Experience-based Rule Base Generation and Adaptation for Fuzzy Interpolation

Jie Li, Hubert P. H. Shum, Xin Fu, Graham Sexton, Longzhi Yang

Abstract—**Fuzzy modelling has been widely and successfully applied to control problems. Traditional fuzzy modelling requires either complete experts' knowledge or large data sets to generate rule bases such that the input spaces can be fully covered. Although fuzzy rule interpolation (FRI) relaxes this requirement by approximating rules using their neighbouring ones, it is still difficult for some real world applications to obtain sufficient experts' knowledge and/or data to generate a reasonable sparse rule base to support FRI. Also, the generated rule bases are usually fixed and therefore cannot support dynamic situations. In order to address these limitations, this paper presents a novel rule base generation and adaptation system to allow the creation of rule bases with minimal a priori knowledge. This is implemented by adding accurate interpolated rules into the rule base guided by a performance index from the feedback mechanism, also considering the rule's previous experience information as a weight factor in the process of rule selection for FRI. In particular, the selection of rules for interpolation in this work is based on a combined metric of the weight factors and the distances between the rules and a given observation, rather than being simply based on the distances. Two digitally simulated scenarios are employed to demonstrate the working of the proposed system, with promising results generated for both rule base generation and adaptation.**

I. INTRODUCTION

Fuzzy inference is one of the most advanced technologies in control field. It has been widely applied to solve real world problems due to its simplicity and effectiveness in representing and reasoning on human natural language. Examples of such applications include the subway control system in the city of Sendai [1], and the home heating control system [2], amongst others. A typical fuzzy inference system consists of mainly two parts, a rule base (or knowledge base) and an inference engine. A number of inference engines have been developed, with the Mamdani method [3] and the TSK method [4] being the most widely used. The Mamdani model is a particular implementation of the Compositional Rule of Inference [5], which is more intuitive and commonly utilised to deal with human natural language. The TSK approach however can produce crisp values as output and thus is more convenient to be employed when crisp values are required. The traditional fuzzy inference approaches require the entire domains of input variables to be fully covered by rules in a given rule base. Otherwise, no rule can be fired when a given observation does not overlap with any rule antecedent.

A fuzzy rule base is traditionally built upon human expertise, which greatly limits the system modelling as experts may not always be available. Data-driven rule base generation was proposed to minimise the involvement of human expertise, in an effort to automate the generation of rule bases during the system modelling process. Rule base generation from data is usually processed in two steps. Firstly, a raw rule base is generalised from the data, by which the fuzzy partition of variable domains and the number of rules are determined. For instance, neuro-fuzzy system has been employed to initialise membership functions and to extract fuzzy rules from a large data set [6]. Secondly, the raw rule base is fine-tuned using optimisation algorithms, with Genetic Algorithm (GA) being a popular choice [7]. Knowing that the success of data-driven approaches is built on a large amount of training data and complex calculations, the system may perform poorly if the quantity of the available data is not able to reach the minimum requirement for the appropriate use of neural networks or GAs.

Fuzzy rule interpolation (FRI) alleviates the problem of lack of expertise and/or data for rule base generation, as FRI enables the performance of inference upon sparse rule bases [8]. When given observations do not overlap with any rule antecedent values, the traditional fuzzy inference systems cannot be applied. However, fuzzy interpolation through a sparse rule base may still obtain certain conclusions and thus improve the applicability of fuzzy models. FRI also can be used to simplify very complex fuzzy models by removing those rules that can be approximated by their neighbouring ones. Various fuzzy rule interpolation methods have been developed in the literature, including [9], [10], [11], [12], [13]. Thanks to the great flexibility of fuzzy rule bases required by FRI, it has been successfully employed to deal with many real world situations such as [14], [15]. Yet, FRI may still suffer from the extreme sparsity of rule base, which usually leads to poor inference performances.

This paper reports an initial investigation into the feasibility of automatic rule base generation and adaptation from very limited a priori knowledge, which is driven by the grasping and transferring of the proceeding performance experiences. In order to implement such a system, a weight value is associated with each rule, which is the integrated information of the rule's previous experience, including the usage frequency and the performance information. Then, two rules are selected by a combined metric of the weights of rules and the distance from rules to the given input. After a fuzzy interpolation step is performed, a *performance index*, which is generally available for most of the intelligent control problems, is employed to update the weight of rules regarding the previous performance information. Based on the updated usage frequency and previous performance in-

Jie Li, Hubert P. H. Shum, Graham Sexton, Longzhi Yang* are with the Department of Computer Science and Digital Technologies, Faculty of Engineering and Environment, Northumbria University, Newcastle upon Tyne, UK (email: {jie2.li, hubert.shum, g.sexton, longzhi.yang}@northumbria.ac.uk).

Xin Fu is with the Management School, Xiamen University, Xiamen, China (email: xfu@xmu.edu.cn).

formation, the rule base is updated by means of adding a new rule, or deleting an existing rule, which is either out-of-date or proven to be faulty. The experimental results show that the proposed system can start from a very limited number of rules to automatically generate a reasonably complete rule base whilst performing inference, and the generated rule base can be adapted to a new situation by the proposed system after the control model has changed.

The rest of the paper is structured as follows: Section II introduces the theoretical underpinnings of fuzzy rule interpolation (FRI), with a focus on transformation-based FRI upon which this work is built. Section III presents the proposed approach. Section IV details a digital experimentation for demonstration and validation. Section V concludes the paper and suggests probable future developments.

II. FUZZY RULE INTERPOLATION

FRI not only makes fuzzy inference possible when only sparse rule bases are available, but also helps in complexity reduction when very complex rule bases are utilised. The current fuzzy rule interpolation approaches can be categorised into two classes, with a few exceptions (such as type II fuzzy interpolation [16], [17]).

The first class of approaches are able to directly interpolate rules whose antecedent variables are identical to the observed. The most typical approach in this group is the very first proposed fuzzy interpolation [18], denoted as the KH approach, which was developed based on the Decomposition and Resolution Principles [19]. According to these principles, each fuzzy set can be represented by a series of α-cuts ($\alpha \in (0, 1]$). Given a certain α, the α-cut of the consequent fuzzy set is calculated from the α-cuts of the observation and all the fuzzy sets involved in the rules used for interpolation. Knowing the α-cuts of the consequent fuzzy set for all $\alpha \in (0, 1]$, the consequent fuzzy set can be assembled by applying the Resolution Principle. Approaches such as [13], [20], [21], [22] also belong to this group.

The second type of fuzzy interpolation is based on the analogical reasoning mechanism [23] and is referred to as "analogy-based fuzzy interpolation". Methods of this type work by first creating an intermediate rule, such that its antecedent is as 'close' (given a fuzzy distance metric) to the given observation as possible. Then, a conclusion is derived from the given observation by firing the generated intermediate rule through the analogical reasoning mechanism. That is, the shape distinguishability between the resultant fuzzy set and the consequence of the intermediate rule is analogous to the shape distinguishability between the observation and the antecedent of the generated intermediate rule. A number of ways to create an intermediate rule, and then to infer a conclusion from the given observation by the intermediate rule have been developed in the literature, including[9], [10], [24], [25], [26]. In particular, the scale and move transformation-based FRI with triangle fuzzy sets has been employed in this work, this approach is outlined as follows.

Suppose that two neighbouring fuzzy rules for interpolation R_i and R_j ($i \neq j$) are given as:

$$R_i : \textbf{IF} \ \ x \ is \ A_i \ \ \textbf{THEN} \ y \ is \ B_i$$
$$R_j : \textbf{IF} \ \ x \ is \ A_j \ \ \textbf{THEN} \ y \ is \ B_j \ . \tag{1}$$

In this work, each variable value A is represented as a triangular fuzzy set and conveniently denoted as (a_1, a_2, a_3), where (a_1, a_3) is the support of the fuzzy set and a_2 is the normal point. Given observations (A^*), the calculation process of the conclusion using FRI is summarised in the following steps.

Step 1: Calculate the *representative value* of the given observation and each fuzzy set involved in the neighbouring rules for interpolation using the following:

$$Rep(A) = \frac{a_1 + a_2 + a_3}{3}. \tag{2}$$

Step 2: Compute the *relative placement factor* λ based on the relative location of the observation regarding the two antecedents:

$$\lambda = \frac{d(Rep(A_i), Rep(A^*))}{d(Rep(A_i), Rep(A_j))}. \tag{3}$$

Step 3: Obtain the antecedent of the new intermediate rule A' as:

$$A' = (1 - \lambda)A_i + \lambda A_j. \tag{4}$$

Step 4: By comparing the areas of A' and A^*, obtain the *Scale Rate S* using the following:

$$S = \frac{a'_3 - a'_1}{a^*_3 - a^*_1}. \tag{5}$$

Step 5: Apply scale rate S to A' to obtain A'' using the following equations:

$$a''_1 = \frac{a'_1(1 + 2S) + a'_2(1 - S) + a'_3(1 - S)}{3}, \tag{6}$$

$$a''_2 = \frac{a'_1(1 - S) + a'_2(1 + 2S) + a'_3(1 - S)}{3}, \tag{7}$$

$$a''_3 = \frac{a'_1(1 - S) + a'_2(1 - S) + a'_3(1 + 2S)}{3}. \tag{8}$$

Step 6: By comparing the shape difference between A^* and A'', obtain *Move Transformation Rate M*.

$$M = \begin{cases} \frac{3(a''_1 - a^*_1)}{a^*_2 - a^*_1}, & if \ a''_l \geqslant a^*_1 \\ \frac{3(a''_1 - a^*_1)}{a^*_3 - a^*_2}, & if \ c''_l \leqslant a^*_1. \end{cases} \tag{9}$$

Step 7: Compute the consequence B' of the interpolated rule using Equation 4 in the same way as the antecedent of the intermediate rule, as given in Step 3.
Step 8: Obtain the consequence B^* of the given observation by applying S and M to B'.

III. RULE BASE GENERATION AND ADAPTATION

The proposed rule base generation and adaptation system for FRI is introduced in this section, and the system framework outlined in Fig. 1, which comprises of mainly four parts: rule base initialisation, rule selection for interpolation, transaction-based FRI, and rule base revision. Firstly, an initial set of rules is generated from limited a priori knowledge. For a given observation, the system then selects the 'best' two rules from the current rule base for interpolation, based on a particular set of metrics, including the usage frequency/experience information, the previous *performance index* and the distances between the given observation and the rule antecedents. From this, a new rule is interpolated based on the selected 'best' rules from the given observation. Afterwards, the *performance index* will be utilised to support the rule base updating, whenever it is available from the feedback system.
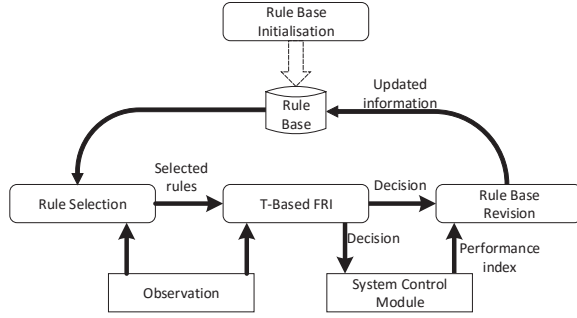

Fig. 1. The framework of the proposed system

A. Rule Base Initialisation

Traditionally, fuzzy rule bases can be generated from either human expertise or historic data, yet neither of them may be sufficiently available during the process of fuzzy modelling. For instance, most existing smart heater control systems require usage data for the rule base generation, as it is very difficult, if not impossible, for human experts to accurately learn the control patterns for different users regarding their living habits. However, collecting such data usually takes a long time, and the control systems cannot make intelligent decisions before this initial modelling progress has been done. This paper addresses such problems to enable fuzzy modelling with very limited a priori knowledge. For simplicity, in this initial investigation, it is assumed that all the rules describing the boundary of the problem space are always available, although fundamentally, the idea underlying the proposed approach can be extended to support the situations where the rule base contains at least 2 rules for any single step of inference (even random ones), which remains for future work.

A weight is assigned to each individual rule in the rule base when the rule is created, which provides a measure to help select the 'best' rules for interpolation in FRI, and will be discussed in Section III-D2. In this work, only rules with single antecedent are considered. Without losing generality, suppose the following two fuzzy rules R_i and R_j are transformed from either human expertise or historic data:

$$R_i : \textbf{IF} \quad x \; is \; A_i \; , \quad \textbf{THEN} \quad y \; is \; B_i \; (w_i, \; EF_i, \; CD_i)$$
$$R_j : \textbf{IF} \quad x \; is \; A_j \; , \quad \textbf{THEN} \quad y \; is \; B_j \; (w_j, \; EF_j, \; CD_j), \tag{10}$$

where w represents the weight of the rule which is introduced in details later; EF stands for *experience factor* and represents the usage and effectiveness information of the particular rule in the previous FRI progresses. It can be increased or decreased during the system running, based on the employability of the rule and the effectiveness of the interpolated results. The rule with greater EF indicates that it is of more experience and is more likely to generate a accurate result. CD stands for the *cooling down factor* and represents the times that the concerned rule has not been selected continuously so far. The introduction of CD allows the control system to identify the rules that are less likely to be selected for interpolation.

In the progress of the rule base initialisation, EF and CD are also initialised. CD is always reconfigured as 0 once the corresponding rule has been selected to perform interpolation, based on its physical meaning. In this initial work, for simplicity, EF is initialised as 50 based on initial investigation through experimentation. Briefly, larger EF usually leads to have a longer convergence time, and a smaller EF may result to rule removal unexpectedly. A further study of the initialisation of this factor remains for future work. These two factors jointly decide the importance or weight of rule R_i in the rule base as:

$$w_i = (\frac{2}{1 + e^{-\frac{EF_i}{n}}} - 1)(1 - \frac{1}{1 + 5e^{-\frac{CD_i}{a}+b}}), \tag{11}$$

where a, b, n are sensitivity factors ($b > 3$, $a, n \neq 0$). Smaller value of b and greater value of a and n make the system less sensitive to the rule weight, and vice versa. The values of these factors need to be determined based on the specific problems, but some early stage experimentation generally suggests $4 \leqslant b \leqslant 10$, $1 \leqslant a \leqslant 100$, and $1 \leqslant n \leqslant 200$.

B. Rule Selection

Fuzzy rule interpolation is utilised in this work to perform fuzzy inferences. In order to enable the utilisation of FRI, two rules need to be selected for interpolation. Dissimilar from traditional FRI approaches that select the two closest rules for interpolation (for a given distance metric), the proposed system selects rules for interpolation based on an importance factor (IF) with regard to a given input, which is a combined metric of rule weights and the distance between the given input and rule antecedents. The rule with the greatest IF value on each side of the observation is selected and used in the process of fuzzy rule interpolation.

Given an input "x is A^*", suppose that there are n rules in the rule base "**IF** x is A_i, **THEN** y is B_i (w_i, EF_i, CD_i)", $i \in \{1, 2, ..., n\}$, then the importance factor for each rule can be calculated as follows:

$$IF_i = \sqrt{\lambda'_i} \; w_i. \tag{12}$$

where w_i is computed using Equation 11, and $\lambda_i^{'}$ is the *inverse distance weighting factor* (IDWF) $\lambda_i^{'}$ for each rule in the rule base using the following equation:

$$\lambda_i^{'} = \frac{\frac{1}{d_i}}{\sum\limits_{l=1}^{n} \frac{1}{d_l}}. \tag{13}$$

In this equation, d_i is the distance between the given observation A^* and rule antecedent A_i, which is calculated as the Euclidean distance between their *representative values* using Equation 2.

C. Rule Interpolation

Once the rules for interpolation have been selected, rule interpolation can be performed. As introduced in Section II, there are main two types of FRI approaches, including the direct rule interpolation approach and the analogical reasoning approach. The system proposed herein can readily work with any analogical reasoning approach, although it can be potentially extended to work with the direct interpolation approach, which is beyond the scope of this paper and remains for future work. The scale and move transformation-based FRI approach is able to handle both interpolation and extrapolation, and also guarantee the uniqueness as well as normality and convexity of the resulting interpolated fuzzy sets. Therefore, this approach is employed in this work, which has been introduced in Section II, and thus the details are omitted here.

D. Rule Base Revision

A feedback mechanism is typically included in an intelligent control system to represent the system performance, which indicates the difference between the actual and desired outputs. Furthermore, a quantitative measure of the performance of a system, generally called *performance index* (PI), is always considered by optimum control systems during the process of parameter configuration or adjustment. Noticing the difficulty in retrieving the accurate desired results in control systems, the feedback system is capable of indicating if the control decision works or not. This feedback is used effectively in this work to support the rule base generation and adaptation. The working progress of rule base revision is outlined in Fig. 2 where n denotes the number of rules in the rule base. Each of its components is elaborated in the rest of this section.

1) Adding Rules:
High quality interpolated rules can be reused in the future, and thus an interpolated rule that has successfully made a decision result will be added into the current rule base. In order to avoid redundant or duplicated rules, the similarity degree between the interpolated and existing rules are calculated. Suppose that the interpolated rule is R^* ("**IF** x is A^*, **THEN** y is B^* (w^*, EF^*, CD^*)"). Given a rule R_i ("**IF** x is A_i, **THEN** y is B_i (w_i, EF_i, CD_i)") in the rule base, then the degree of similarity S_i between the interpolated rule and R_i can be calculated as:

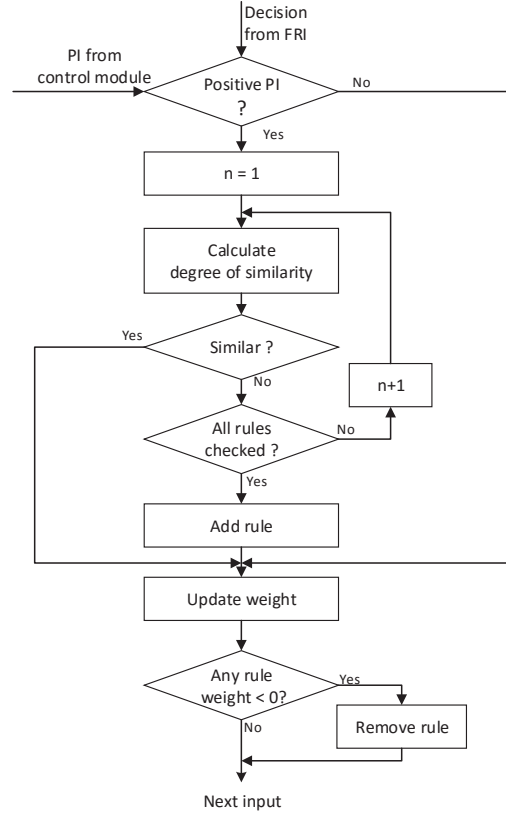$$S_i = \frac{S(A_i, A^*) + S(B_i, B^*)}{2}, \tag{14}$$

Fig. 2. The rule base revision procedure

where $S(A_i, A^*)$ and $S(B_i, B^*)$ are the similarity degrees between the antecedents and the consequences of the interpolated rule and R_i, respectively. The similarity degree between two fuzzy sets A_i and A^* in this work is defined as:

$$S(A_i, A^*) = 1 - \frac{|a_{i1} - a_1^*| + |a_{i2} - a_2^*| + |a_{i3} - a_3^*|}{3}. \tag{15}$$

The similarity between B_i and B^* can be calculated in the same way. Given a threshold, if the similarity degree between the interpolated rule and the existing rule reaches a certain threshold value, the system believes that the interpolated rule is redundant, which will be ignored; otherwise, the interpolated rule will be added into rule base.

Suppose that the neighbouring rules which have been utilised to interpolate rule R^* are R_i and R_j, ($1 \leqslant i, j \leqslant n$ and $i \neq j$), the *experience factor* EF of the new rule is computed as:

$$EF = (1 - \frac{d_i}{d})EF_i + (1 - \frac{d_j}{d})EF_j, \tag{16}$$

where d_i and d_j represents the distances between the observation A^* and antecedents A_i, A_j respectively; d represents the distance between the two antecedents A_i and A_j; and EF_i and EF_j represent the current *experience factors* of rules R_i and R_j, respectively. The value of CD_i is initialised as 0.

2) Updating Weights:

Once a decision is inferred or an interpolated rule is generated, the values of EF and CD for each rule will be updated. In particular, if a rule R_i is not employed for FRI during this step of interpolation performance, the value of CD_i will be increased by 1; and EF_i will remain the same. If the rule has been selected to perform FRI and the generated decision supports the system positively based on the *performance index*, the EF_i value will be increased by 1 for this rule and CD_i value is reset to 0; otherwise if a negative *performance index* is returned, the value of EF_i will be decreased by 1 and the value of CD_i will be reset to 0. These updating operations are summarised in the follow table.

TABLE I.   THE UPDATING OPERATIONS

Situation		Operation
Rule R_i Employed	Positive PI	$EF_i + 1$, $CD_i = 0$
	Negative PI	$EF_i - 1$, $CD_i = 0$
Rule R_i Not Employed		$CD_i + 1$

Based on the updated EF and CD values, the weight factor (w) of each rule will also be accordingly updated using Equation 11. Given rule R_i, if the updated CD_i is equal to 0, the *cooling Down factor* CD_i does not have any effect according to Equation 11. Then, the original weight factor of this particular rule will be:

$$w_i = \frac{2}{1 + e^{-\frac{EF_i}{n}}} - 1. \tag{17}$$

If the updated CD_i is greater than 0, which means that rule R_i has not been selected to perform interpolation for a while, the weight of this rule will be reduced by a certain percentage according to the value of parameter CD_i. The final revised weight w_i' can be computed by:

$$w_i' = w_i(1 - \frac{1}{1 + 5e^{-\frac{CD_i}{a} + b}}). \tag{18}$$

3) Removing Rules:

The rule base adaptation mechanism provides a function allowing redundant or out-of-date rules to be removed from the current rule base. The judgment for the latter situation is made based on their weight factors. In particular, if the weight of rule R_i rule is reduced to less than 0 ($w_i < 0$), then rule R_i will be removed from the rule base immediately. As aforementioned, two situations may lead to the reduction of the weight of rule R_i: 1) rule R_i has been used, but it resulted in incorrect decision, which is indicated by a negative *performance index*; and 2) The rule has not been used for a while, resulting in its weight fading out over time.

The interpolated result may not be accurate enough in the beginning of the deployment of the system, as the initiated rule base may not be sufficiently accurate and of sufficient rules to support FRI. Despite this, the system is still able to generate results and make decisions. However, the patterns will be adaptively learned by the system along with the performance of fuzzy interpolation, and thus the generated results will increasingly better reflect the real situation. If the current situation has changed, the decision making system will gently adapt to the new situation by removing incorrect

rules and adding new high quality interpolated rules in the rule base. Although this may take a while, the system is able to generate a new rule base to reflect the new situation in time.

IV. EXPERIMENTATION

In order to validate and evaluate the proposed approach, a non-linear function is employed in this section to demonstrate the rule base generation functionality. Another similar yet different function is utilised to illustrate the adaptation ability of the proposed approach.

A. Rule Base Generation

Suppose the pattern to be modeled can be represented by a smooth curve of Equation 19, shown in Fig. 3, where x represents the input domain, and y represents the output domain. Assume that the system inputs are vague values that are simulated by randomly generated fuzzy sets within the input domain of x. As the system inputs are usually linguistic values, in order to preserve the comprehensibility and for easy interpretation, the generated fuzzy sets for simulation are always normal and convex.

$$y = 2\sin(\frac{2x}{3}) + 5, x \in [2, 12]. \tag{19}$$
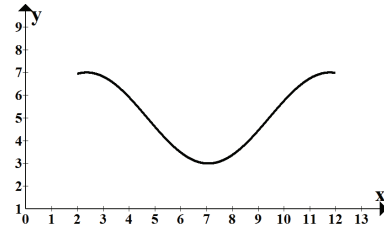

Fig. 3.   The scenario curve to be modelled (Equation 19)

The very first step to utilise the proposed modelling approach is to build an initial rule base, which is usually implemented by a very limited number of the most general rules. Theses rules may be provided by domain experts or simply a good guess of the most typical situations and thus the initialised rules may not be very accurate. In this initial work, for simplicity, suppose 3 initial rules are available in the format of "R_i : **IF** x is A_i , **THEN** y is B_i (w_i, EF_i, CD_i)", ($i = \{1, 2, 3\}$) with all the involved fuzzy sets listed in Table II. EF_i, $i = \{1, 2, 3\}$, are initially configured as 50

TABLE II.   THE INITIALISED RULE BASE

i	A_i	B_i	w_i	EF_i	CD_i
1	(2.00, 2.50, 3.00)	(6.50, 7.00, 7.50)	0.12	50	0
2	(6.50, 7.00, 7.50)	(2.50, 3.00, 3.50)	0.12	50	0
3	(11.00, 11.50, 12.00)	(6.50, 7.00, 7.50)	0.12	50	0

(based on brute force trying), and then the weight of each rule is accordingly initialised as $w_1 = w_2 = w_3 = 0.12$, as shown in Table II. In the implementation of this work, the weight factor calculation function is implemented as follows:

$$w_i = (\frac{2}{1 + e^{-\frac{EF_i}{200}}} - 1)(1 - \frac{1}{1 + 5e^{-\frac{CD_i}{100} + 5}}). \tag{20}$$

To better illustrate the proposed system, this section focuses on a typical step of inference and rule generation, rather than the very first step from the initial rule base, due to space limit. After performing 50 interpolation inferences, the rule base includes 8 rules (denoted as $R_i, i \in \{1, 2, ..., 8\}$) as shown in Fig. 4(b), and listed in the Table III.

TABLE III. THE EVOLVED RULE BASE USED IN THE EXAMPLE

i	A_i	B_i	w_i	EF_i	CD_i
1	$(2.00, 2.50, 3.00)$	$(6.50, 7.00, 7.50)$	0.057	23	10
2	$(3.98, 4.38, 5.29)$	$(4.73, 5.13, 5.74)$	0.049	19	41
3	$(4.53, 4.87, 5.06)$	$(4.68, 4.88, 5.02)$	0.033	13	20
4	$(4.97, 5.58, 7.39)$	$(2.83, 3.37, 4.30)$	0.050	20	20
5	$(6.50, 7.00, 7.50)$	$(2.50, 3.00, 3.50)$	0.129	53	0
6	$(9.23, 10.11, 10.54)$	$(4.81, 5.32, 5.59)$	0.061	24	41
7	$(10.50, 11.00, 11.50)$	$(5.90, 6.50, 6.70)$	0.105	43	0
8	$(11.00, 11.50, 12.00)$	$(6.50, 7.00, 7.50)$	0.087	35	10

(a) The initial rule base

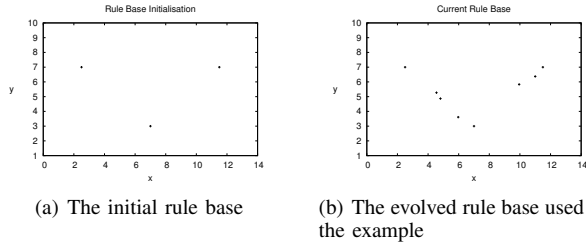(b) The evolved rule base used in the example

Fig. 4. Rule bases at different stages

From this, the next step of rule generation is elaborated as follows:

[Step 1] System input: For each individual step of inference performance, the system starts from taking an observation as system input. In this example, suppose that an observation $A^* = (7.00, 7.30, 9.00)$ is given as system input.

[Step 2] Rules selection: Based on the given observation and the current rule base, the system calculates the importance factor of each rule regarding the given input by Equation 12. The details of the intermediate and final results of the calculation are shown in the Table IV. According to the calculated result, the rule with the greatest importance factor on each side of the given observation will be selected to perform FRI. In this particular example, rules R_5 and R_7 are selected by the proposed approach rather than the closest rules R_5 and R_6 according to the existing FRI approaches.

TABLE IV. THE CALCULATION FOR RULE SELECTION

i	d_i	λ_i'	w_i	EF_i	CD_i	IF_i	Left rule or Right rule?
1	5.27	0.05	0.057	23	10	0.0129	L
2	3.22	0.08	0.049	19	41	0.0142	L
3	2.95	0.09	0.033	13	20	0.0100	L
4	1.79	0.15	0.050	20	20	0.0194	L
5	0.77	0.35	0.129	53	0	0.0764	L
6	2.19	0.12	0.061	24	41	0.0212	R
7	3.23	0.08	0.105	43	0	0.0303	R
8	3.73	0.07	0.087	35	10	0.0234	R

[Step 3] Transformation-based FRI: When the two rules for interpolation are determined with regard to the given observation, the HS approach [9], [10], which is a transformation based fuzzy rule interpolation approach, is employed to generate the inference result. Firstly, based on the values

of rule antecedents and the given observation, the relative placement factors (λ) is calculated, which is $\lambda = 0.07$. Secondly, the scale rate is obtained as $s = 0.5$. Then, the move ratio is calculated: $m = 0.7$. Finally, the interpolation result is achieved as $B^* = (2.76, 3.26, 3.74)$. After the defuzzification using the centre of gravity principle, the crisp result $B^* = 3.20$ is generated as system output.

[Step 4] Performance index: Based on the given observation ($A^* = (7.00, 7.30, 9.00)$, $A^* = 7.7$ after defuzzification) and the simulated model as entailed in Equation 19, the desired result should be 3.20. As the system output is equal to the desired output, a positive *performance index* is returned. Note that the desired result is usually not available or obtainable in most of the control systems at any stage, but it is common that a *performance index* is available after the interpolated result has been utilised. The *performance index* clearly indicates if the interpolated result was acceptable or not. The rule base is then updated next according to the value of the returned *performance index*.

[Step 5] Rule base updating: The rules R_5 and R_7 were used to generate the interpolated result, which supports the system running correctly, and as a result, the *experience factor* and *cooling down factor* of both rules will be accordingly updated. Although the rest of rules in the rule base were not selected to preform this particular FRI, their CD will also be updated. The updating operations of the current rule base is shown in Table V.

TABLE V. THE OPERATIONS OF RULE BASE UPDATING

i	A_i	B_i	w_i	EF_i	CD_i
1	$(2.00, 2.50, 3.00)$	$(6.50, 7.00, 7.50)$	$0.057 \rightarrow 0.057$	23	$10 \rightarrow 11$
2	$(3.98, 4.38, 5.29)$	$(4.73, 5.13, 5.74)$	$0.049 \rightarrow 0.045$	19	$41 \rightarrow 42$
3	$(4.53, 4.87, 5.06)$	$(4.68, 4.88, 5.02)$	$0.033 \rightarrow 0.032$	13	$20 \rightarrow 21$
4	$(4.97, 5.58, 7.39)$	$(2.83, 3.37, 4.30)$	$0.050 \rightarrow 0.049$	20	$20 \rightarrow 21$
5	$(6.50, 7.00, 7.50)$	$(2.50, 3.00, 3.50)$	0.129	$53 \rightarrow 54$	0
6	$(9.23, 10.11, 10.54)$	$(4.81, 5.32, 5.59)$	$0.061 \rightarrow 0.056$	24	$41 \rightarrow 42$
7	$(10.50, 11.00, 11.50)$	$(5.90, 6.50, 6.70)$	0.105	$43 \rightarrow 44$	0
8	$(11.00, 11.50, 12.00)$	$(6.50, 7.00, 7.50)$	$0.087 \rightarrow 0.087$	35	$10 \rightarrow 11$

Due to the positive performance of the interpolation inference, the interpolated rule will be added into the rule base as a new rule for future use unless a similar rule already exists in the current rule base. The degree of similarity between each of the existing rules and the interpolated rule can be calculated using Equations 14 and 15, and the results are summarised in Table VI. A negative similarity degree indicates that neither the antecedents of two compared rules nor the consequences of them overlap with each other. Define the similarity threshold as 0.7 in this work. It is clear from this table, no similar rule exists in the current rule base regarding the interpolated rule, that is, the degree of similarity between every rule and the interpolated rule is less than 0.7. Therefore, the interpolated rule is added into the current rule base as the R_9. The *experience factor* of this new rule EF_9 is calculated using Equation 16, and the cooling down value CD_9 is set as 0. The details of this rule is shown in Equation 21.

$$\mathbf{R}_9 : \mathbf{IF} \quad x = A^* = (7.00, 7.30, 9.00)$$
$$\mathbf{THEN} \quad y = B^* = (2.76, 3.26, 3.74)(0.324, 52, 0)$$
$$(21)$$

The above exemplar interpolation step demonstrates the situation where a satisfied result is generated by FRI and

TABLE VI. SIMILARITY DEGREE BETWEEN EXISTING RULES AND INTERPOLATED RULE

	Rule1	Rule2	Rule3	Rule4	Rule5	Rule6	Rule7	Rule8
Similarity	-2.74	-1.01	0.61	0.65	0.67	-1.54	-2.11	-2.74

the interpolated rule is added into the rule base. When a negative *performance index* is returned, the system will work differently. For instance, the next observation is $A^* = (6.50, 8.10, 9.30)$. Then, rules R_5 and R_7 are selected to preform FRI instead of the closest rules R_5 and R_6. From this, the generated result $B^* = (3.44, 3.97, 4.39)$ is interpolated, which results in $B^* = 3.93$ after defuzzification. This system output is quite different with the desired value which is 3.36, thus, a negative *performance index* will be returned. Consequently, this interpolated rule will be ignored, and the *experience factors* of these two rules will be decreased by 1, as punishment. Of course, the *experience factor* and *cooling down factor* of all other rules will also be updated based on Table I, with the details omitted here to save space.

The system repeats the above process for every new input, and it will be stabilised after a number of performance iterations. Particularly for the given example, the system rule base becomes stable after 3000 inference performance, resulted in a rule base with 36 rules. The evolvement of the rule base for the running example is illustrated in Fig. 5.

(a) Rule base after 1000 interpolation performance

(b) Rule base after 2000 interpolation performance

(c) Rule base after 2600 interpolation performance

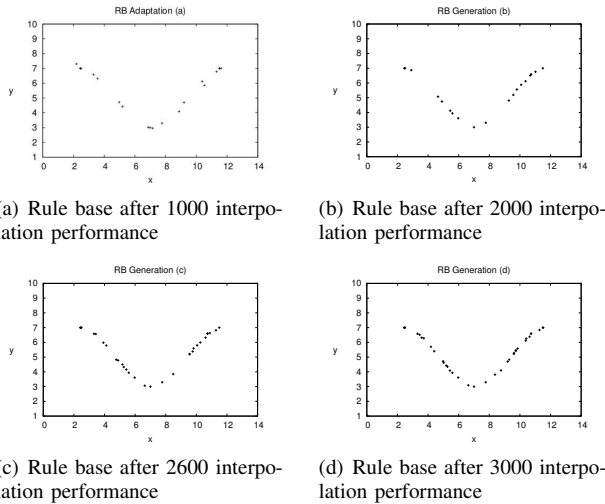(d) Rule base after 3000 interpolation performance

Fig. 5. The processes of rule base generation

B. Rule Base Adaptation

The proposed system is not only able to learn the model pattern whilst performing interpolation inference, but is also able to adapt the current rule base to a changed model pattern. In order to demonstrate how the proposed system handles the changes of the underlying model pattern, assume the ground truth model pattern has changed from the pattern shown in Fig. 3 to the patten shown in Fig. 6(a) (corresponding to Equation 22), after 3,000 interpolation performances by

which the rule base is illustrated in Fig. 5(d).

$$y = \frac{(x-7)^2}{5} + 3, x \in [2, 12]. \tag{22}$$

Due to the change of the underlying pattern to be modeled, the previous rule base will not be able to generate satisfied results from time to time. Therefore, the weight factor of some of the existing rules will be dramatically decreased and they will be gradually removed along with the performance of interpolation inferences. Of course, if a positive *performance index* is returned regarding a certain step of interpolation inference, this particular rule will be added into rule base. These operations are exactly the same with the ones introduced above, and thus the details are omitted here. The overall evolvement progress of the rule base is illustrated in Fig. 6(b)- 6(d).

(a) A changed pattern to be modeled

(b) Rule base adaptation after 3500 of interpolation inferences

(c) Rule base adaptation after 4200 of interpolation inferences

(d) Rule base adaptation after 5000 of interpolation inferences
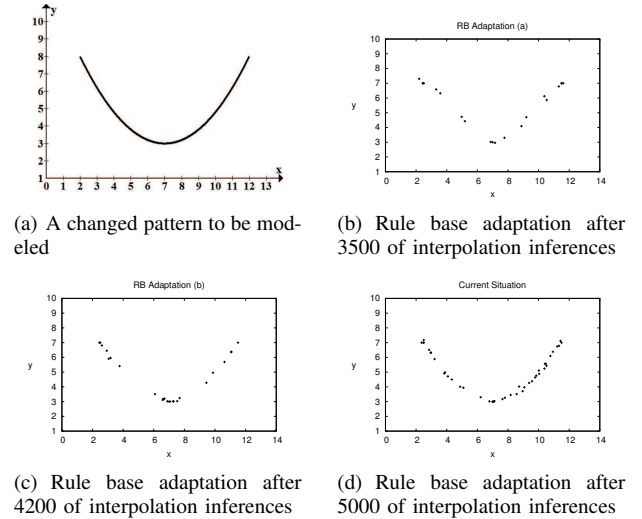
Fig. 6. The processes of rule base adaptation

C. Discussion

The above experimentation results show that the proposed system is able to adaptively generate the rule base and revise it whenever the underlying model has changed. Note that rule base updating and generation approaches for FRI have been proposed in the literature, such as [27], [28]. Although the approaches proposed in [27] is able to promote new rules into the original sparse rule base by collecting and aggregating the interpolated result, a reasonable initial rule base is still required and only a fixed pattern can be modelled. In addition, fuzzy rule interpolation-based Q-learning (FRIQ-learning) was proposed to construct FRI fuzzy model from scratch, based on different environment reward [29]. It is desirable to further compare the performance of these systems and to investigate how FRIQ-learning may be used to support the proposed work herein.

The proposed system may provide solutions for some real world problems, such as smart home control system. FRI has been successfully employed to a smart home heating management system [14], where the rule base was predefined based on the historic data of a particular property and residents and thus it is only able to deal with fixed per-defined

situations. Two benefits will take place if the proposed system is applied. Firstly, only the most general/common a priori knowledge is required to initialise the system, thus the heating management system can be mass-produced (commercialised). Secondly, the system is able to handle changing situation such as change of radiators, residents, or their living styles, thus the model is highly adaptable. The development of this is under progress.

V. CONCLUSION

This paper presented a novel rule base generation and adaptation approach for FRI, which is able to adaptively generate and revise the rule base with limited training data and/or expert knowledge for control problems, as long as a *performance index* is available to indicate if the inference result is acceptable or not. In particular, the system initialises the rule base with very limited rules first. Then, based on the existing rules' usage frequency information, historic performance information, and distances between observation and existing rules, the best two rules are selected for FRI rather than the two closest neighbouring rules as implemented in existing FRI approaches. Finally, the rule base is adaptively generated and revised, which is guided by the *performance index* of the interpolated result and the performance experiences of rules. The simulation experimentation suggests that the proposed system is able to automatically generate and adapt rule bases to enhance FRI.

Although promising, the work can be further extended in the following areas. Firstly, the EF values are arbitrarily given in this work based on some initial experimentation, and thus it would be worthwhile to further study how this parameter can be automatically determined/learned. Secondly, an incremental FRI model creation approach was proposed in [29]. Therefore, it is interesting to compare the two approaches. Thirdly, given that the proposed system is built upon transaction-based FRI, it is worthwhile to investigate how the proposed system may support others FRI approaches such as [13]. Finally, the proposed system can be further validated by real-world applications.

REFERENCES

[1] S. Yasunobu and S. Miyamoto. Automatic train operation system by predictive fuzzy control. *Industrial applications of fuzzy control*, 1(18):1–18, 1985.

[2] C. Von Altrock, B. Krausse, and C. Steffens. Customer-adaptive fuzzy logic control of home heating system. In *Fuzzy Systems, 1994. IEEE World Congress on Computational Intelligence., Proceedings of the Third IEEE Conference on*, pages 1713–1718 vol.3, 1994.

[3] E. H. Mamdani. Application of fuzzy logic to approximate reasoning using linguistic synthesis. *Computers, IEEE Transactions on*, C-26(12):1182–1191, 1977.

[4] T. Takagi and M. Sugeno. Fuzzy identification of systems and its applications to modeling and control. *Systems, Man and Cybernetics, IEEE Transactions on*, SMC-15(1):116–132, 1985.

[5] L. A. Zadeh. The concept of a linguistic variable and its application to approximate reasoningi. *Information Sciences*, 8(3):199 – 249, 1975.

[6] W.A. Farag, V.H. Quintana, and G. Lambert-Torres. A genetic-based neuro-fuzzy approach for modeling and control of dynamical systems. *Neural Networks, IEEE Transactions on*, 9(5):756–767, 1998.

[7] C. C. Chen and C. C. Wong. Self-generating rule-mapping fuzzy controller design using a genetic algorithm. *Control Theory and Applications, IEE Proceedings -*, 149(2):143–148, 2002.

[8] L. T. Kóczy and K. Hirota. Interpolative reasoning with insufficient evidence in sparse fuzzy rule bases. *Information Sciences*, 71(1):169–201, 1993.

[9] Z. Huang and Q. Shen. Fuzzy interpolative reasoning via scale and move transformations. *Fuzzy Systems, IEEE Transactions on*, 14(2):340–359, 2006.

[10] Z. Huang and Q. Shen. Fuzzy interpolation and extrapolation: A practical approach. *Fuzzy Systems, IEEE Transactions on*, 16(1):13–28, 2008.

[11] L.Yang and Q. Shen. Adaptive fuzzy interpolation and extrapolation with multiple-antecedent rules. In *Fuzzy Systems (FUZZ), 2010 IEEE International Conference on*, pages 1–8, 2010.

[12] L.Yang and Q. Shen. Adaptive fuzzy interpolation. *Fuzzy Systems, IEEE Transactions on*, 19(6):1107–1126, Dec 2011.

[13] L.Yang and Q. Shen. Closed form fuzzy interpolation. *Fuzzy Sets and Systems*, 225:1 – 22, 2013. Theme: Fuzzy Systems.

[14] J. Li, L. Yang, H. P. H. Shum, G. Sexton, and Y. Tan. Intelligent home heating controller using fuzzy rule interpolation. In *UK Workshop on Computational Intelligence*, 2015.

[15] G.I. Molnarka, S. Kovacs, and L.T. Kóczy. Fuzzy rule interpolation based fuzzy signature structure in building condition evaluation. In *Fuzzy Systems (FUZZ-IEEE), 2014 IEEE International Conference on*, pages 2214–2221, 2014.

[16] S. Chen and Y. Chang. A new method for fuzzy rule interpolation based on the ratio of fuzziness of interval type-2 fuzzy sets. In *Fuzzy Systems (FUZZ), 2010 IEEE International Conference on*, pages 1–6, 2010.

[17] L. Yang, C. Chen, N. Jin, X. Fu, and Q. Shen. Closed form fuzzy interpolation with interval type-2 fuzzy sets. In *Fuzzy Systems (FUZZ-IEEE), 2014 IEEE International Conference on*, pages 2184–2191. IEEE, 2014.

[18] L. Kóczy and K. Hirota. Approximate reasoning by linear rule interpolation and general approximation. *International Journal of Approximate Reasoning*, 9(3):197–225, 1993.

[19] L.A. Zadeh. Quantitative fuzzy semantics. *Information Sciences*, 3(2):159 – 176, 1971.

[20] Y. Chang, S. Chen, and C. Liau. Fuzzy interpolative reasoning for sparse fuzzy-rule-based systems based on the areas of fuzzy sets. *Fuzzy Systems, IEEE Transactions on*, 16(5):1285–1301, 2008.

[21] W. Hsiao, S. Chen, and C. Lee. A new interpolative reasoning method in sparse rule-based systems. *Fuzzy Sets and Systems*, 93(1):17 – 22, 1998.

[22] S. Chen and W. Hsin. Weighted fuzzy interpolative reasoning based on the slopes of fuzzy sets and particle swarm optimization techniques. *Cybernetics, IEEE Transactions on*, 45(7):1250–1261, 2015.

[23] B. Bouchon-Meunier and L. Valverde. A fuzzy approach to analogical reasoning. *Soft Computing*, 3(3):141–147, 1999.

[24] P. Baranyi, L.T. Kóczy, and T.D. Gedeon. A generalized concept for fuzzy rule interpolation. *Fuzzy Systems, IEEE Transactions on*, 12(6):820–837, 2004.

[25] Q. Shen and L. Yang. Generalisation of scale and move transformation-based fuzzy interpolation. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 15(3):288–298, 2011.

[26] S. Cheng, S. Chen, and C. Chen. Fuzzy interpolative reasoning based on ranking values of polygonal fuzzy sets and automatically generated weights of fuzzy rules. *Information Sciences*, 325:521 – 540, 2015.

[27] N. Naik, R. Diao, and Q. Shen. Genetic algorithm-aided dynamic fuzzy rule interpolation. In *Fuzzy Systems (FUZZ-IEEE), 2014 IEEE International Conference on*, pages 2198–2205, 2014.

[28] T. D. Gedeon, S. Kovacs, and L.T. Kóczy. Sparse fuzzy systems generation and fuzzy rule interpolation: a practical approach. In *Fuzzy Systems, 2003. FUZZ '03. The 12th IEEE International Conference on*, volume 1, pages 494–499 vol.1, 2003.

[29] D. Vincze and S. Kovács. *Computational Intelligence in Engineering*, chapter Incremental Rule Base Creation with Fuzzy Rule Interpolation-Based Q-Learning, pages 191–203. Springer Berlin Heidelberg, Berlin, Heidelberg, 2010.