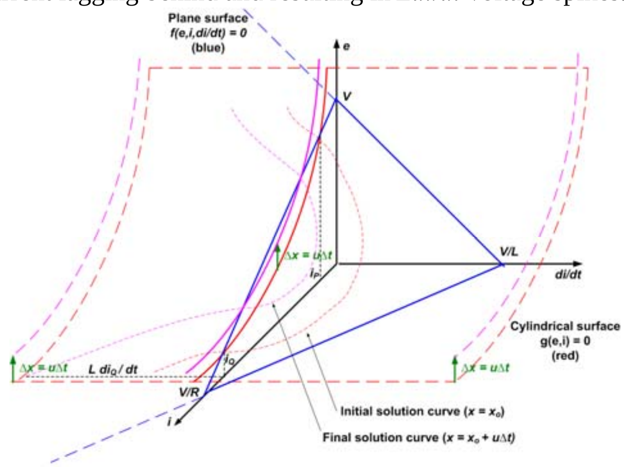

Figure 2: Region of stability for the arc in the circuit of Figure 1 shown in the *i-(di/dt)-e* space, analogous to the conventional *x-y-z* space.

Substituting (4) into (1), we obtain the following single equation with the current as the unknown:

$$\tau \frac{di}{dt} = 2\lambda(t) - \frac{\psi(t)}{i^{\alpha}} - i \tag{5}$$

where

$$\tau = \frac{L}{R} \qquad \text{inductive time constant} \tag{6}$$

$$\lambda = \frac{V - a - bx}{2R} \tag{7}$$

$$\psi = \frac{c + dx}{R} \tag{8}$$

In general, a closed form solution to (5) cannot be found. We can, however, examine some special cases.

A. Case 1: Constant Gap

For example, we could consider the case of $\alpha = 1$, as is commonly done in much of the literature. Furthermore, we can assume that the separation of the electrodes is constant ($u = 0$) or that the mechanical motion of the moving gap electrode is slow compared to the electrical response of the system (quasi-static approximation) and assume, therefore, that $x = x_o =$ constant. In summary, we could solve the problem

$$\tau \frac{di}{dt} = 2\lambda_o - \frac{\psi_o}{i} - i \tag{9}$$

with

$$\lambda_o, \psi_o = \text{const. (corresponding to } x = x_o = \text{const.)} \\ i(t=0) = i_o \ \text{(initial condition)} \tag{10}$$

Equation (9) is separable and can be solved as

$$\int \frac{i\,di}{i^2 - 2\lambda_o i + \psi_o} = -\int \frac{dt}{\tau} \tag{11}$$

The result of the first integral depends on the discriminant of the denominator, namely on $\lambda_o^2 - \psi_o$. In our case, it can be shown that this discriminant is always positive for any combination of parameters that makes physical sense, thus

$$\lambda_o^2 - \psi_o > 0 \tag{12}$$

resulting in the roots i_1 and i_2 of the polynomial in the denominator of (11) being real and distinct and given by

$$i_{1,2} = \lambda_o \pm \sqrt{\lambda_o^2 - \psi_o} \tag{13}$$

where $i_1 > i_2$. When (12) is developed with the definitions in (7) and (8), it leads to

$$x_o < x_{o\max} = \frac{V-a}{b} + \frac{2R}{b}\left[\frac{d}{b} - \sqrt{\left(\frac{d}{b}\right)^2 + \frac{V-a}{R}\left(\frac{d}{b}\right) + \frac{c}{R}}\right] \tag{14}$$

This simply states the anticipated result that, for a given supply voltage V, load resistance, and arc characteristics (a, b, c, d), the arc can exist only if the electrode separation does not exceed a maximum value.

Therefore, on the basis of (12), we can now write the solution of (9) by way of (10) and (11) as

$$t = \tau\left[\frac{i_2}{i_1 - i_2}\ln\left(\frac{i - i_2}{i_o - i_2}\right) - \frac{i_1}{i_1 - i_2}\ln\left(\frac{i - i_1}{i_o - i_1}\right)\right] \tag{15}$$

The solution has been found in terms of the inverse function $t(i)$ instead of $i(t)$, but this is still useful in establishing some interesting results. We shall restrict ourselves to the case of positive current, as assumed in Figure 1. Therefore, in order for (15) to yield a real solution, the following conditions must be satisfied at the same time:

$$\frac{i - i_1}{i_o - i_1} > 0 \\ \text{AND} \\ \frac{i - i_2}{i_o - i_2} > 0 \tag{16}$$

which can be translated into

$$\begin{aligned} &\text{IF } i_o > i_1 && \rightarrow && i > i_1 \\ &\text{IF } i_2 < i_o < i_1 && \rightarrow && i_2 < i < i_1 \\ &\text{IF } i_o < i_2 && \rightarrow && i < i_2 \end{aligned} \tag{17}$$

Thus, i will be confined to one of the three regions (0, i_2), (i_2, i_1), (i_1,∞) and, in fact, to the very same region where the initial current i_o happens to fall.

A consideration of the first two derivatives of the function $t(i)$ will allow us to sketch the expected function $i(t)$. Thus,

$$\frac{dt}{di} = -\tau\frac{i}{(i-i_1)(i-i_2)} \\ \frac{d^2t}{di^2} = \tau\frac{i^2 - \sqrt{i_1 i_2}}{\left[(i-i_1)(i-i_2)\right]^2} \tag{18}$$

from which we derive

	$i < i_2$	$i_2 < i < i_1$	$i > i_1$
dt/di	< 0	> 0	< 0
d^2t/di^2	< 0	< 0 for $i < \sqrt{i_1 i_2}$, otherwise > 0	> 0

This functional study is completed by the observation that if $i_o = i_1$, the function degenerates to the constant $i = i_1$, and, likewise, if $i_o = i_2$, also $i = i_2$. Therefore, noting also that $\psi_o = \sqrt{i_1 i_2}$, the expected evolution in time of the current is as shown in Figure 3.

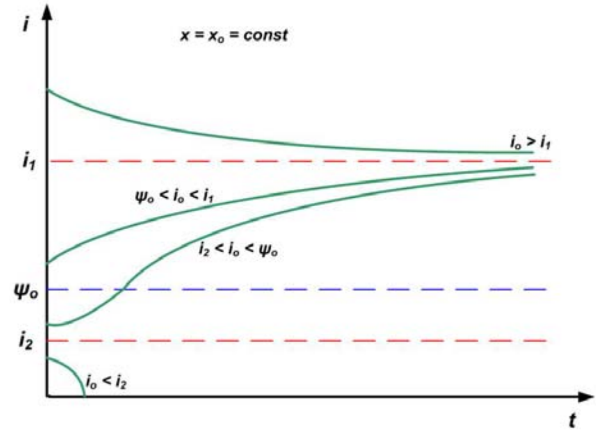

Figure 3: Sketch of function *i(t)* for constant gap and for different values of i_o.

It is interesting to compare the above predictions with test data in the case of an actual experiment. Several experiments on series faults on a dc bus have been run at UT-CEM [5]. A typical one, for example Figure 4, shows the case of a series fault with the electrodes held at a fixed gap distance of 0.25” (6.35 mm). In this test, the current, starting from an initial value of about 175 A, settled at a constant value of about 160 A (the small negative slope for $t > 20$ s can be attributed to electrode wear, and the high frequency spikes to random fluctuations of the environmental conditions in the gap). In this case, due to excessive electrode erosion, the gap was forced open to stop the fault after about 35 s. Using the result in (15) with the following parameters (the values for a, b, c, d were obtained from [1])

$$\begin{aligned} V &= 680 \text{ V} \\ L &= 6.2 \text{ mH} \\ R &= 3.8\ \Omega \\ i_o &= 175 \text{ A} \\ a &= 15.2 \text{ V} \\ b &= 10.7 \text{ V/mm} \\ c &= 21.4 \text{ VA} \\ d &= 3 \text{ VA/mm} \end{aligned} \tag{19}$$

one obtains the values

$$
\begin{aligned}
i_1 &= 157 \text{ A} \\
i_2 &= 0.04 \text{ A} \\
\text{x}_{o\max} &= 57.1 \text{ mm}
\end{aligned}
\tag{20}
$$

and the plots shown in Figure 5. The curve that applies in this case is the one marked "i_o = 175 A" for which the initial current matches the value of that in the test. The calculations predict a stable limit current of 157 A that compares well with the average limit current resulting from the test of approximately 160 A. Both calculations and test indicate that the arc steady state current will be reached very quickly (in just a few ms) and maintained as long as conditions in the arc region remain constant.

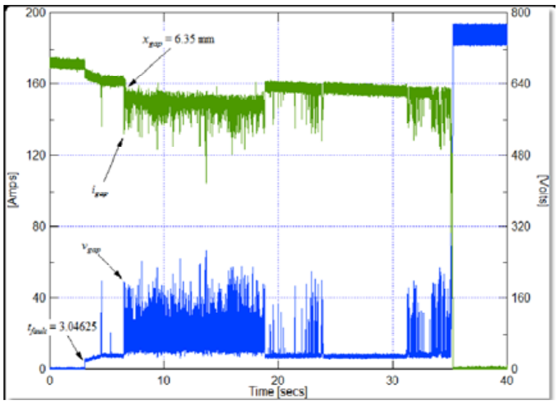

Figure 4: Experiment 4D at UT-CEM on a series fault voltage and current at a fixed gap distance of 0.25".

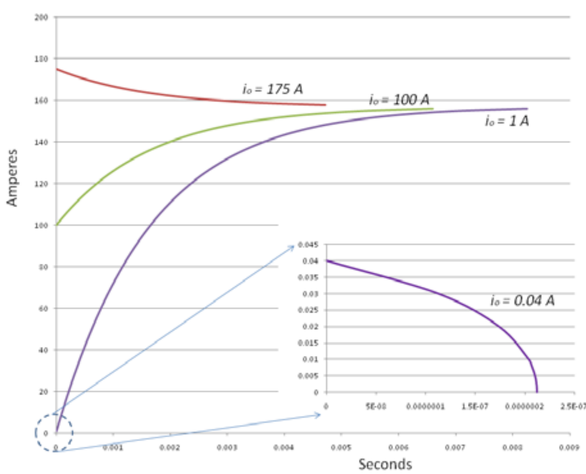

Figure 5: Calculated time evolution of the current for the series fault test 4D at UT-CEM for different cases of initial current (the inset shows arc extinction if the initial current is too low).

To summarize, the analysis of the constant gap arc gives the following results:

1. For a given supply voltage, load resistance, and arc characteristics, the gap must be less than a maximum separation distance in order for an arc to be supported (14).
2. For an arc to be present, the initial current must be at least as large as a minimum value i_2 (13), otherwise the arc self-extinguishes.
3. An arc satisfying the conditions above settles at a steady current given by i_1 (13) (or i_2 only if the initial current is equal to i_2) that will be maintained forever unless other phenomena change the arc environment (e.g., electrode erosion, random fluctuations in the arc structure, etc.).

B. Case 2: Gap Opening at Constant Speed

The case is described by the following equations (again we assume that α = 1):

$$\tau \frac{di}{dt} = 2\lambda(t) = \frac{\psi(t)}{i} - i \tag{21}$$

$$
\begin{aligned}
u &= \text{const.} \\
\lambda &= \frac{V - a - but}{2R} \\
\psi &= \frac{c + dut}{R}
\end{aligned}
\tag{22}
$$

where u is the opening velocity of the gap.

Equation (21) is nonlinear and does not have an exact solution expressible in closed form; thus, it must be solved by some approximation or numerically. The issue is not avoided if, instead of an arc model in terms of its $e-i$ characteristic, a model in terms of an equivalent arc resistance is used, as has been done in the literature [6-9].

For our purposes, some insight can be gained by simply noticing that, for a given minimum resolvable gap length increment of interest in our problem x_r, and for electrode velocities u that are not too large, the system time constant τ is likely to be much smaller than the transit time $t_r = x_r / u$. Thus, using the values in (19) and assuming for example x_r = 1 mm and u = 2.54 mm/s, we find

$$\tau = \frac{L}{R} = 1.6 \text{ ms} << t_r = \frac{x_r}{u} = \frac{1}{2.54} = 394 \text{ ms} \tag{23}$$

This inequality is strong enough to allow quite liberal margins for the choice of x_r and u within most practical limits. This allows us to extend, to a first order approximation, the results found in case 1 under a quasi-static assumption and conclude that at all times during the opening of the gap the current will have stabilized at the value of i_1 corresponding to the gap length at that particular time. Thus, the evolution in time of i will coincide with that of i_1 and be given by

$$i = i_1 = \lambda + \sqrt{\lambda^2 - \psi} \tag{24}$$

with λ and ψ given by (22).

This function has been plotted for the case of u = 2.54 mm/s and is shown in Figure 6. It can be seen that the evolution of voltage and current is almost linear and that the arc extinguishes after about 22.5 s. The corresponding test data is reported in Figure 7. It can be noticed immediately that the theoretical and experimental plots agree well in regard to the slopes of the curves and the general, almost linear trend. The test data, however, show an abrupt arc extinction after about 7 s, namely one third the duration predicted theoretically.

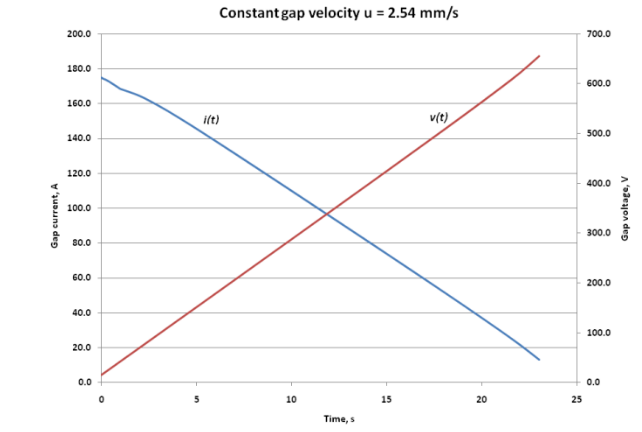

Figure 6: Plot of $i(t)$ when u = 2.54 mm/s under a quasi-static approximation.

This discrepancy between calculated and tested arc extinction times can be attributed to several reasons as, for example, the following:

1. Random fluctuations in the actual arc path not accounted for in the theory that assumes an ideal geometrical straight line for it
2. Thermodynamic effects in the arc plasma not reflected in the simple circuit arc model
3. Electrodynamic effects in the arc plasma also not reflected in the arc circuit model

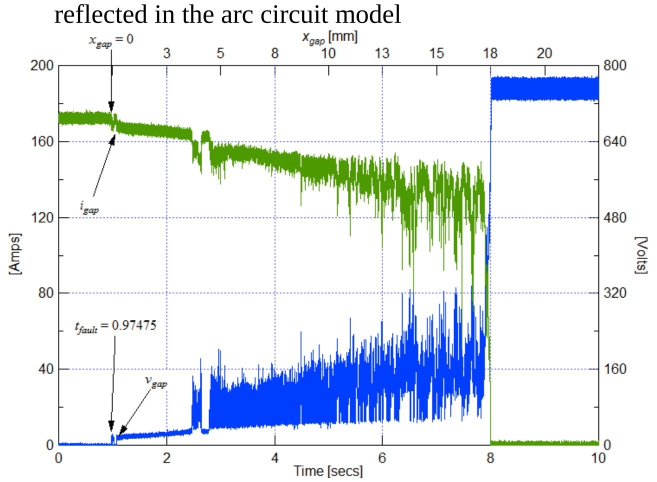

Figure 7: Experimental data for the case of u = 2.54 mm/s.

C. Case 3: Gap Opening at Constant Acceleration

In this case we have

$$\tau \frac{di}{dt} = 2\lambda(t) - \frac{\psi(t)}{i} - i \tag{25}$$

$$x = \frac{gt^2}{2}$$
$$u = gt$$
$$g = \text{const.} \tag{26}$$
$$\lambda = \frac{V - a - bgt^2/2}{2R}$$
$$\psi = \frac{c + dgt^2/2}{R}$$

where g is the constant acceleration with which the gap opens. In this case we can follow exactly the same procedure as for the case of constant velocity and examine under what conditions the quasi-static approximation still holds. Thus, assuming g = 9.8 m/s^2, which represents the case of a conductor falling freely under the influence of gravity, if we allow a ratio of at least 10 in an equivalent relationship to (23) we can write

$$\tau = \frac{L}{R} = 1.6 \text{ ms} << 16 \text{ ms} \le t_r = \frac{x_r}{u_{\max}} = \frac{1}{9800 t_{\max}} \tag{27}$$

Therefore, our approximation is valid for times less than $t_{\max}$ given by

$$t < t_{\max} = 6.4 \text{ ms} \tag{28}$$

If this condition is verified, we can still use (24). Calculations using this equation yield the plot shown in Figure 8. It is clear that the current in the arc extends well beyond the time limit set by (28), thus, the use of the quasi-static approximation and of (24) is no longer warranted. The corresponding experimental data is shown in Figure 9, and, not surprisingly, the measured arc extinction time is about four times longer than that calculated from (24). For this case, the only procedure available is that of a numerical solution to (25) or an analytical solution based on an approximation of the nonlinear terms.

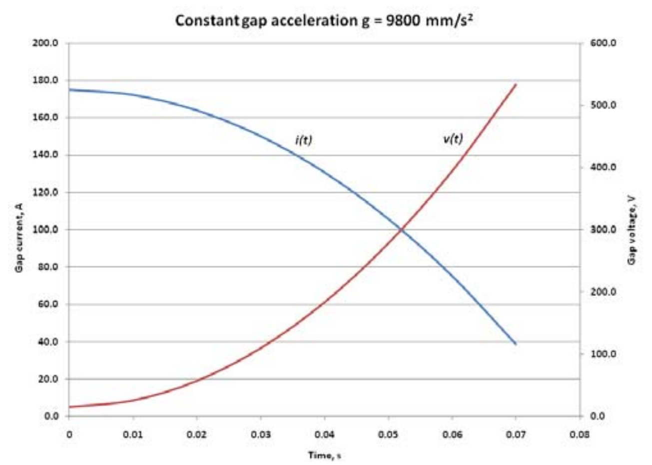

Figure 8: Calculated current and voltage for a gap opening under constant acceleration.

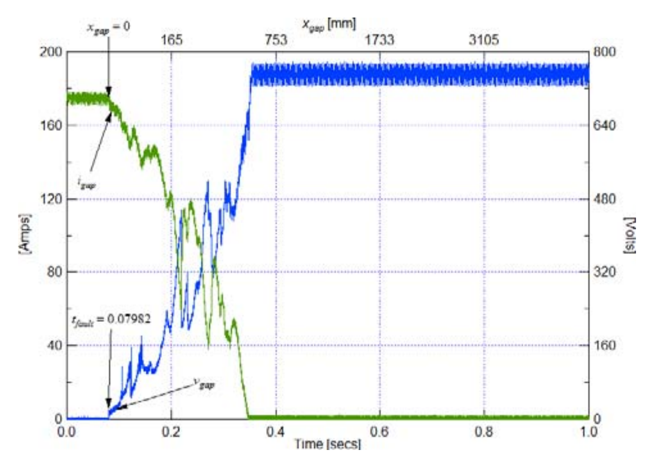

Figure 9: Experimental results for a gap opening with constant acceleration of 9.8 m/s^2.

III. Conclusion

The analytical solution for the circuit with an opening gap shown in Figure 1 was given for the case of constant gap under the assumption that the gap can be represented by the Ayrton model. The results for a constant gap were extended to the cases of a gap opening with uniform velocity, and gap opening with constant acceleration, under the assumption of a quasi-static approximation, for which the limits of applicability were also found. In general, the expressions given are expected to yield arc extinguishing times in excess of those observed in actual experiments since ideal conditions at the electrodes have been assumed with no random fluctuations of the arc path. These fluctuations will tend to extinguish the arc sooner than calculated from the theoretical formulas given. It is clear that the Ayrton model can be used, but that electrode motion, as would occur in many practical situations, introduces complexities that are not addressed by this simple model and that further analytical and experimental work are needed to develop a more accurate model.

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V. Biographies

Angelo L. Gattozzi, Ph.D., is a Research Associate at the Center for Electromechanics of the University of Texas at Austin where he has been working on power modules for the electric gun program, resonant converters for high speed motors/generators with flywheel energy storage, energy harvesting from sea waves, and simulations for the electric system of the DDG51 class Navy destroyer.

Prior to joining the University of Texas, Dr. Gattozzi was responsible, at the Lincoln Electric Company in Cleveland, Ohio, for the development of a complete new line of induction motors, achieving the highest efficiency levels in the industry.

John D. Herbst joined the Center for Electromechanics in 1985. He is currently the Principal Investigator for research programs involving high power rotating electric machines and power converters. Prior to his current position, Mr. Herbst served as Principal Investigator for the ONR Megawatt Power Module for Ship Service program, an effort to explore high speed generators and energy storage flywheels to reduce fuel consumption on the DDG51 class of Navy warships. He was also Co-Principal for the Advanced Locomotive Propulsion System (ALPS) project, a $30M effort to demonstrate an advanced hybrid electric propulsion system for high speed passenger locomotives.

Fabian M. Uriarte, Ph.D., joined the Center for Electromechanics in 2010. He obtained his BS and MS in electrical engineering from Virginia Tech, and his Ph.D. in electrical engineering from Texas A&M University in the area of parallel simulation of power systems. His research interests are in parallel computing, object-oriented programming, and modeling and simulation of power systems, power electronic converters, and microgrids.

Robert E. Hebner, Ph.D., (S'70-M'71-SM'83-F'93) is Director of the Center for Electromechanics at the University of Texas at Austin. The Center develops technology, primarily novel motors, generators, and suspension components, and teams with companies to get the technology into the market.

Previously, Dr. Hebner was the acting Director of the U.S. National Institute of Standards and Technology (NIST). In addition, he has directed NIST's Electronic and Electrical Engineering Laboratory, a laboratory with a staff of more than 250. He also worked at the Defense Advanced Research Projects Agency where he developed programs to improve semiconductor manufacturing.

Throughout his career, Dr. Hebner has been active in having authored or coauthored more than one hundred technical papers and reports. He has extensive experience in international technology programs. This work included the modernization of the measurement systems needed to support global trade and the assessment of the effectiveness of government technology programs in stimulating domestic economies.