Title: Multi-scale Pore-network Representation of Heterogeneous Carbonate Rocks

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Key points:

- This work investigates the multi-scale porosity of a carbonate rock.
- We explore representativeness of multi-scale networks by comparing simulated and laboratory measured capillary-pressure vs saturation curves.
- We present a workflow for selecting number and length scales of networks to generate multi-scale pore-networks.

Abstract:

A multi-scale network integration approach introduced by *Jiang et al.* [2013] is used to generate a representative pore-network for a carbonate rock with a pore-size distribution across several orders of magnitude. We predict the macroscopic flow parameters of the rock utilising i) 3D images captured by X-ray computed micro-tomography and ii) pore-network flow simulations. To capture the multi-scale pore-size distribution of the rock we imaged four different rock samples at different resolutions and integrated the data to produce a pore-network model that combines information at several length-scales that cannot be recovered from a single tomographic image. A workflow for selection of the number and length-scale of the required input networks for the network integration process, as well as fine tuning the model parameters is presented. Mercury injection capillary-pressure data were used to evaluate independently the multi-scale networks. We explore single-scale, two-scale, and three-scale network models and discuss their representativeness by comparing simulated capillary-pressure versus saturation curves with laboratory measurements. We demonstrate that for carbonate rocks with wide pore-size distributions, it may be required to integrate networks extracted from two or three discrete tomographic data sets in order to simulate macroscopic flow parameters.

1. Introduction

Pore-scale fluid flow simulation is a useful tool for calculation of the macroscopic transport properties of porous media, specifically for homogeneous systems [*Meakin and Tartakovsky*, 2009; *Joekar-Niasar and Hassanizadeh*, 2012; *Blunt et al.*, 2013; *Wildenschild and Sheppard*, 2013]. For complex porous material realistic representation of the pore structure is essential for successful simulations but has proven to be challenging. High-resolution X-ray computed micro-tomography (μCT) imaging allows detailed study of porous media in 3D (reviews by *Wildenschild and Sheppard* [2013], *Fusseis et al.* [2014], and *Bultreys et al.* [2016]). Reliable application of this technique requires sufficient image quality to allow discrimination of the important features of the sample. Pore-network models (PNM) extracted from binarised μCT images (see review by *Dong* [2007]) are simple representations of pore

50 structures to simulate flow at a reduced computational cost. Although appealing, PNM
51 predictions often do not agree well with the corresponding laboratory measurements,
52 specifically for complex pore systems such as carbonate rocks [*Sorbie and Skauge*, 2012].
53 The trade-off between the sample size and μCT image resolution means that a single μCT
54 volume may not capture the details of a multi-scale pore structure [*Hebert et al.*, 2014].
55 Capturing a representative image of heterogeneous samples with pore-size distribution (PSD)
56 spanning several orders of magnitude is an outstanding challenge with existing μCT
57 instrumentation. A representative elementary volume (REV) is required that is the smallest
58 sample to which measured properties of the bulk sample can be assigned [*Bear*, 1972].
59 Generating two-scale PNM that are representative of complex pore systems has been studied
60 by *Biswal et al.* [2007]; *Biswal et al.* [2009] and *Ghous et al.* [2008], and more recently by
61 *Prodanović et al.* [2015], *Mehmani and Prodanović* [2014], and *Bultreys et al.* [2015].
62 *Jiang et al.* [2013] introduced a network integration method to generate PNMs of arbitrarily
63 large volumes that can incorporate multi-scale pore systems extracted [*Jiang et al.*, 2007]
64 from images captured at different resolutions. A detailed PNM extracted from a high
65 resolution μCT volume (fine-network) is combined with a network extracted from a coarse
66 resolution volume (coarse-network). For details of how the nodes/bonds of the differently
67 scaled models are interconnected see *Jiang et al.* [2013]. The domain of the fine-network is
68 smaller than that of the coarse-network, hence a larger network that is statistically equivalent
69 to the fine-network is generated and then integrated into the coarse-network. The generated
70 multi-scale networks can be heavier than the original fine-network, with an adverse effect on
71 the computational cost of fluid flow simulations. To address this *Jiang et al.* [2013], reduced
72 the number of fine-network elements by a fraction (f_F) such that $0<f_F\leq1$. It is also possible to
73 scale the size of the domain of the two-scale network (nesting-domain) with respect to that of

74 the original coarse image using a scaling factor (δ) that is arbitrary but is restricted by the
75 available computational power for flow simulations.
76 We investigate generation of a representative PNM for a dolomite using *Jiang et al.* [2013]'s
77 approach and a quasi-static pore-network flow simulator [*Ryazanov et al.* [2009]. We
78 evaluate *Jiang et al.* [2013]'s approach by comparing simulated $P_c(S)$ curves with those
79 measured experimentally using mercury injection capillary-pressure (MICP) tests. A
80 workflow is proposed for multi-scale network generation to address (i) the choice of sample
81 size and, hence, image resolution, and (ii) the tuning of model parameters to reproduce the
82 laboratory $P_c(S)$ curves.

83 **2. Imaging and Mercury Porosimetry**

84 A sucrosic dolomite (Silurian Dolomite (SD), Thornton formation, Chicago, Illinois, US)
85 with pore sizes spanning 3 orders of magnitude was used (Figure 1). Laboratory
86 measurements of a 38mm diameter core displayed porosity and permeability of 17% and
87 ~50mD, respectively (Figure 1b).
88 We imaged, using μCT, four samples of diameter D=2, 5, 25 and 38mm, at resolutions of 2.3,
89 12, 21 and 32.8μm, respectively (Figure 1b). Both the image voxel sizes and the corrected
90 image resolutions are reported in Table 1, the correction accounts for the unsharpness caused
91 by the X-ray source spot size [*Feser et al.*, 2008]. The values presented throughout the rest of
92 the manuscript are the corrected image resolution values. As currently worded, this is still a
93 bit unclear. Methods are detailed in the SI and *Pak et al.* [2013]. Sizes were selected because
94 38mm is a standard petrophysical plug, 25mm is the largest sample the MICP could analyse,
95 and 5mm and 2mm provide details of the finer porosity. Figure 1c displays the PSD derived
96 from these four data volumes (Figure 1b) using a sphere fitting method [*Jiang et al.*, 2007].
97 Each volume has captured only a portion of the PSD. In SD recrystallization to dolomite has
98 produced a structure in which the pores are lined by the apices and edges of dolomite rhombs

99 with well sorted crystals sizes. Microscope images (Figure S1.) show that smaller pores are
100 formed in between a few crystals, are uniformly present throughout the sample, suggesting
101 little or no spatial correlation of the micro-porosity. Hence, we uniformly integrate the fine-
102 network in the entire nesting-domain.
103 MICP measurement was performed on three 25 mm diameter SD core plugs imaged at
104 resolution 21μm (Figure S2). MICP provides $P_c(S)$ curves from which pore-throat size
105 distribution can be extracted [*Ritter and Drake*, 1945]. The MICP-based porosity for these
106 samples is 18.03, 16.03 and 18.1%. The image-based porosities at this resolution are 8.05,
107 6.03 and 8.12%, respectively, hence the 21μm represent a coarse-scale resolution.

3. Multi-scale PNM Reconstruction

We explore representing the SD pore structure using multi-scale networks to incorporate features extracted from the four μCT volumes shown in Figure 1b. For 3D renderings of these networks see Figure S3. The physical size of these networks can be found from the image dimensions and voxel resolutions (Table 1). The two coarse-scale networks are globally disconnected while the fine-scale networks are connected with simulated absolute permeability k_{abs}>0mD. The total (and connected) porosity shows an increasing trend with resolution enhancement, as does the calculated permeability. The two fine-networks display calculated permeability values higher than the laboratory measurement i.e. 50 mD. The small samples do not contain a REV and the calculated permeability values reflect the local connectivity of the pore system at these scales. However, even for the 2.3μm resolution network about 11% of the pore volume is disconnected, reflecting that the resolution is insufficient to capture the smallest pore-throats, or that isolated pores exist in the sample.

3.1. Two-scale PNMs

Six possible combinations for generating two-scale networks are 32.8-21μm, 32.8-12μm, 32.8-2.3μm, 21-12μm, 21-2.3μm, and 12-2.3μm, referring to image resolution. Essentially, we need (i) a sufficiently large coarse-network that represents the largest pores, (ii) a fine-network with sufficient resolution that captures the smaller pores, and (iii) a sufficient overlap of the PSD of the two networks to ensure representation of the intermediate-size pores. Hence, 12-2.3μm and 32.8-21μm combinations are not acceptable. In what follows other combination are discussed.

3.1.1. Combination of 21 and 2.3μm PNMs

Figure 2a shows ten networks extracted from sub-domains (δ=0.1-1) of the 21μm coarse-network. At δ=0.4 the domain is sufficiently large to accommodate a cluster of large pores,

hence the network porosity displays a sharp increase, Figure 2b. At δ=0.5 the domain accommodates a connected cluster of pores, therefore, k_{abs}>0. However, as the sub-domain grows (δ>0.5) the networks become again disconnected, suggesting that only a locally connected cluster is captured at δ=0.5 which is not representative of the connectivity of larger pores at this scale.

Figure 2c displays the sensitivity of these two-scale networks to f_F. Porosity increases linearly with the inclusion of increasing fractions of the fine-network, while the permeability starts to converge for f_F>0.05. Thus even a reduced version of a fine-network can provide the required connectivity to two-scale network.

The calculated porosity and permeability display convergence at δ>0.6 (i.e. total porosity ~6% and k_{abs}=0). Figure 2d, however, shows that the $P_c(S)$ curves, calculated using these two-scale PNMs (f_F=0.01, 0.6<δ<1), do not converge. The curves display jumps at different saturations depending on the nesting-domain size. Examination of Figure 2a reveals that the coarse-network contains two connected clusters of elements (pores and nodes) that are disconnected from each other. Network integration provides connectivity between these clusters through elements of the fine-network. Cluster 1 is connected to the inlet and can be invaded by the injected phase directly. Cluster 2 is only accessible through cluster 1 using the fine-network connections. The jump occurs at saturation S^* that is equal to the ratio of the volume of cluster 1 to the total volume of the connected pore space.

As δ increases, cluster 2 grows more significantly compared to cluster 1, hence S^*approaches 0.8. Hence, the selected domain from the 21μm resolution image does not contain a REV. However, the MICP core plugs were approximately 60% larger than the box selected for pore-network simulations, and the MICP-driven $P_c(S)$ curves for three 25mm diameter SD plugs are very close (Figure S2a), this suggests that the MICP plugs themselves are representative for this rock.

3.1.2. Combination of 32.8 and 2.3μm PNMs

For this combination the two-scale network porosity converges for δ>0.6 (Figure 3a) while k_{abs}=0 for all sub-domains. Further, the simulated $P_c(S)$ curves for this combination (f_F=0.01) also converge for δ>0.6 (Figure 3b). This indicates that sub-domains of the 38.2μm image with δ>0.6 contain a REV of SD. The $P_c(S)$ curves, however, show a jump at S_{air}~0.8 causing an overestimation of the experimental capillary-pressure values for S_{air}<0.8. The PSD of the 2.3μm network (Figure 1b) shows a peak at ~6μm, while pores larger than 10μm display less than 20% probability (i.e. volume fraction). Since the coarse-network is globally disconnected, the two-scale networks are connected only through fine-network elements. The initial displacement (S_{air}>0.8) corresponds to the pores accessible from the inlet. At S_{air}~0.8 the pressure needs to increase sufficiently before elements from the fine-network can be invaded. For the converged set of $P_c(S)$ curves, on Figure 3b, the f_F is equal to 0.01. Including a larger f_F suppresses the jump in the capillary-pressure curve but does not totally remove it. The $P_c(S)$ curve for the δ=0.6 and f_F=0.04 network shows a small jump at S_{air}=0.8 from ~21kP to ~47kP corresponding to invasion of pore-throats with radius of 40μm to 16μm. However, this size range is insufficiently represented in both 32.8 and 2.3μm networks. In particular, a small f_F makes a reduced network less representative of the two ends of the PSD. In practice, the difference in the domain sizes of the coarse and fine-networks causes higher f_F values to make the two-scale network substantially more computationally expensive.

3.1.3. Combination of 32.8 and 12μm PNMs

For this combination the calculated $P_c(S)$ curves show a good match with the laboratory measurements at S_{air}>0.04 (Figure 4). However, the simulations stop at capillary-pressure values around 276kP, due to fine-network resolution limitations. Using a higher resolution

fine-network may not provide significant additional connectivity, but may prove essential for calculation of saturation values.

3.2.Three-scale PNMs

We have demonstrated that a finer length-scale needs to be integrated into the 32.8–12μm combination, or an intermediate length-scale must be added to the 32.8–2.3μm combination. To integrate three PNMs of different length-scales first the two smaller networks are integrated into a two-scale network that is subsequently integrated into the largest scale network. This involves selecting two sets of δ and f_F. As the number of length-scales increases, sensitivity analysis on the effect of δ and f_F becomes increasingly time consuming. Figure 5 compares the closest simulated three-scale network $P_c(S)$ curve with the laboratory measurement. This three-scale PNM is generated by integration of the 2.3 and 12μm networks (δ=1, f_F=0.004) followed by integration of that network with the 32.8μm network (δ=0.7, f_F=0.006). The nesting-domain sizes for the 2.3 and 12μm networks are close, hence selecting δ=1 is feasible. To keep the three-scale network computationally manageable while ensuring representativity, we selected δ=0.7 for the second step.

Sensitivity analysis was performed on the value of f_F within the range of 0.001 to 0.1 for the first step and 0.001 to 0.006 for the second step. The calculated $P_c(S)$ curves show that keeping the first f_F value at 0.004 enables the inclusion of a sufficient number of elements from the 2.3μm network for simulations to predict the higher capillary-pressures while eliminating artificial jumps in capillary-pressure.

We analysed the goodness of fit for the $P_c(S)$ curves for the single (at 2.3μm and 12μm), two-scale, and three-scale PNMs using the Chi-squared measure. Single-scale networks yield the poorest capillary-pressure predictions while the three-scale network and the two-scale (32.8-12μm) show only minor deviations from the laboratory measurements (Figure 5). Chi-squared for the 32.8-12μm network is marginally better that of the 32.8-12-2.3μm network,

since the Chi-squared test only compares model predictions against the laboratory measurements, but does not capture that the model does not predict capillary-pressures at low saturations for the 32.8-12μm combination.

Although the calculated absolute permeability values (Table 2) do not accurately match the experimental observation (50mD), the calculated values are close and within the same order of magnitude. The deviation could stem from (i) the fact that the simulations are conducted on a subsection of the acquired μCT images, (ii) errors introduced in noise filtering and subsequent segmentation steps, and (iii) image properties, in particular, the partial volume effect. This can cause features (e.g. pore-throats) smaller than one voxel to appear as large as a voxel in the segmented images. Larger pore-throats display less resistance to the flow and hence a higher absolute permeability is calculated.

No laboratory *kr(S)* measurements were available, hence simulated *kr(S)* curves for single and multi-scale PNMs are compared in SI. In summary, we show that the parameters involved in the network integration, as well as the number and resolution of the input single-scale networks can have a substantial impact on the shape of the *kr(S)* curves calculated based on the multi-scale PNMs. Future studies could aim to compare the calculated *kr(S)* curves with laboratory measurements to establish the effect of network integration on predicting *kr(S)* curves.

4. Multi-scale Network Generation Workflow

Figure 6 is a practical guide for selection of length-scales for generating a multi-scale PNM from μCT data and is additionally informed by thin-section imaging and MICP techniques. This information assists sample size selection, the choice of length-scales, and resolutions for multi-scale network generation.

5. Summary and Conclusions

A network integration approach introduced by *Jiang et al.* [2013] is used to generate representative models for a dolomite with multi-scale porosity. PNMs extracted from μCT images acquired at different length-scales are integrated into single multi-scale PNMs incorporating all the porosity features that exist in the rock. The main contribution of this paper is to present a workflow for effective selection of appropriate length-scales for generation of the multi-scale network. The choice of images to be combined is informed by MICP throat-size distributions. Additionally, there are two free parameters (δ and f_F) in the model. The integrated networks are used to reproduce $P_c(S)$ curves, evaluated against MICP laboratory measurements. We demonstrate that $P_c(S)$ curves calculated based on the two-scale and three-scale PNMs provide significantly closer comparisons to the laboratory measurements compared to a single-scale network model. Future studies need to validate calculated *kr(S)* curves against experimental measurements to optimise multi-scale network generation for predictions of multiphase fluid behaviour.

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310 Nomenclature Table:

S	saturation
S^*	the saturation at which the jump occurs
S_{air}	air saturation
$\boldsymbol{f_F}$	Fraction of fine network
$\boldsymbol{\delta}$	Nesting-domain fraction
$P_c(S)$	relative permeability
$k_r(S)$	relative permeability
$krnw(S)$	non-wetting phase relative permeability
$krw(S)$	wetting phase relative permeability
k_{abs}	absolute permeability
r	radius
χ^2	Chi-squared

311 Tables:

312 **Table 1: Network parameters for PNM shown in Figure S3.**

Image Resolution (μm), corrected for spot size	Voxel Size (μm)	Domain Dimensions (voxels)	Number of Nodes	Number of Bonds	Connected Porosity %	Total imaged Porosity %	k_{abs} (mD)

32.8	32	881×901×901	10318	12736	0	3.52	0
21	21	821×860×720	19897	24093	0	6.03	0
12	5	500×500×500	3418	4127	6.15	9.23	75.63
2.3	2	570×450×380	1733	2089	12.72	14.32	99.75

Table 2: Network parameters for multi-scale PNMs

Image Resolution (μm)	Number of Nodes	Number of Bonds	Connected Porosity %	Total Porosity %	k_{abs} (mD)	Average coordination number
32.8-12-2.3	13956	22954	3.81	3.96	128.14	3.44
32.8-2.3	439131	534791	3.31	3.36	178.02	3.02
32.8-12	76916	100743	4.28	4.43	90.55	2.94

Figure Captions:

Figure 1: (a) Backscattered SEM image of a polished thin-section of SD in three different magnifications, acquired at SEM facility at Heriot-Watt University (resolution ~0.7μm). The left hand side image represents a tessellated image constructed from 400 fields of view stitched together, (b) Four μCT images of SD rock samples (D=2, 5, 25 and 38mm) captured at respective resolutions of 2.3, 12, 21 and 32.8μm. The scanned samples were not generated sequentially by cutting the same core, (c) PSD derived from the four images shown in (b) using a sphere fitting method [Jiang et al., 2007].

Figure 2: (a) Networks extracted from ten sub-domains with δ ranging from 0.1 to 1 for the coarse-network (resolution 21μm), blue colour shows connected pink is globally disconnected elements, (b) Static properties of the sub-domain networks shown in (a), the blue box corresponds to the jump in the total porosity captured for δ=0.4 while the red box shows the locally connected cluster captured in the sub-domain with δ=0.5, (c) Effect of fraction of fine-network (f_F) on porosity and permeability of two-scale networks with δ=0.7 (see 3D renderings in Figure S4), and (d) Effect of nesting-domain size on the two-scale (21 and 2.3μm, f_F=0.01) $P_c(S)$ curves in comparison with laboratory measurements.

Figure 3: (a) The porosity of the sub-domains of the 32.8μm network as a function of the domain fraction, (b) The $P_c(S)$ curves for the combined 32.8 – 2.3μm networks compared with the MICP laboratory measurements. The 32.8μm network becomes sufficiently representative for δ>0.6 displaying converged porosity and $P_c(S)$ curves. δ: Nesting-domain fraction, f_F: Fraction of fine-network.

Figure 4: The $P_c(S)$ curves for the two-scale 32.8–12µm (δ=0.7) network for various f_F. The model predictions show a good fit to the experimental data, however, the model does not predict the higher pressure end of the curve.

Figure 5: The $P_c(S)$ curves for the three-scale network compared with the laboratory data, two-scale and single-scale simulations. The plot also shows the Chi-squared (χ^2) measure for these networks. The three-scale network model displays the best fit to the MICP data.

Figure 6: Multi-scale network generation workflow and tuning.

Figure 1. Figure

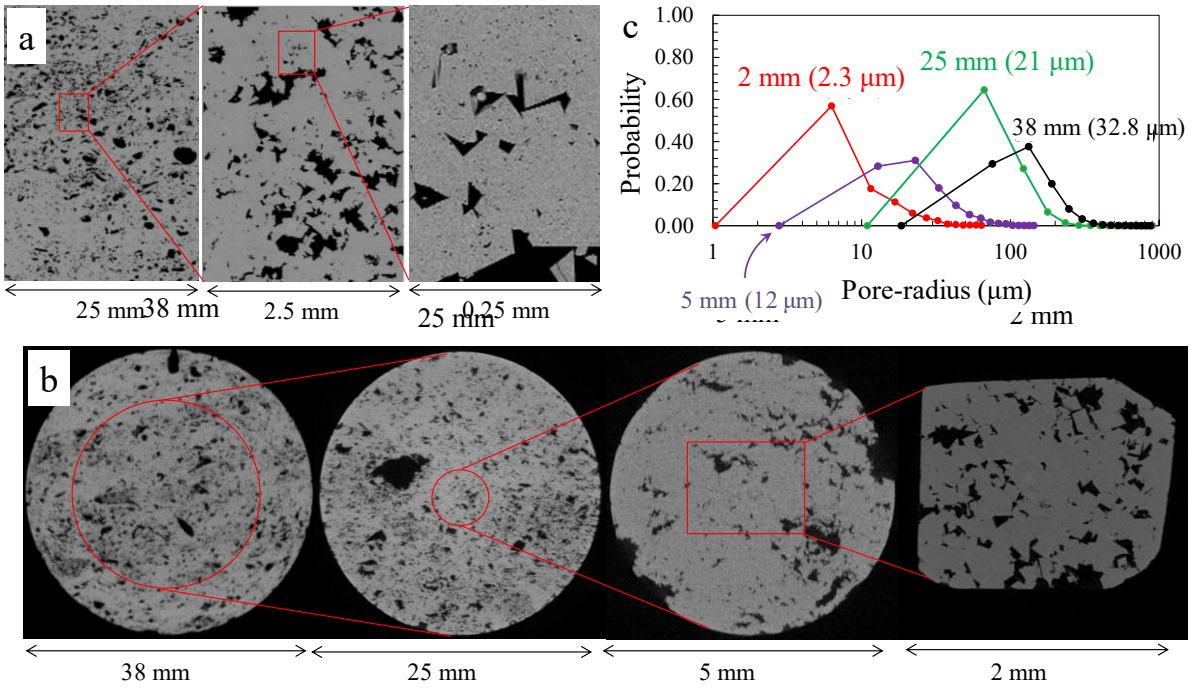

a
25 mm
38 mm
2.5 mm
0.25 mm
25 mm
c
Probability
1.00
0.80
0.60
0.40
0.20
0.00
2 mm (2.3 μm)
25 mm (21 μm)
38 mm (32.8 μm)
5 mm (12 μm)
1
10
100
1000
Pore-radius (μm)
2 mm
b
38 mm
25 mm
5 mm
2 mm

Figure 2. Figure

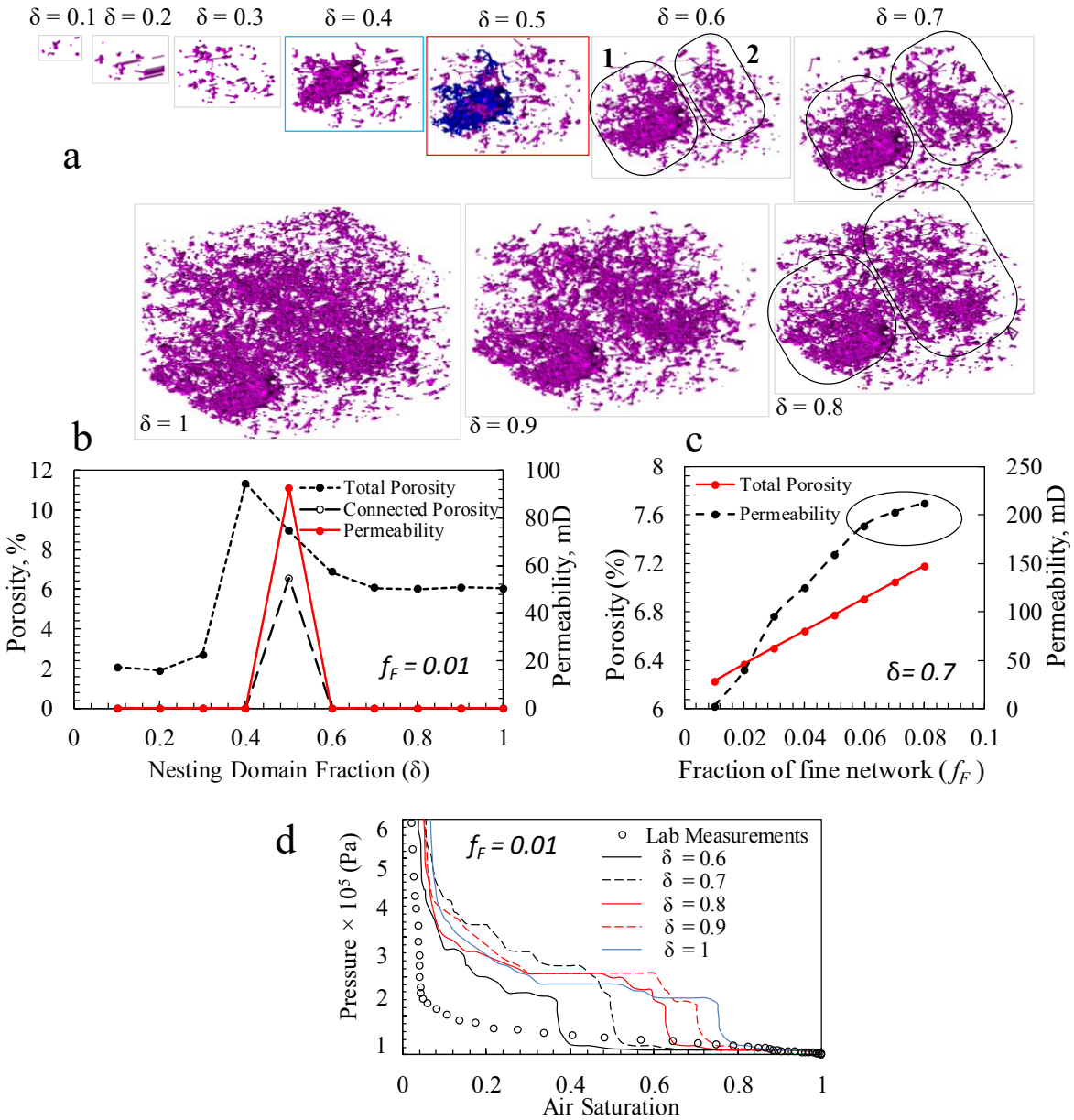

δ = 0.1
δ = 0.2
δ = 0.3
δ = 0.4
δ = 0.5
δ = 0.6
δ = 0.7
1
2
a
δ = 1
δ = 0.9
δ = 0.8
b
Total Porosity
Connected Porosity
Permeability
f_F = 0.01
Porosity, %
Permeability, mD
Nesting Domain Fraction (δ)
c
Total Porosity
Permeability
δ= 0.7
Porosity (%)
Permeability, mD
Fraction of fine network (f_F)
d
f_F = 0.01
Lab Measurements
δ = 0.6
δ = 0.7
δ = 0.8
δ = 0.9
δ = 1
Pressure × 10^5 (Pa)
Air Saturation

Figure 3. Figure

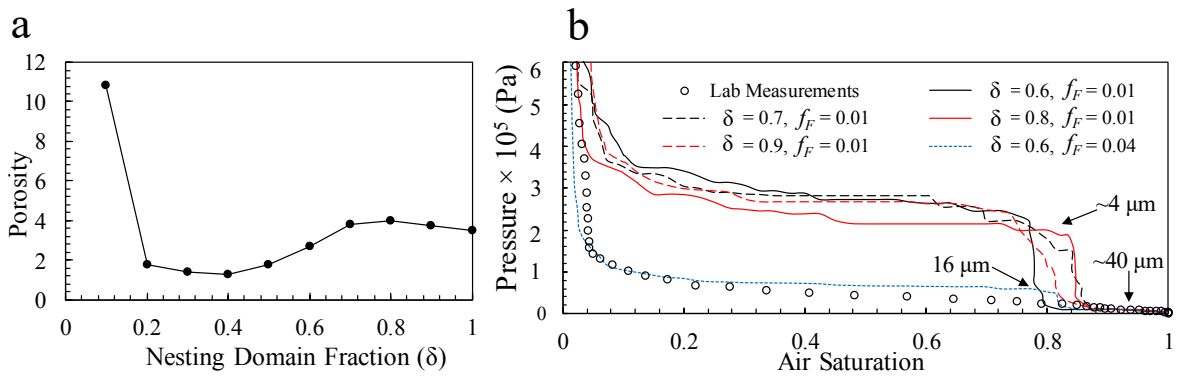

a
Porosity
Nesting Domain Fraction (δ)
b
Pressure × 10^5 (Pa)
Air Saturation
Lab Measurements
δ = 0.7, f_F = 0.01
δ = 0.9, f_F = 0.01
δ = 0.6, f_F = 0.01
δ = 0.8, f_F = 0.01
δ = 0.6, f_F = 0.04
~4 μm
16 μm
~40 μm

Figure 4. Figure

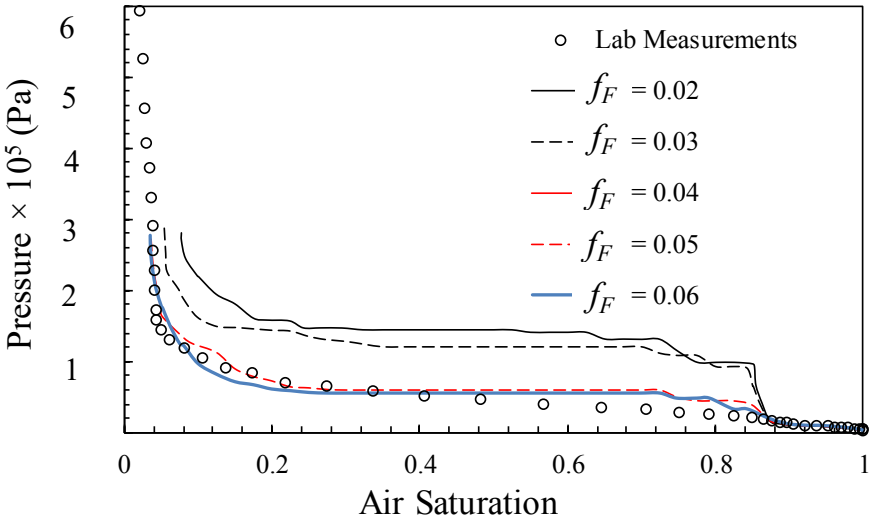

Lab Measurements
f_F = 0.02
f_F = 0.03
f_F = 0.04
f_F = 0.05
f_F = 0.06
Pressure × 10^5 (Pa)
Air Saturation
0
0.2
0.4
0.6
0.8
1
1
2
3
4
5
6

Figure 5. Figure

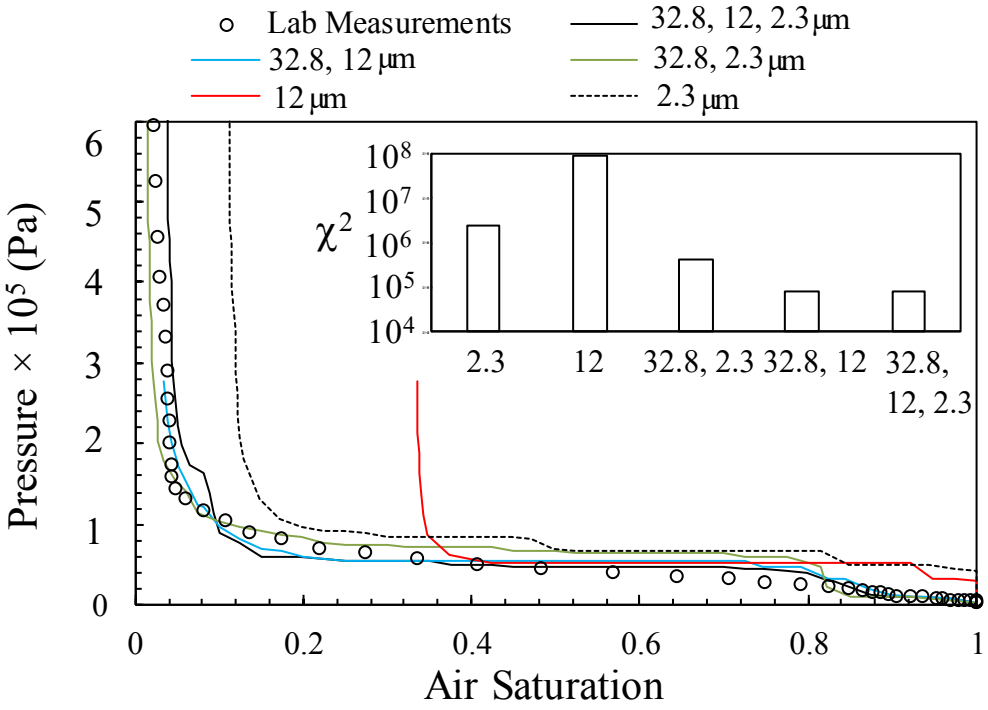
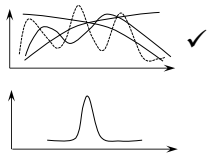

Lab Measurements
32.8, 12 µm
12 µm
32.8, 12, 2.3 µm
32.8, 2.3 µm
2.3 µm
Pressure × 10^5 (Pa)
Air Saturation
χ^2
2.3
12
32.8, 2.3
32.8, 12
32.8, 12, 2.3

Figure 6. Figure

Multi-scale Network Generation Workflow

Does the Rock Have Multi-scale Porosity?

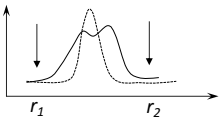
- High resolution thin section imaging
- Pore size distribution analysis



Yes

Pore-space Connectivity Analysis

- Take the largest plug that MICP instrument can take (D ~ 1 inch), run MICP
- Plot the pore-throat size distribution



- Measure the total, effective and isolated rock porosity.

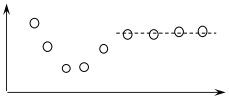
Sample Size Selection

Take a minimum of two samples from the rock such that µCT images can be captured with r_1 and r_2 resolutions, to represent coarse and fine porosity.

Pore network extraction from each scale

REV Analysis

Sensitivity on the nesting domain size: Analyse properties (ϕ, k_{abs}, Pc(S)) of ten sub-domains of the coarse network with δ = 0.1 to 1.

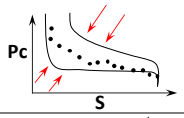


No

A single scale image with enough resolution can capture details of the pore structure.

Multi-scale Network Generation

Tune the fraction of fine network (f_F) to match the experimental $P_c(S)$ curve as close as practically possible.



Yes

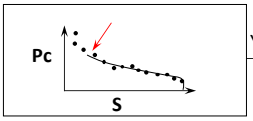
Properties converged?

No

Scan a larger sample to capture at least one REV

Does the Calculated Pc-Sat Curve match the Laboratory Data?

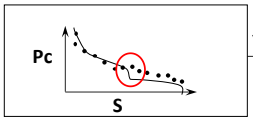
No



Yes

The rock has smaller pores that are not represented by this network.

A higher resolution image is required for the fine network.



Yes

Under-representation of a range middle size pores insufficient overlap range.

A third, middle-scale network is required.

Yes

Proceed to Relative Permeability Calculations and Other Pore-network Flow Simulations

Journal of Water Resources Research

Supporting Information for

Multi-scale Pore Network Representation of Heterogeneous Carbonate Rocks

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Contents of this file

- Figures S1 to S5
- Methodological details
- A comparison of simulated *kr(S)* curves for single or multi-scale pore-networks.

Introduction

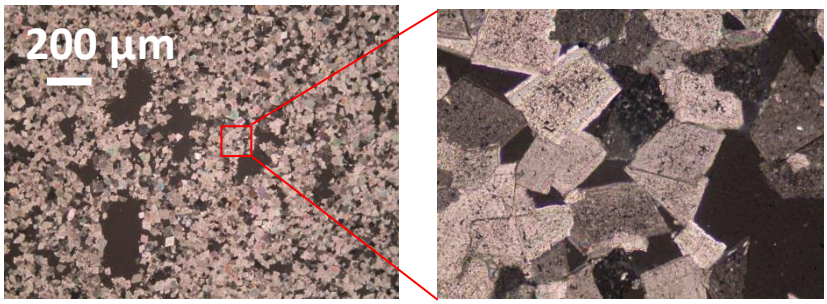

Figure S1: Microscope images for SD showing well sorted crystal sizes and uniform spread of the finer porosity in the rock.

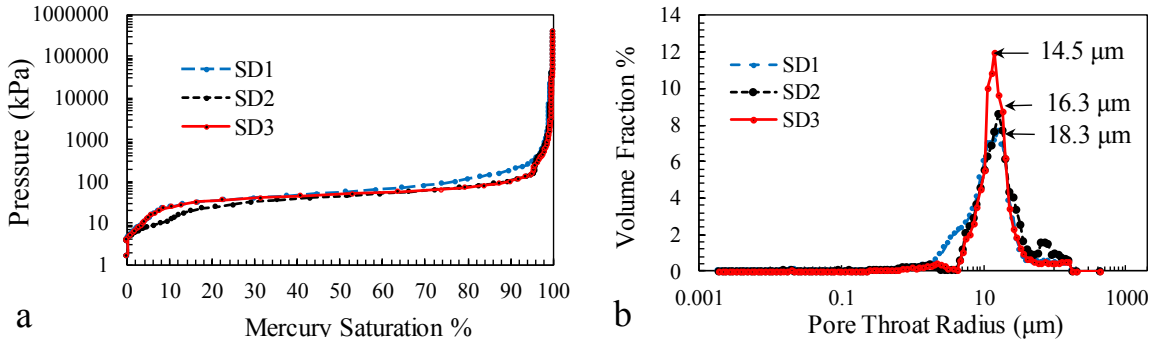

Figure S2: (a) $P_c(S)$ curve for three SD plugs (D = 25 mm) from which pore-throat size distributions (b) are extracted .

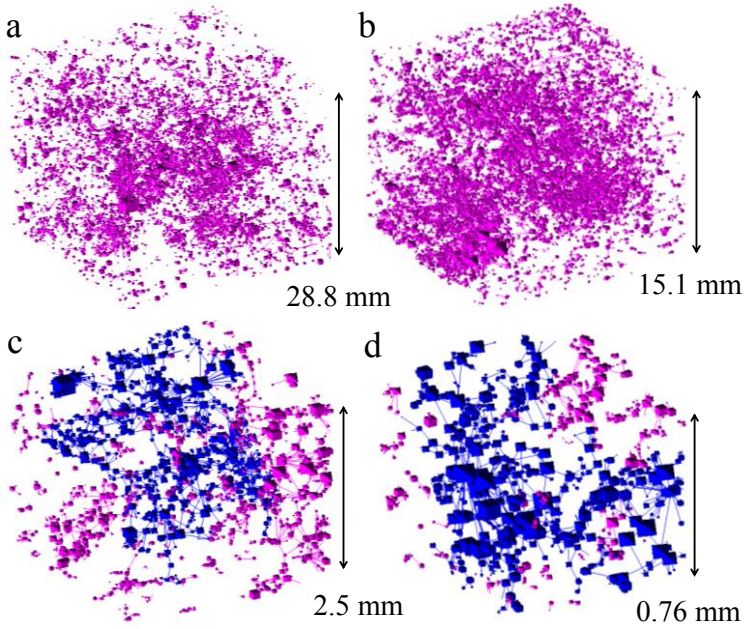

Figure S3: Pore-networks extracted from the four µCT images of the SD samples shown in Figure 3c. Pink and blue indicate the disconnected and connected elements, respectively. See Table 1 for more properties of these four networks.

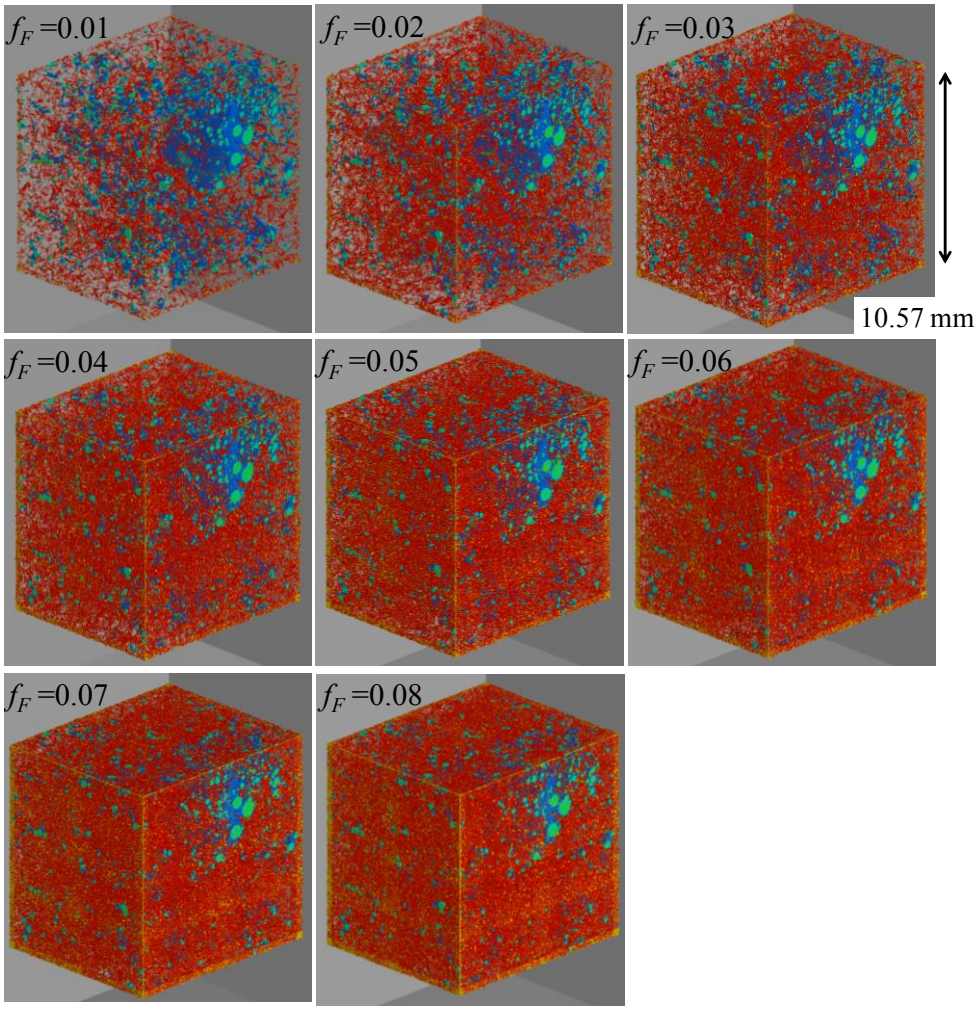

Figure S4: 3D renderings of the two-scale networks with δ = 0.7 and 0.01 ≤ f_F ≤ 0.08. Blue/green shows the coarse-network elements and red shows the fine-network elements.

Methodological Details

X-ray μCT imaging was performed using a μCT instrument built in-house at the University of Edinburgh with a cone-beam configuration. Data were reconstructed using Octopus v 8.5 and processed using Avizo-Fire (v8). Images were filtered using an anisotropic diffusion filter, and segmented using the watershed segmentation method.

Simulation of Relative Permeability Curves

Comparison of experimental *kr(S)* curves and model predictions for complex (multi-scale) pore structures has not been addressed in many publications (as an example see *Bultreys et al. [2015]*) mainly because of (i) limited availability of experimental data, and (ii) the challenging nature of generating pore-networks that are representative of complex porous media at the scale at which the experiment was performed. What is most commonly reported is the comparison

of simulated *kr(S)* curves for single or multi-scale pore-networks with a range of the model inputs. For instance, *Prodanović et al. [2015]*. compared the *kr(S)* curves calculated based on single-scale and two-scale pore-network models and explained the curves display a shift in cross-over point saturation as a result of including micro-porosity. No laboratory *kr(S)* measurements were available to this research. Therefore, only calculated *kr(S)* curves based on two-scale and three-scale networks are compared here, (Figure S5). The *kr(S)* curves are calculated for a drainage process, here we use mercury invading a fully air saturated network. The curve calculated for the mercury/air system can be modified to generate oil/brine curves, by accounting for the difference in the interfacial tension and contact angles.

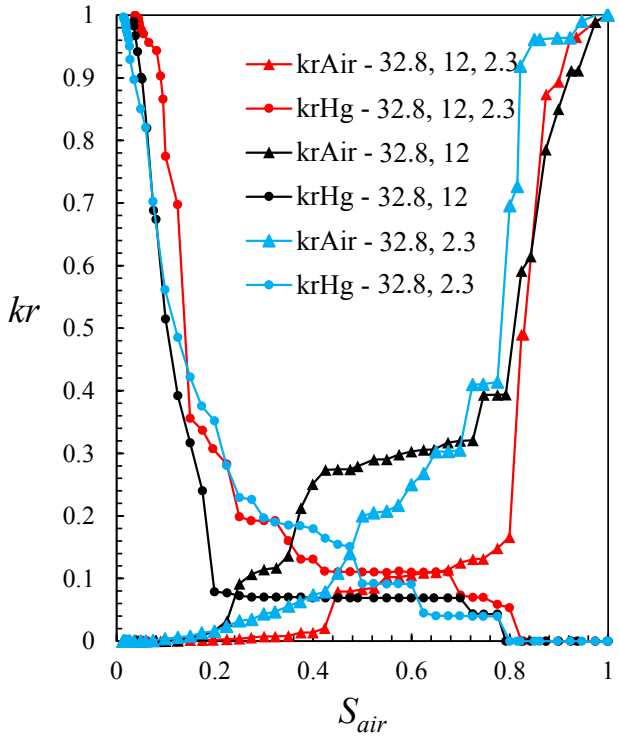

Figure S5: *kr(S)* curves are calculated for a drainage process, mercury invading a fully air saturated network.

One important parameter to study on relative permeability curves is the saturation where the non-wetting phase (here mercury) breaks through, i.e. the saturation where *krnw(S)* > 0. The two-scale network models appear to have very close breakthrough saturations (S_{air} ~ 0.8) while the three-scale network displays a slightly earlier breakthrough (S_{air} ~ 0.82). This is probably due to the higher average coordination number (3.44) of the three-scale network compared to that of the two-scale network (~3) (Table 2). Coordination number is defined as the number of pore-throats connected to each pore. The irreducible air saturation mostly reflects the proportion of the isolated pores present in each network, which are small

This work suggests that the parameters involved in the network integration, as well as the number and resolution of the input single-scale networks, can have a substantial impact on the shape of the *kr(S)* curves calculated based on the multi-scale network models. As suggested by *Prodanović et al. [2015]* the saturation of the cross-over point displays considerable shifts for different multi-scale pore-networks discussed in this study.