

Modelling viscosity of liquid dropout near wellbore region in gas condensate reservoirs using modern numerical approaches.

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Abstract

Liquid dropout occurs in gas condensate reservoirs below the dew point pressure around near wellbore region as a result of depletion from production of such reservoirs. Forecasting production as well as optimizing future recoveries of gas condensate reservoirs are highly desirable. This is not possible to achieve without accurate determination of liquid dropout viscosity (μ_c) below the dew point. The focus of research in past decades has been on the development of accurate viscosity prediction models below the dew point pressure to ensure accurate condensate production forecast. Gas condensate production forecast and optimisation around this region and condition are complicated due to unique gas condensate behaviour that violates thermodynamic laws.

Current methods are based on correlation estimation, however the accuracy of these correlations are less than satisfactory, and root cause is due to the miscapturing of complex behaviour of gas condensate reservoir near the wellbore region. These motivated the consideration of modern numerical approaches such as the Least Square Support Vector Machine (LSSVM) and Artificial Neural Network (ANN) used in this paper. These methods are considered as more data behaviour oriented, with the capability of capturing the fluid complexity of gas condensate in such conditions.

In this study viscosity of condensate phase near the wellbore region was modelled using machine learning techniques including ANN and LSSVM. For this purpose, over 300 viscosity data sets were collected from published literature and experimental studies worldwide. This databank includes API gravity, reservoir temperature, solution gas to oil ratio (Rs), specific gas gravity, fluid compositions and reservoir pressure.

Six well known previously published viscosity correlations refined using least-square approach to match the experimental data. Qualitative and quantitative error analysis of developed LSSVM and ANN showed their performance superiority over refined literature correlations. The new proposed models can be embedded as an extra feature of commercial reservoir simulation packages for optimization and future recoveries of gas condensate reservoirs.

Keywords:

Condensate viscosity, Gas condensate, Machine Learning (ML), Least Square Support Vector Machine (LSSVM), Artificial Neural network (ANN), Correlations

38 Nomenclature and units

39		
40	API	Oil API gravity
41	ANN	Artificial Neural Network
42	LSSVM	Least Square Support Vector Machine
43	ML	Machine Learning
44	RMSE	Root Mean Square Error
45	GOR	Gas to oil Ratio (scf/STB)
46	R_s	Solution gas to oil ratio (scf/STB)
47	μ_d	Dead oil viscosity (cp)
48	μ_{ob}	Live oil viscosity (cp)
49	μ_c	Condensate viscosity (cp)
50	HPHT	High pressure high temperature
51	N	Number of data points
52	P	Reservoir pressure (psia)
53	T	Temperature (°F)
54	cp	Centipoise

55 1. Introduction

56 As reservoir pressure reduces in gas condensate to below the dew point due to
57 production, the liquid evolved from gas phase and creates multi-phase flow near the
58 wellbore region. Accumulation of the liquid in aforementioned region is increasing with
59 time and is usually very high in rich gas condensate reservoirs. This phenomenon is
60 called “liquid banking” and can cause severe productivity declines (Wheaton and
61 Zhang, 2007). To understand this complex behaviour in depleting gas condensate
62 reservoirs for forecasting production and optimizing future recoveries viscosity
63 determination of the condensate liquid below the dew point is essential (Audonnet and
64 Pádua, 2004; Kashefi et al., 2013).

65 In fact, inaccurate estimation of condensate liquid viscosity below the dew point has
66 detrimental effect on cumulative production and can lead to large errors in reservoir
67 performance. Previous studies show 1% error in reservoir fluid viscosity resulted in a
68 1% error in cumulative production (Al-Meshari et al., 2007; Whitson et al., 1999; Yang
69 et al., 2007).

70 Measurement of condensate viscosity in gas condensate reservoirs is not made in a
71 routine laboratory test and it may be very difficult to obtain due to unavailability of the
72 samples, lack of high pressure high temperature (HPHT) facilities, small volume cell

73 viscometers and time and cost required for the measurements. Consequently this
74 makes use of theoretical correlation more attractive (Al-Meshari et al., 2007; Hemmati-
75 Sarapardeh et al., 2014; Whitson et al., 1999).

76 Depending on the input variables the correlations can be divided to two classes: 1).
77 semi-empirical models that use reservoir fluid composition, critical temperature,
78 acentric factor, pour point temperature, molar mass and boiling point. 2). the
79 correlations which use field data such as reservoir temperature, pressure, API gravity
80 and solution gas to oil ratio “Rs” (Chew and Connally, 1959; Khan et al., 1987). These
81 correlations are deployed for three different conditions of under saturated, saturated
82 and dead oil viscosity.

83 Condensate liquid viscosity is typically low for depleted gas condensate reservoirs,
84 ranging from 0.1 to 1cp, in the near wellbore region (Al-Nasser and Al-Marhoun, 2012;
85 Whitson et al., 1999). The API gravity of condensate reservoirs are between 40 to
86 60°API with gas to oil ratio (GOR) between 3000 – 150000scf/STB and temperature
87 between critical temperature (127°C) and cricondentherm temperature (250°C)
88 (Ahmed, 2010; Whitson et al., 2000). The above conditions were our constraint in
89 selecting existing literature viscosity correlations for this study.

90 Variation of the condensate viscosity with reservoir composition is estimated using the
91 correlation proposed by Lohrenz et al., (1964). This correlation is the most widely used
92 viscosity model, especially in many commercial compositional simulators (ECLIPSE,
93 2014). Lohrenz et al., (1964), known commonly as LBC, is developed for predicting
94 viscosity of dense gas mixture based on the original work of Jossi et al., (1962) for
95 pure substances using corresponding state principle. Prediction performance of LBC
96 model for viscosity prediction of gas phase in gas condensate reservoirs is reasonable,
97 while prediction of condensate liquid viscosity by this method is very poor (Yang et al.,
98 2007). Consequently, it is necessary to tune the LBC correlation by adjusting its
99 coefficients to match the experimental data. This method is selected because it is
100 taking into account compositional changes based on reduced density, which is
101 characteristic of gas condensate reservoirs below the dew point (Fevang and Whitson,
102 1996; Mott, 2003).

103 Gas-saturated-oil (live oil) viscosity correlations are another alternative in literature
104 that can be used to determine the condensate oil viscosity. Yang et al., (2007)
105 suggested to use live oil (μ_{ob}) viscosity correlations to predict condensate liquid
106 viscosity if the measured data is not available. These correlations are function of

107 solution gas to oil ratio R_s , reservoir pressure, reservoir temperature, fluid API gravity
108 and gas specific gravity (γ_{Gas}). Subsequently these parameters are classed as input
109 variables for developing our ANN and LSSVM models (Fig. 5). R_s is often the most
110 significant component of the PVT correlations, which have big influence on the oil
111 viscosity and should be precisely measured in any selected correlations (Hemmati-
112 Sarapardeh et al., 2014). The solution gas to oil ratio is the amount of gas dissolved
113 in the oil at any pressure. It increases linearly with pressure and it is a function of
114 reservoir fluid composition (Fevang and Whitson, 1996; Jokhio et al., 2002).

115 The commonly used literature correlations for estimating gas-saturated-oil viscosities
116 and comply with our defined constrained mentioned earlier are Beggs and Robinson,
117 (1975), Kartoatmodjo and Schmidt, (1991), De Ghetto et al., (1994), Elsharkawy and
118 Alikhan, (1999) and Bergman, (2000). The detailed formula of each correlation is given
119 in Table 3. Further description of each correlation include their advantage and
120 disadvantage is given in Appendix 1. These empirical correlations are used to estimate
121 gas-saturated-oil viscosity as a direct function of dead oil viscosity. A brief discussion
122 of each correlation is presented in following.

123 Beggs and Robinsons, (1975) developed a live oil viscosity correlation based on 2073
124 observations. The average error of -1.83% have been recorded during testing for
125 proposed correlation. Their correlation is covering solution gas to oil ratio (R_s) within
126 the range of 20 to 2070 scf/STB, oil gravity of 16 to 58°API, pressure range of 0 to
127 5250 and temperature of 70 to 295°F (Beggs and Robinson, 1975; El Aily et al., 2019).

128
129 Using 5321 gas-saturated-oil samples collected globally Kartoatmodjo and Schmidt,
130 (1991) developed a gas-saturated-oil viscosity correlation as a function of dead oil
131 viscosity and R_s . Their correlation can be applied for crude oils in the range of 14.4 to
132 59°API gravity, temperature range of 80 to 320°F, R_s range of 0 to 2890scf/STB and
133 live oil viscosity range of 0.098 to 586cp (Kartoatmodjo and Schmidt, 1991).

134
135 De Ghetto et al., (1994) developed a correlation for light oil viscosity with gravity of
136 API > 31.1 as a function of solution gas to oil ratio (R_s) and dead oil viscosity. His
137 correlation is based on 195 data points collected globally. Their correlation is able to
138 predicts live oil viscosity with less than 10% error within the temperature range of 80.6
139 to 334.6 °F, R_s of 8.61 to 3299scf/STB and $0.07 < \mu_{ob} < 295.9$ cp (De Ghetto et al.,
140 1994).

141 Elsharkawy and Alikhan, (1999) developed their gas-saturated-oil viscosity correlation
142 utilizing 254 datasets from Middle East oil samples. They concluded their research
143 with 18.6% average absolute relative error obtained from proposed correlation. Their
144 correlation covers the data range of 10 to 3600 for (Rs) and 0.05 to 20.89cp (μ_{ob})
145 (Elsharkawy and Alikhan, 1999).

146 Bergman, (2000) developed a gas-saturated crude oil viscosity using 2048 data points
147 collected from worldwide. Bergman's correlation can be used in the range of 5 to
148 2890scf/STB solution gas to oil ratio (Rs) and live oil viscosity (μ_{ob}) range of 0.125 to
149 123cp with absolute average error of 9% (Whitson et al., 2000).

150 All aforementioned correlations developed from crude oil, which has compositional
151 differences with gas condensate fluid composition. Moreover, they are direct function
152 of dead oil viscosity, which is one of the most unreliable properties to be predicted by
153 correlations due to the large effect that oil type (paraffinicity, aromaticity and
154 asphaltene content) has on viscosity (Aily et al., 2019; Whitson et al., 2000).
155 Condensate liquid viscosity in near wellbore region can change significantly during
156 depletion in gas condensate reservoirs (Al-Meshari et al., 2007; Fevang, 1995;
157 Whitson et al., 2000). Consequently, empirical and semi-empirical correlations do not
158 fully reflect the viscosity changes with pressure in gas condensate reservoirs near
159 wellbore region. Therefore, the utilized correlations in this study have tuned to match
160 the experimental condensate liquid viscosity data.

161 The recent development and success of machine learning techniques in solving
162 complex engineering problems has drawn attention to their various application in
163 petroleum industry (Ahmadi et al., 2014; Ahmadi and Ebadi, 2014a; Ghiasi et al., 2014;
164 Hemmati-Sarapardeh et al., 2014; Kamari et al., 2013; Naderi and Khomehchi, 2019;
165 Shokir, 2008). For gas condensate reservoirs Ahmadi and Ebadi (2014), Elsharkawy
166 and Foda (1998), Jalali et al. (2007) and Nowroozi et al. (2009) were using machine
167 learning (ML) approach for predicting dew point pressure. Zendehboudi et al. (2012)
168 used ML approach to model condensate-to-gas ratio (CGR) of gas condensate
169 reservoirs. Recently Ghiasi et al. (2014) employed LSSVM to predict compressibility
170 factor of gas condensate reservoirs.

171 Although the aforementioned studies modelled some aspects of gas condensate
172 reservoirs such as dew point pressure, CGR and compressibility factor, however there
173 is a gap in literature for modelling viscosity of gas condensate reservoirs using ML
174 approaches. In fact, to the best of the author's knowledge, there is not any published

175 work on modelling condensate liquid viscosity of gas condensate reservoirs using any
 176 ML approach. Therefore, the aim of this study is to develop novel models for prediction
 177 of condensate viscosity in gas condensate reservoirs based on machine learning
 178 techniques, namely, Least Squares Support Vector Machine (LSSVM) and Artificial
 179 Neural Network (ANN). For this purpose, more than 300 data sets from 13 PVT reports
 180 and experimental study were collected and a data bank was created. To establish
 181 accuracy of the proposed models an error analysis in terms of coefficient of
 182 determination (R^2), root-mean square error (RMSE) and mean square error (MSE) is
 183 carried out. In addition, in order to evaluate the performance of the newly proposed
 184 models against the existing empirical correlations, graphical and statistical error
 185 analysis are utilized (Hagan and Menhaj, 1994).

186 2. Methodology

187 2.1 Data acquisition

188 A database was developed in order to ascertain the accuracy of the proposed methods
 189 and examine the suitability of published viscosity correlations. Data from gas
 190 condensate PVT reports and also experimental investigation of gas condensate fluid
 191 is the base of our data bank. More than 300 data sets have been utilized for developing
 192 and testing the models. This data bank includes API gravity, gas specific gravity,
 193 reservoir fluid compositions, reservoir pressure, reservoir temperature and initial gas
 194 to oil ratio (GOR). Various techniques were used to measure viscosity of the
 195 condensate phase such as using electromagnetic pulse technology viscometer, rolling
 196 ball viscometer and capillary viscometer.

197 Ranges, sources and their corresponding statistical parameters of the data are
 198 presented in Table 1. The data base represents a comprehensive wide range of gas
 199 condensate systems obtained worldwide. Hence, the developed models in this study
 200 should be reliable to use in prediction of condensate viscosity below the dew points
 201 globally within the specified pressure and temperature.

Author	Source of data	Pressure(psia)	Tem (°F)	Solution GOR(Rs)	μ (cp)
Al-Meshari et al., (2007)	Saudi Arabia	0 – 5000	243	334 – 6759	0.264 – 0.561
Yang et al., (2007)	Norway	630 – 7014	338	1889 – 10279	0.178 – 0.271

Kashefi et al., (2013)	Binary Mixture	6011 – 20023	122 – 302	8125 – 25067	0.034 – 0.199
Thomas and Bennion, (2009)	Recombined fluid	2900 – 10600	246	2985 – 11812	0.076 – 0.62
Chen et al., (1995)	North Sea	4520 – 5733	259	7195 – 9264	0.1175 – 0.1572
Wheaton and Zhang, (2007)	Mixture C1-C7	304 – 2393	160	283 – 2661	0.04 – 0.141
Saeedi and Rowe, (1981)	US	253 – 2730	109 – 189	1889 - 10279	0.171 – 0.271
Gozalpour et al., (2005)	Binary Mixture	549 – 5019	100	6859 – 8592	0.0386 – 0.042
Guo et al., (1997)	Binary Mixture	2610 – 5366	110 – 262	5551 – 6000	0.45 – 0.67
O'Dell and Miller, (1967)	US, Texas	1500 – 3500	Unknown	2027 – 4731	0.075 – 0.27
Fetkovich et al., (1986)	North Sea	2827 – 6791	155	3686 – 9180	0.171 – 0.332
Ghahri et al., (2011)	Binary Fluid	800 – 5255	Unknown	1081 – 7103	0.0261 – 0.1411
Audonnet and Pádua, (2004)	Binary Fluid	14 – 10877	76 – 247	18 – 14703	0.086 – 1.672

Table 1. The origin and the ranges of data used for condensate liquid viscosity study.

202

203 2.2 Prediction of liquid dropout viscosity using literature correlations

204 The (Lohrenz et al., 1964) correlation shown in Eq. (1) is one of the most common
205 methods in petroleum industry for estimating the viscosity of petroleum fluid and

206 commonly known as LBC method. The LBC is based on generalised relationship
 207 between viscosity and fourth degree polynomial of the reduced density.

$$[(\mu - \mu^*)\zeta + 10^{-4}]^{\frac{1}{4}} = A_0 + A_1\rho_r + A_2\rho_r^2 - A_3\rho_r^3 + A_4\rho_r^4 \quad (1)$$

208 Where ζ is the viscosity reducing parameter shown in Eq. (2), ρ_{pr} is reduced density
 209 calculated by Eq. (3), μ^* is low pressure gas mixture viscosity defined by Eq. (4), A_{0-4}
 210 are LBC coefficients of 0.1023, 0.023364, 0.058523, -0.040758 and 0.0093324
 211 respectively.

$$\zeta = 5.35 \left(\frac{T_{pc}}{Mi^3 P_{pc}^4} \right)^{1/6} \quad (2)$$

$$\rho_{pr} = \frac{\rho}{\rho_{pc}} = \frac{\rho}{M} v_{pc} \quad (3)$$

$$\mu^* = \frac{\sum_{i=1}^N z_i \mu_i}{\sum_{i=1}^N z_i \sqrt{M_i}} \quad (4)$$

212 Kay's mixing rule (Kay, 1936) is utilized to calculate the pseudocritical properties of
 213 temperature T_{pc} , pressure P_{pc} and volume v_{pc} . In Eq. (4) z_i is the mole fraction of
 214 each pure components i and M_i is molecular weight of each component.

215 To establish special relation between C_{7+} fractions and critical volume Eq. (5)
 216 suggested by (Lohrenz et al., 1964) is used.

$$v_{cC_{7+}} = 21.573 + 0.015122 M_{C_{7+}} - 27.65\gamma_{C_{7+}} + 0.070615 M_{C_{7+}}\gamma_{C_{7+}} \quad (5)$$

217 Where $v_{cC_{7+}}$ is the critical molar volume, $M_{C_{7+}}$ is molecular weight and $\gamma_{C_{7+}}$ is specific
 218 gravity of C_{7+} fraction.

219 The component viscosities, μ_i in Eq. (4) is calculated using (Stiel and Thodos, 1962)
 220 expression as follows.

$$\begin{cases} \mu_i \zeta_i = (34 \times 10^{-5}) Tr^{0.94} & \text{for } Tr \leq 1.5 \\ \mu_i \zeta_i = (17.78 \times 10^{-5})(4.58 Tr - 1.67)^{5/8} & \text{for } Tr > 1.5 \end{cases} \quad (6)$$

222
 223 In LBC correlation viscosity unit ' μ ' is in centipoise (cp), viscosity reducing parameter
 224 ' ζ ' is in cp^{-1} , ρ is in lbm/ft^3 , specific volume ' v_c ' is in $\text{ft}^3/\text{lbm mol}$, temperature ' T ' is in
 225 Rankine ($^{\circ}\text{R}$), pressure ' P ' is in psia, and molecular weight of each component ' M_i ' is
 226 in $\text{lbm}/\text{lbm mol}$.

227 The prediction capability of the LBC for viscosity measurement of the hydrocarbon
 228 liquid especially in gas condensate reservoirs below the dew point is very poor and
 229 rapid increase in liquid viscosity cannot be represented by original LBC correlation (Ali,
 230 1991; Hernandez; et al., 2002; Yang et al., 2007). The result of this study illustrated in
 231 Fig. (1a) also indicates LBC performance in predicting condensate liquid viscosity is
 232 very poor. Hence LBC correlation has been regressed using least-square approach to
 233 match the experimental viscosity data. The procedure for tuning of the LBC correlation
 234 recommended by Yang et al., (2007) followed in this study. The coefficients of A_{0-4} in
 235 LBC correlation Eq. (1) has tuned and new coefficient values are presented in Table.
 236 2.

Coefficients	New values
A0	0.11364
A1	0.02173
A2	-0.20666
A3	0.06283
A4	0.17139

237 **Table 2.** The new coefficients for LBC correlations.

238 Fig (1a) depicts the prediction performance of LBC correlation with default and
 239 regressed values in predicting condensate viscosity. As it can be seen the
 240 performance of the LBC correlation improved significantly after tuning the coefficients.
 241 The second types of the empirical correlations, used in this study correlate gas-
 242 saturated-oil viscosity as a function of deal oil viscosity and solution gas to oil ratio.
 243 Six well known published literature correlations were selected for this purpose. The
 244 prediction performance of gas-saturated-oil correlations found to be poor in forecasting
 245 viscosity of condensate liquid and the results associate with large error. Therefore, in
 246 this study these correlations have been refined to match the experimental
 247 measurements. Table 3 depicts the original and tuned form of the utilized correlations
 248 for predicting condensate liquid viscosity.

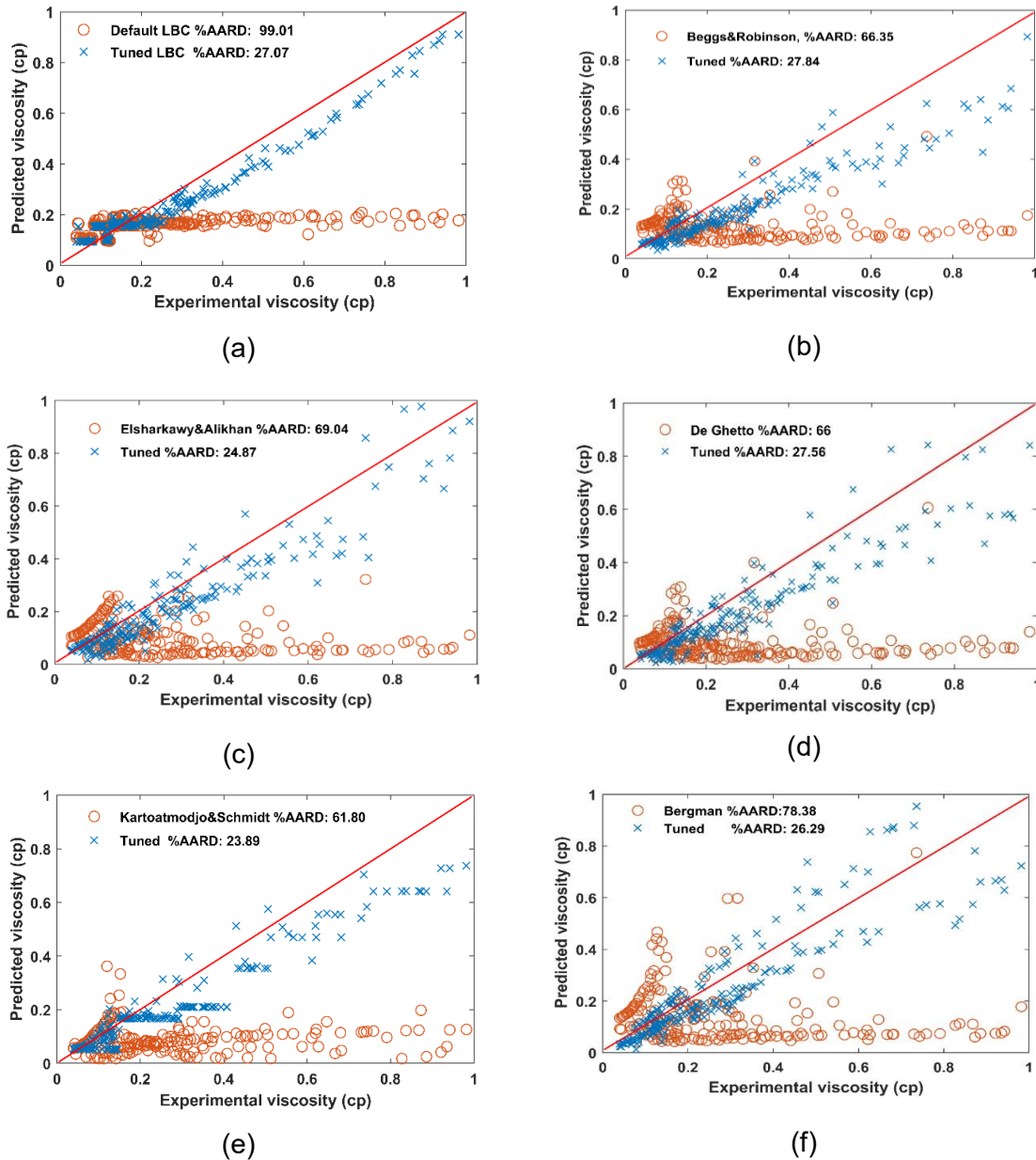
249 Graphical error analysis of the refined literature correlations in predicting condensate
 250 viscosity is presented in Fig (1b-1f). The slope line of 45° in aforementioned figures
 251 representing zero error line in matching between measured and calculated values
 252 (Mansour et al., 2013). Qualitative error analysis in terms of coefficient of
 253 determination (R^2), absolute average relative deviation percentage (AARD%), mean
 254 square error (MSE) and root-mean square error (RMSE) has been applied. From Fig
 255 (1b-1f), and also quantitative error analysis in Table 6, Kartoatmodjo and Schmidt,
 256 (1991) outperforms other methods followed by Elsharkawy and Alikhan, (1999),

257 Bergman, (2000), De Ghetto et al., (1994), Beggs and Robinson, (1975) and LBC,
 258 (1964) correlation. The results of tuned correlations compared to the proposed LSSVM
 259 and ANN numerical methods, which will be discussed later.

Author	Correlation	Tuned correlation
Beggs & Robinson, (1975)	$\mu_{ob} = A(\mu_{od})^B$ $A = \frac{10.715}{(R_s + 100)^{0.515}}$ $B = \frac{5.44}{(R_s + 150)^{0.338}}$	$\mu_c = A(\mu_{od})^B$ $A = \frac{17.99}{(R_s + 100)^{0.515}}$ $B = \frac{4.056}{(R_s + 150)^{0.338}}$
Kartoatmodjo & Schmidt, (1991)	$\mu_{ob} = -0.06821 + 0.9824X_1 + 4.034 \times 10^{-4}X_2^2$ $X_1 = 0.43 + 0.5165 \times 10^{(-8.1 \times 10^{-4}R_s)}$ $X_2 = [0.2001 + 0.8428 \times 10^{(-8.1 \times 10^{-4}R_s)}] \mu_{od}^{X_1}$	$\mu_c = -0.30612 + 1.174X_1 + 4.034 \times 10^{-4}X_2^2$ $X_1 = 0.43 + 0.5165 \times 10^{(-8.1 \times 10^{-4}R_s)}$ $X_2 = [0.2001 + 0.8428 \times 10^{(-8.1 \times 10^{-4}R_s)}] \mu_{od}^{X_1}$
De Ghetto, (1994)	<p>For ($^{\circ}$API > 31.1)</p> $\mu_{ob} = A(\mu_{od})^B$ $A = \frac{25.192}{(R_s + 100)^{0.6487}}$ $B = \frac{2.7516}{(R_s + 150)^{0.2135}}$	$\mu_c = A(\mu_{od})^B$ $A = \frac{62.96}{(R_s + 100)^{0.6487}}$ $B = \frac{2.1334}{(R_s + 150)^{0.2135}}$
Elsharkawy & Alikhan, (1999)	$\mu_{ob} = A(\mu_{od})^B$ $A = 1241.932(R_s + 641.026)^{-1.12410}$ $B = 1768.84(R_s + 1180.335)^{-1.06622}$	$\mu_c = A(\mu_{od})^B$ $A = 3978.167(R_s + 641.026)^{-1.12410}$ $B = 1361.93(R_s + 1180.335)^{-1.06622}$
Bergman, (2000)	$\mu_{ob} = A(\mu_{od})^B$ $A = e^{[4.768 - 0.8359 \ln(R_s + 300)]}$ $B = 0.555 + \frac{133.5}{R_s + 300}$	$\mu_c = A(\mu_{od})^B$ $A = e^{[4.6792 - 0.7772 \ln(R_s + 300)]}$ $B = 0.555 + \frac{133.5}{R_s + 300}$

260 **Table 3.** The original and tuned form of the employed literature correlations for predicting condensate
 261 liquid viscosity.

262



263 **Fig. 1.** Cross plot of the experimental viscosity versus predicted viscosity using employed correlations
 264 and their tuned results.

265 **2.3 Least square support vector machine (LSSVM)**

266 The support vector machine (SVM) has been identified as an efficient and powerful
 267 strategy developed from the machine-learning community (Cortes and Vapnik, 1995;
 268 Curilem et al., n.d.; Suykens et al., 2002). SVM is a tool for a set of related supervised
 269 learning methods that analyse data and recognize pattern using regression analysis
 270 and it is identified as a non-probabilistic binary linear classifier. The objective of this
 271 study is to develop a nonlinear relationship between the available experimental data
 272 considered as inputs (pressure, temperature, API gravity, gas to oil ratio and gas
 273 specific gravity) and the desired output (liquid dropout or condensate liquid viscosity)

274 (Ahmadi and Ebadi, 2014; Eslamimanesh et al., 2012; Hemmati-Sarapardeh et al.,
275 2014; Kamari et al., 2013).

276 SVM method has many advantages over other machine learning techniques as
277 follows: they are more likely to converge to the global optima, prior determination of
278 the network is not required in this model and can be automatically determined as the
279 training ends. Furthermore, the number of hidden layers and hidden nodes should not
280 be determined and this algorithm has fewer adjustable parameters compared to ANN
281 network (Eslamimanesh et al., 2012; Suykens et al., 2002).

282 Original SVM algorithm requires implementing set of nonlinear equations using
283 quadratic programming, which is very hard to implement. Also the obtained outputs
284 using SVM algorithm is much scattered for both linear and nonlinear regressions
285 (Eslamimanesh et al., 2012; Suykens et al., 2002; Suykens and Vandewalle, 1999).

286 To overcome abovementioned problems Suykens and Vandewalle, (1999) suggested
287 a modification to the original SVM algorithm named Least-Squares Support Vector
288 Machine (LSSVM). The LSSVM only requires solving set of linear equations, makes it
289 easier to implement and faster alternative to the original SVM method (Eslamimanesh
290 et al., 2011; Pelckmans et al., 2002; Suykens and Vandewalle, 1999). Suykens and
291 Vandewalle, (1999) defined the cost function (J) for LSSVM by Eq. (7).

$$J = \frac{1}{2}w^T w + \frac{1}{2}\gamma \sum_{k=1}^N e^2_k \quad (7)$$

292 Eq. (7) is subjected to the following constraint:

$$y_k = [w^T \varphi(x_k) + b + e_k], \quad k = 1, \dots, N. \quad (8)$$

293 Where, x_k is input vector containing the input parameters (pressure, temperature,
294 solution gas to oil ratio and gas specific gravity), y_k is output vector (condensate liquid
295 viscosity), b stands for intercept of linear regression in LSSVM method, w stands for
296 regression weight, e_k is the regression error for N training objects in least-squares
297 error approach, γ is relative weight of the summation of the regression errors
298 compared to the regression weight (right hand side of Eq. (7), φ is the feature map,
299 mapping the feasible input region to the high dimensional feature space and transcript
300 T stands for transposing the matrix.

301 Applying Lagrangian function, the regression weight w can be defined in Eq. (9).

$$w = \sum_{k=1}^N \alpha_k x_k \quad (9)$$

302 Where

$$\alpha_k = 2\gamma e_k \quad (10)$$

303 α_k denotes to the Lagrange multiplier, that may be either positive or negative, since
 304 LSSVM has equality restrictions. Assuming linear regression between the inputs and
 305 output parameters of LSSVM algorithm, Eq. (8) is re-written as follows (Pelckmans et
 306 al., 2002; Suykens et al., 2002; Suykens and Vandewalle, 1999).

$$\alpha_k = \frac{(y_k - b)}{x_k^T x + (2\gamma)^{-1}} \quad (11)$$

307 The linear regression in Eq. (11) can be converted to a nonlinear using the Kernel
 308 function in Eq. (12)

$$f(x) = \sum_{k,l=1}^N \alpha_k K(x, x_k) + b \quad (12)$$

309 Where $K(x, x_k)$ represents dependency of Kernel function to the inner values of two
 310 vectors x and x_k in the feasible region built by the inner product of the vectors $\phi(x)^T$
 311 and $\phi(x_k)$ as follows: (Cortes and Vapnik, 1995; Eslamimanesh et al., 2012; Fazeli et
 312 al., 2013; Suykens et al., 2002; Suykens and Vandewalle, 1999).

$$K(x, x_k) = \phi(x)^T \phi(x_k) \quad (13)$$

313 The radial basis function (RBF) Kernel defined in Eq. (14) has been executed. (Cortes
 314 and Vapnik, 1995; Eslamimanesh et al., 2012; Pelckmans et al., 2002; Suykens et al.,
 315 2002):

$$K(x, x_k) = \exp\left(-\frac{\|x_k - x\|^2}{\sigma^2}\right) \quad (14)$$

316 Where σ in Eq. (14) and γ in Eq. (7) are tuning parameters of LSSVM and can be
 317 determined by any external optimization algorithm. Robust Simulated Annealing (SA)
 318 algorithm in MATLAB optimization toolbox has been used to find the optimum values
 319 of these parameters. The root mean square error (RMSE) between the developed
 320 LSSVM model obtained results and experimental values, defined by Eq. (15), was
 321 considered as an objective function during the SA computation.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Vis_{esti} - Vis_{expi})^2}{ns}} \quad (15)$$

322 Where V_{is} represents condensate viscosity, subscripts est and exp represent the
 323 predicted and actual value, ns is number of data points from the initial assigned
 324 population of 144 data sets. The optimized values of γ and σ^2 using SA optimization
 325 method for predicting the condensate liquid viscosity presented in Table 4.

LSSVM model	Input parameters	Model parameters	
		γ	σ^2
Condensate phase viscosity	Reservoir pressure, Temperature, API, gas SG, Rs	5625.256	23.65

326 **Table 4.** The optimum values of the LSSVM parameters.

327 In this study the data is divided into three subsets of “Training”, “Optimization” and
 328 “Testing”. Training set is used for generating the model structure, optimization is used
 329 for minimization of the error in trained model and test data is used to investigate the
 330 prediction capability of the developed model.

331 The database was randomly split into three sub data sets of 80% training, 10% testing
 332 and 10% validation. The allocation percentage of the data is selected according to the
 333 recommendations by Ahmadi and Ebadi, (2014) and Eslamimanesh et al., (2012).
 334 During the training of the model cross validation has been performed where, the
 335 training data sets into several folds and accuracy of each fold checked. Table 5 is
 336 presenting the statistical error analysis of the LSSVM in each stage of training,
 337 optimizing and testing.

338 Input variables for this model are as pressure, temperature, API gravity, gas specific
 339 gravity and solution gas to oil ratio “Rs”. The acceptable distribution of the data is one
 340 with homogeneous accumulations of the data on the domain of the three data sets
 341 (Eslamimanesh et al., 2011; Gharagheizi et al., 2014).

342 The MATLAB code for trained LSSVM model generated and prediction capability of
 343 the trained model was tested for new data sets. The graphs in Fig. 2 and Fig. 3 are
 344 indicating the performance of LSSVM model in training stage and in predicting new
 345 experimental set of data (testing stage), respectively. The majority (73%) of the data
 346 points in this study are within lower viscosity range of 0 – 0.4cp. Therefore, the testing
 347 of the data is toward lower viscosity region, which is more realistic characterisation of
 348 gas condensate viscosity below the dew point near wellbore region (Whitson et al.,
 349 1999; Yang et al., 2007). The viscosity of condensate liquid in near wellbore region,
 350 where condensate liquid in mobile is very low. This is due to existence of more lighter

351 C₇₊ fractions in mobile condensate liquid composition in aforementioned region
 352 (Fevang, 1995, p. 44). Even though the higher viscosity prediction in Fig. (3) has
 353 higher error than the lower viscosity prediction, the AARD% is still reasonably small
 354 because the majority (73%) of the values are in lower viscosity region.

355 Fig. 4 is representing residual plot of LSSVM trained data. Ability of the trained LSSVM
 356 in predicting new data sets are also analysed by presenting graph of standard
 357 deviation error in Fig. 5 and standard error from the mean in Fig. 6.

358

Stage of the process	R^{2a}	RMSE ^b	MSE ^c	AARD% ^d
Training set	0.9139	0.10845	0.01176	13.96
Optimization set	0.87256	0.111121	0.012348	14.12
Testing set	0.7723	0.121037	0.01465	14.25

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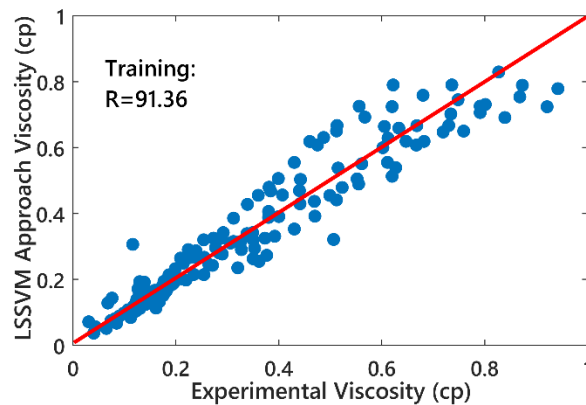
Table 5. Statistical error performance of the LSSVM.

360 a
$$R^2 = 1 - \frac{\sum_i^N (cal.(i)/Est.(i) - exp.(i))^2}{\sum_i^N (cal.(i)/Est.(i) - average(exp.(i)))^2}$$

361 b
$$RMSE = \left(\frac{\sum_i^N (cal.(i)/Est.(i) - exp.(i))^2}{N} \right)^{0.5}$$

362 c
$$MSE = \left(\frac{\sum_i^N (cal.(i)/Est.(i) - exp.(i))^2}{N} \right)$$

363 d
$$AARD\% = \frac{100}{N} \sum_i^N \frac{|cal.(i)/Est.(i) - exp.(i)|}{exp.(i)}$$

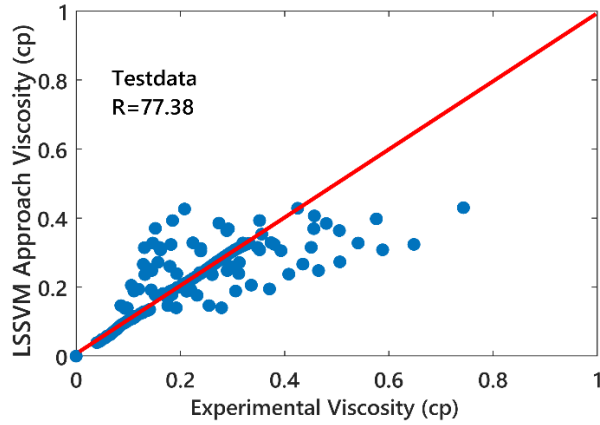


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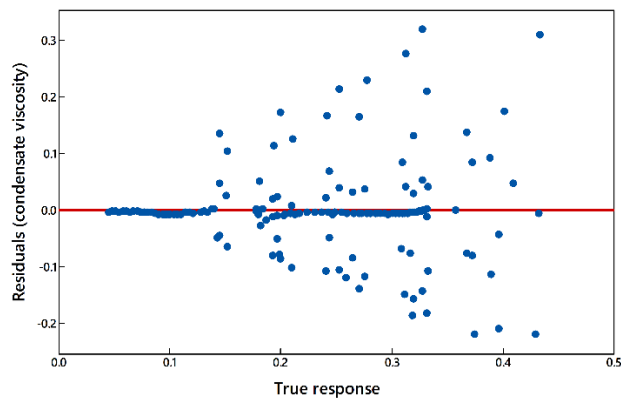
Fig. 2. Performance of the LSSVM trained model ($R^2=0.9136$).



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Fig. 3. Performance of LSSVM in predicting new data ($R^2=0.7738$).

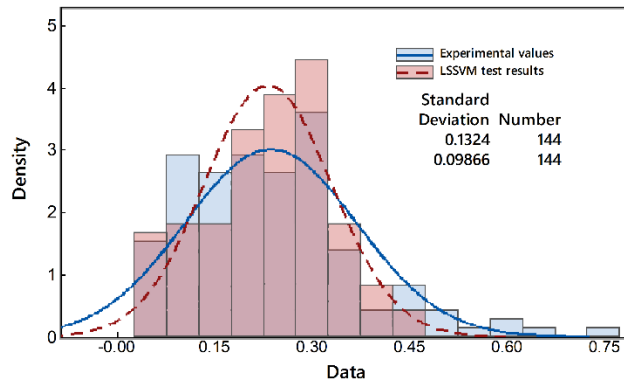
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Fig. 4. Residual plot of LSSVM trained data.



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Fig. 5. Graph of standard deviation of LSSVM method against experimental data.

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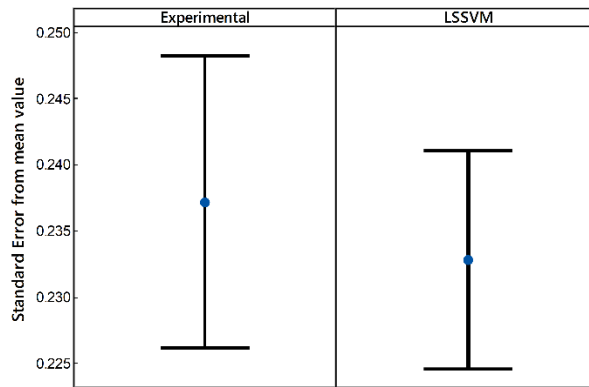
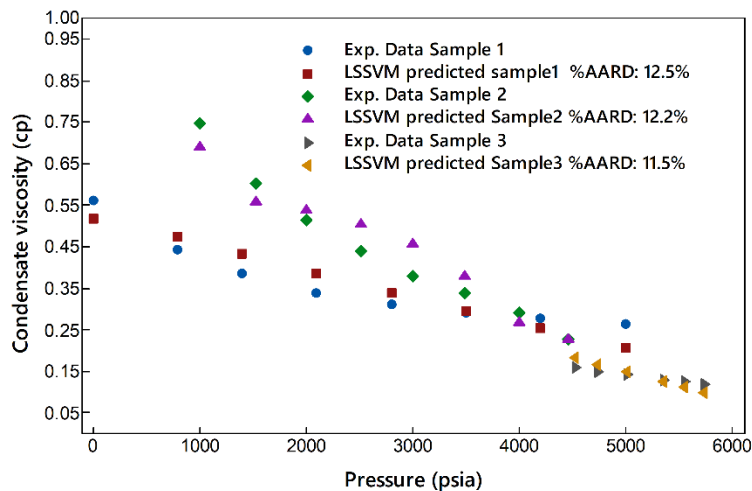


Fig. 6. Interval plot of experimental data against LSSVM approach.



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Fig. 7. Comparison between experimental and predicted values provided by LSSVM for three samples of condensate fluid viscosity.

380 The primary aim of this study was to develop a model that predicts viscosity of the
381 condensate liquid in depleted gas condensate reservoirs with high accuracy using
382 machine learning techniques. The results indicate that LSSVM is performing better
383 than tuned literature correlations. However, the error is still high, approximately about
384 23% in testing stage, where the capability of the model assessed using new data sets.
385 Therefore, to certify the effectiveness and accuracy of the suggested LSSVM model
386 for estimation of condensate viscosity among smart approaches in another attempt an
387 Artificial Neural Network (ANN) was developed, which is presented in following
388 section.

389 2.4 Artificial Neural Networks (ANN)

390 A detailed description of neural networks can be found in Cios and Shields (1997),
391 Dreyfus (2005) and Haykin (1994). ANN is a computational technique in artificial
392 intelligence that uses complex computation system for predicting the output
393 responses. ANNs are inspired by biological networks, performing in a massive parallel

394 connection between nonlinear, parametrized, and bounded functions called neurons
395 (Cios and Shields, 1997; Mesbah et al., 2017).

396 Such a network is a massively parallel-distributed processor that has a natural
397 tendency for storing experimental knowledge and making it available for future use. In
398 ANN system knowledge is acquired by the network through a learning process and
399 synaptic weights will store this knowledge (Haykin, 1994). Hence, mathematical
400 interpretation of the problem does not required. Neurons in such a system coordinate
401 their work, and they transfer information by using synapses “electromagnetic
402 communications” (Ghaffari et al., 2006). Through a set of known input (5 in this study)
403 and output data (1 in this study), the network will be trained. Through a learning
404 process the network monitors the error between the predicted and desired outputs and
405 continue to adjust the weights until the optimization criteria are reached. This process
406 is usually carried out in two stages: first the input variables are linearly combined, then
407 the result is used as argument of non-linear *activation function* (a). The activation
408 function must be non-decreasing and differentiable function; the most common
409 choices are either the identity function ($y = x$), or bounded sigmoid (s-shaped)
410 function, as the logistic [$y = 1/(1 + e^{-x})$] (Eslamimanesh et al., 2011; Ghaffari et al.,
411 2006; Haykin, 1994; Hippert et al., 2001).

412 The neurons are organized in a way that define the network architecture. We used
413 multilayer perception (MLP) type, in which the neurons are organized in layers Fig.
414 (8). The neurons in each layer may share same inputs, but they are not connected to
415 each other. The neural networks consist of hidden layers, output layer, inputs and bias
416 units. Number of hidden layers and number of neuron of each layers can be arbitrary
417 (Khosrojerdi et al., 2016). However, increasing number of neurons may cause
418 overfitting while decreasing their numbers may result on poor performance of the
419 network. The main advantage of ANN is ability to process large amount of data sets
420 (Ghaffari et al., 2006; Khosrojerdi et al., 2016; Mesbah et al., 2017; Hippert et al.,
421 2001).

422 Fig. (8) depicts the schematic diagram of ANN structure for predicting viscosity of
423 condensate liquid fluid. This design has one layer for inputs consists of five input
424 parameters, one hidden layer, two bias units and one output unit. This architecture
425 recommended by Hagan et al. (2014), Hagan and Menhaj (1994) and Hippert et al.
426 (2001) as an efficient and the most popular multilayer feed-forward architecture.
427 Nevertheless, there is large number of other designs, which might be considered

428 suitable for other applications. Further information about ANN network architecture
429 used in this study is presented in Appendix B.

430 The network is designed in MATLAB and calculations carried out by implementing
431 different number of neurons in hidden layer (layer 2). To select the best architecture
432 in terms of number of neurons in a hidden layer a trial and error procedure was
433 implemented. The performance of each structure was assessed by comparing
434 coefficient of determination (R^2) and root man square error (RMSE). We came up with
435 the proposed structure in Fig. (8) (5 neurons in layer 2) as the best topology.

436 The aforementioned architecture performance evaluation is required to determine the
437 complexity of a neural network as one of the important factors. Hagan et al, (2014)
438 and Soroush et al, (2015) highlighted importance of level of complexity in neural
439 network structure to ovoid overfitting with higher number of neurons and poor
440 performance with not enough number of neurons.

441 Our input parameters are API gravity, solution gas to oil ratio (R_s), pressure,
442 temperature and gas specific gravity. The output layer is viscosity of condensate fluid
443 calculated by the ANN network. There are many algorithms available to train the
444 network and minimize the error and find the optimum values of the weights and biases;
445 including Levenberg–Marquardt (LM), scaled conjugate gradient (SCG), and resilient
446 back propagation (Hippert et al., 2001; Soroush et al., 2015).

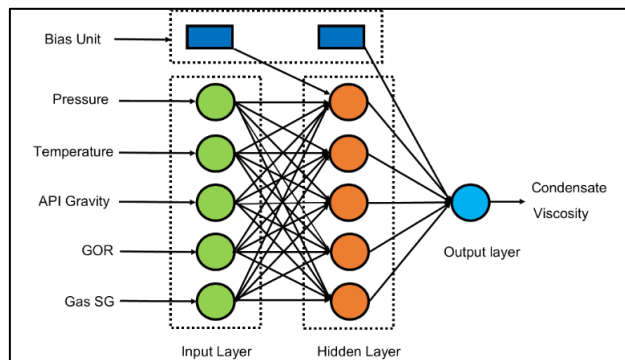
447 The LM backpropagation algorithm introduced by Kenneth, (1944) and recommended
448 by Behera and Chattopadhyay, (2012) as one of the fastest and most popular
449 backpropagation algorithm was used for adjusting the weights in this study. The
450 tangent sigmoid transfer functions set for the neurons in hidden layer.

451 For training of the model 70% of whole data bank (210 data points) randomly selected
452 and split to three data sets of 80% (168 data points) for training, 10% (21 data points)
453 for validation and 10% (21 data points) for testing.

454 The ANN network is trained to map input data by iterative adjustment of the weight
455 function. Information from inputs feed forwarded through the network to optimize the
456 weight between the neurons. Optimization of the weight function is carried out by back
457 propagation of the error during training or learning stages. The ANN reads the inputs
458 and output values in training stage and changes the value of weight functions to
459 minimise the difference in predicted and the target (observed) values. The error in
460 prediction is minimized across training iterations (epochs) and training continues to
461 the point that the network reaches a specified level of accuracy (Ghaffari et al., 2006).

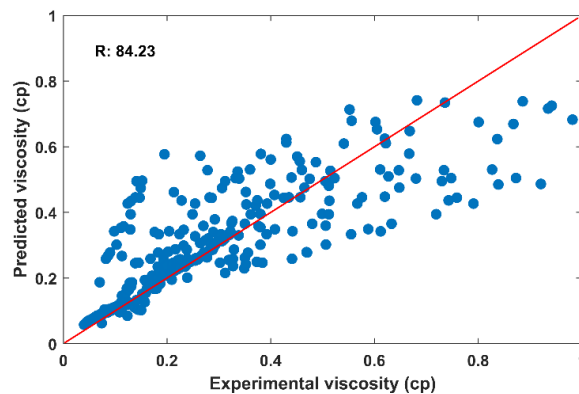
462 Once the model has reached satisfactory accuracy or the model is converged, the
463 training will stop. The performance of the ANN trained model for the training stage is
464 presented in Fig. (9) and Fig. (10).

465 Fig. (7) and Fig. (11) depict the performance of the developed LSSVM and ANN
466 models respectively in predicting the condensate viscosity data. As it can be seen from
467 the aforementioned figures both LSSVM and ANN network predict the independent
468 sample data with satisfactory accuracy. This will be discussed in details in results
469 section.



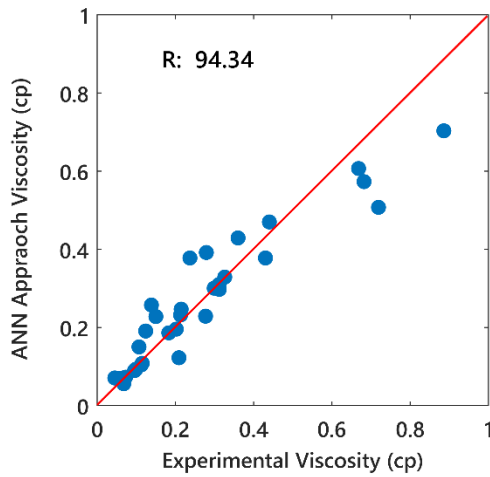
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Fig. 8. Developed ANN model architecture for prediction of condensate liquid viscosity.



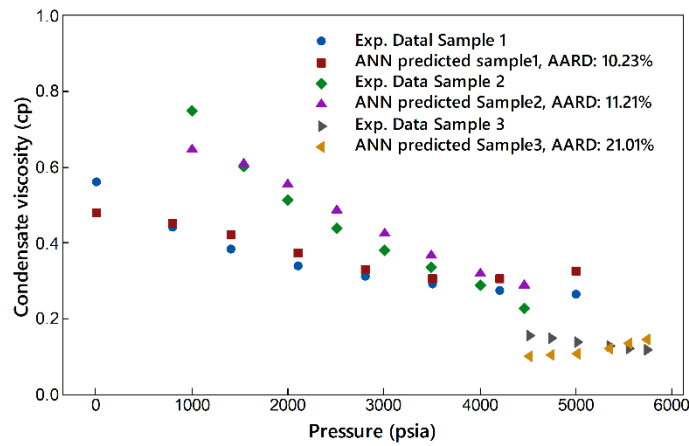
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Fig. 9. Prediction performance of developed ANN network for condensate liquid viscosity in training stage.



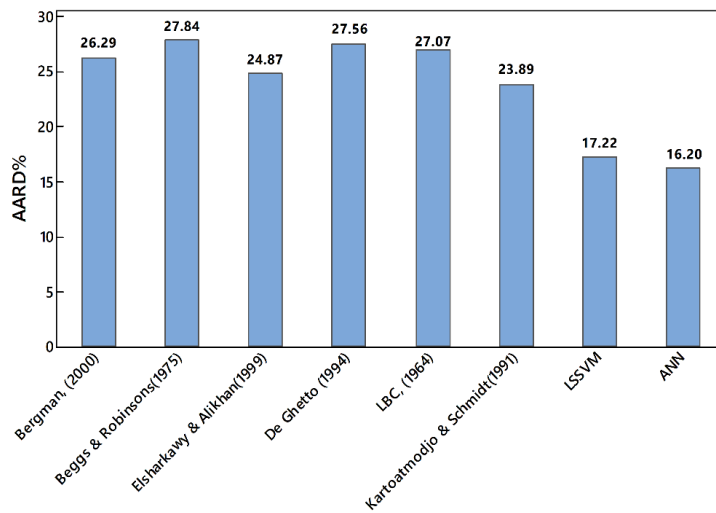
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Fig. 10. Prediction performance of ANN network for condensate liquid viscosity in testing stage.



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Fig. 11. Prediction performance of ANN model for 3 condensate liquid viscosity samples as a function of pressure.



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Fig. 12. Performance comparison of employed methods in this study in predicting experimental condensate liquid viscosity.

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Method	R ²	RMSE	MSE	AARD%
LBC (1964)	0.7241	0.1240	0.0154	27.07
Bergman (2000)	0.7297	0.1236	0.0153	26.29
Beggs and Robinson (1975)	0.7207	0.1244	0.0155	27.84
Elsharkawy and Alikhan (1999)	0.7344	0.1228	0.0151	24.87
De Ghetto (1994)	0.7243	0.1240	0.0154	27.56
Kartoatmodjo and Schmidt (1994)	0.7412	0.1220	0.0149	23.89
LSSVM	0.7738	0.1208	0.0146	17.22
ANN	0.8423	0.1144	0.0131	16.20

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490

Table 6. Statistical parameters of developed models and utilized correlation for prediction of condensate liquid viscosity.

491 3 Results and discussion

492 In this study two intelligent based models of LSSVM and ANN were developed to
493 predict condensate liquid viscosity in depleted gas condensate reservoirs near
494 wellbore region.

495 In first phase of this study prediction performance of LBC compositional model and 5
496 gas-saturated-oil empirical literature correlations were investigated for prediction of
497 condensate viscosity.

498 The prediction performance of the compositional method of LBC, (1964) in predicting
499 condensate liquid viscosity is very poor (Yang et al., 2007) and adjustment of LBC
500 coefficients are usually necessary to match the experimental condensate viscosity
501 (Fevang and Whitson, 1996; Whitson et al., 1999; Yang et al., 2007). The statistical
502 analysis of the results shown in Fig. (1a) confirm the poor performance of
503 compositional based LBC model. The reason for this is might due to the sensitivity of
504 LBC method to mixture density and critical volumes of the heavy components. Hence,
505 in this study the coefficients of the LBC correlation have tuned using least-square
506 approach to match the experimental condensate viscosity data. Fig. (1a) representing
507 the prediction performance of LBC, (1964) with default and adjusted coefficients.

508 The coefficients of five well-known gas-saturated-oil viscosity literature correlations
509 regressed to match the condensate experimental data. The results of these
510 regressions presented in Fig. (1b – f). These empirical correlations are function of
511 dead oil viscosity and solution gas to oil ratio. It should be noted that dead oil viscosity

512 is one of the most “difficult” properties to be estimated by correlations due to its
513 dependency to paraffin, aromatic, naphthalene and asphaltene content (Hemmati-
514 Sarapardeh et al., 2014; Whitson et al., 2000). This might be one of the reasons for
515 poor performance of the default empirical gas-saturated-oil viscosity correlations.
516 Moreover, these correlations were originally developed using crude oil samples, which
517 its properties are fundamentally different from condensate liquid.

518 Poor performance of the published literature correlations in predicting liquid dropout
519 viscosity, motivated to develop two machine leaning models of LSSVM and ANN
520 network in this study. The performance of the newly proposed models LSSVM and
521 ANN were compared against refined previously published correlations through
522 graphical and statistical error analysis. The statistical error analysis results carried out
523 in terms of coefficient of determination (R^2), Root Mean Square Error (RMSE),
524 Average Absolute Relative Deviation (AARD%) and Mean Square Error (MSE). The
525 result of this error analysis is tabulated in the Table 6. Graphical representation of
526 AARD% is also provided in Fig. (12). The results in Table 6 and Fig. (12) indicate ANN
527 model outperforms other methods with AARD of 16.20%, R^2 of 0.8423, RMSE of
528 0.1144 and MSE of 0.0131. ANN followed by LSSVM, Kartoatmodjo and Schmidt
529 (1994), Elsharkawy and Alikhan (1999), Bergman (2000), LBC (1964), De Ghetto et
530 al. (1994) and Beggs and Robinson (1975).

531 The results show using either compositional model of LBC or gas-saturated-oil
532 viscosity literature correlations require significant tuning of coefficients for viscosity
533 prediction of condensate liquid. Whereas developed two intelligent approaches were
534 able to monitor condensate liquid viscosity with appropriate precision and integrity.

535 Non-linear relationship between the available experimental data and the desired
536 outputs created using developed LSSVM model. The optimum values of two important
537 tuning parameters of LSSVM include σ^2 and γ are presented in Table 2. Simulated
538 Annealing optimization (SA) algorithm was applied to achieve these two optimum
539 values.

540 The ability of proposed LSSVM and ANN models for calculating condensate liquid
541 viscosity as a function of changing pressure has been investigated for three gas
542 condensate samples from literature. Fig. (7) and Fig. (11) are demonstrating
543 experimental and predicted condensate liquid viscosities using LSSVM and ANN
544 models respectively. The results show that both models are able to forecast physical
545 trend of experimental condensate viscosity. The accuracy of the models for predicting

546 condensate viscosity of independent samples determined by AARD%. The error
547 analysis show that both models perform well with acceptable level of accuracy.
548 From Fig. (7) and Fig. (11) it is evident that increasing pressure decreases the
549 condensate viscosity. The pressure changes due to depletion in gas condensate
550 reservoirs can have significant effect on condensate viscosity variation near wellbore
551 region (Fevang and Whitson, 1996). This changes can be due to the complex
552 behaviour of gas condensate reservoir below the dew points, which violate
553 thermodynamic laws. The developed LSSVM and ANN models successfully captured
554 the trend of condensate viscosity while utilized correlations were not accurate enough
555 in tracking these changes.
556 Although the prediction performance of the LSSVM was better than published
557 literature correlations, however the error was still high with R^2 of 0.7738 and AARD of
558 17.22%. Therefore, Artificial Neural Network (ANN) method was used aiming for more
559 accurate ML modelling approach. Performance prediction of ANN network is a function
560 of number of neurons that is used in hidden layer (layer 2 in Fig. 8). A trial and error
561 approach were implemented to find the optimum number of neurons. For this study
562 the ANN architecture with five neurons provide the most satisfying results with least
563 RMSE and the highest R^2 .

564 4 Conclusion

565 Better modelling of condensate viscosity is very important for optimizing future
566 recoveries, simulation studies, PVT calculations and accurate production performance
567 forecast of gas condensate reservoirs. Current techniques in literature are providing
568 poor prediction performance of condensate viscosity in near wellbore region. Hence
569 in this study efforts have been made to model this liquid dropout viscosity using
570 numerical artificial intelligence based methods including Least Square Support Vector
571 Machine (LSSVM) and Artificial Neural Network (ANN). Both LSSVM and ANN models
572 are capable of simulating the actual physical trend of the condensate viscosity in gas
573 condensate reservoirs with variation of condensate API gravity, reservoir pressure,
574 reservoir temperature, solution gas to oil ratio (R_s) and gas specific gravity. The
575 advantage of LSSVM is that overfitting is not possible with this method. The robust
576 simulated annealing optimizer implemented to find two important tuning parameters
577 σ^2 and γ and tune LSSVM method.

578 The results of this study indicated that proposed ANN and LSSVM are more robust,
579 efficient and reliable than literature correlations. In ANN approach care should be
580 taken to not over fit the data. This can be done by designing a network with appropriate
581 level of complexity such as number of neurons and hidden layers.
582 Tuning the evolved LSSVM and ANN approach with other optimization method such
583 as Genetic Algorithm (GA) or Coupled Simulated Annealing (CSA) to reduce the error
584 can be considered for future studies.
585 Simplicity and flexibility of the developed model make them a good candidate to
586 determine the viscosity of the condensate liquid in depleted gas condensate
587 reservoirs. The developed models can be implemented in PVT calculation of gas
588 condensate reservoirs for more accurate and reliable modelling of such reservoirs.

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810

Appendix A

Researcher	Number of data points	Fluid sample	Reported error	Advantages and applicability	Disadvantages
Lohrenz-Bary-Clark (1964)	520 data points used to develop oil viscosity and 300 data samples used to develop dense gas viscosity.	Black to highly volatile oil samples. High pressure gas mixture.	16% of average error for oil and 4% of average error for gases.	<ul style="list-style-type: none"> • Can be used to determine both gas and hydrocarbon liquid viscosity. • The LBC correlation uses reservoir fluid composition to determine the fluid viscosity. • Most widely used correlation due to its simplicity and flexibility. • Take account of compositional changes in reservoirs fluids. 	<ul style="list-style-type: none"> • Very sensitive to mixture density and critical volume of heavy components. • Prediction performance of the LBC is poor for oil viscosity. • The tuning of coefficients is usually required to match the experimental data. • The tuning procedure is not straight forward especially for gas condensate fluids. • Heavy tuning of LBC coefficients can cause non-monotonic relations between viscosity and reduced density.
Bergman (2000)	2048 data points from worldwide used to develop gas-saturated-oil viscosity.	Crude oil	9% absolute average error and 11.58% standard deviation.	<ul style="list-style-type: none"> • Ability to predict the wide range of crude oil viscosity 0.125 – 123cp. • Simple and flexible to use. • One of the most accurate method over wide range of conditions (Bergman and Sutton, 2007). 	<ul style="list-style-type: none"> • Limited range of solution gas to oil ratio 5 – 2890scf/STB. • Applicable to crude oil and need tuning for other type of hydrocarbon liquids such as condensate liquid. • Inaccurate dead oil calculation can reduce the accuracy.

Elsharkawy and Alikhan (1999)	254 crude oil samples from Middles East	Crude oil	Average relative error of 2.8% an average absolute error of 18.6%.	<ul style="list-style-type: none"> • Ability to predict the gas-saturated-oil viscosity in lower range 0.05 – 20.89cp. • Less input parameters in computation process (API, reservoir pressure and reservoir temperature). 	<ul style="list-style-type: none"> • Limited applicability to specific geographical region. • Function of dead oil viscosity, which reduce the accuracy. • Require accurate solution gas to oil ratio.
Beggs and Robinson (1975)	2073 data points used in development of correlation	Crude oil	Average error of - 1.83% and standard deviation of 27.25.	<ul style="list-style-type: none"> • Covers good range of solution gas to oil ratio (Rs) of 20 – 2070scf/STB. • Widely used in industry. • Simple calculation procedure. 	<ul style="list-style-type: none"> • Unknown applicability to the specific region. • Unknown ability of predicting different viscosity ranges.
Kartoatmodjo & Schmidt (1991)	5321 crude oil data from Indonesia, America, Middles East & Latin America	Crude oil	Absolute error of 0.08% and 16.08% absolute average deviation	<ul style="list-style-type: none"> • Comprehensive data bank has been used in developing the correlation. • Covers a wide range of viscosity between 0.096 – 586cp. 	<ul style="list-style-type: none"> • Cannot accurately predict the viscosity at low gas-oil ratio when reservoir pressure becomes atmospheric.

De Ghetto et al. (1994)	195 oil samples from Mediterranean Basin, Africa, Persian Gulf, North America (3700 data points)	Light crude oil API >31.1	Absolute error of 15.2% and standard deviation of 14.8% for oil with API > 31.1	<ul style="list-style-type: none"> • Developed for light crude oil with API >31.1, which its properties are close to condensate fluid. • Able to predict the gas-saturated-oil viscosity within the range of 0.1 – 120cp. • Simple calculation procedure. 	<ul style="list-style-type: none"> • Function of dead oil viscosity, which is hard to predict accurately. • Poor performance for predicting of gas condensate reservoirs.
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Table A1. Description of utilized empirical gas-saturated-oil correlations.

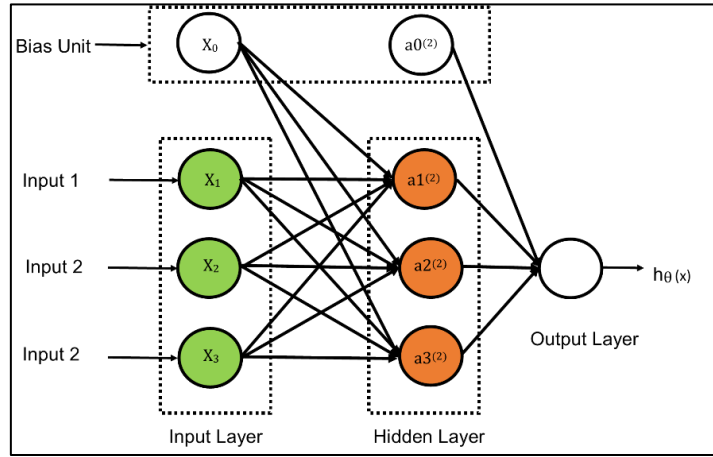
813

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815

816 **Appendix B**

817 This section covers mathematical hypothesis of simple neural network architecture
 818 shown in Fig. B1, where superscripts are values associated with each layer.
 819



820
 821 **Fig. B1.** Schematic illustration of the ANN structure and computational steps to measure any output.

822
 823 In graph shown in Fig. B1:
 824

825 $a_i^{(j)}$ = activation" of unit i in layer j

826 $\theta^{(j)}$ = matrix of weights controlling function mapping from layer j to layer $j+1$

827 In order to calculate each activation function (a) a sigmoid function (g) is multiplied by
 828 sum of linear combination of inputs for each neuron; these inputs include
 829 (x_1, x_2, x_3 and bias unit x_0) in hidden layer. Eq. (B1) to Eq. (B3) are representing the
 830 calculation of the activation functions.

831 Then the output function $h_\theta(x)$ shown in Eq. (B4) is a sigmoid function of sum of each
 832 neuron's weight multiplied by activation function of same neuron in layer 2. The
 833 neurons of the output layer have linear transfer functions.

834
$$a_1^{(2)} = g\left(\theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3\right) \quad (B1)$$

835
$$a_2^{(2)} = g\left(\theta_{20}^{(1)} x_0 + \theta_{21}^{(1)} x_1 + \theta_{22}^{(1)} x_2 + \theta_{23}^{(1)} x_3\right) \quad (B2)$$

836
$$a_3^{(2)} = g\left(\theta_{30}^{(1)} x_0 + \theta_{31}^{(1)} x_1 + \theta_{32}^{(1)} x_2 + \theta_{33}^{(1)} x_3\right) \quad (B3)$$

837
$$h_\theta(x) = a_1^{(3)} = g\left(\theta_{10}^{(2)} a_0^{(2)} + \theta_{11}^{(2)} a_1^{(2)} + \theta_{12}^{(2)} a_2^{(2)} + \theta_{13}^{(2)} a_3^{(2)}\right) \quad (B4)$$

838 In above equation g is a sigmoid type function and can be evaluated from Eq. (B5).

839
$$g(z) = \frac{1}{(1+e^{-z})} \quad (B5)$$

840 To vectorise the above mathematical definition of neural network presented in
 841 Equation (B1) to (B3), the following relations can be defined:

842 If:

$$843 \left\{ \begin{array}{l} \theta_{10}^{(1)} x_0 + \theta_{11}^{(1)} x_1 + \theta_{12}^{(1)} x_2 + \theta_{13}^{(1)} x_3 = Z_1^{(2)} \\ \theta_{20}^{(1)} x_0 + \theta_{21}^{(1)} x_1 + \theta_{22}^{(1)} x_2 + \theta_{23}^{(1)} x_3 = Z_2^{(2)} \\ \theta_{30}^{(1)} x_0 + \theta_{31}^{(1)} x_1 + \theta_{32}^{(1)} x_2 + \theta_{33}^{(1)} x_3 = Z_3^{(2)} \end{array} \right\} \quad (B6)$$

844

845 Substituting Equation (B6) into Eq. (B1) to Eq. (B3) defines the activation functions in
 846 Equation (B7).

$$847 \left\{ \begin{array}{l} a_1^{(2)} = g(Z_1^{(2)}) \\ a_2^{(2)} = g(Z_2^{(2)}) \\ a_3^{(2)} = g(Z_3^{(2)}) \end{array} \right\} \quad (B7)$$

848

849 And If:

$$850 \left\{ \begin{array}{l} x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ Z^{(2)} = \begin{bmatrix} Z_1^{(2)} \\ Z_2^{(2)} \\ Z_3^{(2)} \end{bmatrix} \end{array} \right\} \quad (B8)$$

851 And then input functions substitute with $a^{(1)}$ in layer one:

$$852 \left\{ \begin{array}{l} Z^{(2)} = \theta^{(1)} x = \theta^{(1)} a^{(1)} \\ a^{(2)} = g(Z^{(2)}) \end{array} \right\} \quad (B9)$$

853 In equation B9, $a^{(2)}$ is [3x3] matrix without bias function, and if $a_0^2 = 1$ for bias unit in
 854 layer 2, $Z^{(3)}$ defined as follow:

$$855 Z^{(3)} = \theta^{(2)} a^{(2)} \quad (B10)$$

856 The value of the final function or output layer is sigmoid function of $Z^{(3)}$, as shown in
 857 Eq. (B11).

$$858 h_{\theta}(x) = g(Z^{(3)}) \quad (B11)$$

859

860 The values of x are considered as input of activation function. The above calculation
 861 was carried out and completed in MATLAB, to determine the output values.

862