Performance evaluation of the deconvolution techniques used in analyzing multicomponent transient signals

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Performance Evaluation of the Deconvolution Techniques used in Analyzing Multicomponent Transient Signals

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Abstract: Deconvolution is an important preprocessing procedure often needed in the spectral analysis of transient exponentially decaying signals. Three deconvolution techniques are studied and applied to the problem of estimating the parameters of multiexponential signals observed in noise. Both the conventional and optimal compensated inverse filtering approaches produce data which are further analyzed by SVD-based autoregressive moving average (ARMA) modeling techniques. The third procedure is based on homomorphic filtering and it is implemented by fast Fourier transform (FFT) technique. A comparative study of the performance of the above deconvolution techniques in analyzing multicomponent exponential signals with varied signal-to-noise ratio (SNR) is examined in this paper. The results of simulation studies show that the homomorphic deconvolution technique is most computationally efficient, however, it produces inaccurate estimates of signal parameters even at high SNR, especially with closely related exponents. Simulation results show that the optimal compensation deconvolution technique is indeed a generalized form of the conventional inverse filtering and has the potential of producing accurate estimates of signal parameters from a substantial wide range of SNR data.

Key words: Exponentially decaying signal, Gardner transformation, Optimal and homomorphic deconvolution

1. Introduction

The analysis of transient signals of exponentially decaying nature, arises in many areas of electrical engineering, physical sciences and medicine. For instance, such signals occur in solving system identification or characterization problems in communication and control engineering [1-3], electronic component reliability study [4], deep-level transient spectroscopy [5], nuclear magnetic resonance in medical diagnosis[6,7], compartment analysis in physiology [8], and pharmokinetics [9], to name just a few. In most cases the response $S(\tau)$ from these systems can be expressed as

$$S(\tau) = \sum_{k=0}^{M} A_k \exp\{-\lambda_k \tau\} + w(\tau) \quad \text{........(1)}$$

where M, A_k and λ_k represent respectively the number of components, amplitude and the real-valued decay rate; $w(\tau)$ denotes the additive white noise with variance σ_w^2. The problem is to determine the unknown parameters M, A_k, and λ_k from the measured data. This problem is different from that encountered in spectral analysis of sinusoidal signals since λ_k is a real constant here.

Many techniques have been suggested in the literature for solving the aforementioned problem; however, the frequency-domain method is often preferable since the desired parameters are obtained directly from the power spectral estimates of the signal. In this method the parameter estimation problem is posed first as a deconvolution problem by using the Gardner transformation [7-10] method to convert (1) into a discrete convolution model whose input is a train of weighted delta function that contains the signal parameters to be determined. The impulse response function of the model is derived from a known basis function p(t) = exp(-t). A set of deconvolved data, consisting of complex sinusoids in noise, is generated from this model by using discrete Fourier transformation and either the conventional (uncompensated) or the optimal (compensated) [11,12] deconvolution technique. These data are analyzed by using an ARMA model whose parameters are determined via the singular value decomposition algorithm. Estimates of the decay rates are then obtained from the ARMA model spectrum. This approach produces high-resolution estimate of the decay constants that can be graphically displayed; however, it is computationally demanding. Another technique that relies on the FFT algorithm is the homomorphic filtering [13]. Though this method is computationally efficient, it produces mixed results. This paper examines

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the relative performance of the above deconvolution techniques in the analysis of both clean and noisy multicomponent exponential signals. Results of analysis show that the conventional technique is appropriate for only high signal-to-noise (SNR) data. Though the homomorphic deconvolution procedure is computationally efficient, it produces inaccurate results even at high SNR, especially with closely related exponents. Though the optimal compensation deconvolution method involves laborious and complex computations, it produces much more accurate results at both high and low SNR.

2. Convolution model

Analysis of a sum of exponential signals poses some difficulties as result of the nonorthogonal nature of the exponential function. The problem becomes much more difficult as the level of noise in the signal increases. In fact the aforementioned factors are amongst the reasons for the failure of some of the time-domain techniques such as the method of moments [14], the Prony technique and its variants [15]. It is therefore necessary to apply some preprocessing techniques on $S(\tau)$ so as to alleviate the above-mentioned problems.

Equation (1) is a particular case of [7]

$$s(\tau) = \sum_{k=1}^{M} A_k p(\lambda_k \tau) + n(\tau) \quad \text{.......................(2)}$$

where the basis function $p(\tau) = \exp(-\tau)$. This equation can be expressed as

$$s(\tau) = \int_0^{\infty} g(\lambda) p(\lambda\tau) d\lambda + n(\tau) \quad \text{......................(3)}$$

where

$$g(\lambda) = \sum_{k=1}^{M} A_k \delta(\lambda - \lambda_k) \quad \text{.....................................(4)}$$

and this contains all the signal parameters to be determined.

Multiplying both sides of (3) by τ^{α} and applying the nonlinear transformation $\tau = e^t$ and $\lambda = e^{-r}$ results in a convolution integral

$$y(t) = \int_{-\infty}^{\infty} x(\lambda) h(t-\lambda) d\lambda + v(t) \quad \text{...................(5)}$$

where $y(t) = \exp(\alpha t)\, s\{\exp(t)\}$, $x(t) = \exp\{(\alpha-1)t\} g(e^{-t})$, $h(t) = \exp(\alpha t) p(e^t)$, and $v(t) = \exp(\alpha t) n(e^t)$. Sampling $y(t)$ at a rate of $1/\Delta t$ Hz converts (5) into a discrete-time convolution

$$y[n] = \sum_{m=-n_{min}}^{n_{max}} x[m] h[n-m] + v[n] \quad \text{..................(6)}$$

where n_{min} and n_{max} represent respectively the upper and lower data cut-off points. N is the total number of samples for both y[n] and h[n] such that $N = n_{max} - n_{min} + 1$. The problems associated with the selection of these sampling conditions have been well-highlighted in [15] and will not be discussed here.

Equation (6) forms the basis for estimating the signal parameters since ideally x(n) can be recovered from the observed data by deconvolution. In the frequency domain, the convolution term becomes a product of $H(\omega)$ and $X(\omega)$. Considering noiseless case, it follows that x(n) can be recovered by either dividing $Y(\omega)$ by $H(\omega)$ or subtracting $log[H(\omega)]$from $\log[Y(\omega)]$ followed by inverse transformation. A brief summary of these procedures is consequently discussed.

3. Inverse Filtering

Taking the DFT of both sides of (8) yields $Y(k) = X(k)H(k) + V(k)$ and on dividing this by H(k) results in $X'(k) = X(k) + V'(k)$. This is termed the conventional, direct or uncompensated deconvolution method. For y(n) with high SNR this approach produces good results, however the method suffers from performance degradation for decreasing SNR. In fact, at moderately low SNR, the number of samples corresponding to the good portion of Y(k) are too few for use in any subsequent analysis. Using either a single parameter or multi parameter optimal compensation deconvolution technique can alleviate the above shortcoming. The concept of optimal compensation deconvolution [11] has been discussed in some other applications, but its use in solving the above parameter estimation problem has only been recently investigated. This involves the design of a compensating transfer function $H_c(k)$ which when applied to Y(k) produces the deconvolved data $X_{os}(k)$, where

$$X_{os}(k) = \frac{Y(k) H^*(k)}{\left[|H(k)|^2 + \lambda \right]} \quad \text{.....................(7)}$$

and λ is an optimizing parameter that is selected according to the SNR of the data. Note that a relatively small value of λ has little effect in the frequency ranges where $|H(k)|^2$ is significant, but has a great influence in ranges where $|H(k)|^2$ is very small. Thus the parameter λ puts limit to the noise amplification because the denominator becomes lower-bounded. Through an experimental testing, an optimum value of $\lambda = \lambda_{opt}$ is sought which can be used in the above deconvolution procedure so that reasonable large samples of $X_{os}(k)$ with high SNR are obtained. The main drawback of this method is the determination of λ_{opt}. Also as highlighted in [12] that under certain conditions, deconvolution with one free parameter does not guarantee a minimum error between the actual and estimated data. A multi parameter optimal compensation procedure is suggested in improving the performance of the above deconvolution techniques.

The inverse filter from equation (7) is given by

$$H_I(k) = \frac{H^*(k)}{|H(k)|^2 + \lambda} \quad \text{.................................(8)}$$

If the regularization operator is the second-order backward difference operator, the inverse filter is of the form

$$H_I(k) = \frac{H^*(k)}{|H(k)|^2 + \alpha |L(k)|^2}, \quad \text{....................(9)}$$

where

$$|L(k)|^2 = 16 \sin^4\left(\frac{\pi k}{N}\right),$$

and L(k) is the DFT of the second order backward difference sequence. This same procedure can be used to generate any derivative of higher even order that can be used to generate high quality deconvolved data. Combining (8), (9) and the fourth-order difference operator gives an inverse filter of the form [12]

$$H_I(k) = \frac{H^*(k)}{|H(k)|^2 + \lambda + \alpha |L(k)|^2 + \beta |L(k)|^4} \quad \text{..(10)}$$

This equation represents, in essence, a generalized expression for inverse filtering where the associated parameters λ, α, and β can be used to control the quality of the deconvolved data. It follows that the set values of these parameters will depend on the SNR of y(n). The effects of λ, α, β, and sampling conditions on the accuracy of estimated signals are discussed under the simulation studies.

4. Homomorphic Deconvolution

The above procedures are computationally very demanding and may not be suitable for real-time analysis of multiexponential signals. Considering (4) and (6) it appears that homomorphic deconvolution technique can also be used to estimate the decay rates. Though homomorphic deconvolution is a nonlinear processing it requires less complex computation since it can be implemented using the FFT algorithm. Cepstral analysis is one example of Homomorphic filtering that leads to two kinds of cepstra: the real cepstrum and complex cepstrum. The real cepstrum is derived from the power spectrum of the signal whereas the complex cepstrum is obtained from the signal complex spectrum. Both cepstra possess some properties that make them effective for the analysis of a variety of signals. Furthermore the ability of the cepstrum to detect periodic structures in logarithmic spectrum makes it suitable for use in estimating the exponential constants as given in (4). Real cepstrum analysis is used here because it requires a much simpler calculation.

Consider v(n) =0 in (6) so that the complex cepstrum, in the z-transform domain, is given by

$$\log Y(z) = \log H(z) + \log X(z).$$

Taking the inverse z-transform of this equation gives

$$y_c(n) = h_c(n) + x_c(n).$$

It is known that for a minimum-phase sequence the amplitude and phase spectra form a Hilbert transform pair [17]. Consequently, the real cepstrum, c_x of x(n) is obtained from the inverse z-transform of log | X(z) |, where

$$\log|X(z)| = \log|Y(z)| - \log|H(z)| \quad \text{..............(11)}$$

Thus, c_x is given by

$$c_x(n) = \begin{cases} c_p(n) & n=0 \\ 2\,c_p(n) & 1 \le n \le N_0 \\ 0 & N_0+1 \le n \le N-1 \end{cases}$$

N_0 is the cepstrum cut-off point. This must be carefully selected to get good results. Taking the z-transform of $c_x(n)$, followed by exponentiation and inverse z-transform yields $x_r(n)$ with dominant peaks at $\ln\lambda_k$.

Both the real and complex cepstra form stable sequences and as such the region of convergence in the above analysis must contain the unit circle. Consequently, the above analysis is implemented by DFT via the FFT algorithm and this accounts for the computational efficiency of this method.

There are a lot of problems in the implementation of this technique. In many practical cases the measurement will be contaminated by noise. As expected the performance of this technique deteriorates as the SNR of y(n) decreases since ln{1+V(z)/(H(z)X(z)} differs significantly from zero. Thus at high SNR the above analysis should give reasonably good estimates of the decay rates. Another important consideration is the choice of the sampling rate since this affects the quality of the cepstrum. Furthermore a singularity may occur during the implementation of the cepstrum processing. Some factors that can lead to this are discussed in [18]. This problem is overcome by smoothing the resultant spectrum; that is, a small additive perturbation is introduced to the spectrum at the singularity points. This processing is detrimental to the cepstrum performance. The value to be added is selected as the smallest possible value whose logarithm does not yield an overflow. Some experimental testing is carried out to obtain this value. It is noted that the same smoothing effect can be achieved when v(n) $\neq$ 0, but at the expense of poor or inaccurate estimates. The detection of the peaks may be difficult either as a result of factors stated above or whenever the spectrum obtained in (13) is less than unity. This latter problem causes oscillation that masks the peaks. A possible solution is to apply either an appropriate windowing or a low-pass filtering after the nonlinear operation in the cepstrum. The latter procedure is used in this paper. Thus, filtering $c_x(n)$ in the forward and backward directions produces $c_{xm}(k)$. That is, for $n \geq 1$

$$q(1) = c_x(1), q(n) = \gamma c_x(n) + (1-\gamma) q(n-1);$$

whereas for $n \leq N$,

$$w(N) = c_x(N), w(n) = \gamma c_x(n) + (1-\gamma) w(n+1)$$

so that

$$\bar{c}_{xm}(n) = 0.5\big(q(n) + w(n)\big)$$

where γ is the filtering coefficient such that $0 < \gamma < 1$.

5. Signal Parameter Estimation

From the preceding analysis it is observed that both M and $\ln\lambda_k$ are determined directly from the homomorphic deconvolution. However, in the case of inverse filtering, further analysis is needed in order to obtain the same signal parameters.

Denoting the deconvolved data by f(k) then

$$f(k) = \sum_{i=1}^{M} A_i \exp\{jk\Delta\omega \ln\lambda_i\} + \varepsilon(k), \quad ....(12)$$

where $\Delta\omega = 2\pi\Delta f$ and the variance of the output noise is given by $\sigma_\varepsilon^2 = \sigma_n^2/[H_I(k)]^2$. The DFT processing of f(k) should theoretically yield its power spectrum with i^{th} peak corresponding to $\ln\lambda_k$. However, only limited samples of f(k) are available especially at low SNR and as such the estimated power spectrum has large sidelobes with poor resolution. Consequently, DFT processing of the deconvolved data is of limited use especially when M is large and $\ln\lambda_k$ are closely related.

To overcome the above shortcomings of the DFT technique, a signal modeling approach is used in estimating the signal parameters. In this method an ARMA (p, q)model is fitted to f(k) so that

$$\sum_{n=0}^{p} a_n f(k-n) = \sum_{n=0}^{q} b_n \varepsilon(k-n); a_0 = 1, ...(13)$$

where a_n and b_n respectively represent the AR and MA coefficients; p and q correspond to the AR and MA model order respectively. The power spectral density associated with this model is given by

$$S_f(z) = \left|\frac{B(z)}{A(z)}\right|^2 \quad ....(14)$$

where A(z) and B(z) are the z-transform of a_n and b_n respectively. This expression provides the decay rates estimates if $S_f(z)$ is evaluated on the unit circle $z = \exp\{j2\pi t/N\Delta t\}$, that is,

$$P_x(t) = S_f(z)\Big|_{z=\exp\{\frac{j2\pi t}{N\Delta t}\}} = \sum_{k=1}^{M} A_k^2 \delta(t - \ln\lambda_k).$$

where $P_x(t)$ is the power distribution of x(t).

6. Simulation studies

The performance of the above deconvolution techniques in estimating the decay rates of exponential signals observed in noise is examined in this section. To achieve this a multicomponent exponential signal, $S(\tau)$ is considered, where

$$S(\tau) = 0.5e^{-0.5\tau} + e^{-\tau} + 2e^{-2\tau} + 5e^{-5\tau}$$
$$.......... + 10e^{-10\tau} + A_0 + w(\tau)$$

and the number of exponential signals, their amplitudes and rate constants are to be determined. The algorithm for estimating A_k is given in [19] hence only the determination of M and λ_k are considered here. In this study, the SNR of the data due to the dc offset, A_0 and the random noise $w(\tau)$ is expressed respectively as SNR1 = 20 $\log_{10}\{y(\tau_p)/A_0\}$ and SNR2 = 20 $\log_{10}\{y(\tau_p)/\sigma_w\}$, where $y(\tau_p)$ is the peak value of the transformed data .

Applying the processing procedures given in section 2 leads to the convolution expression in (6) from which the distribution function

$$x(t) = 0.5\delta(t + \ln 2) + \delta(t) + 2\delta(t - \ln 2)$$
$$............ + 5\delta(t - \ln 5) + 10\delta(t - \ln 10),$$

is obtained. The results obtained with cepstrum processing, direct inverse filtering and optimal compensation deconvolution are denoted as REHOM, REDIF, and REOCD respectively and are depicted in Table I for various SNR. It should be noted that REHOM gives estimates of x(t) whilst both REDIF and REOCD are the results of the computation of power distribution, $P_x(t)$. The results of homomorphic deconvolution are least accurate for all the considered cases. First only four strong components with biased values of $\ln\lambda_k$ are produced. Secondly, some weak components can also be detected and some of these produce more accurate estimates of the decay rates. Figure 1 shows the estimated input signal x(t) at high SNR (125 dB). The effect of cepstrum filtering in improving the resolution, especially at the peaks, is evident in both Figure 1 and Figure 2. This technique suffers from performance degradation as the SNR decreases. In fact the resolution becomes so poor at SNR = 50 dB that no components can be detected. The poor results of the homomorphic deconvolution are probably due to the additive perturbation to avoid singularity of the cepstrum. Further work on how to improve these results is still under investigation.

As expected, the performance of the direct inverse filtering and optimal compensation deconvolution is similar at high SNR, however the latter technique produces better and high-resolution estimates of signal parameters at both low and high SNR. Here it is shown how α, β, and λ can be effectively used to derive good estimates of both M and $\ln\lambda_k$, see Figures 3 and 4. The optimal deconvolution technique provides a way of identifying the true peak form a false (spurious) one since as the values of α, β, and λ are changed the dynamic range of the peaks increase at the expense of the spurious peaks.

7. Conclusion

The performance of three deconvolution techniques for use in the analysis of multicomponent exponential decaying signals has been discussed in this paper. It is observed that the homomorphic deconvolution approach is easy to implement but produces inaccurate results at both high and low SNR. The conventional inverse filtering approach is appropriate for analyzing high SNR data. Though the optimal compensation approach produces accurate estimates of signal parameters, it involves laborious computation.

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