Load Forecasting and Dispatch Optimisation for Decentralised Co-generation Plant with Dual Energy Storage

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Abstract

Environmental concerns combined with the liberalisation of the energy markets has led to the emergence of small to medium-scale decentralised generation equipment embedded within transmission and distribution networks. Commonly, such plant is operated by small to medium private enterprises and dispatched independently from centralised resources. The liberalisation of energy markets has also brought about the rise of variable wholesale electricity markets, in the form of the spot (day-ahead) market and the balancing (intra-day) markets across the EU and beyond. As such, there is much interest in how decentralised generation equipment can be most profitably operated in this context. This paper focuses on short-term forecasting of both heat and electrical loads, along with unit commitment scheduling and economic dispatch optimisation, for a small/medium scale decentralised combined heat and power (CHP) plant. In the work presented the plant is assumed to be equipped with local heat and electricity storage and operating in the presence of fluctuating wholesale energy prices and local loads. The approach adopted builds on recent research employing Mixed Integer Linear Programming (MILP) models and non-linear boiler efficiency curves, and extends this work into a rolling horizon context. Results are presented which demonstrate the efficiency of the proposed approach and investigate the sensitivity of the results with respect to CHP model accuracy and load prediction accuracy. The results indicated that profit is much more sensitive to the accuracy of load predictions than indicated by previous work in the area. The findings also challenge those of recent work in the field, which suggest that a strategy of interacting with the spot (day-ahead) market only is the most profitable for small/medium scale decentralised energy producers. The results presented in this paper indicate that when load prediction inaccuracies are also considered in the CHP optimisation framework, a strategy interacting with both the spot (day-ahead) market and the balancing (intra-day) market is significantly more profitable than a strategy interacting with the spot market only.

Keywords: Short term heat and electricity load forecasting; Decentralised Combined Heat and Power (CHP) plant optimisation; Economic Dispatch; Unit Commitment; Convex optimisation; Mixed Integer Linear Programming (MILP).

Nomenclature

Indices

Name	Description (units)
t	Discrete-time step index (hours)
i	Index of slack variables (-)
j	Index of co-efficient in polynomials (-)
k	Step-ahead index for predictions (-)

Parameters

Name	Description (units)
λ	Forgetting factor of recursive least-squares identifier (-)
A_B	Co-efficient of boiler efficiency function (-)
B_B	Co-efficient of boiler efficiency function (-)
C_B	Co-efficient of boiler efficiency function (-)

A_E	Autoregressive polynomial for electricity demand prediction (-)
B_E	Exogenous input polynomial for electricity demand prediction (-)
C_E	Moving average polynomial for electricity demand prediction (-)
A_H	Autoregressive polynomial for heat demand prediction (-)
B_H	Exogenous input polynomial for heat demand prediction (-)
C_H	Moving average polynomial for heat demand prediction (-)
ah_j	Coefficient j of polynomial A_H (-)
bh_j	Coefficient j of polynomial B_H (-)
ch_j	Coefficient j of polynomial C_H (-)
ae_j	Coefficient j of polynomial A_E (-)
be_j	Coefficient j of polynomial B_E (-)
ce_j	Coefficient j of polynomial C_E (-)
η_E	Electrical efficiency of CHP plant (-)
η_H	Thermal efficiency of CHP plant (-)
β_{Max}	Maximum heat-to power ratio (-)
β_{Min}	Minimum heat-to power ratio (-)
$C_{E\alpha}$	Equivalent electrical store holding efficiency (-)
$C_{E\beta}$	Equivalent electrical store holding efficiency (-)
C_F	CHP plant unit fuelling cost (€ per unit of fuel/hour)
C_{Fmin}	Minimum fuelling constraint for CHP plant (€ per unit of fuel/hour)
$C_{H\alpha}$	Thermal store hourly holding efficiency (-)
$C_{H\beta}$	Thermal store conversion efficiency (-)
C_{Off}	Cost savings per time step when CHP plant is switched off (€)
C_{SS}	Startup/shutdown costs of the CHP plant (€)
C	Unit commitment horizon (time steps)
fa_j	Coefficient fa of cost function affine segment j (-)
fb_j	Coefficient fb of cost function affine segment j (-)
H	Dispatch horizon (time steps)
M	Prediction model horizon (time steps)
PQ_{Max}	Maximum limit of combined power (electrical and thermal) generated by CHP plant (kW)

Variables

Name	**Description (units)**
$\beta(t)$	Heat-to power ratio at time step t (-)
$\Delta C_E(t)$	Charge/discharge power of equivalent electrical energy store at time step t (kW)
$\Delta C_H(t)$	Charge/discharge power of thermal energy store at time step t (kW)
$Q(t)$	Thermal power generated by CHP plant at time step t (kW)
$C_E(t)$	Equivalent amount of electrical energy held in storage at time step t (kWh)
$C_H(t)$	Thermal energy held in storage at time step t (kWh)
$C_{HB}(t)$	Price for buying thermal energy at time step t (€ per kWh)
$C_{HS}(t)$	Price for selling thermal energy at time step t (€ per kWh)
$C_{EB}(t)$	Price for buying electrical energy at time step t (€ per kWh)
$C_{ES}(t)$	Price for selling electrical energy at time step t (€ per kWh)
$D_E(t)$	Measured electrical demand at time step t (kW)
$\hat{D}_E(t+k\,|\,t)$	k–step ahead prediction of electrical demand made at time step t (kW)
$D_H(t)$	Measured thermal demand at time step t (kW)
$\hat{D}_H(t+k\,|\,t)$	k–step ahead prediction of thermal demand made at time step t (kW)
$e_H(t)$	Value of white noise sequence in heat demand prediction model at time step t
$e_E(t)$	Value of white noise sequence in electricity demand prediction model at time step t
$I(t)$	Plant On/Off indicator variable at time step t (-)
$J(t)$	Weighted sum of quadratic electricity and thermal prediction errors up to time step t (-)

$P(t)$	Electrical power generated by CHP plant at time step t (kW)
$PQ(t)$	Combined power (electrical and thermal) generated by CHP plant at time step t (kW)
$T(t)$	Average ambient temperature at time step t (°C)
$\hat{T}(t+k\mid t)$	k–step ahead prediction of ambient temperature made at time step t (°C)
$X_{EB}(t)$	Amount of electrical energy bought at time step t (kWh)
$X_{ES}(t)$	Amount of electrical energy sold at time step t (kWh)
$X_{HB}(t)$	Amount of thermal energy bought at time step t (kWh)
$X_{HS}(t)$	Amount of thermal energy sold at time step t (kWh)
$z_i(t)$	Slack variable i at time step t (-)

Functions

Name	Description (units)
$\eta_B(L)$	Boiler efficiency (%) at load L (%)
$F(PQ)$	Approximate fuelling cost function for combined CHP plant output level PQ

1 Introduction

1.1 Context

Traditionally, for economic and safety reasons, the two most commonly consumed worldwide forms of energy – heat and electricity – have been generated by large fossil-fuelled generators and transported to consumers via one-way transmission and distribution networks (typically through hot water or steam pipe work and copper wires) [1]. Typically generation has been distributed over several large generating stations operating in parallel, with transmission interconnections possibly spanning several countries. For electrical grids with these generators and interconnections have been under the control of a small number of public and private bodies [1, 2]. However the liberalisation of the energy markets - combined with environmental concerns and the need for a low-carbon economy - has forced a rethink in the way that energy is generated and distributed to consumers [2]. In particular, the emergence of small and medium scale generation equipment (typically driven by renewable or alternative forms of energy conversion) embedded within transmission and distribution networks is becoming increasingly commonplace. In addition, technological improvements to Energy Storage Systems (ESSs) are enabling an increase in their use and capacity. ESSs provide an effective means to help supply meet demand with unpredictable daily and seasonal variations, and offers additional energy arbitrage opportunities: buying or generating energy when it is comparatively inexpensive, and reselling it at a later time at a higher price [3, 4].

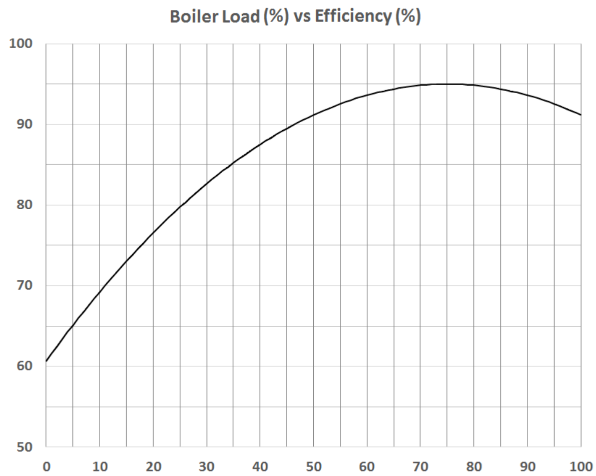
In the above context this paper presents a timely and novel approach to the repetitive cost-optimal balancing of supply with forecasted demand for a decentralised small/medium-scale Combined Heat and Power (CHP) cogeneration plant (typically of rating 10 ~ 50 MW [5]), along with experiments to explore the sensitivity of key configuration parameters on the achievable economic costs.

1.2 Technology Overview

Cogeneration systems can be broadly defined as the coincident (parallel) generation of multiple forms of energy from a single fuel source: for CHP, the energy produced is the combined production of electric power and useable heat [7]. Cogeneration using CHP is an increasingly important component of energy production technology in Europe and other continents [7, 8]. Combined generation via CHP is highly efficient (typically ≅ 80%) when compared to a traditional power-only steam generator (typically ≅ 35% - 40%), and plant may be fired using a variety of sustainable low-carbon fuels, such as biomass [7].

A CHP plant is normally one of two distinct types of construction: a backpressure unit or extraction-type unit [10, 16, 22, 23]. The former typically has a fixed ratio β of thermal power generation (Q) to electrical power generation (P), and operates by reclaiming heat from the steam exiting the turbine. The heat/power ratio β is approximately constant over the majority of its working operating range. The latter, however, allows some independence in the levels of heat and electricity generated by allowing a varying level of high-pressure steam extraction. This gives a variable heat/power ratio which is adjustable over a prescribed operating range. For the large majority of CHP units the allowed operating region is a convex polytope, and this can be captured in a model using inequalities connecting its extreme points [16, 22].

A variety of models including linear, mixed-integer linear and non-linear have been developed to model the economic costs for short-term optimisation of CHP plant [10, 16]. Although non-linear and mixed-integer models give more accurate results, extensive investigation has found that the optimality gap between the two is very small, typically ≤ 3% in representative cases [10]. Linear models – whilst producing less accurate solutions than fully non-linear models – are more computationally efficient and less sensitive to input parameter changes [10]. The loss of accuracy is quite small and mainly occurs due to poor modelling of efficiency loss in the low-load region of boiler operation, typically < 30% full load. A typical efficiency curve for a medium-scale CHP plant (reproduced from [10]) is shown in Figure 1 (left). The relationship is concave, increasing to a maximum efficiency of typically 90% – 95%. From Figure 1 (right), it can be seen that this produces a sigmoidal relationship between the required input fuelling rate (and hence the fuelling costs) and the load, both shown as a percentage. The upper part of the relationship is convex, while the lower part of the relationship (for loads less than ≈ 30%) is concave.



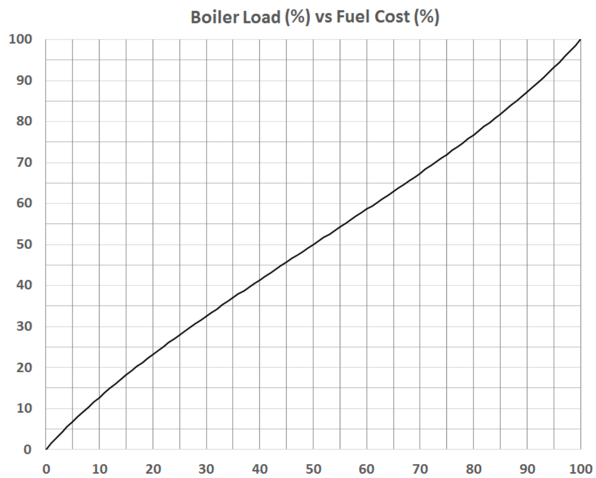


Figure 1. *(Left): Boiler efficiency as a function of load, (Right): required fuelling as a function of boiler load in black*

The relationship between load and fuelling cost is sometimes modelled as a single linear function with the efficiency of the boiler assumed constant over the load range [10]. With such a simplified model, the plant is then assumed to be operable efficiently at very low load which is not the case in reality; minimum load constraints can be employed to enforce operation in the efficient boiler regions, but the CHP plant is then forced to operate continuously in the model regardless of the need for power. Both assumptions lead to inaccuracies [10].

In reality, there are numerous operational and legislative reasons why CHP boiler units are constrained to operate with some minimum fuelling or loading constraints. This can be for operational and safety reasons, such as the need to maintain combustion of the fuel bed and ensure adequate circulation of steam and water in the boiler drum [10, 24]. Most boilers for the size of plant considered have an operational lower load of ≈ 20% - 30% full load [5, 10]. There are also legislative constraints, such as the need to maintain flue gas temperatures at minimum levels to comply with emissions regulations. In addition fiscal incentives often require a threshold boiler efficiency level (typically 70%) is achieved as a precursor to acquiring a 'CHP quality index' [25]. For a modern CHP plant with a co-ordinated control system for the boiler, turbine and heat exchanger, operation at low output power and heat levels is nevertheless achievable with a steam bypass valve [24]. A co-ordinated control system can use such a valve to allow some of the generated steam to bypass the turbine/extractor and divert it straight to the condenser (or cooling stack), such that at very low output levels the boiler fuelling is held at the minimum required level.

In the studies presented, it is assumed that the CHP plant is equipped with technology allowing thermal and electrical energy to be stored (directly or indirectly) and a gas fired boiler for peak load handling. As there is no efficient technology allowing the direct storage of electrical energy at the current time, it is assumed that Battery Energy Storage Systems (BESS), Compressed Air Energy Storage (CAES) or flywheels allow indirect storage of electricity, i.e. electrical energy is converted into another form (chemical or mechanical) and stored over short to medium time periods, and later converted back to useable electrical energy [3,4]. The approach adopted in this paper employs a MILP model incorporating a non-linear boiler efficiency curve for the main plant which uses piecewise approximations in a similar manner to the work of Milan [6], extending this work into a rolling horizon context. In the simulations presented it is assumed that the plant is serving a local load (heat and electricity) and operating in the presence of a wholesale de-regulated energy market with fluctuating energy prices.

1.3 Related Work

Rolfsman [9] considered the optimisation of CHP plant with heat storage and secondary boilers in the presence of a deregulated electricity market. In this work, time is divided into discrete 3 hour periods and two optimisation strategies for dealing with unknown market prices are compared to a baseline case in which all prices are assumed to be accurately known. Simulation results obtained over the course of a year indicated the strategy interacting with both the spot (day-ahead) market and the balancing (intra-day) market strategy was significantly less profitable than the strategy interacting with the spot market only, by a value of approximately 1.4 MSEK (≈ €150,000 using recent conversion rates). These findings suggested that additional costs incurred from interaction with the balancing market are enough to render the storage valueless and therefore the latter (simpler) strategy is preferable. The study assumed that both heat and electricity demand are explicitly and accurately known beforehand, and that the CHP plant has a fixed (constant) efficiency at part load. Neither of which are the case in a real world situation.

Ommen et al [10] in 2014 investigated CHP plant non-linearities within the context of a rolling horizon short-term optimisation framework, using representative district heating data from Copenhagen and surrounding district. This work explores cost, optimal solution structure and computation time for simple linear, non-linear and mixed integer models for economic dispatch and unit commitment. The findings suggest that although differences in the structure of the optimal solutions were found (especially related to the part-load operation of generators), cost differences are low, typically < 3%. In the simple linear model, fixed CHP plant efficiency is assumed over the full load range; in the Mixed Integer Linear Programming (MILP) model, a fixed efficiency is considered over a partial load range (≈ 30% - 100%), with the plant not capable of low-load operation and needing to be switched off.

More detailed modelling of non-linear efficiency curves within a MILP framework is considered by Milan et al. [6]. They develop and compare two solver strategies for the resulting model. The focus in this work is on medium-term planning of required plant capacity, as opposed to short-term operational optimisation [6].

Wang et al. [11] consider the economic dispatch optimisation of a CHP and boiler-based district heating system with renewable electricity generation and both heat and electricity storage [11]. The optimisation is formulated as a Linear Programming (LP) model over a planning horizon of one month in hourly steps. Non-linear (convex) characteristics of CHP units are piecewise linear modelled. The length of the planning horizon employed by Wang et al [11] is markedly longer than in other research in the field. This allows both capacity and operations planning to be considered in a single detailed framework. Unit commitment for the CHP plants within this framework is not considered by Wang et al. [11] due to the excessive numbers of binary variables that would be required for the Mixed Integer LP (MILP) model [11]. The LP model employed by Wang et al [11] is utilised in a computational study based upon multiple CHP plant, boilers and solar thermal collectors located in the south of Finland. Again, the results of this study assume both heat and electricity demand is explicitly and accurately known. Nevertheless, the conclusions indicate that amongst other things, heat storage is utilised in proportion to the level of daily load variability.

Gadd and Werner [12] examine daily and seasonal heat load variations in 20 Swedish district heating systems. They found that variations in the daily load are typically 3% – 5% of the average daily heat delivered with significant differences between seasons. This study found peak-to-peak variation and average demand are at their lowest during summer months and at their highest during winter months, although midday drops in demand are largest and most pronounced during autumn and spring months. Taken together with the conclusions of Wang, et al. [11], discussed earlier, this suggests that to eliminate variations, total daily volumes of heat stored would need to be at their highest during the winter months and at their lowest during summer months. Although, the rate of daily heat store loading and unloading will be at its largest during the autumn and winter months.

Gruber [13] describe and test a two-stage receding horizon optimisation problem for building energy management. The focus here is principally upon electrical energy management in the building. The main decision variables considered in the optimisation are controllable loads (both deferrable and proportionally controllable), dispatchable generation, battery storage and grid power exchange. A novel feature of Gruber's [13] work is the separation of the optimisation problem into two distinct phases and timescales to enable consideration of a longer horizon (24 hour) with high temporal resolution (5 minutes). In the first phase a rough-grained optimisation is solved over a 24 hour ahead period using an hourly time resolution. In the second phase, the results of the first step of the first phase are employed to initialise a second fine-grained optimisation over a 1-hour ahead period using a 5-minute time resolution. The first phase is repeated hourly, and the second phase repeated every 5 minutes. Both optimisation problems are formulated as MILPs and the strategy is validated using power-hardware-in-the-loop experiments. The two-stage approach are shown to be an effective means to consider longer optimisation horizons with relatively high temporal resolutions, and the computational overheads are shown to be low (< 1 second per phase) using a modern numerical solver on a modern PC. Gruber [13] states that prediction accuracy has an impact on the quality of the optimisation, but does not provide a solution as to how to obtain the required demand predictions.

Clearly, future load and demand information is not known accurately and any receding horizon optimisation strategy must employ predictions for these unknown quantities. Therefore reliable forecasting is required to provide predictions for the plant heat and electricity load allowing the scheduling and economic dispatch to take place. Previous work on prediction of electricity loads is numerous and a large variety of strategies are reported in the literature. Taylor [14] compares various models and strategies for day ahead prediction of half-hourly electricity loads for both British and French national grids (of typical daily load ≈ 40 GW and ≈ 50 GW) and reports MAPE levels between 1.5% and 6%. There are comparatively much fewer studies related to prediction of district heating loads. Dotauzer [15] reports MAPE figures of 10.90% and 9.85% for predictions of hourly heat loads for two district heating systems (of typical daily Winter load ≈ 700 MW and ≈ 300 MW) in Stockholm, Sweden, and reports that these error levels are commensurate with those obtained using commercial prediction software. Combined prediction of both heat and electricity loads using a unified model structure does not seem to have been considered.

1.4 Contributions

In the light of the previous work discussed above, three novel contributions to the area of study are presented in this paper.

First, mathematical models and supporting software components for load prediction that enable the operator of a CHP plant to automatically generate hourly load predictions of both heat and electricity using a unified model structure with minimum intervention. The load forecasting mechanisms have a linear structure of varying complexity and are adaptive in nature. They employ Exponentially Weighted Extended Recursive Least Squares (EWE-RLS) to obtain minimum variance parameter estimates and track any environmental changes that would otherwise require manual model re-calibration. The models have an adjustable number of parameters represented by a model horizon, allowing the possibility to reduce computational overheads and fine-tune the prediction accuracy. Based upon extensive experiments, it is shown that even when a large number of parameters are included, the overheads are small, and the proposed models seem to be competitive with - and in some cases may even be superior to - existing linear methods and commercial prediction software in terms of prediction accuracy.

Secondly, mathematical models and supporting software components for CHP plant optimisation in a rolling horizon framework. The MILP model incorporates non-linear boiler efficiency curves using piecewise approximations in a similar manner to those used in [6], extending this work into a rolling horizon context. The optimisation model also has an adjustable number of binary variables representing the plant on/off states represented by a unit commitment horizon, allowing the possibility to reduce computational complexity and fine-tune the optimisation overheads while using a longer horizon for economic dispatch. Based upon extensive experiments, the results indicate that the introduction of separate horizons for economic dispatch and unit commitment is more effective in terms of costs and solution structure than considering heuristics over a longer horizon. The suggested approach provides a useful alternative to [13] when considering trade-offs between complexity and horizon length in rolling-horizon energy optimisation problems, and provides a useful contribution to this area. In addition, the results obtained suggest that the optimisation problem is significantly harder to solve during Summer months than in Winter months due to the increased chance of the plant being un-committed. This provides useful addendum to the results of [11] and [12], and suggests that future work aimed at testing the efficiency of CHP optimisation should focus configurations to include plentiful data corresponding to these months.

Thirdly, a sensitivity analysis using a typical configuration of plant is carried out. In this sensitivity analysis, the impact of model accuracy and load predictions on the results of the optimisation strategy are investigated. The results indicated that, although increasing accuracy in the plant model leads to increased profit, diminishing returns are seen; in addition, the latter is much more sensitive to the accuracy of load predictions. In addition, the results also indicated that when load prediction inaccuracies are considered in the CHP optimisation framework, from a financial perspective a strategy interacting with both the spot (day-ahead) market and also the balancing (intra-day) market strategy is significantly better than a strategy interacting with the spot market only, and in fact leads to cost reductions of approximately €70,000 over the course of the year. This differs from conclusions drawn in previous work [9] which considered perfect load predictions, and can be attributed to the fact that shorter-term predictions are inherently more accurate than longer-term predictions. Without interaction in a balancing market, the updated strategy based upon these improved predictions cannot be put into place effectively; as the system state evolves and more reliable or accurate information is revealed or acquired, the results of the rolling horizon optimisation must be acted upon accordingly [16, 17, 18].

The paper concludes that the software components presented, when used in conjunction with other freely-available open-source software (e.g. SCADA database management) and web services (e.g. weather forecast APIs), could enable a plant operator to assemble a fully integrated environment for decentralised CHP plant management and optimisation at very little cost. The findings also suggest that this approach for CHP plant management can reduce the relative cost of acquiring energy which translates into significant financial savings over the course of a year.

The remainder of this paper is structured as follows. Section 2 presents the methodology employed. It describes an overview of the rolling horizon optimisation concept for a decentralised CHP plant, an ICT framework for its implementation, discusses the forecasting methods and models for heat and electricity load prediction, and presents the compact dispatch and unit commitment optimisation models. Section 3 describes a case study consisting of a prototype implementation of the proposed components, their integration into a test platform and an experimental configuration that has been employed along with representative source data to enable detailed investigations of the proposed methodology to take place. Section 4 presents the results obtained from these investigations and provides detailed discussions. Conclusions and a summary are given in Section 5.

2 Methodology

2.1 Rolling Horizon Optimisation for CHP Plant

The emergence of generation equipment embedded within energy transmission and distribution networks has helped to push forward the concept of the smart grid: an energy distribution network that not only allows for the physical transfer of energy but also features heavy automation and ICT support [2]. This ICT infrastructure offers many opportunities for improved monitoring, control and optimisation of energy generation and distribution when compared to a traditional grid. One such improvement is the optimal economic balancing of energy supply with demand, as defined for a CHP plant as in the previous Section. Costs for purchasing and installing equipment such as the CHP plant itself are considered as sunk costs which cannot be optimised. The economic dispatch problem for a decentralised CHP plant must necessarily employ predictions for unknown loads and prices, and must also be reactive in the sense that when the state of the system evolves from one hour to the next and more reliable information is revealed, this information must be employed to update the previous dispatch plan to a new one. As such, the concepts of adaptive control (to track and adapt to changing conditions and supply/demand trends) [17] and receding-horizon predictive control (to re-calculate cost-optimal corrective strategies) [18] can be employed.

It is assumed that time is indexed by the non-negative integer t which counts the number of elapsed time units since system switch-on. The time units will typically have a one-hour, half-hour or quarter hour resolution. For the remainder of this paper an hourly resolution for time units is assumed as this is commonly used in wholesale energy markets. At hour t, a dispatch optimisation problem and (optionally) a related unit commitment problem are to be solved considering a future planning horizon of H hours. In other words, the decisions which are possible for each hour in the range $[t, t+H)$ are optimised using the knowledge available at time t. The optimal decisions for the hour t are then applied. At time $t+1$, the optimisation is then repeated for the time range $[t+1, t+H+1)$ using the knowledge available at time $t+1$, and this process repeats indefinitely.

In a typical short-term energy market, there are three types of prices for the purchase and sale of energy: day-ahead (spot) prices, intra-day (real-time) prices and regulation prices. In this paper, it is assumed that the plant may actively operate in both the day-ahead and intra-day markets and is subject to retrospective regulation charges for deviations from planned commitments. At any point in time, only the intra-day (real-time) prices can be accurately known before market closure, as they are advertised electronically in the exchange. It is assumed that transactions can be taken in this market until some fixed time before actual delivery (typically one time step, but often can be less), but not during the same time step as delivery. It is assumed that spot prices are fixed once the market has closed around 12 hours before first delivery, and that regulation prices are set retrospectively by the area TSO. Price prediction is not the focus of the current paper, and it is assumed that existing forecasting methods may be employed for these spot prices and regulation charges [19, 20]. A generic structure for such a CHP plant optimisation framework is as shown in Figure 2 below:

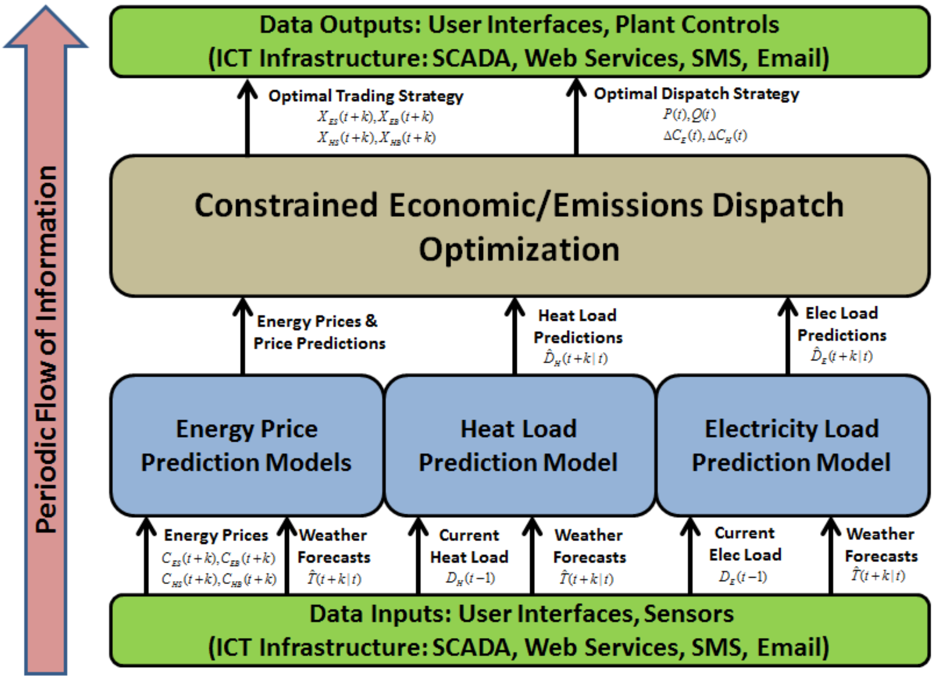


Figure 2: *Prediction and optimisation architecture*

As illustrated in Figure 2, the data flow is periodic; the operations highlighted require data acquisition to enable the prediction phase. In the data acquisition phase, measurements of the current state of the system (e.g. aggregated loads) and related data (e.g. weather forecasts) are first acquired, either locally and/or remotely (e.g. via a SCADA system, Web Services, email or SMS). The current and historical state measurements are then employed to predict the future energy demand evolution across the horizon along with the future wholesale market prices. During the optimisation phase, the optimal corrective strategy to balance supply and demand across the horizon using the available options for generating, storing, buying or selling energy are determined. During the post-processing phase, data extraction and post-processing is carried out and decision support information is then distributed locally and/or remotely (e.g. via a SCADA system, Web Services, email or SMS). Key input and output data flows are indicated in the Figure; descriptions of the data and how they are obtained is contained in subsequent Sections. Descriptions of a suitable underlying PC- and Internet-based ICT platform to support such an optimisation may be found in related work by the authors [8]. The platform essentially comprises of software components to support communications such as SCADA and Web services, along with components for data management/visualisation and a generic optimisation solver support. Aspects of each of these main components are discussed below.

2.1.1 Web Services/HTTP

A Web service is a software system designed to support interoperable machine-to-machine interaction over a network. It provides a network address over the Web, and in its simplest form is a Website providing an Application Programming Interface (API). A Web API is a programmatic request-response interface using a standard method of encoding structured information such as XML. Web services can be accessed using simple HTTP client interfaces, which are supported as standard (along with email) in most operating systems. As shown in Figure 1, the main information requirements for forecasting models (in addition to historic data) are typically weather forecasts. Predictions of hourly temperatures and wind speeds are often the main aspects required: in the absence of specialised weather forecast information, free-to-use Web services for forecasting can be used (e.g. www.yrno.api and similar sites).

2.1.2 SCADA

A SCADA (supervisory control and data acquisition) system is an industrial communication system which provides telemetry and tele-control services, typically between a central Control Centre (CC) and a number of field devices such as remote terminal units (RTUs), Programmable Logic Controllers (PLCs) and other embedded controllers, and also Human Machine Interfaces (HMIs) such as operator displays. In the context of the current paper, the central CC is the main prediction and optimisation platform, and the main RTUs in the field are the local co-ordinated control system of the CHP plant and the energy storage unit local controllers. Given that the CC may be located in offices some distance from the CHP plant itself, a SCADA system is required to provide the connectivity needed to send the dispatch commands from the CC to the plant and to send aggregated loads and other measured data (such as outdoor temperature) and alarms from the plant to the CC.

Although many SCADA protocols have been developed, each with particular advantages and disadvantages with respect to electrical and thermal plant automation, the Modbus protocol is one of the most popular [21]. The Modbus-RTU application protocol was originally designed as a simple, flexible and open Master-Slave protocol that can be used over serial communication links such as RS-232. Due to its popularity and widespread use, more recently it has been mapped for use with the Internet Protocol using a Client-Server model, leading to Modbus TCP/IP. Many local CHP controller and ESS controllers support Modbus-TCP/IP (either directly or through a TCP/IP-RTU gateway). An ICT infrastructure for CHP plant management may make use of one of the many commercial or open-source components which are available for this protocol (e.g. the open source library available from http://libmodbus.org/ or similar) for interaction with the plant and ESSs.

2.1.3 Data Management and Visualisation

Database functionality is required to store and access both historic data associated with plant operation and the configuration settings for the plant and its environment. Since the configuration settings are either static or very infrequently changed, they are best handled using simple storage in a flat-file or structured XML file. The data related to plant operation however, consists of potentially very large arrays of numbers. These arrays are indexed by the time variable t and include hourly energy prices, hourly heat and electricity loads and average hourly temperatures. As such they fall into the category of time series data, and although it can be managed by traditional relational database software, this is not very efficient and a time series database (TSDB) is a preferred (optimised) solution. A TSBD is often a core component of an operational historian, which also features functionality for compressing the data, recording summary statistics over receding timelines and data visualisation. An ICT infrastructure for CHP plant management may make use of one of the many commercial or open-source TSDB/operational historian components and environments which are available (e.g. openTSDB available from http://opentsdb.net/ or similar).

2.1.4 Optimisation Software

In terms of generic support for optimisation, many commercial or open-source optimisation components and environments are available for solving the Linear Programs (LPs), Mixed Integer LPs, Quadratic Programs (QPs) and Mixed Integer QPs that typically arise in power system dispatch problems. In the context of the research presented, the CHP economic dispatch problem is formulated as a MILP. An ICT infrastructure for CHP plant management may make use of one of the many commercial or open-source optimisation components and environments for LP and MILP which are available (e.g. the open source LP Solve available from http://lpsolve.sourceforge.net/5.5/ or similar).

Having outlined the systems architecture in this Section, the following Section describes the adaptive models suitable for load forecasting of both electricity and heat loads and CHP optimisation models for use with such an ICT infrastructure.

2.2 Load Forecasting

Research suggests that for short-term heat and electricity prediction, simple but well-calibrated model structures often perform almost as well as complex models (e.g. see [14, 15]), and strong load correlations with temperature can normally be found. Simpler models require less computational effort and data requirements are easier to manage. A principal goal was therefore to keep the load prediction model structure very simple and to employ temperature as the only fundamental model input. In addition, to remove the need for extensive calibration and re-calibration of the model, a secondary goal was to use recursive adaptive parameter estimation for model parameter identification. The structure of the load prediction model is presented below.

2.2.1 Load prediction models

Let the variable $D_H(t)$ represent the load (demand) for heat (in kW) during time slot t, and the variable $T(t)$ represent the average ambient temperature for the hour starting at time t, where t is a non-negative integer variable. Assuming that the demand has Auto-Regressive Integrated Moving-Average (ARIMA) white-noise 'dynamics' with temperature as an exogenous input, the model structure below is obtained:

$$A_H(z)D_H(t) = B_H(z)T(t) + C_H(z)\frac{e_H(t)}{\Delta} \tag{1}$$

In this model $e_H(t)$ is a zero-mean white noise sequence, z^{-1} is the backshift (delay) operator, $\Delta = 1\text{-}z^{-1}$ (i.e. the differencing operator), $A_H(z)$, $B_H(z)$ and $C_H(z)$ are polynomials in z^{-1} with both A_H and C_H monic. The integrated white sequence implies that the load is effectively experiencing unknown disturbances that are akin to Brownian motion or a random walk. In particular, this provides resilience against non-constant means (which demand data seems to exhibit). Let the variable $D_E(t)$ represent the demand for electricity (in kW) during the time slot t. Then an Equation analogous to (1) may also be written for electricity demand, replacing $D_H(t)$ with $D_E(t)$, $e_H(t)$ with $e_E(t)$ and the polynomials $A_H(z)$, $B_H(z)$ and $C_H(z)$ with polynomials $A_E(z)$, $B_E(z)$ and $C_E(z)$. In a separate study of representative data [8] for hourly heat and electrical loads from Sweden and the UK, plus hourly electricity loads from the US, it was found that the load at hour t was mainly correlated to the previous short term hourly loads, i.e. t-1, t-2, t-3, …, however the correlation weakens over the course of the previous day before increasing once more at hour t-24, i.e. one full day before. In addition, a correlation was also found at t-168, i.e. one full week before. The correlations with the short term load history along with seasonal behaviours are consistent with the findings of previous studies [14]. Although it has been suggested that there may also be monthly and/or yearly seasonal correlations in load data [14], these were omitted from the current models for the sake of simplicity; in addition, as will shortly be described the adaptive parameter estimation employed removes the need for including long term load correlations. In terms of the temperature correlations, it was found that in the heat data, the load at hour t is inversely correlated with the short-term average ambient temperature in the hour t and previous hours, i.e. t-1, t-2, t-3, … , however the correlation weakens for increasing time shifts. For electricity, the situation is similar but correlations are positive. M parameters are included in the model to capture the effects of the short term load history, temperature effects and moving-average errors. Setting $\Delta D_H(t) = D_H(t) - D_H(t\text{-}1)$ and $\Delta T(t) = T(t) - T(t\text{-}1)$ as the first difference of the heat demand and temperature respectively, incorporating the M parameters and seasonal behaviours in the A_H, B_H and C_H polynomials allows (1) to be written in a more convenient incremental form; for example, in the case that $M = 3$, we may write:

$$\begin{aligned}\Delta D_H(t) &= ah_1\Delta D_H(t-1) + ah_2\Delta D_H(t-2) + ah_3\Delta D_H(t-3)\\ &+ ah_{24}\Delta D_H(t-24) + ah_{168}\Delta D_H(t-168)\\ &+ bh_0\Delta T(t) + bh_1\Delta T(t-1) + bh_2\Delta T(t-2)\\ &+ e_H(t) + ch_1 e_H(t-1) + ch_2 e_H(t-1) + ch_3 e_H(t-1)\end{aligned} \tag{2}$$

Where ah_j is the j^{th} coefficient of the A_H polynomial, bh_j is the j^{th} coefficient of the B_H polynomial and ch_j is the j^{th} coefficient of the C_H polynomial. Since the error terms are assumed zero-mean, at time t it holds that $E[e_H(t+1)] = 0$ and as $D_H(t+1) = D_H(t) + \Delta D_H(t+1)$, the one-step head prediction equation for heat demand using all information available up to and including time t becomes:

$$\begin{aligned}\hat{D}_H(t+1\,|\,t) &= D_H(t) + ah_1\Delta D_H(t) + ah_2\Delta D_H(t-1) + ah_3\Delta D_H(t-2)\\ &+ ah_{24}\Delta D_H(t-23) + ah_{168}\Delta D_H(t-167)\\ &+ bh_0\Delta\hat{T}(t+1\,|\,t) + bh_1\Delta T(t) + bh_2\Delta T(t-1)\\ &+ ch_1\hat{e}_H(t) + ch_2\hat{e}_H(t-1) + ch_3\hat{e}_H(t-2)\end{aligned} \tag{3}$$

Where a hat upon a variable indicates its predicted value is to be used. A two-step ahead prediction can be formed from recursion upon (3):

$$\begin{aligned}\hat{D}_H(t+2\,|\,t) = {} & \hat{D}_H(t+1) + ah_1\Delta\hat{D}_H(t+1) + ah_2\Delta D_H(t) + ah_3\Delta D_H(t-1) \\ & + ah_{24}\Delta D_H(t-22) + ah_{168}\Delta D_H(t-166) \\ & + bh_0\Delta\hat{T}(t+2\,|\,t) + bh_1\Delta\hat{T}(t+1\,|\,t) + bh_2\Delta T(t) \\ & + ch_1\hat{e}_H(t-1) + ch_2\hat{e}_H(t-2)\end{aligned} \tag{4}$$

Similarly, k-step ahead predictions across the full horizon are obtained by recursion upon the known and predicted load and load increments obtained up to step k-1, plus the forecasted temperature increments and estimates of the previous errors. In the model, at step t an estimate of the error $e_H(t)$ can be obtained as the difference between the measured and predicted change in load, i.e.:

$$\hat{e}_H(t) = (\Delta D_H(t) - \Delta\hat{D}_H(t\,|\,t)) \tag{5}$$

Prediction Equations of a similar form to (3), (4) and (5) may also be written for the electricity demand, replacing $D_H(t)$ with $D_E(t)$, $e_H(t)$ with $e_E(t)$ and the polynomials $A_H(z)$, $B_H(z)$ and $C_H(z)$ with polynomials $A_E(z)$, $B_E(z)$ and $C_E(z)$ as needed.

2.2.2 Adaptive parameter updating

Since the distribution of $e_H(t)$ and $e_E(t)$ become approximately Gaussian for large sample sizes due to the Central Limit Theorem, unknown model parameters may be estimated on-line using minimum-variance (least squares) techniques. At each time step t, when the actual loads from step t-1 become known, before predictions are formed the parameters are first updated online in an identification step to minimise the following objective function J (the final weighted quadratic error):

$$J(t) = \sum_{i=0}^{t-1} \lambda^{(t-i)} \Big[\big(\Delta D_H(i) - \Delta\hat{D}_H(i)\big)^2 + \big(\Delta D_E(i) - \Delta\hat{D}_E(i)\big)^2 \Big] \tag{6}$$

With $0 < \lambda \leq 1$ acting as an exponentially receding weighting factor known as the forgetting factor. The objective function in (6) contains two separable squared error sums, one related to heat demands and one related to electricity demands; hence the problem may be separated into two distinct weighted least squares problems. The parameter estimation to optimally solve these weighted least squares problems and hence minimize $J(t)$ at each time step can be efficiently carried out using the method of EW-ERLS as described by Astrom and Wittenmark [17], with a complexity that is quadratic in the number of estimated regression parameters. Following the regression updates to estimate the unknown model co-efficients at time t using the available measured data, the predictions for both heat and electricity demands required for optimisation are then carried out recursively as detailed above. The 'model horizon' M has an influence on both the computational complexity and predictive performance of the method. Computational experiments to explore the influence of this configuration parameter are described in a later Section.

2.3 Optimisation Models

2.3.1 CHP plant

In this paper we assume that the heat to power ratio β is adjustable on an hourly basis, and during hourly time step t let $\beta(t)$ denote the employed ratio which is assumed to satisfy the relationship $\beta_{\mathrm{Min}} \leq \beta(t) \leq \beta_{\mathrm{Max}}$ [22]. Let $P(t)$ and $Q(t)$ be the electrical and thermal power outputs during hour t respectively, and let $PQ(t)$ be the combined plant output during hour t. The constant PQ_{Max} represents the maximum output of the plant at full boiler load. In addition to the basic constraints that $P(t) \geq 0$ and $Q(t) \geq 0$, the following two inequalities are sufficient to describe the convex region of plant operation:

$$P(t)\beta_{Min} \leq Q(t) \leq P(t)\beta_{Max} \tag{7}$$
$$P(t)+Q(t) \leq PQ_{Max} \tag{8}$$

For a backpressure unit with fixed β, we may simply set $\beta_{\text{Min}} = \beta_{\text{Max}} = \beta$ in the above. With respect to fuelling costs, it is well known that for a suitable choice of the constants A_B, B_B, and C_B the relationship between boiler load L (expressed as % of full scale) and boiler efficiency $\eta_B(L)$ (expressed as %) is well modelled by a quadratic equation of the form [10, 23]:

$$\eta_B(L) = A_B + B_B L + C_B L^2 \tag{9}$$

As discussed in the introduction, this efficiency curve produces a sigmoidal relationship between the required input fuelling rate (and hence the hourly fuelling costs) and the load. The upper part of the relationship is convex, and since hourly fuelling costs should be minimized may be approximated by piecewise affine functions. In addition, there are numerous reasons why CHP boiler units are constrained to operate with some minimum fuelling or loading constraints, and a modern CHP plant with co-ordinated control system for the boiler, turbine and heat exchanger can operate at low output power and heat levels using a steam bypass valve. As such it is proposed to approximate the hourly fuelling costs as a function of the combined plant output PQ with three piecewise affine functions as follows:

$$F(PQ) = \max\{fa_1 + fb_1 PQ, fa_2 + fb_2 PQ, fa_3 + fb_3 PQ\} \tag{10}$$

Such that $fb_3 \geq fb_2 \geq fb_1$ to ensure convexity. The first affine function has a slope of zero and enforces a minimum fuelling and boiler loading. The second is drawn from the point of maximum efficiency tangential to the lower part of the cost curve, while the third connects the point of maximum efficiency with the point of full load. In Figure 3 below, the convex fuelling costs as shown in Figure 1 (right) are approximated by 3 such piecewise affine sections.

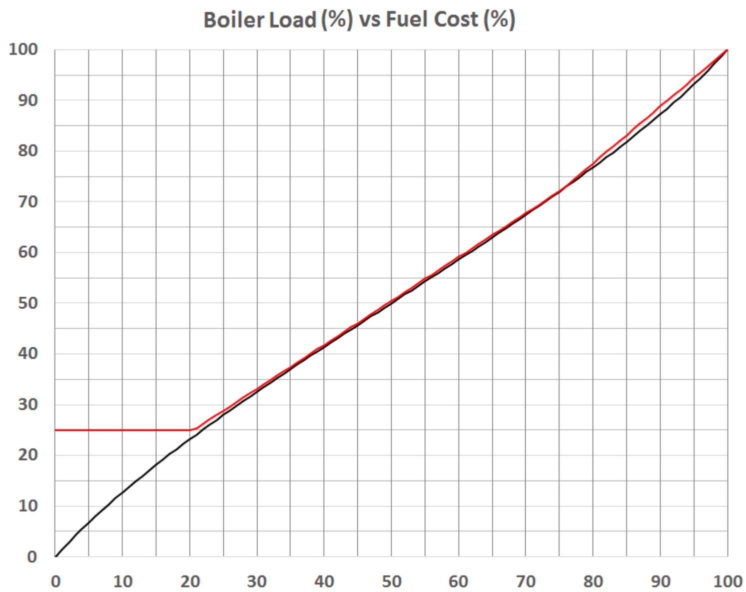


Figure 3: *Piecewise approximation of fuelling costs*

The fuelling function may then be modelled in the LP by using a slack variable $z_1(t)$ plus the following three inequalities for each considered hour t:

$$z_1(t) - fb_j P(t) - fb_j Q(t) \geq fa_j, \quad j = 1,2,3 \tag{11}$$

For the cost curve shown in Figure 2 (right), the resulting approximation using a minimum fuelling constraint of 25% (to ensure boiler load > 20% and efficiency > 75%) is obtained with $\{fa_j, fb_j\}$ values as: {25.00, 0.000}, {7.200, 0.865} and {-12.000, 1.120}. The piecewise approximation has an average absolute error of 1.20 % over the working range of the boiler. Defining the unit hourly fuelling cost as C_F, then the term $z_1(t)\ C_F$ is incorporated into the cost function to represent the hourly economic costs of the CHP plant.

While the operation at low and zero output levels can be modelled, it is relatively inefficient to do so (in both the model and in reality), and it is also beneficial to be able to shut down the plant when it is economically justified. In the optimisation model this allows the 'unit commitment' scheduling problem to be integrated with the CHP economic dispatch problem. To facilitate such operations, the model includes, hourly binary indicator variables $I(t) \in [0, 1]$ with the interpretation that for $I(t)$ = '1', the plant is switched on for hour t and for $I(t)$ = '0' the plant is switched off. By modifying constraint (6) to the following form:

$$PQ(t) - I(t)PQ_{Max} \leq 0 \tag{12}$$

Then both the electrical power and rate of heat transfer are forced to zero when $I(t) = 0$. The latter is ensured as when $I(t) = 0$ the condition $PQ(t) \leq 0$ must hold, with both terms also being non-negative. In terms of operating costs, let the savings that occur when the plant is switched off (e.g. due to reduction in labour cost) be denoted as C_{Off}, and let the minimum fuelling costs be denoted as $C_{Fmin} = C_F\ af_1$. Adding the terms:

$$I(t)(C_{Off} + C_{F\min}) - C_{F\min} \tag{13}$$

Into the cost function ensures that when $I(t)$ = '0' the operating cost is driven to zero, and when $I(t)$ = '1' the cost is equal to C_{Off} plus that incurred for the fuel used. If costs of C_{SS} are incurred every time the plant is shutdown or started up, then since the plant state at the previous time step is represented by $I(t\text{-}1)$, this can be modelled by using the slack variable $z_2(t)$ and by adding the following constraints:

$$z_2(t) - I(t) + I(t-1) \geq 0 \tag{14}$$
$$z_2(t) + I(t) - I(t-1) \geq 0 \tag{15}$$

(14) and (15) ensure that when $I(t) \neq I(t\text{-}1)$, $z_2(t)$ is forced to '1' and zero otherwise; the term $z_2(t)\ C_{SS}$ is then incorporated into the cost function. The final aspects to be considered are those related to hourly constraints on the rate at which the combined power output of the plant can be changed. As the combined output of the plant at time t is given by $PQ(t)$, the hourly rate of change of the combined output of the plant is given by:

$$PQ(t) - PQ(t-1) \tag{16}$$

Assuming that the maximum and minimum rate limits on the combined output are ΔPQ_{Max} and ΔPQ_{Min} respectively, constraints of the form:

$$PQ(t) - PQ(t-1) \leq \Delta PQ_{Max} \tag{17}$$
$$PQ(t) - PQ(t-1) \geq \Delta PQ_{Min} \tag{18}$$

Ensure that the hourly change in output adheres to the physical constraints. The overall model for each hour of CHP plant operation is summarized in Table I below (note that since $PQ(t) = P(t) + Q(t)$, $PQ(t)$ has been eliminated to reduce the number of required constraints and variables). The efficiencies of the turbine / generator and heat exchanger (η_E and η_H respectively) are not explicitly included, and should be added as factors in additional constraints related to energy balances. Before such constraints are considered, models for energy storage are discussed in the next Section. Note that if unit commitment scheduling is not required, the $I(t)$ variables can be fixed to '1' and effectively eliminated from the representation to leave a basic LP.

Table I: Summary of CHP model

Costs:	$z_1(t)\ C_F + z_2(t)\ C_{SS} + I(t)\ (C_{Off} + C_{Fmin}) - C_{Fmin}$;
Decision Variables:	$P(t),\ Q(t),\ I(t)$;
Slack Variables:	$z_1(t),\ z_2(t)$;
Constraints:	$P(t) \geq 0,\ Q(t) \geq 0,\ z_1 \geq 0, z_2 \geq 0, I(t) \in [0, 1]$ and integer;
	$z_1(t) \geq fa_j + fb_j\ (P(t) + Q(t)), j = 1, 2, 3$;
	$P(t) + Q(t) \leq I(t)\ PQ_{\text{Max}}$;
	$P(t)\beta_{\text{Min}} \leq Q(t) \leq P(t)\beta_{\text{Max}}$;
	$z_2(t) \geq (I(t) - I(t\text{-}1)), z_2(t) \geq -(I(t) - I(t\text{-}1))$;
	$\Delta PQ_{Min} \leq (P(t) + Q(t)) - (P(t\text{-}1) + Q(t\text{-}1)) \leq \Delta PQ_{Max}$;

2.3.2 Linear storage and additional model elements

To address the issue of linear storage it is assumed that the energy stored in the heat accumulator at hour t is denoted as $C_H(t)$, and the (equivalent) amount of electrical energy 'stored' at hour t is denoted as $C_E(t)$. It is acknowledged that the latter is a slight abuse of notation as there is no efficient technology allowing the direct storage of electrical energy at the current time. However as discussed in the introduction, technologies such as BESS, CAES and flywheels allow electrical energy to be converted into another form (chemical or mechanical) and stored over short to medium time periods, and later converted back to useable electrical energy. Let the charging/discharging power of the heat accumulator at hour t be denoted as $\Delta C_H(t)$, and the charging/discharging power of the equivalent electrical store at hour t be denoted as $\Delta C_E(t)$, with the convention in both cases that positive values imply storage (charging) and negative values imply extraction (discharging). It is assumed that there is an energy conversion (efficiency) loss associated with each store, and also a leakage (discharge) loss of energy associated with each store over time. It is assumed that the limits of thermal energy storage and the limits of charging/discharging are constrained to be as follows:

$$0 \leq C_H(t) \leq C_{HMax} \tag{19}$$
$$\Delta C_{HMin} \leq \Delta C_H(t) \leq \Delta C_{HMax} \tag{20}$$

And similarly for the equivalent electrical energy store:

$$0 \leq C_E(t) \leq C_{EMax} \tag{21}$$
$$\Delta C_{EMin} \leq \Delta C_E(t) \leq \Delta C_{EMax} \tag{22}$$

An auto regressive model with controlled input [23] is assumed to describe the evolution of the energy content of each of the storage facilities at hour t, such that the following relationships hold:

$$C_H(t) = C_{H\alpha} C_H(t-1) + \Delta C_H(t) \tag{23}$$
$$C_E(t) = C_{E\alpha} C_E(t-1) + \Delta C_E(t) \tag{24}$$

Where the parameters $C_{H\alpha} \in (0, 1]$ and $C_{E\alpha} \in (0, 1]$ represent the capacitive loss factors of the store. To capture the net effect of charging and discharging losses, let $C_{H\beta} \in (0, 1]$ and $C_{E\beta} \in (0, 1]$ represent the conversion loss factors of the stores: a proportion of energy is assumed lost during both the storage and retrieval processes. Two additional slack variables $z_3(t)$ and $z_4(t)$ are employed along with the following constraints:

$$z_3(t) - \frac{1}{C_{H\beta}} \Delta C_H(t) \geq 0 \tag{25}$$

(26)

$$z_3(t) - C_{H\beta}\Delta C_H(t) \geq 0$$

$$z_4(t) - \frac{1}{C_{E\beta}}\Delta C_E(t) \geq 0 \tag{27}$$

$$z_4(t) - C_{E\beta}\Delta C_E(t) \geq 0 \tag{28}$$

The constraints are configured such that when $\Delta C_H(t)$ is positive, the net charging power that is required is equivalent to $\Delta C_H(t)/C_{H\beta}$; but when $\Delta C_H(t)$ is negative, the net discharging power that is available is equivalent to $\Delta C_H(t)C_{H\beta}$. A similar relationship is used to relate $\Delta C_E(t)$ and $C_{E\beta}$. The overall model for each hour of operation of the energy stores is summarised in Table II.

Table II: Summary of storage models

Decision Variables:	$\Delta C_H(t)$, $\Delta C_E(t)$;
Slack Variables:	$C_H(t)$, $C_E(t)$, $z_3(t)$, $z_4(t)$;
Constraints:	$C_H(t) = C_{H\alpha}\, C_H(t\text{-}1) + C_{H\beta}\, \Delta C_H(t)$, $C_E(t) = C_{E\alpha}\, C_E(t\text{-}1) + C_{E\beta}\, \Delta C_E(t)$;
	$0 \leq C_H(t) \leq C_{HMax}$, $0 \leq C_E(t) \leq C_{EMax}$;
	$\Delta C_{HMin} \leq \Delta C_H(t) \leq \Delta C_{HMax}$, $\Delta C_{EMin} \leq \Delta C_{EMax} \leq \Delta C_{EMax}$;
	$z_3(t) \geq 1/C_{H\beta}\, \Delta C_H(t)$; $z_3(t) \geq C_{H\beta}\, \Delta C_H(t)$;
	$z_4(t) \geq 1/\, C_{E\beta}\, \Delta C_E(t)$; $z_4(t) \geq C_{E\beta}\, \Delta C_E(t)$;

Considering the above, the constraints related to the area electrical and thermal power balances can be written. Let the (predicted or known) demands for heat and electricity during hour t be denoted as $D_H(t)$ and $D_E(t)$, and the associated (predicted or known) prices for buying/selling heat and electrical energy during hour t be denoted as $C_{HB}(t)$, $C_{HS}(t)$, $C_{EB}(t)$ and $C_{HS}(t)$ respectively. Let the amount of heat and electrical energy that is to be bought/sold by the plant operator during hour t be denoted as $X_{HB}(t)$, $X_{HS}(t)$, $X_{EB}(t)$ and $X_{HS}(t)$. Then for each hourly time step t the following additional costs and constraints in Table III are employed in the optimisation to ensure that both electrical and heat supply and demand are balanced.

Table III: Summary of power balance models

Costs:	$C_{HS}(t)\, X_{HS}(t) + C_{ES}(t)\, X_{ES}(t) - C_{HB}(t)\, X_{HB}(t) + C_{EB}(t)\, X_{EB}(t)$;
Decision Variables:	$X_{HB}(t)$, $X_{HS}(t)$, $X_{EB}(t)$, $X_{HS}(t)$;
Constraints:	$X_{HB}(t) > 0$, $X_{HS}(t) > 0$, $X_{EB}(t) > 0$, $X_{HS}(t) > 0$;
	$Q(t)\,\eta_H + X_{HB}(t) - X_{HS}(t) - z_3(t) = D_H(t)$;
	$P(t)\,\eta_E + X_{EB}(t) - X_{ES}(t) - z_4(t) = D_E(t)$;

The complete model as presented above has 15 H decision variables (of which H are binary) and 34 H constraints (of which 10 H are simple non-negativity of variables). Further constraints such as limits on the amount of imported/exported power due to line thermal limitations can be added in the optimisation model as required. The full optimisation model is constructed at time t by first forming the objective function cost coefficients for each hour and adding the hourly constraints. The known energy storage levels $C_H(t\text{-}1)$ and $C_E(t\text{-}1)$, plus the CHP plant electrical and thermal power outputs $P(t\text{-}1)$ and $Q(t\text{-}1)$ along with on/off status of the plant $I(t\text{-}1)$ are employed to initialise those constraints which are temporally linked between successive time-steps. Note that in a case in which the time step t is not equal to an hour, small scaling adjustments are required to relate stored and traded quantities of energy (expressed in kWh) to power (kW).

3 Case Study

In this Section, a prototype implementation of the methods and models proposed for CHP plant management described in the previous Section is described.

3.1 Prediction and Optimisation Modules

The procedures described for adaptive load prediction and plant optimisation have been coded as modular, configurable C++ libraries. The prediction models and related adaption code is implemented as a code library. The EWE-RLS algorithm is implemented as a sub-library, and is based upon a standard Kalman-filter style algorithm to recursively update the underlying weighted least squares expression [17]. To implement the optimisation software, the open-source linear optimisation solver software package LP Solve was employed. The package can be included into the host software as a linkable library implementing the LP/MILP solver and associated header file [26]; an additional code library to define and implement the optimisation models described in the previous Section was created. The structure of the software environment is as shown in Figure 4. The main configuration parameters (such as the lengths of prediction and model horizons H and M, etc) are set using definitions in the prediction and optimisation header files.

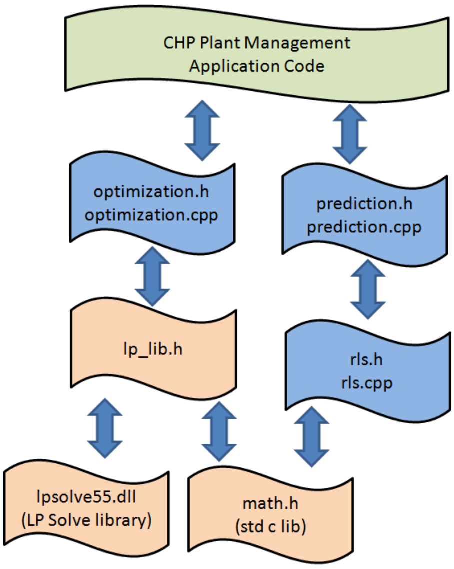


Figure 4: Library structure of the developed software components.

The MILP solver employed in LP Solve uses the Branch and Bound (B&B) algorithm as the core solving procedure [26]. For unit commitment optimisation, the employed solver configuration in this work branches on the most fractional binary variable, and is 'warm started' by first rotating the optimal settings for the indicator variables found in the previous step. The new indicator variable entering the horizon is assumed to have the same state as in the previous time step. This gives an initial integer-feasible solution of reasonable quality to assist in pruning the search tree - initial testing indicated this approach gave promising results. Nevertheless the B&B algorithm can still be forced (in the worst-case) to explore the full search tree given by the binary variables, with $O(2^H)$ leaf nodes; the optimisation horizon H has a strong influence on the computational overheads of the method.

Motivated by the fact that in constrained predictive control schemes, an effective means to reduce computational overheads that is often employed is to enforce certain constraints over only an initial portion of the 'control horizon', the following scheme was considered: supposing that economic dispatch is to be optimised over the entire horizon H, a separate horizon of $C \leq H$ steps for unit commitment can be employed. In this approach the first C indicator variables are free to be optimised by the solver, with the remaining ($H - C$) variables fixed at '1' to relax the problem and consider dispatch only. This allows the complexity to be reduced at the expense of possible loss of optimality; the extreme case is to set $C = 0$, and solve only the economic dispatch problem.

One further option to reduce complexity is the use of a heuristic instead of the full MILP solution. The heuristic considered in this paper takes $C = H$, and first solves the master LP at the root node of the B&B tree. Subsequently, the heuristic greedily fixes the largest fractional binary variable to either '0' or '1' based upon the estimated change in the final objective function that occurs in each case. Once a variable is fixed, a corresponding equality is added to the LP which is then re-solved; the search terminates at the first feasible solution found. Since at most H LPs are required to be solved, a fixed bound on the computation time can be achieved but again this is at the expense of possible loss of solution optimality. Results of computational experiments to explore the influence of the configuration parameter C, and also the performance of the heuristic, are described in Section 4.

3.2 Test Platform

The above libraries for CHP plant management have been integrated, along with a simple graphical interface, into a test software package to be run on a standard IBM PC. In the experiments described in the next Section, a PC with 4 cores running at 2.0 GHz with 3 GB RAM was employed in all cases. The Borland C++ Builder© Environment was employed to compile the executable.

For experimental purposes, an anonymised set of data have been obtained for the hourly thermal load on a district heating system located in Angelholm, Sweden along with hourly ambient temperature measurements for the area for the entire year 2009[1] [27]. Summarising the data and the details presented in [27], the average hourly demand over the course of the year is 24 MW and the peak demand is 70 MW. There are approximately 2,000 buildings connected to the district heating system, the majority of which are one- or two-dwelling buildings, with multi-dwelling buildings, small industrial manufacturing units and commercial / public administration buildings making the remainder. Electricity demands were also obtained for the Sweden SW4 control area (in which Angelholm resides) from the Nordpool spot website. This latter data is publically available. The electricity data for the whole control area have been scaled such that the average demand for heat and electricity matches an assumed plant capacity of 42 MW, which is typical of a medium sized CHP plant [5]. The electricity and heat demand data time series are visualised in Figure 5 below. For both cases, daily and weekly cycling effects can be observed along with seasonal variations. Of particular note is that the demand for heat drops to a very low base load level during the main Summer months (June, July and August) and peaks during the main Winter months (December, January and February), which is a typical feature of heat demands in this part of Europe [12]. A similar, but much less pronounced, effect can be observed in the electricity demand data. Simple flat-file storage was used for data management in this test platform.

[1] It must be noted that a very small number (< 10) of samples were either omitted or invalid (outliers due to system-wide faults) in the dataset. Interpolation of values between the nearest neighbours was employed to restore these samples.

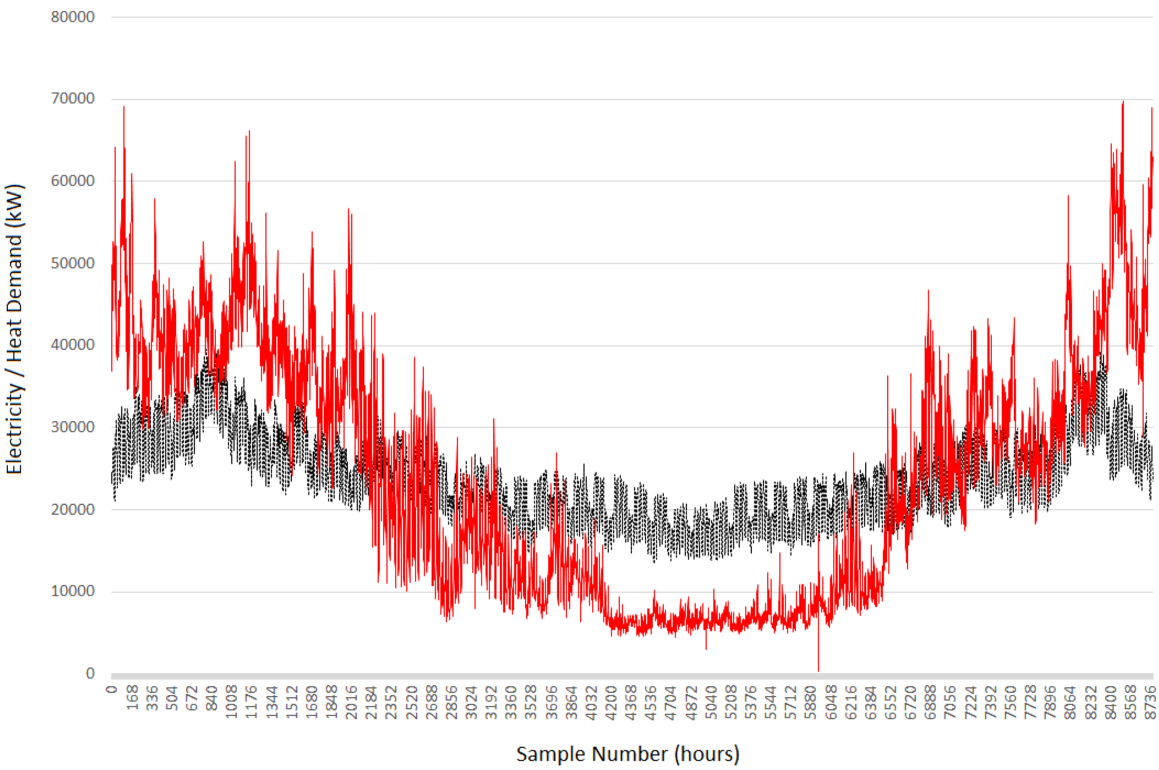


Figure 5: Visualisation of load data. Black/dashed: Electricity demand profile, Red/Solid: Heat Demand profile

In terms of the CHP plant configuration, it was assumed that the heat to power ratio satisfies $0.4 \leq \beta(t) \leq 0.8$ and the accurate efficiency curve (c.f. the piecewise approximation) in Figure 2 (scaled to the maximum plant output) was used to calculate actual operating cost in the cost model. Constraints on the rate of change of the combined heat and power output were set to 25% of capacity per hour. Fuel cost was taken at 0.016 € per unit of fuel/hour, assuming a biomass type of fuel. A fast boiler start-up / shutdown sequence was assumed available [24][25], incurring net fuel and labour costs of 100 €. For computational efficiency, the efficiencies η_E and η_H are both taken to be unity. Local heat storage in the form of a 40 MWh capacity lagged tank with a maximum rate of charge/discharge of 10 MW is assumed available. Electricity storage in the form of a 10 MWh battery bank with a maximum rate of charge/discharge of 2.5 MW is assumed available. Holding efficiencies of these stores is taken as $C_{H\alpha} = 0.95$ and $C_{E\alpha} = 1$, with conversion loss factors taken as $C_{H\beta} = 1$ and $C_{E\beta} = 0.85$. The price of natural gas to produce additional heat is taken to be 0.055 € / kWh for every hour of the day. The selling of heat is assumed not to be an option. The hourly price to purchase electricity is taken as 110% of the corresponding spot price, and the hourly price to sell electricity taken as 90% of the corresponding spot price. Post-hoc regulation charges for electricity imbalances were taken as 150% of the corresponding spot price for both up and down regulation, which is a representative assumption [20].

Using this test platform and experimental configuration, a series of detailed experiments has been performed to investigate aspects of the proposed methodology. The results of these experiments is described in the next Section.

4 Experimental Results

The procedures for prediction and optimisation, described in the previous Sections, have several adjustable parameters and configuration options which can influence the performance and run-time complexity of the software component implementation. For the software components to be of practical use within a rolling-horizon framework, they must be able to produce meaningful results within appropriate timescales relative to the base time units employed. In addition, the configuration of the optimisation and its sensitivity to key perturbations such as prediction and cost function accuracy requires investigation. This Section describes computational experiments and results carried out to determine the impact and influence of these parameters, to assess the suitability of the components in these respects, and to explore the sensitivity of the optimisation and how configuration choices impact upon performance and economic costs. The experiments described below were all performed using the test facility configuration as described in Section 3 of the paper.

4.1 Prediction Model Experiments

A detailed series of investigations were performed to investigate the influence of the prediction model horizon M upon the Mean Absolute Prediction Error (MAPE) across the optimisation horizon and also the average CPU execution time required for a full adaption/prediction step update. Execution times were recorded using the CPU performance counter. A total of 23 experiments were performed, using a full year of data in each case. In each experiment, the prediction horizon H was set to 24 hours and the exponential forgetting factor λ was set to 0.994.This value of λ was given to ensure that the regression in each step is based upon approximately one month of previous data. In the experiments, M is varied between 1 and 23 and the MAPE for electricity and heat load was recorded for each prediction step value k between 1 and 24, i.e. between one hour ahead and one day ahead prediction. In each case the first month of predictions are not included in the MAPE calculations to allow time for the estimator to converge. Figure 5 displays the average values of MAPE obtained across the prediction horizon as M was varied in the specified range. Figure 6 displays the corresponding average CPU execution times (in microseconds) obtained (note that the worst-case execution time was commensurate with this average, as the code does not have major conditional branches).

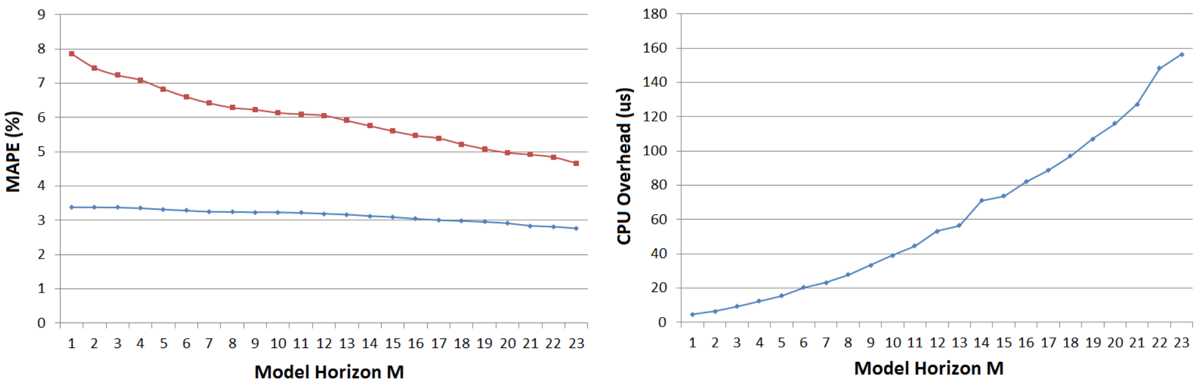


Figure 6: Left: Comparison of Model Horizon M and MAPE across the horizon; Right: Comparison of Model Horizon M and CPU execution time.

From Figure 6, it can be observed that the increase in model complexity has a beneficial effect on both heat and electricity prediction errors, as may be expected. However the effect of increased model complexity on heat errors is much more pronounced than that of electricity errors; the error reduces steadily from 7.85 % to 4.67 % in the former case, and from 3.38 % to 2.76 % in the latter. This is indicative that the heat load benefits more from introducing additional parameters in the underlying model, and the electricity load experiences diminishing returns. Nevertheless, some improvement is still evident in the electricity load case, and even small improvements may translate into potentially large savings over time. Hence it is worthwhile to examine the increase in CPU load that results from increasing M.

Consulting Figure 6 again it is evident that the rise in computational demand is quadratic in the number of parameters which increases linearly in M. However one may observe that even in the most complex case for $M = 23$, the overheads are still < 160 μs for a full prediction step, including updating the EWE-RLS algorithm and calculating the predictions recursively over the full horizon. As such, it is evident that the additional overheads resulting from utilising the more complex model are justified in terms of the potential reduction in MAPE in both heat and electricity loads on a standard PC-based platform. For other computing platforms (e.g. small resource—constrained processors) this overhead increase may not be acceptable, and a trade-off between accuracy and overhead will have to be made. In this respect, Figure 7 shows a plot of the recorded MAPE vs model complexity M for each individual step along the horizon for heat load predictions (a similar - but less pronounced - effect was observed for electricity and the details are omitted for space reasons).

From these data it can be observed that for less complex model structures, the MAPE is first rising in k and then falling; it seems inherently more difficult to predict the load at points around 12-19 hours ahead. As the model complexity increases, the effect is to gradually flatten the peak of the prediction errors. At the point when M = 23, the MAPE rises to a value ≈ 5 % at the 5 hours ahead point and remains approximately constant thereafter. This is indicative that, if a reduced complexity model is required, then adding additional seasonal components around the 12-19 hours ahead point to the simpler model structure may be beneficial. In addition, the forgetting factor may have an additional influence upon the MAPE. These points suggest a need for future work to further explore these issues. Finally, although direct comparisons to previous works are clearly difficult to make, it is observed that Dotauzer [15] reports MAPE figures of 10.90 % and 9.85 % for predictions of hourly heat loads for two district heating systems (of typical daily load in Winter ≈ 700 MW and ≈ 300 MW) in Stockholm, Sweden, and reports that these error levels are commensurate with those obtained using commercial prediction software. Taylor [14] compares various models for day ahead prediction of half-hourly electricity loads for both British and French national grids (of typical daily load ≈ 40 GW and ≈ 50 GW) and reports MAPE levels between 1.5 % and 6 %. Therefore our models seem to be competitive with - and in some cases may even be superior to – existing methods and commercial prediction software based upon linear models in terms of predictive accuracy.

However, it must be cautioned that non-linear models and methods such as neural networks have been applied successfully to short-term load prediction for both electrical and thermal systems [28, 29]. Although non-linear methods inevitably lead to larger computational overheads and possess more complex calibration requirements, the achievable performance can be significantly better than their linear model counterparts. For example lower one-step-ahead MAPE figures than those reported in this paper have been reported by Eriksson [29] when predicting district heating demands using data of a similar profile to that considered in this paper (MAPE figures for further than one-step predictions were not reported in [29], but may be reasonably expected to be also smaller). Any choice of load prediction method should factor this aspect into account.

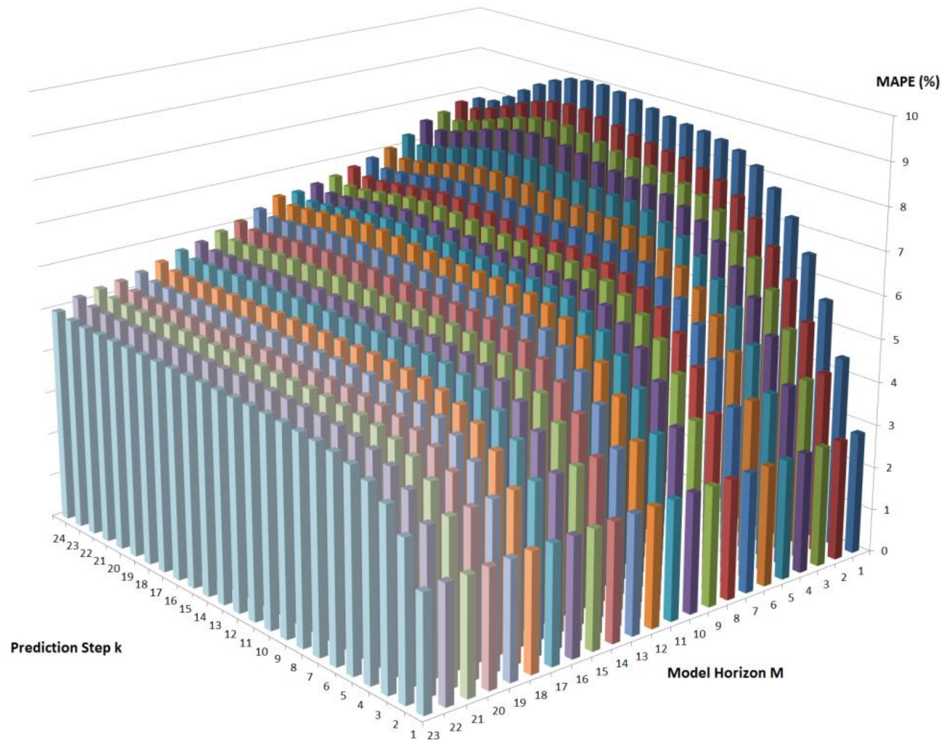


Figure 7: Illustration on the effect of model horizon M on heating prediction errors

4.2 Optimisation Model Experiments

To explore the effects on the quality of the solutions found and computational overheads when using the heuristic version (HEUR) of the optimisation software and the full (MILP) version of the software with various values for the commitment horizon C were conducted. In each experiment, the solver ran for each and every hour of the full year of data, using the load prediction models with $M = 23$ to obtain the demand forecast over the prediction horizon of $H = 24$ hours at each step. This approach follows previous work on short-term co-generation dispatch planning, which investigated the impact of different choices of H on the optimal objective function value when optimising a district heating system with multiple power plant and heat storage in Denmark [10]. In this earlier work it was found that a 24 hour horizon leads to an almost negligible (≈ 0.01 %) impact on optimal objective function value in comparison to a ‘perfect’ (i.e. infinite) horizon [10]. In addition to considering the case $C = 0$ (corresponding to dispatch only, DISP), the cases $C = 24$ (MILP24), $C = 12$ (MILP12) and $C = 8$ (MILP8) were considered to explore the impact of varying the length of the unit commitment horizon. The resulting solution in each case was applied for the next hour, and the net costs calculated (including regulation charges) and added to a running total. Solver execution times were recorded using the CPU performance counter at each step. Table IV displays summary statistics for each solution method. The relative cost increase is defined as the percentage cost increase incurred with the concerned method (MILP24, MILP12, MILP8, HEUR or DISP) over the optimal cost found by the MILP24 solver. Also reported as the absolute cost increase (in €) that this corresponds to, and the average cost for acquiring (generating or buying) one unit (kWh) of energy incurred by each method. The number of times the plant was shut down (uncommitted) during the course of the experiment is also listed for each method, along with the obtained average and worst-case solver times, in seconds.

Table IV: Comparative summary of the solution results

Metric	MILP24	MILP12	MILP8	HEUR	DISP
Relative Cost Increase (%)	0.000	0.004	0.009	0.362	0.552
Absolute Cost Increase (€)	0.000	430.4	848.3	34,994.0	53,355.8
Net Acquisition Cost (€ / kWh)	0.026259	0.026260	0.026261	0.026354	0.026404

Shutdown Events (#)	115	109	102	35	0
Ave Solver Time (s)	3.316	0.301	0.172	0.150	0.128
Max Solver Time (s)	156.896	1.894	0.495	0.477	0.410

From Table IV, it can be observed that the optimal consideration of plant commitment by the MILP24 approach has a measurable effect on the cost of acquiring one unit of energy over the course of the year, when compared to economic dispatch (DISP) approach only. Whilst the increase is relatively small in percentage terms, this nevertheless translates into savings of over €53,355 over the course of the year; a not insignificant figure which could impact upon profit margins and/or the price charged to end consumers for energy. Under optimal commitment with C = 24, it can be observed that the savings made are due to the plant being shut down 115 times during the course of the year (and remaining off for an average of 4 hours each time, with longest shut down being 6 hours). The cost savings that this scheduling bring about are at the expense of increase in average- and worst-case computational effort by the solver: the average case sees an almost 25 fold increase, while the worst-case sees a 380-fold increase. This is not surprising given the added complexity of the introduction of additional binary variables in the model. Although these large increases may be observed, all optimal results were delivered in less than 160 seconds by the MILP24 B&B solver over the course of the year. This is a good indication that the model proposed is efficient and has a reasonably tight LP relaxation.

Considering the Heuristic (HEUR) solution, it can be seen that in this case the plant is shut down only 35 times during the course of the year (again remaining off for an average of 4 hours each time), leading to a cost increase of nearly €34,994 over the full MILP24 approach. However this still represents a cost reduction of over €18,361 compared to the consideration of dispatch only; consideration of average- and worst-case solver times reveals that there is an almost negligible increase (≈ 17%) in overheads between the HEUR and DISP methods; the cost savings are achieved at almost no cost.

For the MILP12 and the MILP8 approaches, the influence of the reduction in unit commitment horizon can be observed in the data. For the MILP12 approach, the plant was shut down 109 times during the course of the year (again remaining off for an average of 4 hours each time, with longest plant shut down 6 hours). This lead to a negligible cost increase of just over €430 when compared to the MILP24 approach; average- and worst-case solver times were reduced by considerable factors (over 10-fold and 80-fold respectively). In the case of the MILP8 approach, the plant was shut down 102 times during the course of the year (again remaining off for an average of 4 hours each time, with longest plant shut down 6 hours). This lead to a slightly larger, but still practically negligible cost increase of just over €848 when compared to MILP24 approach; average- and worst-case solver times were reduced by even bigger factors (over 19-fold and 315-fold respectively). Both choices gave considerable reductions to the CPU overhead (< 1 second) at almost negligible cost; this gives an indication of the effectiveness of the approach. The choice of C = 8 seems to be a particularly good trade-off between overheads and cost impact. Observing that that the average- and worst-case solver times are only marginally larger than the HEUR case (see Table IV), it would seem that the commitment horizon approach is potentially a better approach. Although further investigations into improved heuristics for plant commitment will, no doubt, enable further cost reductions in the HEUR case, these results strongly indicate that it is preferable to consider the exact optimisation of a smaller unit commitment horizon than the heuristic optimisation of a larger one. Again, there is a need for future work to further explore the issue further. Although commitment is not considered in the DISP approach, it can be noted that there were 17 hours during the course of the experiments in which the commanded plant heat and power was zero in the optimal dispatch. This suggests that, in the absence of any further guidance, a plant operator may use a simple lower load threshold on the optimal dispatch to assist with manual commitment decisions.

From the obtained data, it may be observed that there is a comparatively much larger gap between the average-case and worst-case overheads incurred by the MILP24 solver, which is indicative of high variance in solving times over the course of the year. To investigate this aspect in more depth Figure 8 displays the corresponding CPU execution times (in seconds) obtained for each hour over the course of the year for this method. In which it can be observed that, the MILP24 approach has a similar computational overhead to that of the other approaches for around three quarters of the simulated year. This time period corresponded to the Winter and most of the Spring and Autumn months. During this time the plant is mostly committed and is fully operational, and the optimal solution is quickly discovered and verified in the B&B solver. The remaining times – most notably during the Summer months when there is a lack of heat demand – the plant is not required to be fully committed and a comparatively much larger time is spend fathoming the optimal commitment and dispatch plan in the solver. Comparing Figures 5 and 8 directly, the inverse relationship between heat demand and problem complexity (as indicated by the required solver time) is easy to observe. Nevertheless, during these times of higher problem complexity, as mentioned optimal solutions were delivered within a timescale of 160 seconds in each and every case. These data do suggest, however, that the CHP optimisation problem is significantly harder to solve during Summer months than in Winter months, and suggests that future work aimed at testing the efficiency of such optimisation platforms should focus test configurations to include plentiful data corresponding to these months.

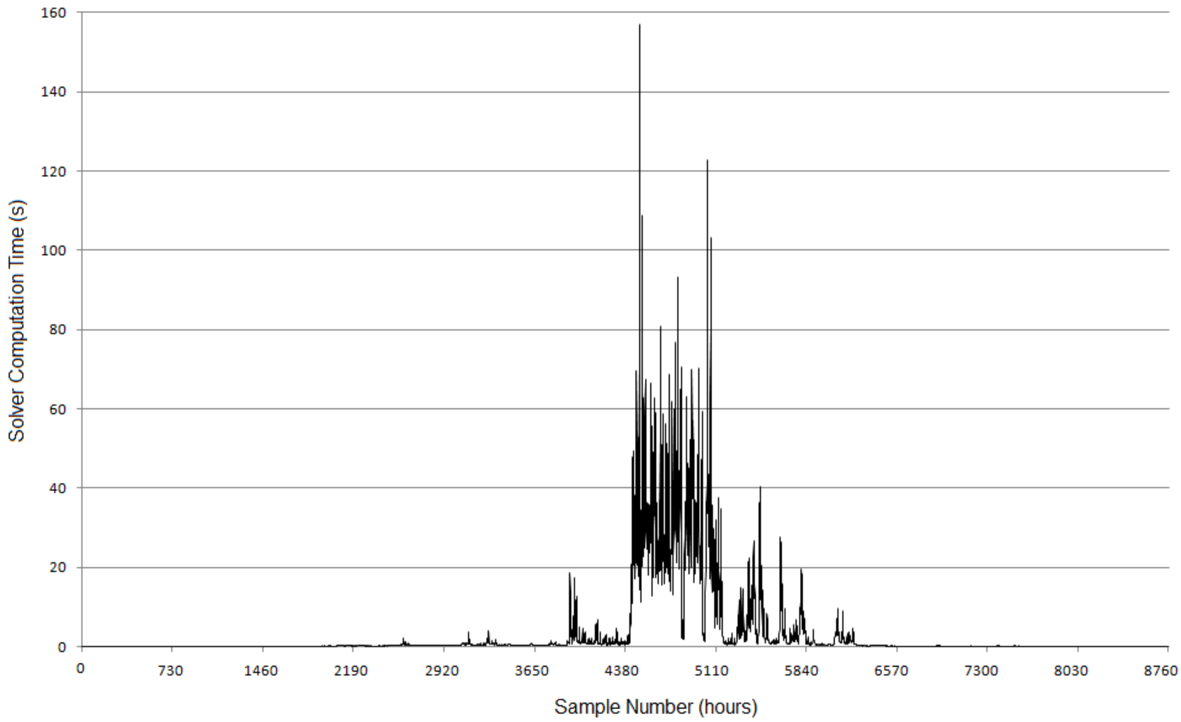


Figure 8: Evolution of CPU execution times over the course of the simulated year

Overall the additional overheads resulting from utilising the more complex MILP model in the solver on a standard PC-based platform are justified in terms of the potential reduction in costs that may be obtained; however the results also suggest that the approach making use of a reduced horizon for optimal unit commitment combined with optimal economic dispatch over the remaining time steps has a negligible impact upon the achievable costs and a potentially large impact upon the computational overhead. When combined with the overheads incurred from updating the prediction models, as discussed in the previous Section, the combined computation time for a full prediction-optimisation step using the MILP8 approach remained at the sub-one second level on our test setup.

It can be commented that the use of commercial MILP software along with specialised solving techniques such as cutting planes and delayed column generation would likely see further decreases on the computational overheads of MILP24 approach. However, as the main concern in this paper is with the use of open source components running on typical PC hardware in standard configurations, only the relevant benchmarks for this are presented. Of course, it must be cautioned that changes to the proposed cost, plant and storage models and solver configuration will likely lead to slightly different results in each case. Nevertheless, these results provide a useful indication of the likely performance for a typical realistic plant configuration running on a standard PC-based computing platform. Using the MILP8 model, two further sets of experiments were considered to explore the sensitivity of the solution to both model and prediction accuracy.

4.3 Sensitivity Analysis

In the first set of additional experiments, the investigations of [9] were reconsidered; however using predictions in place of assumed (known) demand data. Results were obtained considering a strategy interacting with only the spot (day-ahead) market, and a deterministic 'baseline' case in which optimisation was carried out using exactly known future demand data for both heat and electricity. This data was compared to the previous MILP8 result, which corresponds to the strategy interacting with both a spot and balancing market. Results are as shown in Table V below.

Table V: Comparative summary of optimisation strategies using MILP8 solver

Metric	Deterministic	Spot/Balance	Spot Only
Relative Cost Increase (%)	0.000	1.943	2.864
Absolute Cost Increase (€)	0.000	184110.4	271409.2
Net Acquisition Cost (€ / kWh)	0.025671	0.026261	0.026499

From the results obtained, it can be observed that - as expected - the deterministic case (with perfect load predictions) can reduce the net acquisition costs over the other two cases by significant factors. This is in general agreement with the previous work [9]. However, a relative reduction in cost of over 32% can now be observed in the strategy interacting with both the balancing market as well as the spot market, translating to savings of over €87,298 over the course of the year. This is not in agreement with previous work [9]. Observing Figure 6, this can be attributed to the fact that shorter-term predictions are inherently more accurate than longer-term predictions, and without interaction in a balancing market, the updated strategy based upon these improved predictions cannot be put into place as system evolves and more reliable or accurate information is acquired. Interestingly, these result also show that despite having heat and electricity predictions as accurate as those obtainable with commercial load prediction software (as reported in the previous Section), the most favourable solution (i.e. interacting with both spot and balancing markets) leads to cost increases in excess of €184,110. Since this is well in excess of the difference reported in the previous Section between the MILP24 and the MILP8 solutions, it suggests that model accuracy may not be as important as load prediction accuracy.

In the second set of experiments, this aspect was explored further, and results were obtained considering two further optimisation models acting upon exactly known future demand data for both heat and electricity. In these models, the number of affine sections comprising the approximation of the cost function (10) was varied in the approximation of the boiler cost curve shown in Figure 1 (right). A constraint horizon $C = 8$ was employed in both cases. In the first model (P2MILP8), a minimum fuelling constraint of 25% was employed, however now the boiler efficiency was assumed constant over the entire load range and hence only two line segments were employed to model the fuel costs. The following $\{fa_j, fb_j\}$ values were employed: {25.00, 0.000}, {3.993, 0.960}. This piecewise approximation had an average absolute error of 3.43 % over the working range of the boiler. In the second model (P4MILP8), a minimum fuelling constraint of 25% was employed, and four line segments were employed to model the fuel costs. The following $\{fa_j, fb_j\}$ values were employed: {25.00, 0.000}, {5.662, 0.885}, {-2.396, 0.992}, {-23.545, 1.235}. This piecewise approximation had an average absolute error of 0.43 % over the working range of the boiler. This data was again compared to the deterministic MILP8 results described in the previous Section (having three line segments employed to model the fuel costs, denoted as P3MILP8). Comparison of the costs achieved gives an indication of the sensitivity of the model accuracy on achievable economic costs. Results are as shown in Table VI below. The table indicates the accuracy of the fuel cost functions over the boiler working range in each case, the relative and absolute cost increases over the cheapest case, and three indications of the optimal solution structure.

Table VI: Comparative summary of cost sensitivity

Metric	P2MILP8	P3MILP8	P4MILP8
Model Average Error (%)	3.43	1.20	0.43
Relative Cost Increase (%)	0.089	0.034	0.000
Absolute Cost Increase (€)	8309.7	3192.5	0.00
Shutdown Events (#)	63	79	80
Plant Uptime (%)	96.815	96.073	96.027
Average Output (%)	72.813	73.483	73.547

From the results obtained, it can be observed that the P4MILP8 model having four segments in the cost approximation gave the lowest costs over the course of the year, which is as expected given the increased accuracy over the others. Using either two or three segments in the cost approximation lead to cost increases of over €8,309 or €3,192 over the course of the year. Other interesting observations that can be made relate to the optimal solution structure; as the accuracy of cost function approximation improved, the plant experienced more shutdown events, operated with a lower uptime and also with an average out when switched on. The results suggest that improving accuracy of the cost functions employed in the optimisation leads to improved results, and that decreasing the average error in the cost function from 3.43 % to 0.43 % over the working range of the boiler reduced the economic costs over the course of the year by over €8,309. This is, however, a very small improvement considering that a reduction of load prediction errors across the prediction horizon from ≈ 5.00 % to 0 % led to a decrease in costs of over €184,110.

4.4 Discussion and Implications

Key findings are that CHP solver overheads increased 380-fold during low heat load periods, but solving times at the sub-second level were regained by considering a shorter unit commitment horizon than economic dispatch; this approach had an almost negligible impact upon achievable economic costs. The findings also illustrate that cost reductions of just under one third were observed through balancing market interaction when load prediction inaccuracies were present. Economic costs were significantly more sensitive to the accuracy of load predictions than the accuracy of the plant fuelling cost approximation in the experiments presented.

These finds have obvious implications for the potential profitability of CHP powered district heating and increasing the efficiency of operation of CHP plant. Cost reductions of almost a third on the potential payback period for investments in CHP and associated district heating networks also have implications related to the penetration and uptake of CHP. Current payback following plant investment varies significantly, with major influences including the site requirements, technology, type of fuel and level of demand for the heat produced [30]. However for a medium or large scale CHP and district heat network currently they are greater than five years [30]. This is often problematic when the investment is not considered core business, as typically it is necessary to meet much tougher financial hurdles [31]. In this context the ability to reduce running costs by a third, and therefore the payback period by a third, has significant implications for increasing local governments' and the utilities industries abilities and likelihood to invest in CHP powered district heating: as it offers the potential to reduce payback to a level which is more realistic in the current economic climate. Since distributed CHP plants are often fuelled by low emission, sustainable fuels such as biomass, this in turn has the potential to help contribute towards carbon emission reductions.

5 Summary and Conclusions

In this paper, open software components that enable a CHP plant operator to obtain reliable load predictions and efficiently optimise the short-term dispatch and unit commitment of the plant are presented. Extensive computational studies are described, and the effectiveness of the software components is validated using one year of representative data; a typical configuration with low overheads and minimal impact on solution structure has been suggested. The obtained results are indicative that, when used in conjunction with open-source SCADA and timeseries database software, the proposed software components can enable a plant operator to assemble an integrated environment for decentralised CHP plant management at virtually no cost. The findings of a representative case study also suggest that this approach for CHP plant management can reduce the relative cost of acquiring energy, which translates into significant financial savings over the course of a year. The findings also suggest that future work should emphasise improvements to load predictions over that of model accuracy, as the former had a comparatively larger impact upon the economic costs than the latter. Future work in this area will concentrate upon further refinements to the components to increase performance and efficiency.

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