A Refined Approach for Forecasting based on Neutrosophic Time Series

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Abstract

This research introduces a neutrosophic forecasting approach based on neutrosophic time series (NTS). Historical data can be transformed into neutrosophic time series data to determine their truthiness, indeterminacy and falsity functions. The basic for the neutrosophication process is the score and accuracy functions of historical data. In addition, neutrosophic logical relationship groups (NLRG) are determined and a deneutrosophication method for NTS is presented. The objective of this research is to suggest an idea of first and high-order NTS. By comparing our approach with other approaches, we conclude that the suggested approach of forecasting gets better results over the other existing approaches of fuzzy, intuitionistic fuzzy and neutrosophic time series.

Keywords Neutrosophic time series, Triangular neutrosophic number, Neutrosophic logical relationship, Neutrosophic logical relationship groups.

1. Introduction

There are different methods in the literature on fuzzy and intuitionistic fuzzy time series methods to forecast the future values. The major difference between traditional and fuzzy time series is that, the values of traditional time series are presented in numbers, but values in fuzzy time series are fuzzy sets or linguistic values with real meanings. In intuitionistic fuzzy time series the values are intuitionistic fuzzy sets or linguistic values. The first method in literature for forecasting the future values based on fuzzy time series introduced by Song and Chissom [1]. They also applied time-variant and time-invariant models for forecasting the enrollments data at the University of Alabama [1,2]. The identification of fuzzy relationship and the defuzzification process in both models were the main steps for calculating forecasted values. They supposed that the autocorrelation is dependent in time variant, but independent in time-invariant fuzzy time series.

The term “fuzzy relationship” means collection of fuzzy sets which caused only by other sets. Also, the “defuzzification” process means converting the fuzzy values into crisp ones. Further, a straight forward approach for time series forecasting was presented by Chen [3] by using uncomplicated arithmetic computations. For enhancing the accuracy of forecasted outputs, some papers suggested various methods on fuzzy time series (FTS) forecasting [4-7]. A high-order FTS method was also presented by Chen [8] and Singh [9], and a method of bivariate fuzzy time series analysis for the forecasting of a stock index was introduced by Hsu et al. [10]. Furthermore, a framework developed for the evaluation and forecasting based on the fuzzy NEAT F-PROMETHEE method presented by Ziemba and Becker [11] for taking into account the uncertainty of input data, which is particularly burdened with the forecast values of the ICT development indicators.

The concept of fuzzy set was introduced by Zadeh [12], and it was generalized by Atanassov [13] to intuitionistic fuzzy set (IFS) for making it more suitable to handle ambiguity. The IFS considers both the membership (truthiness) and non-membership (falsity) degrees. However, the fuzzy set considers only the membership degree. Recently, the IFS was used for handling the fuzzy time series forecasting by Gangwar and Kumar [14] and Wang et al. [15]. In addition, the notion of intuitionistic fuzzy time series (IFTS)was employed in forecasting, as in[16-18]. Several researchers [19, 20] proposed forecasting models using genetic algorithm, or suggested a method of forecasting based on aggregated FTS and particle swarm optimization [21]. A novel method of forecasting based on hesitant fuzzy set was proposed by Bisht and Kumar [22], and fuzzy descriptor models for earthquake was introduced by Bahrami and Shafiee [23]. A heuristic adaptive-order IFTS forecasting model was presented by Wang et al. [24]. Further on, Abhishekh et al. [25, 26] presented a weighted type 2 FTS and score function-based IFTS forecasting approach. Moreover, Abhishekh and Kumar [27] suggested an approach for forecasting rice production in the area of FTS.

Since the accuracy rates of forecasting in the previous approaches are not good enough in the field of fuzzy and intuitionistic fuzzy time series, we introduce the notion of first and high-order neutrosophic time series data for this research. Additionally, with the growing need to represent vague and random information, neutrosophic sets (NSs) theory [28] is an effective extension of fuzzy and intuitionistic fuzzy set theories. Smarandache [29] suggested NSs, which consist of truth membership function, indeterminacy membership function, and falsity membership function, as a better representation of reality. Neutrosophic sets received wide attention, as well as benefitting from various practical applications in diverse fields [30-39]. However, there are only two recent research papers published in forecasting field, e.g. for stock market analysis. Guan et al.[40] proposed a new forecasting model based on multi-valued neutrosophic sets and two-factor third-order fuzzy logical relationships to forecast the stock market. Further on, Guan et al.[41] proposed a new forecasting method based on high order fluctuation trends and information entropy.

The aim of this research is to enhance accuracy rates of forecasting in the area of fuzzy, intuitionistic fuzzy, and neutrosophic time series (NTS). In this research, we present the notion of forecasting based on first and high-order NTS data by determining the suitable length of neutrosophic numbers that influence on expected values. We also suggest a neutrosophication of the historical time series data, based on the biggest score function (i.e. the maximum value of score function), and define neutrosophic logical relationship groups for obtaining forecasted outputs. The suggested approach of neutrosophic time series forecasting has been validated and compared with different existing models for showing its superiority.

The remaining parts of this research are organized as follows: The essential concepts of neutrosophic set and neutrosophic time series are briefly presented in Section 2. Section 3 presents the proposed neutrosophic time series method for forecasting process. Section 4 validates the proposed method by applying it on two numerical examples for showing its effectiveness; a comparison with other existing methods is presented. Finally, Section 5 concludes the research and determines future trends.

2. Some basic definitions of neutrosophic set and neutrosophic time series

Neutrosophic time series is a concept for solving forecasting problems using neutrosophic concepts. In this section, we present the basic concepts of the neutrosophic set and of the neutrosophic time series (NTS).

Definition 1. Let X be a finite universal set. A neutrosophic set N in X is an object having the following form: $N=\{\langle x, T_N(x), I_N(x), F_N(x)\rangle | x \in X\}$, where $T_N(x): X \rightarrow [0,1]$ determines the degree of truth membership function, $I_N(x): X \rightarrow [0,1]$ determines the degree of indeterminacy, and function $F_N(x): X \rightarrow [0,1]$ determines the degree of non-membership or falsity function. For every $x \in X, 0^- \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+$. [29]

Definition 2. A single valued triangular neutrosophic number $\tilde{N} = \langle (n_1, n_2, n_3); T_{\tilde{N}}, I_{\tilde{N}}, F_{\tilde{N}} \rangle$ is a special neutrosophic set on the real number set R whose truth (membership), indeterminacy and falsity (non-membership) degrees are as follows [29]:

$$T_{\tilde{N}}(x) = \begin{cases} T_{\tilde{N}}\left(\frac{x-n_1}{n_2-n_1}\right) & (n_1 \leq x \leq n_2) \\ T_{\tilde{N}} & (x = n_2) \\ T_{\tilde{N}}\left(\frac{n_3-x}{n_3-n_2}\right) & (n_2 < x \leq n_3) \\ 0 & \text{otherwise} \end{cases}, \quad (1)$$

$$I_{\tilde{N}}(x) = \begin{cases} \frac{\left(n_2-x+I_{\tilde{N}}(x-n_1)\right)}{(n_2-n_1)} & (n_1 \leq x \leq n_2) \\ I_{\tilde{N}} & (x = n_2) \\ \frac{\left(x-n_2+I_{\tilde{N}}(n_3-x)\right)}{(n_3-n_2)} & (n_2 < x \leq n_3) \\ 1 & \text{otherwise} \end{cases}, \quad (2)$$

$$F_{\tilde{N}}(x) = \begin{cases} \frac{\left(n_2-x+F_{\tilde{N}}(x-n_1)\right)}{(n_2-n_1)} & (n_1 \leq x \leq n_2) \\ F_{\tilde{N}} & (x = n_2) \\ \frac{\left(x-n_2+F_{\tilde{N}}(n_3-x)\right)}{(n_3-n_2)} & (n_2 < x \leq n_3) \\ 1 & \text{otherwise} \end{cases}. \quad (3)$$

where $0 \leq T_{\tilde{N}} \leq 1$, $0 \leq I_{\tilde{N}} \leq 1$, $0 \leq F_{\tilde{N}} \leq 1$, $0 \leq T_{\tilde{N}}+I_{\tilde{N}}+F_{\tilde{N}} \leq 3, n_1, n_2, n_3 \in R$,and being the lower, median, and upper values of the triangular neutrosophic number.

Definition 3. Let X and Y be two finite universal sets. A neutrosophic relation R from X to Y is a neutrosophic set in the direct product space X to Y:

$R=\{\langle(x,y), T_N(x,y), I_N(x,y), F_N(x)\rangle|(x,y) \in X \times Y\}$, where $0^- \leq T_N(x,y) + I_N(x,y) + F_N(x,y) \leq 3^+$, $\forall(x,y) \in X \times Y$ for $T_N(x,y) \rightarrow [0,1]$, $I_N(x,y) \rightarrow [0,1]$, and $F_N(x,y) \rightarrow [0,1]: X \times Y \rightarrow [0,1]$.

Definition 4. Let $X(t)(t = 1,2, \dots ,)$, a subset of R, be the universe of discourse on which neutrosophic sets $f_i(t) = \langle T_N(x,y), I_N(x,y), F_N(x)\rangle (i = 1,2, \dots)$ are defined. $F(t)=\{f_1(x), f_2(x), \dots\}$ is a collection of $f_i(t)$ and it defines a neutrosophic time series on $X(t)(t = 0,1,2, \dots)$.

Definition 5. If there exists a neutrosophic relationship $R(t-1,t)$, such that $F(t) = F(t-1) \times R(t-1,t)$, where '×' represents an operator, then $F(t)$ is said to be caused by $F(t-1)$. The relationship between $F(t)$ and $F(t-1)$ is symbolized by $F(t-1) \rightarrow F(t)$.

Definition 6. Let $F(t)$ caused by $F(t-1)$ only and symbolized by $F(t-1) \rightarrow F(t)$;consequently, a neutrosophic relationship exists between $F(t)$ and $F(t-1)$ that is denoted as $F(t) = F(t-1) \times R(t-1,t)$, since R is a first-order model of $F(t)$. The $F(t)$ is a time invariant neutrosophic time series if $R(t-1,t)$ is independent of time t , $R(t,t-1) = R(t-1,t-2) \forall\, t$. Otherwise, $F(t)$ is called a time-variant neutrosophic time series.

Definition 7. Let $F(t-1) = \tilde{N}_i$ and $(t) = \tilde{N}_j$; a neutrosophic logical relationship (NLR) can be defined as $\tilde{N}_i \rightarrow \tilde{N}_j$, where $\tilde{N}_i$, $\tilde{N}_j$ are the current and next state of NLR. Since $F(t)$ is occurred by more than one neutrosophic sets $F(t-n), F(t-n+1), \dots F(t-1)$, then the neutrosophic relationship is represented by $\tilde{N}_{i1}, \tilde{N}_{i2}, \dots, \tilde{N}_{in} \rightarrow \tilde{N}_j$, where $F(t-n) = \tilde{N}_{i1}$, $F(t-n+1) = \tilde{N}_{i2}$. The relationship is called high-order neutrosophic time series model.

3. Neutrosophic time series forecasting algorithm

Because neutrosophic set plays a significant role in decision making and data analysis problems via handling vague, inconsistent and incomplete information [30-39], we propose in this section an enhanced approach of forecasting using the concept of neutrosophic time series (NTS).

The stepwise method of the suggested algorithm of neutrosophic time series forecasting is depended on historical time series data.

3.1. The proposed method of forecasting based on first order NTS data

Step 1: By depending on range of existing data set, determine the universe of discourse U as follows:

- Select the largest D_l and the smallest D_s from all available data D_v, then

  $U=[D_s - D_1,\ D_l + D_2]$ (4) ,

  where D_1 and D_2 are two proper positive numbers assigned by experts in the problem domain. So, we can define $\mathrm{D_1},\ \mathrm{D_2}$ as the values by which the range of the universe of discourse be less than the specified value of $\mathrm{D_s}$ for the first ($\mathrm{i.e.\,D_1}$) or greater than the specified value of D_l for the later (i.e. $\mathrm{D_2}$) .

Step 2: Create a partition of the universe of discourse, to m triangular neutrosophic numbers as it follows:

- Decide the suitable length (Le) of available time series data:
  - Among the value D_{v-1},D_v, calculate all absolute differences and take average of these differences.
  - Consider half the average as the initial length.
  - According to obtained result, use base mapping table [42] to determine the base for length of intervals.
  - Round the result to determine the appropriate length of neutrosophic numbers.
  - For example: if we have these time series data $30,50,80,120,100,70$, then the absolute differences will be $20,30,40,20,30$, and the average of these values $= 28$. Then the half of average will be 14 and this is the initial value of length. By using base mapping table [42], the base for length $=\ 10$ because 14 locates in the range $[11-100]$ and by rounding the length 14 by the base

ten the result will equal 10. Here, the appropriate length of neutrosophic numbers equals 10.

- Compute the number of triangular neutrosophic numbers (m) as follows:

$$m = \frac{D_l + D_2 - D_s + D_1}{le} \quad (5)$$

Step 3: According to the numbers of triangular neutrosophic numbers on the universe of discourse and determined length (le), begin to construct the triangular neutrosophic numbers. The triangular neutrosophic numbers are $\widetilde{N}_1, \widetilde{N}_2, \dots, \widetilde{N}_m$.

As we illustrated in Definition 2, each triangular neutrosophic number consists of two parts which are the value of triangular neutrosophic number (lower, median, upper) and the degree of confirmation (truth/membership degree T, indeterminacy degree I , falsity/ non-membership degree F). The initial value of T, I, F must be determined by experts according to existing problem.

Step 4: Make a neutrosophication process of the existing data:

For $i, j = 1,2, \dots, v$ (the end of data):

Rule 1: Use this equation to calculate score degree, and if the score degree of two neutrosophic numbers is not equal for any data, then choose the maximum value of score degree:

$$SC_{\widetilde{N}_j}(x_i) = 2 + T_{\widetilde{N}_j}(x_i) - I_{\widetilde{N}_j}(x_i) - F(x_i) \quad (6)$$

Then, select $SC_{\widetilde{N}_k} = max\ (SC_{\widetilde{N}_k}, SC_{\widetilde{N}_k}, \dots, SC_{\widetilde{N}_k})$ for x_i , $i = 1,2 \dots, n$, $1 \leq k \leq n$, and assign the neutrosophic number $\widetilde{N}_k$ to x_i.

Rule 2: If two neutrosophic numbers have the same score degree, then use the following equation to calculate score degree, and select minimum accuracy degree:

$$AC_{\widetilde{N}_j}(x_i) = 2 + T_{\widetilde{N}_j}(x_i) - I_{\widetilde{N}_j}(x_i) + F(x_i) \quad (7)$$

Furthermore, $AC_{\widetilde{N}_k} = min\ (AC_{\widetilde{N}_k}, AC_{\widetilde{N}_k}, \dots, AC_{\widetilde{N}_k})$ for x_i, $i = 1,2 \dots, n$, $1 \leq k \leq n$; assign the neutrosophic number $\widetilde{N}_k$ to x_i.

Step 5: Construct the neutrosophic logical relationships (NLR) as follows:

If $\widetilde{N}_j$,$\widetilde{N}_k$ are the neutrosophication values of year $n$and year $n + 1$ respectively, then the NLR is symbolized as$\widetilde{N}_j \rightarrow \widetilde{N}_k$.

Step 6: Based on NLR, begin to establish the neutrosophic logical relationship groups (NLRG).

Step 7: Calculate the forecasted values as follows:

Rule 1: If the neutrosophication value of $data_i$ is $\widetilde{N}_k$ and it is not caused by any other neutrosophication values, and by looking at the NLRG of this value, you cannot find the value which it depends on (i.e. $\neq\rightarrow \widetilde{N}_k$), then the forecasted value in this case will equal $-$ (i.e. leave it empty). The $\neq$ symbol means no value.

Rule 2: If the neutrosophication value of $data_i$ is $\widetilde{N}_k$ and it is caused by $\widetilde{N}_j(\widetilde{N}_j \rightarrow \widetilde{N}_k)$, then look at NLRG of $\widetilde{N}_j$, and

- If NLRG of $\widetilde{N}_j$ is empty i.e. $\widetilde{N}_j \rightarrow \emptyset$, or $\widetilde{N}_j \rightarrow \widetilde{N}_j$, then the forecasted value is the middle value of $\widetilde{N}_j$.
- If NLRG of $\widetilde{N}_j$ is one-to-one i.e. $\widetilde{N}_j \rightarrow \widetilde{N}_k$, then the forecasted value is the middle value of $\widetilde{N}_k$.
- If NLRG of $\widetilde{N}_j$ is one-to-many i.e. $\widetilde{N}_j \rightarrow \widetilde{N}_{k1}, \widetilde{N}_{k2}, \dots, \widetilde{N}_{kn}$, then the forecasted value is the average of middle values of $\widetilde{N}_{k1}, \widetilde{N}_{k2}, \dots, \widetilde{N}_{kn}$.

Step 8: Use the following equations to calculate forecasting error:

Root mean square error (RMSE) = $\sqrt{\frac{\sum_{i=1}^{n}(\text{Forecast}_i - \text{Actual}_i)^2}{n}}$ (8)

Forecasting error $= \frac{|\text{ Forecast} - \text{Actual}l|}{Actual} \times 100$ (9)

Average forecasting error (AFE) (%) $= \frac{\text{Sum of forecasting error}}{number\ of\ errors} \times 100$ (10)

3.2. The proposed method of forecasting based on high order NTS data

We can also apply the proposed method of forecasting based on high order NTS data:

- All steps from 1 to 4 are the same as previously, but in step 5 we begin to construct the neutrosophic logical relationships (NLR) of n^{th} order NTS where $n \geq 2$.
- Based on NLR of n^{th} order, NTS begin to establish the neutrosophic logical relationship groups (NLRG).
- Calculate the forecasted values as follows:
  - Rule 1: If the neutrosophication values of $data_i$ is $\widetilde{N}_l$ and it is not caused by any other neutrosophication values, then by looking at the NLRG of this value, you cannot find the values which it depends on (i.e. $\neq\rightarrow \widetilde{N}_l$) then the forecasted value in this case will equal $-$ (i.e. leave it empty). The $\neq$ symbol means no value.

- Rule 2: If the neutrosophication value of $data_i$ is $\widetilde{N}_l$ and it is caused by $\widetilde{N}_{in}, \widetilde{N}_{i(n-1)}, \ldots, \widetilde{N}_{ik}$ (i.e. $\widetilde{N}_{in}, \widetilde{N}_{i(n-1)}, \ldots, \widetilde{N}_{ik} \rightarrow \widetilde{N}_l$), then look at NLRG of $\widetilde{N}_{in}, \widetilde{N}_{i(n-1)}, \ldots, \widetilde{N}_{ik}$, and
  - If $\widetilde{N}_{in}, \widetilde{N}_{i(n-1)}, \ldots, \widetilde{N}_{ik} \rightarrow \emptyset$, then the forecasted value at this year is the average of middle value of $\widetilde{N}_{in}, \widetilde{N}_{i(n-1)}, \ldots, \widetilde{N}_{ik}$.
  - If $\widetilde{N}_{in}, \widetilde{N}_{i(n-1)}, \ldots, \widetilde{N}_{ik} \rightarrow \widetilde{N}_j$, then the forecasted value at this year is the middle value of $\widetilde{N}_j$.
  - If $\widetilde{N}_{in}, \widetilde{N}_{i(n-1)}, \ldots, \widetilde{N}_{ik} \rightarrow \widetilde{N}_j, \widetilde{N}_{j1}, \widetilde{N}_{j2}$ then the forecasted value at this year is average of the middle value of $\widetilde{N}_j, \widetilde{N}_{j1}, \widetilde{N}_{j2}$.

4. Numerical examples

We solved in this section two numerical examples and compared outputs with other existing methods for verifying the applicability and superiority of the suggested method.

4.1. Numerical example 1

In this example, the suggested approach is implemented on the bench marking time series data of student enrollments at the University of Alabama from year 1971–1992 adopted from [26]. The steps are as follows:

Step 1: Let the two proper positive numbers D_1 and D_2 be 5 and 13, determined by the expert. By selecting the largest and the smallest observation from all available data which presented in Table 1, then $D_l = 19337$, $D_s = 13055$ respectively. Consequently, the universe of discourse $U = [13055 - 5, 19337 + 13] = [13050, 19350]$.

Step 2: Create a partition of the universe of discourse, to m triangular neutrosophic numbers, as it follows:

- Determine the suitable length (Le) of available time series data:
  - From Table 1, the average of absolute differences $= 510.3$.
  - The initial length $= \frac{510.3}{2} = 255.15$.
  - By using base mapping table [42], the base for length of intervals $= 100$, since it is located in the range [101,1000] .
  - By rounding 255.15 regarding to base 100, then the appropriate length of neutrosophic numbers = 300.

- Compute the number of triangular neutrosophic numbers (m) as follows:

  $m = \frac{19350-13050}{300} = 21.$ Then, we can partition U into 21 triangular neutrosophic number with length $= 300.$

Step 3: According to the number of triangular neutrosophic numbers on the universe of discourse and determined length (le), begin to construct the triangular neutrosophic numbers as follows:

$\widetilde{N}_1 = \langle 13050, 13350, 13650; 0.90, 0.10, 0.10 \rangle$,

$\widetilde{N}_2 = \langle 13350, 13650, 13950; 0.80, 0.20, 0.10 \rangle$,

$\widetilde{N}_3 = \langle 13650, 13950, 14250; 0.90, 0.20, 0.10 \rangle$, $\widetilde{N}_4 = \langle 13950, 14250, 14550; 0.85, 0.15, 0.10 \rangle$,

$\widetilde{N}_5 = \langle 14250, 14550, 14850; 0.75, 0.10, 0.30 \rangle$,

$\widetilde{N}_6 = \langle 14550, 14850, 15150; 0.90, 0.10, 0.10 \rangle$,

$\widetilde{N}_7 = \langle 14850, 15150, 15450; 0.60, 0.30, 0.40 \rangle$,

$\widetilde{N}_8 = \langle 15150, 15450, 15750; 0.80, 0.20, 0.20 \rangle$,

$\widetilde{N}_9 = \langle 15450, 15750, 16050; 0.70, 0.20, 0.30 \rangle$,

$\widetilde{N}_{10} = \langle 15750, 16050, 16350; 0.90, 0.10, 0.30 \rangle$,

$\widetilde{N}_{11} = \langle 16050, 16350, 16650; 0.85, 0.10, 0.15 \rangle$,

$\widetilde{N}_{12} = \langle 16350, 16650, 16950; 0.80, 0.20, 0.20 \rangle$,

$\widetilde{N}_{13} = \langle 16650, 16950, 17250; 0.90, 0.10, 0.30 \rangle$,

$\widetilde{N}_{14} = \langle 16950, 17250, 17550; 0.90, 0.10, 0.30 \rangle$,

$\widetilde{N}_{15} = \langle 17250, 17550, 17850; 0.75, 0.10, 0.30 \rangle$,

$\widetilde{N}_{16} = \langle 17550, 17850, 18150; 0.65, 0.20, 0.35 \rangle$,

$\widetilde{N}_{17} = \langle 17850, 18150, 18450; 0.90, 0.10, 0.10 \rangle$,

$\widetilde{N}_{18} = \langle 18150, 18450, 18750; 0.90, 0.10, 0.10 \rangle$,

$\widetilde{N}_{19} = \langle 18450, 18750, 19050; 0.60, 0.20, 0.30 \rangle$,

$\widetilde{N}_{20} = \langle 18750, 19050, 19350; 0.90, 0.10, 0.10 \rangle$,

$\widetilde{N}_{21} = \langle 19050, 19350, 19350; 0.90, 0.10, 0.10 \rangle$.

Step 4: Make a neutrosophication of the available time series data:

The first value of actual enrollments is 13055 which is located only in the range of triangular neutrosophic number $\widetilde{N}_1$, then the neutrosophication value of 13055 is $\widetilde{N}_1$ as appears in Table 1.

Also, the second value of actual enrollments, i.e. 13563, locates in the range of triangular neutrosophic numbers $\widetilde{N}_1 = \langle 13050,13350,13650; 0.90,0.10,0.10 \rangle$ and $\widetilde{N}_2 = \langle 13350,13650,13950; 0.80,0.20,0.10 \rangle$.

Then we must select the highest score degree of 13563 as follows:

The membership degree, indeterminacy and non-membership degrees of this value are calculated by using Eq.(1), Eq.(2), Eq.(3) as follows:

$T_{\widetilde{N}_1}(13563) = 0.261$, $I_{\widetilde{N}_1}(13563) = 0.739, F_{\widetilde{N}_1}(13563) =$ 0.739. We must also calculate membership, indeterminacy and non-membership degrees of 13563 according to $\widetilde{N}_2 = \langle 13350,13650,13950; 0.80,0.20,0.10 \rangle$ as follows:

$T_{\widetilde{N}_2}(13563) = 0.568, I_{\widetilde{N}_2}(13563) =0.432$, $F_{\widetilde{N}_2}(13563) = 0.361$.

In this case, we must calculate the score degree of 13563 in both $\widetilde{N}_1$ and $\widetilde{N}_2$ and select the maximum value.

$SC_{\widetilde{N}_1}(13563) = 2 + 0.262 - 0.739 - 0.739 = 0.783,$

and

$SC_{\widetilde{N}_2}(13563) = 2 + 0.568 - 0.432 - 0.361 = 1.775.$

Since the score degree of 13563 in $\widetilde{N}_2$ is greater than $\widetilde{N}_1$, then the neutrosophication value of 13563 is $\widetilde{N}_2$, as in Table 1.

We will apply the previous steps on the remaining data as follows:

The value 13867 locates in the range of $\widetilde{N}_2 = \langle 13350,13650,13950; 0.80,0.20,0.10 \rangle$, and $\widetilde{N}_3 = \langle 13650,13950,14250; 0.90,0.20,0.10 \rangle$.

Then $T_{\widetilde{N}_2}(13867) = 0.221$, $I_{\widetilde{N}_2}(13867) = 0.156, F_{\widetilde{N}_2}(13867) = 0.751$.

$T_{\widetilde{N}_3}(13867) = 0.651, I_{\widetilde{N}_3}(13867) = 0.421, F_{\widetilde{N}_3}(13867) = 0.349,$

$SC_{\widetilde{N}_2}(13867) = 2 + 0.221 - 0.156 - 0.751 = 1.314,$

and $SC_{\widetilde{N}_3}(13867) = 2 + 0.651 - 0.421 - 0.349 = 1.881.$

So, the neutrosophication value of 13867 is $\widetilde{N}_3$.

Also, the value of 14696 locates in the range of $\widetilde{N}_5 = \langle 14250,14550,14850; 0.75,0.10,0.30\rangle$,

$\widetilde{N}_6 = \langle 14550,14850,15150; 0.90,0.10,0.10\rangle$, then $T_{\widetilde{N}_5}(14696) = 0.385, I_{\widetilde{N}_5}(14696) = 0.538, F_{\widetilde{N}_5}(14696) = 0.641$.

$T_{\widetilde{N}_6}(14696) = 0.438, I_{\widetilde{N}_6}(14696) = 0.562, F_{\widetilde{N}_6}(14696) = 0.562$.

$SC_{\widetilde{N}_5}(14696) = 2 + 0.385 - 0.538 - 0.641 = 1.206$,

and $SC_{\widetilde{N}_6}(14696) = 2 + 0.438 - 0.562 - 0.562 = 1.314$.

So, the neutrosophication value of 14696 is $\widetilde{N}_6$.

The value 15460 locates in the range of $\widetilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$, and $\widetilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$, then

$T_{\widetilde{N}_8}(15460) = 0.773, I_{\widetilde{N}_8}(15460) = 0.226, F_{\widetilde{N}_8}(15460) = 0.226$.

$T_{\widetilde{N}_9}(15460) = 0.023, I_{\widetilde{N}_9}(15460) = 0.973, F_{\widetilde{N}_9}(15460) = 0.973$.

$SC_{\widetilde{N}_8}(15460) = 2 + 0.773 - 0.226 - 0.226 = 2.321$, and

$SC_{\widetilde{N}_9}(15460) = 2 + 0.023 - 0.973 - 0.976 = 0.074$.

So, the neutrosophication value of 15460 is $\widetilde{N}_8$.

The value of 15311 locates in the range of $\widetilde{N}_7 = \langle 14850,15150,15450; 0.60,0.30,0.40\rangle$ and $\widetilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$, then

$T_{\widetilde{N}_7}(15311) = 0.278, I_{\widetilde{N}_7}(15311) = 0.675, F_{\widetilde{N}_7}(15311) = 0.722$.

$SC_{\widetilde{N}_7}(15311) = 2 + 0.278 - 0.675 - 0.722 = 0.881$.

$T_{\widetilde{N}_8}(15311) = 0.429, I_{\widetilde{N}_8}(15311) = 0.570, F_{\widetilde{N}_8}(15311) = 0.570$.

$SC_{\widetilde{N}_8}(15311) = 2 + 0.429 - 0.570 - 0.570 = 1.289$.

So, the neutrosophication value of 15311 is $\widetilde{N}_8$.

The value of 15603 locates in the range of $\widetilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$ and $\widetilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$, then

$T_{\widetilde{N}_8}(15603) = 0.392, I_{\widetilde{N}_8}(15603) = 0.608, F_{\widetilde{N}_8}(15603) = 0.608$.

$SC_{\widetilde{N}_8}(15603) = 2 + 0.392 - 0.608 - 0.608 = 1.176$.

$T_{\widetilde{N}_9}(15603) = 0.357, I_{\widetilde{N}_9}(15603) = 0.592, F_{\widetilde{N}_9}(15603) = 0.643$.

$SC_{\widetilde{N}_9}(15603) = 2 + 0.357 - 0.592 - 0.643 \ = \ 1.122$.

So, the neutrosophication value of 15603 is $\widetilde{N}_8$.

The value of 15861 locates in the range of $\widetilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$, and $\widetilde{N}_{10} = \langle 15750,16050,16350; 0.90,0.10,0.30\rangle$, then

$T_{\widetilde{N}_9}(15861) = 0.441, I_{\widetilde{N}_9}(15861) = 0.496, F_{\widetilde{N}_9}(15861) = 0.559.$

$SC_{\widetilde{N}_9}(15861) = 2 + 0.441 - 0.496 - 0.559 \ = \ 1.386.$

$T_{\widetilde{N}_{10}}(15861) = 0.333, I_{\widetilde{N}_{10}}(15861) = 0.667, F_{\widetilde{N}_{10}}(15861) = 0.741.$

$SC_{\widetilde{N}_{10}}(15861) = 2 + 0.333 - 0.667 - 0.741 \ = \ 0.925.$

So, the neutrosophication value of 15861 is $\widetilde{N}_9$.

The value of 16807 locates in the range of $\widetilde{N}_{12} = \langle 16350,16650,16950; 0.80,0.20,0.20\rangle$, $\widetilde{N}_{13} = \langle 16650,16950,17250; 0.90,0.10,0.30\rangle$ then, $T_{\widetilde{N}_{12}}(16807) = 0.381, I_{\widetilde{N}_{12}}(16807) = 0.618, F_{\widetilde{N}_{12}}(16807) = 0.618.$

$SC_{\widetilde{N}_{12}}(16807) = 2 + 0.381 - 0.618 - 0.618 \ = \ 1.145.$

$T_{\widetilde{N}_{13}}(16807) = 0.471, I_{\widetilde{N}_{13}}(16807) = 0.529, F_{\widetilde{N}_{13}}(16807) = 0.634.$

$SC_{\widetilde{N}_{13}}(16807) = 2 + 0.471 - 0.529 - 0.634 \ = \ 1.308.$

So, the neutrosophication value of 16807 is $\widetilde{N}_{13}$.

The value of 16919 locates in the range of $\widetilde{N}_{12} = \langle 16350,16650,16950; 0.80,0.20,0.20\rangle$, $\widetilde{N}_{13} = \langle 16650,16950,17250; 0.90,0.10,0.30\rangle$, then $T_{\widetilde{N}_{12}}(16919) = 0.063, I_{\widetilde{N}_{12}}(16919) = 0.917, F_{\widetilde{N}_{12}}(16919) = 0.917.$

$SC_{\widetilde{N}_{12}}(16919) = 2 + 0.063 - 0.917 - 0.917 \ = \ 0.229.$

$T_{\widetilde{N}_{13}}(16919) = 0.807, I_{\widetilde{N}_{13}}(16919) = 0.193, F_{\widetilde{N}_{13}}(16919) = 0.372.$

$SC_{\widetilde{N}_{13}}(16919) = 2 + 0.807 - 0.193 - 0.372 = \ 2.24.$

So, the neutrosophication value of 16919 is $\widetilde{N}_{13}$.

The value of 16388 locates in the range of $\widetilde{N}_{11} = \langle 16050,16350,16650; 0.85,0.10,0.15\rangle$, $\widetilde{N}_{12} = \langle 16350,16650,16950; 0.80,0.20,0.20\rangle$, then

$T_{\widetilde{N}_{11}}(16388) = 0.742, I_{\widetilde{N}_{11}}(16388) = 0.214, F_{\widetilde{N}_{11}}(16388) = 0.257.$

$SC_{\widetilde{N}_{11}}(16388) = 2 + 0.742 - 0.214 - 0.257 \ = \ 2.271.$

$T_{\widetilde{N}_{12}}(16388) = 0.101, I_{\widetilde{N}_{12}}(16388) = 0.898, F_{\widetilde{N}_{12}}(16388) = 0.898.$

$SC_{\widetilde{N}_{12}}(16388) = 2 + 0.101 - 0.898 - 0.898 = 0.305.$

So, the neutrosophication value of 16388 is $\widetilde{N}_{11}$.

The value of 15433 locates in the range of $\widetilde{N}_7 = \langle 14850,15150,15450; 0.60,0.30,0.40\rangle$, and $\widetilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$, then $T_{\widetilde{N}_7}(15433) = 0.034, I_{\widetilde{N}_7}(15433) = 0.960, F_{\widetilde{N}_7}(15433) = 0.966.$

$SC_{\widetilde{N}_7}(15433) = 2 + 0.034 - 0.960 - 0.966 = 0.108.$

$T_{\widetilde{N}_8}(15433) = 0.754, I_{\widetilde{N}_8}(15433) = 0.245, F_{\widetilde{N}_8}(15433) = 0.245.$

$SC_{\widetilde{N}_8}(15433) = 2 + 0.754 - 0.245 - 0.245 = 2.264.$ So, the neutrosophication value of 15433 is $\widetilde{N}_8$.

The value of 15497 locates in the range of $\widetilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$ and $\widetilde{N}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$ then,

$T_{\widetilde{N}_8}(15497) = 0.674, I_{\widetilde{N}_8}(15497) = 0.325, F_{\widetilde{N}_8}(15497) = 0.325$.

$SC_{\widetilde{N}_8}(15497) = 2.024.$

Also, $T_{\widetilde{N}_9}(15497) = 0.109, I_{\widetilde{N}_9}(15497) = 0.874, \ F_{\widetilde{N}_9}(15497) = 0.890$.

$SC_{\widetilde{N}_9}(15497) = 0.345$. So, the neutrosophication value of 15433 is $\widetilde{N}_8$.

The value of 15145 locate in the range of $\widetilde{N}_6 = \langle 14550,14850,15150; 0.90,0.10,0.10\rangle$, and $\widetilde{N}_7 = \langle 14850,15150,15450; 0.60,0.30,0.40\rangle$, then $T_{\widetilde{N}_6}(15145) = 0.015, I_{\widetilde{N}_6}(15145) = 0.985, \ F_{\widetilde{N}_6}(15145) = 0.985$. $SC_{\widetilde{N}_6}(15145) = 0.045$.

Also, $T_{\widetilde{N}_7}(15145) = 0.59, I_{\widetilde{N}_7}(15145) = 0.311, \ F_{\widetilde{N}_7}(15145) = 0.41$.

$SC_{\widetilde{N}_7}(15145) = 1.869$. So, the neutrosophication value of 15145 is $\widetilde{N}_7$.

The value of 15163 locates in the range of $\widetilde{N}_7 = \langle 14850,15150,15450; 0.60,0.30,0.40\rangle$, $\widetilde{N}_8 = \langle 15150,15450,15750; 0.80,0.20,0.20\rangle$, then $T_{\widetilde{N}_7}(15163) = 0.6, I_{\widetilde{N}_7}(15163) = 0.330, F_{\widetilde{N}_7}(15163) = 0.426$. $SC_{\widetilde{N}_7}(15163) = 1.844$.

Also $T_{\widetilde{N}_8}(15163) = 0.034$, $I_{\widetilde{N}_8}(15163) = 0.965, F_{\widetilde{N}_8}(15163) = 0.965$.

$SC_{\widetilde{N}_8}(15163) = 0.104$. So, the neutrosophication value of 15163 is $\widetilde{N}_7$.

The value of 15984 locates in the range of $\widetilde{\mathrm{N}}_9 = \langle 15450,15750,16050; 0.70,0.20,0.30\rangle$, $\widetilde{\mathrm{N}}_{10} = \langle 15750,16050,16350; 0.90,0.10,0.30\rangle$, then

$T_{\widetilde{N}_9}(15984) = 0.154$, $I_{\widetilde{N}_9}(15984) = 0.824$, $F_{\widetilde{N}_9}(15984) = 0.846$.

$SC_{\widetilde{N}_9}(15984) = 0.484$.

Also, $T_{\widetilde{N}_{10}}(15984) = 0.702$, $I_{\widetilde{N}_{10}}(15984) = 0.298$, $F_{\widetilde{N}_{10}}(15984) = 0.454$,

$SC_{\widetilde{N}_{10}}(15984) = 1.95$. So, the neutrosophication value of 15984 is $\widetilde{N}_{10}$.

The value of 16859 locates in the range of $\widetilde{N}_{12} = \langle 16350,16650,16950; 0.80,0.20,0.20\rangle$,

$\widetilde{N}_{13} = \langle 16650,16950,17250; 0.90,0.10,0.30\rangle$,then

$T_{\widetilde{N}_{12}}(16859) = 0.242$, $I_{\widetilde{N}_{12}}(16859) = 0.757$, $F_{\widetilde{N}_{12}}(16859) = 0.757$,

$SC_{\widetilde{N}_{12}}(16859) = 0.728$.

Also,

$T_{\widetilde{N}_{13}}(16859) = 0.627$, $I_{\widetilde{N}_{13}}(16859) = 0.373$, $F_{\widetilde{N}_{13}}(16859) = 0.512$,

$SC_{\widetilde{N}_{13}}(16859) = 1.442$. So, the neutrosophication value of 16859 is $\widetilde{N}_{13}$.

The value of 18150 locates in the range of $\widetilde{N}_{16} = \langle 17550,17850,18150; 0.65,0.20,0.35\rangle$, $\widetilde{N}_{17} = \langle 17850,18150,18450; 0.90,0.10,0.10\rangle$, then

$T_{\widetilde{N}_{16}}(18150) = 0$, $I_{\widetilde{N}_{16}}(18150) = 1$, $F_{\widetilde{N}_{16}}(18150) = 1$, $SC_{\widetilde{N}_{16}}(18150) = 0$.

Also, $T_{\widetilde{N}_{17}}(18150) = 0.90$, $I_{\widetilde{N}_{17}}(18150) = 0.1$, $F_{\widetilde{N}_{17}}(18150) = 0.1$,

$SC_{\widetilde{N}_{17}}(18150) = 2.7$. So, the neutrosophication value of 18150 is $\widetilde{N}_{17}$.

The value of 18970 locates in the range of $\widetilde{N}_{19} = \langle 18450,18750,19050; 0.60,0.20,0.30\rangle$,

$\widetilde{N}_{20} = \langle 18750,19050,19350; 0.90,0.10,0.10\rangle$, then

$T_{\widetilde{N}_{19}}(18970) = 0.16$, $I_{\widetilde{N}_{19}}(18970) = 0.786$, $F_{\widetilde{N}_{19}}(18970) = 0.813$.

$SC_{\widetilde{N}_{19}}(18970) = 0.561$.

Also, $T_{\widetilde{N}_{20}}(18970) = 0.66$, $I_{\widetilde{N}_{20}}(18970) = 0.34$, $F_{\widetilde{N}_{20}}(18970) = 0.34$.

$SC_{\widetilde{N}_{20}}(18970) = 1.98$. So, the neutrosophication value of 18970 is $\widetilde{N}_{20}$.

The value of 19328 locates in the range of $\widetilde{N}_{20} = \langle 18750,19050,19350; 0.90,0.10,0.10\rangle$,

$\widetilde{N}_{21} = \langle 19050,19350,19; 0.90,0.10,0.10\rangle$, then

$T_{\widetilde{N}_{20}}(19328) = 0.066$, $I_{\widetilde{N}_{20}}(19328) = 0.992$, $F_{\widetilde{N}_{20}}(19328) = 0.992$.

$SC_{\widetilde{N}_{20}}(19328) = 0.082$. Also,

$T_{\widetilde{N}_{21}}(19328) = 0.834$, $I_{\widetilde{N}_{21}}(19328) = 0.166$, $F_{\widetilde{N}_{21}}(19328) = 0.166$.

$SC_{\widetilde{N}_{21}}(19328) = 2.502$. So, the neutrosophication value of 19328 is $\widetilde{N}_{21}$.

The value of 19337 locates in the range of $\widetilde{N}_{20} = \langle 18750,19050,19350; 0.90,0.10,0.10\rangle$, $\widetilde{N}_{21} = \langle 19050,19350,19; 0.90,0.10,0.10\rangle$, then

$T_{\widetilde{N}_{20}}(19337) = 0.039$, $I_{\widetilde{N}_{20}}(19337) = 0.961$, $F_{\widetilde{N}_{20}}(19337) = 0.961$.

$SC_{\widetilde{N}_{20}}(19337) = 0.117$.

Also, $T_{\widetilde{N}_{21}}(19337) = 0.861$, $I_{\widetilde{N}_{21}}(19337) = 0.139$, $F_{\widetilde{N}_{21}}(19337) = 0.139$.

$SC_{\widetilde{N}_{21}}(19337) = 2.583$. So, the neutrosophication value of 19337 is $\widetilde{N}_{21}$.

Finally, the value of 18876 locates in the range of $\widetilde{N}_{19} = \langle 18450,18750,19050; 0.60,0.20,0.30\rangle$,

$\widetilde{N}_{20} = \langle 18750,19050,19350; 0.90,0.10,0.10\rangle$, then

$T_{\widetilde{N}_{19}}(18876) = 0.348$, $I_{\widetilde{N}_{19}}(18876) = 0.536$, $F_{\widetilde{N}_{19}}(18876) = 0.594$.

$SC_{\widetilde{N}_{19}}(18876) = 1.218$.

Also, $T_{\widetilde{N}_{20}}(18876) = 0.378$, $I_{\widetilde{N}_{20}}(18876) = 0.622$, $F_{\widetilde{N}_{20}}(18876) = 0.622$ ..

$SC_{\widetilde{N}_{20}}(18876) = 1.134$. So, the neutrosophication value of 18876 is $\widetilde{N}_{19}$.

Table 1. Actual and neutrosophication values of students enrollments

Years	Actual enrollments	Neutrosophication values of enrollments $\widetilde{N}$
1971	13055	$\widetilde{N}_1$
1972	13563	$\widetilde{N}_2$
1973	13867	$\widetilde{N}_3$
1974	14696	$\widetilde{N}_6$
1975	15460	$\widetilde{N}_8$
1976	15311	$\widetilde{N}_8$
1977	15603	$\widetilde{N}_8$
1978	15861	$\widetilde{N}_9$
1979	16807	$\widetilde{N}_{13}$
1980	16919	$\widetilde{N}_{13}$
1981	16388	$\widetilde{N}_{11}$
1982	15433	$\widetilde{N}_8$
1983	15497	$\widetilde{N}_8$
1984	15145	$\widetilde{N}_7$
1985	15163	$\widetilde{N}_7$
1986	15984	$\widetilde{N}_{10}$
1987	16859	$\widetilde{N}_{13}$
1988	18150	$\widetilde{N}_{17}$

1989	18970	$\widetilde{N}_{20}$
1990	19328	$\widetilde{N}_{21}$
1991	19337	$\widetilde{N}_{21}$
1992	18876	$\widetilde{N}_{19}$

Step 5: Construct the neutrosophic logical relationships (NLR) as in Table 2:

Table 2. The neutrosophic logical relationship

$\widetilde{N}_1 \to \widetilde{N}_2$	$\widetilde{N}_2 \to \widetilde{N}_3$	$\widetilde{N}_3 \to \widetilde{N}_6$	$\widetilde{N}_6 \to \widetilde{N}_8$	$\widetilde{N}_8 \to \widetilde{N}_8$
$\widetilde{N}_8 \to \widetilde{N}_9$	$\widetilde{N}_9 \to \widetilde{N}_{13}$	$\widetilde{N}_{13} \to \widetilde{N}_{13}$	$\widetilde{N}_{13} \to \widetilde{N}_{11}$	$\widetilde{N}_{11} \to \widetilde{N}_8$
$\widetilde{N}_8 \to \widetilde{N}_7$	$\widetilde{N}_7 \to \widetilde{N}_7$	$\widetilde{N}_7 \to \widetilde{N}_{10}$	$\widetilde{N}_{10} \to \widetilde{N}_{13}$	$\widetilde{N}_{13} \to \widetilde{N}_{17}$
$\widetilde{N}_{17} \to \widetilde{N}_{20}$	$\widetilde{N}_{20} \to \widetilde{N}_{21}$	$\widetilde{N}_{21} \to \widetilde{N}_{21}$	$\widetilde{N}_{21} \to \widetilde{N}_{19}$	

Step 6: Based on NLR, begin to establish the neutrosophic logical relationship groups (NLRG) as in Table 3.

Table 3. The neutrosophic logical relationship groups of enrollments

$\widetilde{N}_1 \to \widetilde{N}_2$		
$\widetilde{N}_2 \to \widetilde{N}_3$		
$\widetilde{N}_3 \to \widetilde{N}_6$		
$\widetilde{N}_6 \to \widetilde{N}_8$		
$\widetilde{N}_7 \to \widetilde{N}_7$	$\widetilde{N}_7 \to \widetilde{N}_{10}$	
$\widetilde{N}_8 \to \widetilde{N}_7$	$\widetilde{N}_8 \to \widetilde{N}_8$	$\widetilde{N}_8 \to \widetilde{N}_9$
$\widetilde{N}_9 \to \widetilde{N}_{13}$		
$\widetilde{N}_{10} \to \widetilde{N}_{13}$		
$\widetilde{N}_{11} \to \widetilde{N}_8$		
$\widetilde{N}_{13} \to \widetilde{N}_{11}$	$\widetilde{N}_{13} \to \widetilde{N}_{13}$	$\widetilde{N}_{13} \to \widetilde{N}_{17}$
$\widetilde{N}_{17} \to \widetilde{N}_{20}$		
$\widetilde{N}_{20} \to \widetilde{N}_{21}$		
$\widetilde{N}_{21} \to \widetilde{N}_{19}$	$\widetilde{N}_{21} \to \widetilde{N}_{21}$	

Step 7: Calculate the forecasted values as in Table 4:

For calculating forecasted value of 13055 in year 1971, do the following:

- Look at the neutrosophication value of 13055 in year 1971 which is $\widetilde{N}_1$ as it appears in Table 1.
- Go to NLRG which is presented in Table 3, and because $\widetilde{N}_1$ is the first neutrosophication value of data, then it is not caused by any other value (i.e. $\neq\rightarrow \widetilde{N}_1$) as in Table 3. Therefore, the forecasted value of 13055 is $-$ which means leaving it empty, as we illustrated in Step 7, Rule 1 of proposed algorithm.

Also, for calculating forecasted value of 13563 in year 1972, do the following:

- Look at the neutrosophication value of 13563 in year 1972 which is $\widetilde{N}_2$ as it appears in Table 1, and because $\widetilde{N}_2$ is caused by $\widetilde{N}_1$ (i.e. $\widetilde{N}_1 \rightarrow \widetilde{N}_2$) , then
- Go to Table 3, and look at the NLRG which starts with $\widetilde{N}_1$, and we noted that it is $\widetilde{N}_1 \rightarrow \widetilde{N}_2$. Then the forecasted value of 13563 is middle value of $\widetilde{N}_2$.

Another illustrating example for calculating forecasted value of 18876 in year 1992:

- Look at the neutrosophication value of 18876 in year 1992 which is $\widetilde{N}_{19}$ as it appears in Table 1. Since $\widetilde{N}_{19}$ is caused by $\widetilde{N}_{21}$, then
- Go to Table 3, and look at the NLRG which starts with $\widetilde{N}_{21}$ (i.e.$\widetilde{N}_{21} \rightarrow \widetilde{N}_{19}, \widetilde{N}_{21} \rightarrow \widetilde{N}_{21}$). Then the forecasted value of 18876 is average of middle values of $\widetilde{N}_{19}$, $\widetilde{N}_{21}$, and it will equal 19050.

The other forecasted values are calculated in the same manners.

Table 4. Actual and forecasted values of enrollments

Years	Actual enrollments	Forecasted values of enrollments
1971	13055	$-$
1972	13563	13650
1973	13867	13950
1974	14696	14850
1975	15460	15450
1976	15311	15450
1977	15603	15450
1978	15861	15450
1979	16807	16950
1980	16919	17150
1981	16388	17150

1982	15433	15450
1983	15497	15450
1984	15145	15450
1985	15163	15600
1986	15984	15600
1987	16859	16950
1988	18150	17150
1989	18970	19050
1990	19328	19350
1991	19337	19050
1992	18876	19050

The actual and forecasted values of enrollments appears in Fig.1.

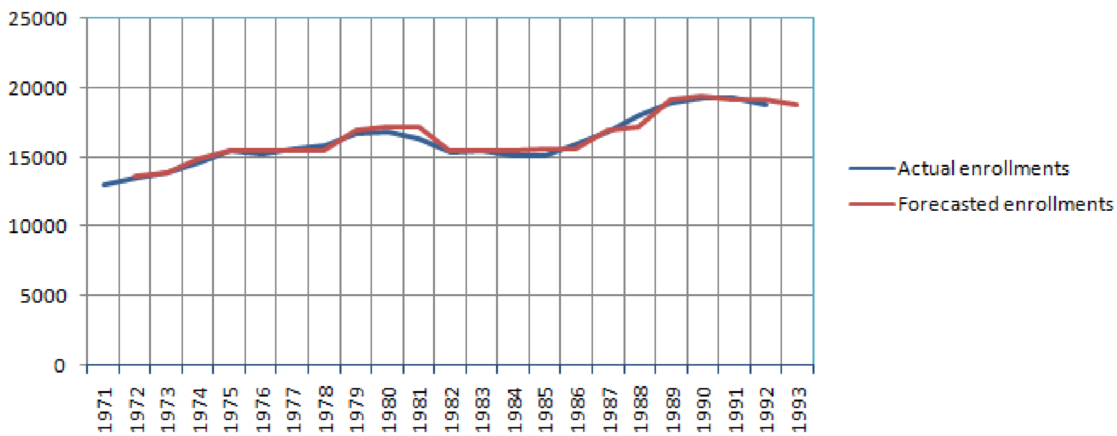

Fig. 1. Forecasted and actual enrollments

The forecasted enrollment data obtained with the suggested method, along with the forecasted data obtained with the models in [43],[44],[45],[46],[14] and [17] are presented in Table 5.

Table 5. Forecasted values by suggested method and other methods

Years	Actual values	Forecasted values						
		Proposed	[43]	[44]	[45]	[46]	[14]	[17]
1971	13055	–	–	–	–	–	–	–
1972	13563	13650	14242.0	14025	13250	14031.35	14586	13693
1973	13867	13950	14242.0	14568	13750	14795.36	14586	13693
1974	14696	14850	14242.0	14568	13750	14795.36	15363	14867
1975	15460	15450	15774.3	15654	14500	14795.36	15363	15287

1976	15311	15450	15774.3	15654	15375	16406.57	15442	15376
1977	15603	15450	15774.3	15654	15375	16406.57	15442	15376
1978	15861	15450	15774.3	15654	15625	16406.57	15442	15376
1979	16807	16950	16146.5	16197	15875	16406.57	15442	16523
1980	16919	17150	16988.3	17283	16833	17315.29	17064	16606
1981	16388	17150	16988.3	17283	16833	17315.29	17064	17519
1982	15433	15450	16146.5	16197	16500	17315.29	15438	16606
1983	15497	15450	15474.3	15654	15500	16406.57	15442	15376
1984	15145	15450	15474.3	15654	15500	16406.57	15442	15376
1985	15163	15600	15474.3	15654	15125	16406.57	15363	15287
1986	15984	15600	15474.3	15654	15125	16406.57	15363	15287
1987	16859	16950	16146.5	15654	16833	16406.57	15438	16523
1988	18150	17150	16988.3	16197	16667	17315.29	17064	17519
1989	18970	19050	19144.0	17283	18125	19132.79	19356	19500
1990	19328	19350	19144.0	18369	18750	19132.79	19356	19000
1991	19337	19050	19144.0	19454	19500	19132.79	19356	19500
1992	18876	19050	19144.0	19454	19500	19132.79	19356	19500

By comparing the proposed method with other existing methods in Table 5, the RMSE and AFE tools confirms that the suggested method is better than others, as it appears in Table 6.

Table 6. Error measures

Tool	Proposed	[43]	[44]	[45]	[46]	[14]	[17]
RMSE	342.68	478.45	781.47	646.67	805.17	642.68	493.56
AFE(%)	1.44	2.39	3.61	2.98	4.28	2.96	2.33

We combined forecasted values with respect to all methods in Fig.2.

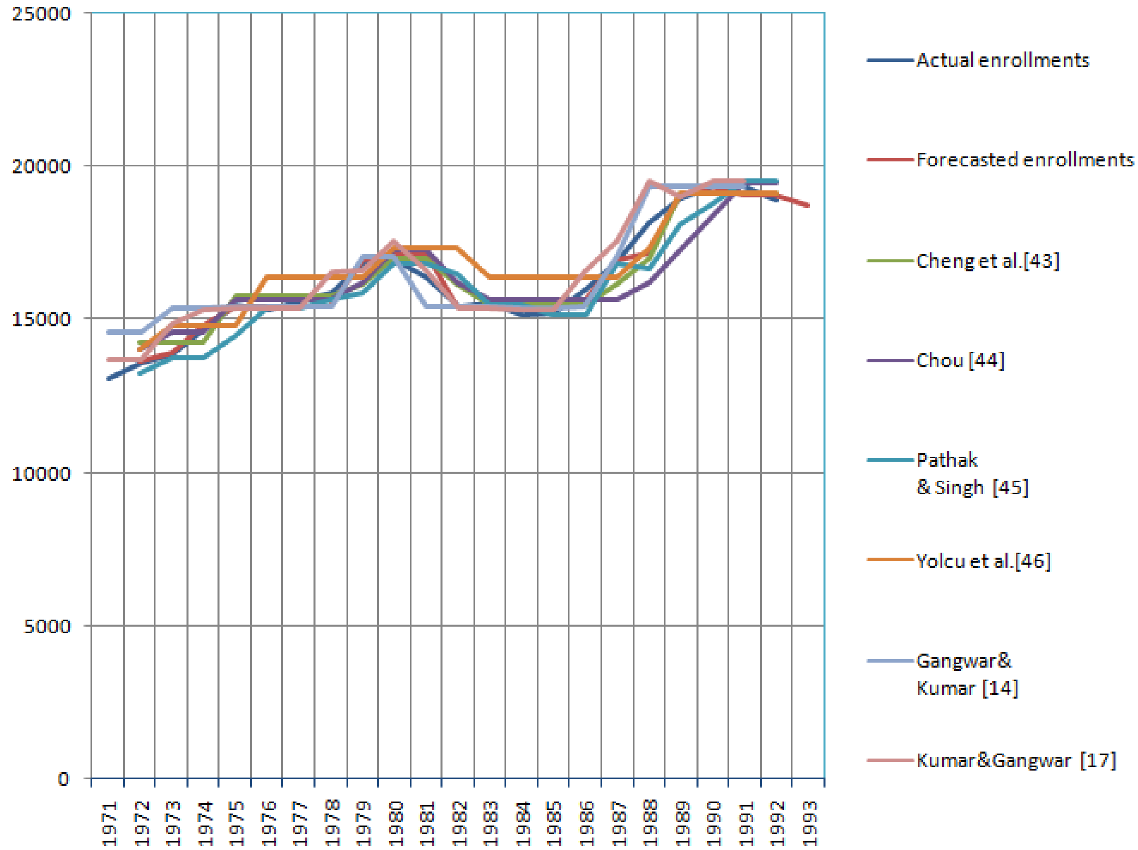

Fig. 2. Comparison figure between all forecasted values

If we plan to find the second order neutrosophic logical relationships of previous example by applying the proposed method of forecasting based on the second order NTS, it will be as in Table 7.

Table 7. Second order-NLR

$\widetilde{N}_1, \widetilde{N}_2 \rightarrow \widetilde{N}_3$
$\widetilde{N}_2, \widetilde{N}_3 \rightarrow \widetilde{N}_6$
$\widetilde{N}_3, \widetilde{N}_6 \rightarrow \widetilde{N}_8$
$\widetilde{N}_6, \widetilde{N}_8 \rightarrow \widetilde{N}_8$
$\widetilde{N}_8, \widetilde{N}_8 \rightarrow \widetilde{N}_8$
$\widetilde{N}_8, \widetilde{N}_8 \rightarrow \widetilde{N}_9$
$\widetilde{N}_8, \widetilde{N}_9 \rightarrow \widetilde{N}_{13}$
$\widetilde{N}_9, \widetilde{N}_{13} \rightarrow \widetilde{N}_{13}$
$\widetilde{N}_{13}, \widetilde{N}_{13} \rightarrow \widetilde{N}_{11}$
$\widetilde{N}_{13}, \widetilde{N}_{11} \rightarrow \widetilde{N}_8$
$\widetilde{N}_{11}, \widetilde{N}_8 \rightarrow \widetilde{N}_8$
$\widetilde{N}_8, \widetilde{N}_8 \rightarrow \widetilde{N}_7$
$\widetilde{N}_8, \widetilde{N}_7 \rightarrow \widetilde{N}_7$

$\tilde{N}_7, \tilde{N}_7 \rightarrow \tilde{N}_{10}$
$\tilde{N}_7, \tilde{N}_{10} \rightarrow \tilde{N}_{13}$
$\tilde{N}_{10}, \tilde{N}_{13} \rightarrow \tilde{N}_{17}$
$\tilde{N}_{13}, \tilde{N}_{17} \rightarrow \tilde{N}_{20}$
$\tilde{N}_{17}, \tilde{N}_{20} \rightarrow \tilde{N}_{21}$
$\tilde{N}_{20}, \tilde{N}_{21} \rightarrow \tilde{N}_{21}$
$\tilde{N}_{21}, \tilde{N}_{21} \rightarrow \tilde{N}_{19}$

The second order neutrosophic logical relationships groups of previous example will be as in Table 8.

Table 8. Second order-NLRG

$\tilde{N}_1, \tilde{N}_2 \rightarrow \tilde{N}_3$		
$\tilde{N}_2, \tilde{N}_3 \rightarrow \tilde{N}_6$		
$\tilde{N}_3, \tilde{N}_6 \rightarrow \tilde{N}_8$		
$\tilde{N}_6, \tilde{N}_8 \rightarrow \tilde{N}_8$		
$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_8$	$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_9$	$\tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_7$
$\tilde{N}_8, \tilde{N}_9 \rightarrow \tilde{N}_{13}$		
$\tilde{N}_9, \tilde{N}_{13} \rightarrow \tilde{N}_{13}$		
$\tilde{N}_{13}, \tilde{N}_{13} \rightarrow \tilde{N}_{11}$		
$\tilde{N}_{13}, \tilde{N}_{11} \rightarrow \tilde{N}_8$		
$\tilde{N}_{11}, \tilde{N}_8 \rightarrow \tilde{N}_8$		
$\tilde{N}_8, \tilde{N}_7 \rightarrow \tilde{N}_7$		
$\tilde{N}_7, \tilde{N}_7 \rightarrow \tilde{N}_{10}$		
$\tilde{N}_7, \tilde{N}_{10} \rightarrow \tilde{N}_{13}$		
$\tilde{N}_{10}, \tilde{N}_{13} \rightarrow \tilde{N}_{17}$		
$\tilde{N}_{13}, \tilde{N}_{17} \rightarrow \tilde{N}_{20}$		
$\tilde{N}_{17}, \tilde{N}_{20} \rightarrow \tilde{N}_{21}$		
$\tilde{N}_{20}, \tilde{N}_{21} \rightarrow \tilde{N}_{21}$		
$\tilde{N}_{21}, \tilde{N}_{21} \rightarrow \tilde{N}_{19}$		

We compared forecasted values of enrollments based on second order of neutrosophic logical relationship groups of the proposed method with the method of second order of Gautam and Singh [47].The results are presented in Table 9.

Table 9. Actual and forecasted values of enrollments based on order 2 of proposed method vs. Gautam and Singh [47] method

Years	Actual enrollments	Forecasted values of order 2 by proposed method	Forecasted values in [47]
1971	13055	–	–
1972	13563	–	–

1973	13867	13950	13800
1974	14696	14850	14400
1975	15460	15450	15300
1976	15311	15450	15300
1977	15603	15450	15600
1978	15861	15450	15600
1979	16807	16950	16800
1980	16919	16950	16800
1981	16388	16350	16200
1982	15433	15450	15300
1983	15497	15450	15300
1984	15145	15450	15000
1985	15163	15150	15000
1986	15984	16050	15900
1987	16859	16950	16800
1988	18150	18150	18000
1989	18970	19050	18900
1990	19328	19350	19200
1991	19337	19350	19200
1992	18876	18750	18600

The MSE and AFE of the two methods are presented in Table 10.

Table 10. Error measures of proposed method and Gautam and Singh method [47]

Tool	Proposed	[47]
MSE	19823.4	24443.4
AFE(%)	0.60	0.81

From Table 10, it appears that our proposed method of second order is also better than the proposed method of second order of Gautam and Singh [47].

In addition, the third order neutrosophic logical relationship groups of previous example is constructed and presented in Table 11.

Table 11. Third order-NLRG

$\tilde{N}_1, \tilde{N}_2, \tilde{N}_3 \rightarrow \tilde{N}_6$
$\tilde{N}_2, \tilde{N}_3, \tilde{N}_6 \rightarrow \tilde{N}_8$
$\tilde{N}_3, \tilde{N}_6, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_6, \tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_8$
$\tilde{N}_8, \tilde{N}_8, \tilde{N}_8 \rightarrow \tilde{N}_9$
$\tilde{N}_8, \tilde{N}_8, \tilde{N}_9 \rightarrow \tilde{N}_{13}$
$\tilde{N}_8, \tilde{N}_9, \tilde{N}_{13} \rightarrow \tilde{N}_{13}$

$$\widetilde{N}_9, \widetilde{N}_{13}, \widetilde{N}_{13} \rightarrow \widetilde{N}_{11}$$
$$\widetilde{N}_{13}, \widetilde{N}_{13}, \widetilde{N}_{11} \rightarrow \widetilde{N}_8$$
$$\widetilde{N}_{13}, \widetilde{N}_{11}, \widetilde{N}_8 \rightarrow \widetilde{N}_8$$
$$\widetilde{N}_{11}, \widetilde{N}_8, \widetilde{N}_8 \rightarrow \widetilde{N}_7$$
$$\widetilde{N}_8, \widetilde{N}_8, \widetilde{N}_7 \rightarrow \widetilde{N}_7$$
$$\widetilde{N}_8, \widetilde{N}_7, \widetilde{N}_7 \rightarrow \widetilde{N}_{10}$$
$$\widetilde{N}_7, \widetilde{N}_7, \widetilde{N}_{10} \rightarrow \widetilde{N}_{13}$$
$$\widetilde{N}_7, \widetilde{N}_{10}, \widetilde{N}_{13} \rightarrow \widetilde{N}_{17}$$
$$\widetilde{N}_{10}, \widetilde{N}_{13}, \widetilde{N}_{17} \rightarrow \widetilde{N}_{20}$$
$$\widetilde{N}_{13}, \widetilde{N}_{17}, \widetilde{N}_{20} \rightarrow \widetilde{N}_{21}$$
$$\widetilde{N}_{17}, \widetilde{N}_{20}, \widetilde{N}_{21} \rightarrow \widetilde{N}_{21}$$
$$\widetilde{N}_{20}, \widetilde{N}_{21}, \widetilde{N}_{21} \rightarrow \widetilde{N}_{19}$$

We also compared the forecasted values of enrollments based on third order of neutrosophic logical relationship groups of proposed method with proposed methods of third order of [47],[8],and [9],and presented the results in Table 12.

Table 12. Actual and forecasted values of enrollments based on order 3 of proposed method vs. [47],[8],[9] methods

Years	Actual enrollments	Forecasted values of order 3 by proposed method	Forecasted values in [47]	Forecasted values in [8]	Forecasted values in [9]
1971	13055	–	–	–	–
1972	13563	–	–	–	–
1973	13867	–	–	–	–
1974	14696	14850	14400	14500	14750
1975	15460	15450	15300	15500	15750
1976	15311	15450	15300	15500	15500
1977	15603	15450	15600	15500	15500
1978	15861	15750	15600	15500	15500
1979	16807	16950	16800	16500	16500
1980	16919	16950	16800	16500	16500
1981	16388	16350	16200	16500	16500
1982	15433	15450	15300	15500	15500
1983	15497	15450	15300	15500	15500
1984	15145	15150	15000	15500	15250
1985	15163	15150	15000	15500	15500
1986	15984	16050	15900	15500	15500
1987	16859	16950	16800	16500	16500
1988	18150	18150	18000	18500	18500

1989	18970	19050	18900	18500	18500
1990	19328	19350	19200	19500	19500
1991	19337	19350	19200	19500	19500
1992	18876	18750	18600	18500	18750

The MSE and AFE of methods presented in Table 13.

Table 13. Error measures of proposed method and [47],[8],[9] methods

Tool	Proposed	[47]	[8]	[9]
MSE	7367.316	25493.6	86694	76509
AFE(%)	0.40	0.82	1.52	1.40

4.2. Numerical example 2

We verified the proposed method by solving the TAIEX2004 example [40], and by putting D_1, D_2 equal 56, and 61 respectively, then U = $[5600.17, 6200.69]$. TAIEX2004 is used as a baseline to compare our method with other competitive methods. The objective is to compare and identify how all the methods can manage error reduction, in which RMSE is a common approach used in financial analysis. For example, Chang [55] has developed a pioneering business intelligence approach in financial stock analysis and used RMSE for error reduction and measurement. To suit our approach, we have devised it, with the aim to calculate the suitable length as illustrated previously and found that it is equal to 40. Therefore, the number of triangular neutrosophic numbers is equal to 12. For these neutrosophic numbers, the decision makers determined the truth, indeterminacy and falsity degrees equal $0.9, 0.1, 0.1$ respectively. The actual and forecasted values of TAIEX2004 example are presented in Table 14 and Fig.3.

Table 14. The actual and forecasted values of TAIEX2004

Dates	Actual values	Forecasted values by proposed method
01/11/2004	5656.17	−
02/11/2004	5759.61	5760.17
03/11/2004	5862.85	5813.5
04/11/2004	5860.73	5900.17
05/11/2004	5931.31	5900.17
08/11/2004	5937.46	5903.02
09/11/2004	5945.2	5903.02
10/11/2004	5948.49	5940.17
11/11/2004	5874.52	5940.17

12/11/2004	5917.16	5903.02
15/11/2004	5906.69	5903.02
16/11/2004	5910.85	5903.02
17/11/2004	6028.68	5940.17
18/11/2004	6049.49	5940.17
19/11/2004	6026.55	5940.17
22/11/2004	5838.42	5830.17
23/11/2004	5851.1	5830.17
24/11/2004	5911.31	5903.02
25/11/2004	5855.24	5830.17
26/11/2004	5778.65	5813.5
29/11/2004	5785.26	5813.5
30/11/2004	5844.76	5860.17
1/12/2004	5798.62	5830.17
02/12/2004	5867.95	5860.17
03/12/2004	5893.27	5900.17
06/12/2004	5919.17	5900.17
07/12/2004	5925.28	5903.02
08/12/2004	5892.51	5903.02
09/12/2004	5913.97	5900.17
10/12/2004	5911.63	5903.02
13/12/2004	5878.89	5903.02
14/12/2004	5909.65	5900.17
15/12/2004	6002.58	5903.02
16/12/2004	6019.23	6040.17
17/12/2004	6009.32	6040.17
20/12/2004	5985.94	6040.17
21/12/2004	5987.85	6040.17
22/12/2004	6001.52	6040.17
23/12/2004	5997.67	6040.17
24/12/2004	6019.42	6040.17
27/12/2004	5985.94	6040.17
28/12/2004	6000.57	6040.17
29/12/2004	6088.49	6040.17
30/12/2004	6100.86	6080.17
31/12/2004	6139.69	6080.17

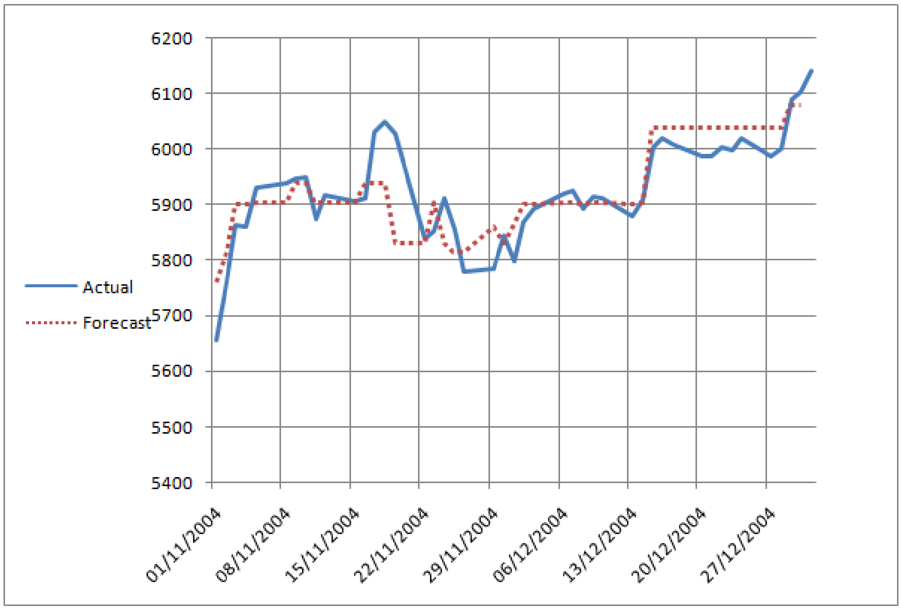

Fig.3. The actual and forecasted values of TAIEX2004

The RMSE and AFE of the proposed method presented in Table 15.

Table 15. Error measures of proposed method

Tool	Proposed
RMSE	42.05
AFE	0.005

For confirming the performance of the suggested method, we compared it with other existing methods and presented the results in Table 16. Compared with the existing methods, our proposed method can offer the least presence of errors since it has the most minimized RMSE. In other words, our method appears to be performing the best in reducing errors and ensuring all our analyses are accurate with insights. This may provide a new insight for business intelligence with artificial intelligence, cloud computing and neutrosophic research.

Table 16. Error measures of proposed method and other existing methods which solved TAIEX2004 example

Methods	RMSE
Guan et al's method [40]	53.01

Huarng et al.’s method [48]	73.57
Chen and Kao’s method [49]	58.17
Cheng et al’s method [50]	54.24
Chen et al’s method [51]	56.16
Chen and Chang’s method [52]	60.48
Chen and Chen’s method [53]	61.94
Yu and Huarng’s method [54]	55.91
Proposed method	42.05

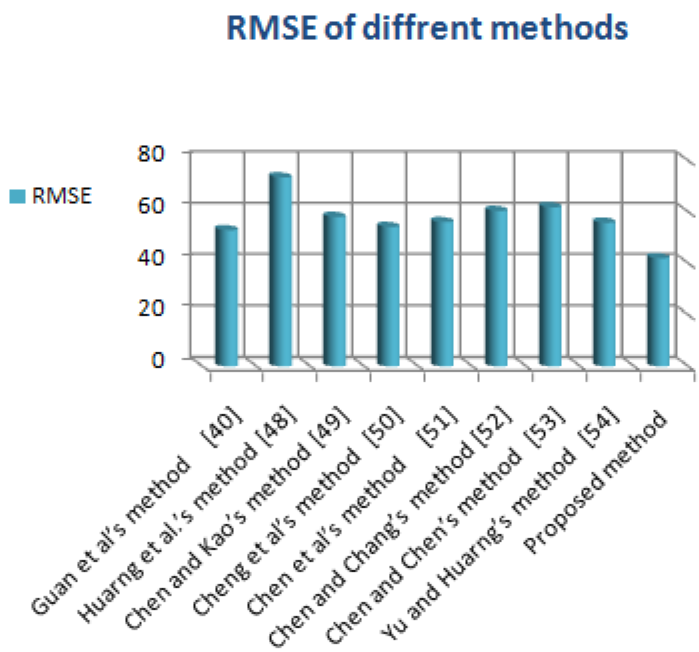

Fig.4. The RMSE of different methods that solved the TAIEX2004 example

By comparing the proposed method with other existing methods as appears in Fig.4 the RMSE tools confirms that our proposed method is better than others.

5. Conclusion and future directions

The objective of this research was to enhance the accuracy rates of forecasting, since the accuracy rates of forecasting in the existing approaches of fuzzy and intuitionistic fuzzy time series were not good enough. Thus, in this research we introduced the notion of first and high-order neutrosophic time series data via defining the fitting length of intervals and proposing a novel method for calculating forecasted values that affect actually in the obtained results. For obtaining truth, indeterminacy and falsity membership degrees of each historical data, we defined triangular neutrosophic numbers. The neutrosophication process of historical time series data depends on the biggest score function of the triangular neutrosophic numbers. For the deneutrosophication process of first and high-order NTS, we used simple arithmetic computations. The suggested approach of first and high-order neutrosophic time series proved its superiority against other existing methods in the field of fuzzy, intuitionistic fuzzy and neutrosophic time series. In the future, we plan to apply meta-heuristic optimization techniques for improving accuracy of the suggested method. **We** will apply this model for predicting other time series, such as demand forecasting, electricity consumption, etc. **Furthermore,** we may consider using other approaches for comparing similarities of historical data, like information entropy.

Authors' Contributions:

All authors have contributed equally to this paper. The individual responsibilities and contribution of all authors can be described as follows: the idea of this whole paper was put forward by Mohamed Abdel-Basset and Mai Mohamed, Victor Chang completed the preparatory work of the paper. Florentin Smarandache analyzed the existing work. The revision and submission of this paper was completed by Florentin Smarandache and Mohamed Abdel-Basset.

Conflicts of Interest: The authors declare no conflict of interest

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