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A New Noise-Tolerant and Predefined-Time ZNN Model for Time-Dependent Matrix Inversion

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Abstract

In this work, a new zeroing neural network (ZNN) using a versatile activation function (VAF) is presented and introduced for solving time-dependent matrix inversion. Unlike existing ZNN models, the proposed ZNN model not only converges to zero within a predefined finite time but also tolerates several noises in solving the time-dependent matrix inversion, and thus called new noise-tolerant ZNN (NNTZNN) model. In addition, the convergence and robustness of this model are mathematically analyzed in detail. Two

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comparative numerical simulations with different dimensions are used to test the efficiency and superiority of the NNTZNN model to the previous ZNN models using other activation functions. In addition, two practical application examples (i.e., a mobile manipulator and a real Kinova JACO^2 robot manipulator) are presented to validate the applicability and physical feasibility of the NNTZNN model in a noisy environment. Both simulative and experimental results demonstrate the effectiveness and tolerant-noise ability of the NNTZNN model.

Keywords:
Zeroing neural network, Recurrent neural network, Time-dependent matrix inversion, Noise tolerance, Finite-time convergence.

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1. Introduction

The matrix inversion has been widely used in practical engineering applications, such as MIMO and robotics [1, 2, 3]. Besides, numerous principles of machine operation in real life can be explained by matrix inversion. For example, in robotics [4, 5, 6], the path tracking is a typical task for any robot manipulators. To achieve this task successfully, the Jacobian matrix of a robot manipulator has to be solved online according to the given path. When the Jacobian matrix has been obtained, the corresponding control law can be solved and described by joint-angle or joint-velocity variables, which can drive the robot manipulator to complete the path-tracking task. Obviously, the matrix inversion is closely related to the age of artificial intelligence that people are yearning for. It is very important to find a better method for matrix inversion with faster convergence and stronger robustness.

In the past, the methods commonly used for matrix inversion were numerical approaches such as methods of iteration [7, 8, 9]. For example, Zhang *et al.* [9] used the Newton iterative method to solve matrix inversion problem and compared it with neural network methods. Although iterative and other numerical methods are effective for solving matrix inversion in some cases, they may have high computational complexity when dealing with large-scale data [10]. In general, most of numerical methods have $O(n^3)$ computational complexity per iteration with n being matrix size [11, 12, 13].

Unlike traditional numerical approaches such as iterative methods, recurrent neural networks are attracting the researchers' interest due to the important features such as parallel processing and hardware implementation, and widely applied in various fields [14, 15, 16, 17, 18]. For example, Coban *et al.* presented a kind of recurrent neural network for dynamic system identification and quadratic optimal controller design [15, 16, 17, 18]. The zeroing neural network (ZNN), as a continuous solving model, is also employed to solve matrix inversions [19, 20, 21, 22]. For example, based on a matrix-valued error, a continuous ZNN model was proposed for matrix inversion, and several computer simulation examples verified the validity of this model for time-dependent matrix inversion [21]. In [22], Guo *et al.* proposed a discrete ZNN model, which was successfully applied to finding the time-dependent matrix inverse. To further improve the performance of the ZNN model, some specially constructed activation functions [such as linear activation function (LAF), hyperbolic sine activation function (HSAF), sgn-bi-power activation function (SBPAF), power-sum activation function (PSAF)] were employed to accelerate the ZNN model [23, 24, 25, 26, 27, 28]. For example, Li *at al.* [24] for the first time explored SBPAF to speedup the ZNN model to finite-time convergence for the Sylvester equation solving. In [27], a finite-time convergent neural network based on SBPAF was further proposed and used to solve the time-dependent complex matrix equations, and the simulation consequences verified its finite-time convergence performance. Unlike the idea of [27] which uses SBPAF to speed up the solution process of ZNN, a novel ZNN model [29] was proposed on the basis of a new design formula for matrix inversion with finite-time convergence also guaranteed.

It is worth pointing out that although some ZNN models have finite-time convergence, they do not consider the impact of external disturbances. That is to say, the above-mentioned ZNN models are conducted in an ideal condition. However, external disturbances are inevitable in the real world. If the denoising capability of the above-mentioned ZNN models is limited and may not solve the problem accurately when there are various noise disturbances. Therefore, some improved ZNN models with the denoising capability are also gradually proposed and employed for practical engineering applications [30, 31, 32, 33, 34, 35, 36, 37]. In [31], an integrated enhanced ZNN model with noise-tolerant performance was proposed by Jin *et al.* to inverse a time-dependent matrix, and this model can tolerate constant noise, time-dependent noise and random noise, etc. In [33], an enhanced discrete-time ZNN model was proposed to find the time-dependent matrix inverse in the presence of bias noise, and the convergence of this model in solving the matrix inversion with biased noise has also been proved. In [34], a new neural network model on the basis of the sign-bi-power nonlinear activation function and an integral formula is proposed for the dynamic Sylvester equation in the presence of noise, and its finite-time convergence and robustness are also analyzed and proved in detail. In [35], a ZNN model with denoising capability and finite-time convergence was proposed by Xiao *et al.* and successfully employed to solve dynamic quadratic minimization problems under noisy interference.

As reviewed above, to improve the convergence speed, the sign-bi-power activation function is presented to modify the performance of ZNNs and able to make them achieve finite-time convergence. However, the upper bound of this finite-time convergence is related to initial states of the corresponding ZNN models, which will lead to a big upper bound if initial errors are relatively large. Based on the above consideration, different from the method of constructing a noise-tolerant ZNN model based on an integral design formula in [30, 31, 32, 34, 35], in the current work, we are devoted to studying a versatile activation function (VAF) to design a new noise tolerant ZNN (NNTZNN) model for time-independent matrix inversion. Compared with the previous ZNN models either finite-time convergence or noise-tolerant property, the proposed NNTZNN model not only has a strong noise capability but also has a predefined finite-time convergence. In addition, the upper bound of the predefined convergence time for the NNTZNN model is independent to its initial states (i.e., the upper bound of the predefined convergence time is known). More detailed comparisons about these models can be seen from Table 1. To the best of authors' knowledge, this is the first time to propose such an NNTZNN model by using VAF, which features inherent noise-tolerance and predefined finite-time convergence when meeting external additive noises. More importantly, the convergence and robustness of the NNTZNN model are mathematically rigorously demonstrated in theorems. In addition, to numerically verify the efficacy and generalization of the proposed NNTZNN model, two different dimensional time-independent matrix examples and a robotic application are presented in the simulation part. The simulation results also demonstrate the efficiency, superiority and applicability of NNTZNN using VAF for solving time-dependent matrix inversion.

At the end of this section, the primary contributions of this work are listed as below

- Different from the methods of constructing a noise-tolerant ZNN model based on an integral design formula in [30, 31, 32, 34, 35], in the current

Table 1: The main differences of the NNTZNN model from other models (i.e., GNN model, ZNN model and EIZNN model) for time-independent matrix inversion [19, 20, 31, 32]

#	Item	GNN	ZNN	EIZNN	NNTZNN
1	Target	static	dynamic	dynamic	dynamic
2	Error	no-zero	zero	no-zero	zero
3	AF	no	yes	no	yes
4	Model	explicitly	implicitly	implicitly	implicitly
5	Speed	asymptotically	finitely	exponentially	predefined
6	Robustness	weaker	weak	strong	stronger

work, we are devoted to proposing and studying a versatile activation function (VAF) to design a new noise tolerant ZNN (NNTZNN) model for time-independent matrix inversion.

- Compared with the traditional ZNN models activated by LAF, PSAF, and SBPAF (including integral-enhanced ZNN models), the proposed NNTZNN model not only has better predefined-time convergence performance, but also achieves robustness against various kinds of noises.

- The upper bound of the predefined convergence time for the proposed NNTZNN model is theoretically calculated under different external disturbances, which shows the superior robustness of the proposed NNTZNN model. In addition, as compared to the finite-time convergence that is related to initial states of existing ZNN models, the predefined-time covnergence is a major theoretical breakthrough for ZNN.

- Two numerical comparative simulations with different dimensions are used as test examples in a noisy environment to validate the efficiency, and superiority of the proposed NNTZNN model. In addition, two practical applications are conducted on different robotic platforms to demonstrate the applicability and physical feasibility of the proposed NNTZNN model.

For convenience, the mathematical notations and the model parameters used in the paper are presented as below.

$L(t)$	coefficient of time-independent matrix inversion
$E(t)$	error function $E(t) = L(t)X(t) - I$
$e_{i,j}(t)$	the i, jth element of $E(t)$
$Y(t)$	additive noises
$\Phi(\cdot)$,	nonlinear activation function arrays
$\phi(\cdot)$	the element of $\Phi(\cdot)$
$\lambda > 0$	design parameter of NNTZNN
$\mathrm{sgn}(\cdot)$	the sign function
$0 < \eta < 1$, $w > 1$	design parameters of VAF
$a_1 > 0$, $a_2 > 0$	design parameters of VAF
$a_3 \geqslant 0$, $a_4 \geqslant 0$	design parameters of VAF
t_c	convergence upper bound of NNTZNN

2. Preliminaries

In this section, in order to make the process of the proof and solution more convenient, some basic preparations for finding the inverse of the time-dependent matrix are given as below.

2.1. Mathematical Preparation

In general, a recurrent neural network can be represented as a differential dynamic system in mathematics, which is formed by

$$\dot{\mathrm{x}}(t) = s(\mathrm{x}(t), t), \ t \in [0, +\infty) \tag{1}$$

where $\mathrm{x}(t) \in \mathbb{R}^n$ represents an appropriately sized system state. Let $\mathrm{x}(0) = \mathrm{x}_0$ represent an appropriately sized initial state for this system, and assume that $\mathrm{x}(t) = 0$ is the equilibrium state of the system. There are some concepts related to the convergence for this system (1), which are presented as follows for completeness of this work [38, 39, 40, 41, 42, 43, 44].

Definition 1. The origin of system (1) is globally finite-time stable if it is globally and asymptotically stable; and there exists a locally bounded settling-time function $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$, such that $\mathrm{x}(t, \mathrm{x}_0) = 0$ for all $t \geqslant T(\mathrm{x}_0)$.

Definition 2. The origin of system (1) is globally predefined-time stable if the system is globally finite-time stable and the settling-time function T is globally bounded, i.e., there exists a constant $t_c \in \mathbb{R}_+$ satisfying $t_c \geqslant T(\mathrm{x}_0)$ for all $\mathrm{x}_0 \in \mathbb{R}^n$.

Lemma 1. If there exists a continuous radially unbounded function U : $\mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that $U(\zeta) = 0$ for $\zeta \in S$ and any solution $\zeta(t)$ satisfies

$$\dot{U}(t) \leqslant -\tau U^{\eta}(\zeta(t)) - \rho U^{w}(\zeta(t)),$$

where parameters $\tau > 0$, $\rho > 0$, $0 < \eta < 1$ and $w > 1$ are constants, then the set S is globally predefined-time attractive for system (1), and the upper bound for the predefined time convergence is

$$t_c = \frac{1}{\tau(1-\eta)} + \frac{1}{\rho(w-1)}.$$

2.2. Problem Formulation

In mathematics, the time-dependent matrix inversion problem is generally formulated as the following dynamic matrix equation:

$$\mathrm{L}(t)\mathrm{X}(t) = I \in R^{n\times n}, \text{or } \mathrm{X}(t)\mathrm{L}(t) = I \in R^{n\times n},\ t \in [0, +\infty) \tag{2}$$

where $\mathrm{L}(t) \in R^{n\times n}$ represents an invertible time-dependent coefficient matrix, $\mathrm{X} \in R^{n\times n}$ represents an unknown time-dependent matrix, and $I \in R^{n\times n}$ represents an appropriately sized identity matrix. Without loss of generality, let $\mathrm{X}^* \in R^{n\times n}$ represent the theoretical solution of (2). This current work focuses on finding an unknown $\mathrm{X}(t)$ using the proposed NNTZNN model within predefined finite time under the interference of various noises (such as constant noise, time-dependent bounded or unbounded noise).

3. NNTZNN Model

In this section, the NNTZNN model will be proposed for time-dependent matrix inversion. The detailed design process is presented as follows.

Considering problem (2), according to the design method of ZNN [19, 20, 21, 22, 45], a time-dependent error function $E(t)$ is defined as follows:

$$\mathrm{E}(t) = \mathrm{L}(t)\mathrm{X}(t) - I \in R^{n\times n}. \tag{3}$$

Then, a design formula for $E(t)$ is given directly as

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = -\lambda\Phi(E(t)), \tag{4}$$

Table 2: Comparisons and differences of commonly used activation functions.

Activation Function	Formulation
LAF	$\phi(x) = x$
BPAF	$\phi(x) = (1 - \exp(-\xi x))/(1 + \exp(-\xi x))$ with $\xi > 1$
PAF	$\phi(x) = x^l$ with $l > 3$ indicating an odd integer
SAF	$\phi(x) = \begin{cases} x^l, & \text{if } \lvert x \rvert \geq 1 \\ \frac{1+\exp(-\xi)}{1-\exp(-\xi)} \cdot \frac{1-\exp(-\xi x)}{1+\exp(-\xi x)}, & \text{otherwise} \end{cases}$
HSAF	$\phi(x) = (\exp(\xi x) - \exp(-\xi x))/2$ with $\xi > 1$
SBPAF	$\phi(x) = (\lvert x \rvert^l + \lvert x \rvert^{1/l})\text{sgn}(x)/2$ with $0 < l < 1$
VAF (this work)	$\phi(x) = (a_1 \lvert x \rvert^\eta + a_2 \lvert x \rvert^w)\text{sgn}(x) + a_3 x + a_4 \text{sgn}(x)$

where $\Phi(\cdot)$ represents an activation function with each element denoted by $\phi(\cdot)$ and $\lambda > 0$ represents a resizable design parameter to adjust the convergence rate of the neural network. Substituting equation (3) into the formula (4), one can get the following initial ZNN model:

$$\mathrm{L}(t)\dot{\mathrm{X}}(t) = -\dot{\mathrm{L}}(t)\mathrm{X}(t) - \lambda\Phi(\mathrm{L}(t)\mathrm{X}(t) - I), \tag{5}$$

where the meanings of λ and $\Phi(\cdot)$ are the same as before. Considering that various noises may exist during the actual problem solving, it is better to study a noise-perturbed ZNN model, which is directly given as follows:

$$\mathrm{L}(t)\dot{\mathrm{X}}(t) = -\dot{\mathrm{L}}(t)\mathrm{X}(t) - \lambda\Phi(\mathrm{L}(t)\mathrm{X}(t) - I) + \mathrm{Y}(t) \tag{6}$$

where $\mathrm{Y}(t)$ represents a universal noise.

Generally speaking, different activation functions for a neural network can lead to different convergence and stability. In the past few years, many activation functions (e.g., BPAF, SBPAF) have been studied to speed up the convergence of neural networks, and some of that even reach finite time convergence. However, the denoising capability of the ZNN models using these activation functions is not considered. That is to say, when perturbed by noises, these models may be no longer effective. In order to overcome this drawback, the following versatile activation function (VAF) will be added to the above presented ZNN model to solve the time-dependent matrix inversion problem under different noise pollution environments [42, 43]:

$$\phi(x) = (a_1|x|^\eta + a_2|x|^w)\text{sgn}(x) + a_3 x + a_4\text{sgn}(x), \tag{7}$$

where design parameters $0 < \eta < 1$, $w > 1$, $a_1 > 0$, $a_2 > 0$, $a_3 \geqslant 0$, $a_4 \geqslant 0$ and $\mathrm{sgn}(\cdot)$ denotes the sign function. For better understanding, if ZNN model (5) is activated by VAF (7), this model is termed as the new noise-tolerant ZNN (NNTZNN) model; and if the noise-perturbed ZNN model (6) is activated by VAF (7), this model is termed as the noise-perturbed NNTZNN model. Besides, the main differences and formulations of the commonly used activation functions are compared and listed in Table 2. In addition, different from other activation functions for ZNN, when the VAF is used, the NNTZNN model can converge to the theoretical solution within a predefined finite time when solving the time-dependent matrix inversion problem, regardless of whether there is bounded vanishing or non-vanishing noise. That is to say, the NNTZNN model has faster convergence speed and stronger robustness, as compared ZNN activated by other activation functions.

4. Theoretical Analysis

In the above section, the NNTZNN model with finite time convergence performance was deduced step by step to find the time-dependent matrix inversion. In this section, the predefined finite-time convergence and robustness of the NNTZNN model will be theoretically analyzed in detail under different noise environments.

4.1. NNTZNN in the Absence of Noises

The following theorem guarantees the predefined finite-time convergence of NNTZNN model (5) activated by VAF (7) in the absence of noises.

Theorem 1. Assume that time-dependant matrix $\mathrm{L}(t)$ in equation (2) is smooth and invertible for $t \in [0, +\infty)$. If VAF (7) is used, then neural-state matrix $\mathrm{X}(t)$ of NNTZNN model (5), starting from an arbitrary initial matrix $\mathrm{X}(0) \in \mathrm{R}^{n\times n}$, converges to the theoretical time-dependent matrix inversion $\mathrm{X}^*(t)$ of $\mathrm{L}(t)$ in predefined time t_c:

$$t_{\mathrm{c}} \leqslant \frac{1}{\lambda a_1(1-\eta)} + \frac{1}{\lambda a_2(w-1)}.$$

Proof. Since $\mathrm{E}(t) = \mathrm{L}(t)\mathrm{X}(t) - \mathrm{I}$, NNTZNN model (5) is simplified as $\dot{\mathrm{E}}(t) = -\lambda\Phi(\mathrm{E}(t))$ which entry-wisely consists of the following n^2 subsystems:

$$\dot{e}_{i,j}(t) = -\lambda\phi(e_{i,j}(t)) \text{ with } i, j \in \{1, 2, \cdots, n\}$$

where matrix $\dot{\mathrm{E}}(t)$ denotes the time derivative of matrix $\mathrm{E}(t)$ and scalars $e_{i,j}(t)$ and $\dot{e}_{i,j}(t)$ are the ijth elements of matrices $\mathrm{E}(t)$ and $\dot{\mathrm{E}}(t)$, respectively. Evidently, dynamics of each element in the error function $\mathrm{E}(t)$ is independent and self-autonomous.

Define a Lyapunov function candidate $u(t) = |e_{i,j}(t)|$ for the ijth subsystem. The time derivative of $u(t)$ is

$$\dot{u}(t) = \dot{e}_{i,j}(t)\mathrm{sgn}(e_{i,j}(t)) = -\lambda\phi(e_{i,j}(t))\mathrm{sgn}(e_{i,j}(t)).$$

When VAF (7) is used, one can obtain

$$\begin{aligned}\dot{u}(t) &= -\lambda(a_1|e_{i,j}(t)|^\eta + a_2|e_{i,j}(t)|^w + a_3|e_{i,j}(t)| + a_4)\\ &\leqslant -\lambda(a_1|e_{i,j}(t)|^\eta + a_2|e_{i,j}(t)|^w)\\ &= -\lambda(a_1u^\eta(t) + a_2u^w(t)).\end{aligned}$$

On basis of *Lemma 1* mentioned in Section 2, one can obtain the convergence time of the ijth subsystem for NNTZNN model (5):

$$t_{i,j} \leqslant \frac{1}{\lambda a_1(1-\eta)} + \frac{1}{\lambda a_2(w-1)}.$$

Since the upper bound is a constant that is independent on the initial conditions of the ijth subsystem of NNTZNN model (5) and time t, the maximum convergence upper bound of NNTZNN model (5) is obtained as

$$t_{\mathrm{c}} = \max(t_{i,j}) \leqslant \frac{1}{\lambda a_1(1-\eta)} + \frac{1}{\lambda a_2(w-1)}.$$

Hence, NNTZNN model (5) activated by VAF (7) exhibits a predefined-time convergence property. The predefined-time convergence of NNTZNN model (5) activated by VAF (7) is theoretically proved.

4.2. NNTZNN in the Presence of Noises

In practical implementation of a neural network model, there always exist unavoidable additive noises. Hence, it is worth investigating the convergence performance of the noise-perturbed NNTZNN model (6).

4.2.1. Dynamic Bounded Gradually Disappearing Noise

When perturbed by a dynamic bounded vanishing noise Y(t), the following result can ensure the predefined-time convergence of the noise-perturbed NNTZNN model (6).

Theorem 2. Assume that time-dependant matrix L(t) in equation (2) is smooth and invertible for $t \in [0, +\infty)$, and NNTZNN model (6) is perturbed by a matrix noise Y(t) with its ijth entry satisfying $|y_{i,j}(t)| \leqslant \delta |e_{i,j}(t)|$ where $\delta \in (0, +\infty)$ and $|e_{i,j}(t)|$ denotes the ijth absolute element of error function (3). If VAF (7) is used with $\lambda a_3 \geqslant \delta$, then neural-state matrix X(t) of the noise-perturbed NNTZNN model (6), starting from an arbitrary initial matrix $\mathrm{X}(0) \in \mathbb{R}^{n\times n}$, converges to the theoretical time-dependent matrix inversion $\mathrm{X}^*(t)$ of L(t) in predefined time t_c:

$$t_{\mathrm{c}} \leqslant \frac{1}{\lambda a_1(1-\eta)} + \frac{1}{\lambda a_2(w-1)}.$$

Proof: As the same as *Theorem 1*, the noise-perturbed NNTZNN model (6) can be simplified as $\dot{\mathrm{E}}(t) = -\lambda\Phi(\mathrm{E}(t)) + \mathrm{Y}(t)$ that entry-wisely consists of the following n^2 subsystems:

$$\dot{e}_{i,j}(t) = -\lambda\phi(e_{i,j}(t)) + y_{i,j}(t) \text{ with } i, j \in \{1, 2, \cdots, n\}, \tag{8}$$

where matrix $\dot{\mathrm{E}}(t)$ denotes the time derivative of matrix E(t) and scalars $e_{i,j}(t)$, $\dot{e}_{i,j}(t)$ and $y_{i,j}(t)$ are the ijth elements of matrices E(t), $\dot{\mathrm{E}}(t)$ and Y(t), respectively.

Define a Lyapunov function candidate $u(t) = |e_{i,j}(t)|^2$ for the ijth subsystem. The time derivative of $u(t)$ is

$$\dot{u}(t) = 2e_{i,j}(t)\dot{e}_{i,j}(t) = 2e_{i,j}(t)(-\lambda\phi(e_{i,j}(t)) + y_{i,j}(t)).$$

When VAF (7) is used with $\lambda a_3 \geqslant \delta$, one can obtain

$$
\begin{aligned}
\dot{u}(t) = &-2\lambda(a_1|e_{i,j}(t)|^{\eta+1} + a_2|e_{i,j}(t)|^{w+1}) - 2\lambda a_4|e_{i,j}(t)| \\
&+2(e_{i,j}(t)n_{i,j}(t) - \lambda a_3|e_{i,j}(t)|^2) \\
\leqslant &-2\lambda(a_1|e_{i,j}(t)|^{\eta+1} + a_2|e_{i,j}(t)|^{w+1}) \\
&+2(\delta|e_{i,j}(t)|^2 - \lambda a_3|e_{i,j}(t)|^2) \\
\leqslant &-2\lambda(a_1|e_{i,j}(t)|^{\eta+1} + a_2|e_{i,j}(t)|^{w+1}) \\
= &-2\lambda(a_1 u^{\frac{\eta+1}{2}}(t) + a_2 u^{\frac{w+1}{2}}(t)).
\end{aligned}
$$

On basis of *Lemma 1* mentioned in Section 2, the convergence time of the noise-perturbed NNTZNN model (6) is

$$t_{\rm c} \leqslant \frac{1}{\lambda a_1(1-\eta)} + \frac{1}{\lambda a_2(w-1)}.$$

Hence, NNTZNN model (6) activated by VAF (7) under a dynamic bounded vanishing noise exhibits a predefined-time convergence property.

4.2.2. Dynamic Bounded Non-Disappearing Noise

When the noise-perturbed NNTZNN model (6) with dynamic bounded non-vanishing noises is involved, one has the following theorem to ensure the predefined-time convergence property.

Theorem 3. Assume that time-dependant matrix $\mathrm{L}(t)$ in equation (2) is smooth and invertible for $t \in [0, +\infty)$, and the noise-perturbed NNTZNN model (6) is perturbed by a matrix noise $\mathrm{Y}(t)$ with its ijth entry satisfying $|y_{i,j}(t)| \leqslant \delta$ where $\delta \in (0, +\infty)$. If VAF (7) is used with $\lambda a_4 \geqslant \delta$, then neural-state matrix $\mathrm{X}(t)$ of the noise-perturbed NNTZNN model (6), starting from an arbitrary initial matrix $\mathrm{X}(0) \in \mathbb{R}^{n\times n}$, converges to the theoretical time-dependent matrix inversion $\mathrm{X}^*(t)$ of $\mathrm{L}(t)$ in predefined time $t_{\rm c}$:

$$t_{\rm c} \leqslant \frac{1}{\lambda a_1(1-\eta)} + \frac{1}{\lambda a_2(w-1)}.$$

Proof. Define a Lyapunov function candidate $u(t) = |e_{i,j}(t)|^2$ for the ijth subsystem depicted in (8). The time derivative of $u(t)$ is

$$\dot{u}(t) = 2e_{i,j}(t)\dot{e}_{i,j}(t) = 2e_{i,j}(t)(-\lambda f(e_{i,j}(t)) + n_{i,j}(t)).$$

When VAF (7) is used with $\lambda a_4 \geqslant \delta$, one would have

$$\begin{aligned}
\dot{u}(t) = & -2\lambda(a_1|e_{i,j}(t)|^{\eta+1} + a_2|e_{i,j}(t)|^{w+1}) - 2\lambda a_3|e_{i,j}(t)|^2 \\
& +2(e_{i,j}(t)n_{i,j}(t) - \lambda a_4|e_{i,j}(t)|) \\
\leqslant & -2\lambda(a_1|e_{i,j}(t)|^{\eta+1} + a_2|e_{i,j}(t)|^{w+1}) \\
& +2(\delta|e_{i,j}(t)| - \lambda a_4|e_{i,j}(t)|) \\
\leqslant & -2\lambda(a_1|e_{i,j}(t)|^{\eta+1} + a_2|e_{i,j}(t)|^{w+1}) \\
= & -2\lambda(a_1 u^{\frac{\eta+1}{2}}(t) + a_2 u^{\frac{w+1}{2}}(t)).
\end{aligned}$$

Therefore, the maximum convergence time for the noise-perturbed NNTZNN model (6) is

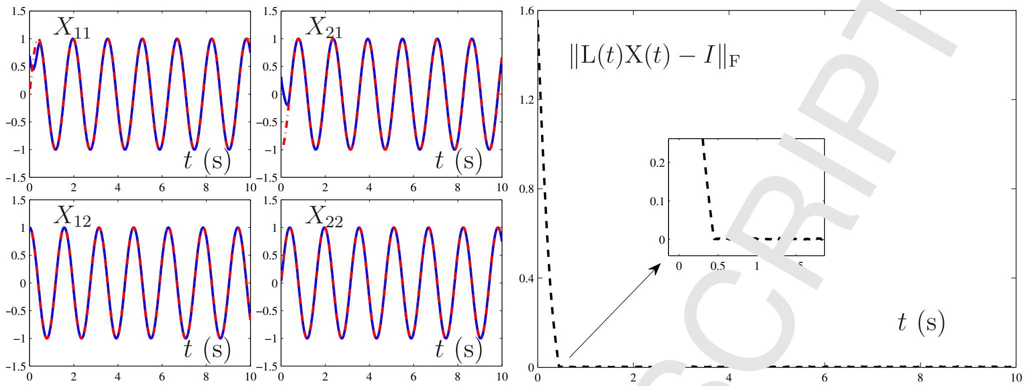
$$t_{\mathrm{c}} \leqslant \frac{1}{\lambda a_1(1-\eta)} + \frac{1}{\lambda a_2(w-1)}.$$

Thus, the noise-perturbed NNTZNN model (6) activated by VAF (7) under a dynamic bounded non-vanishing noise exhibits a predefined-time convergence property. The proof is thus completed.

It is worth pointing out that Theorems 2 and 3 indicate that the noise-perturbed NNTZNN model (6) can not only converge to the theoretical inversion $\mathrm{X}^*(t)$ of $\mathrm{L}(t)$ in a predefined time but also can reject dynamic bounded vanishing and non-vanishing noises simultaneously. This is a remarkable improvement when compared with the previous ZNN models that require infinity long time or finite time to be convergent or cannot handle dynamic bounded noises completely.

5. Comparative Verification

In Section 3, NNTZNN model (6) with the versatile activation function (VAF) is proposed for time-dependent matrix inversion. In Section 4, the predefined finite-time convergence and robustness of the NNTZNN model (6) for time-dependent matrix inversion are theoretically analyzed in detail under various noises. In this part, two illustrative numerical examples and one robotic application will be used to authenticate the efficacy and prominent convergence of NNTZNN model (6) for solving time-dependent matrix problem (2). For the purpose of comparison, several commonly used activation functions (such as LAF, PSAF, SBPAF) are also used to construct the ZNN models for solving time-dependent matrix inversion problems (2) under the same noise-contaminated environment.



(a) state solutions (b) residual error

Figure 1: Transient behavior of NNTZNN model (6) activated by VAF for solving time-dependent matrix inversion (9) without noise.

5.1. *Example 1: Two-Dimensional Coefficient*

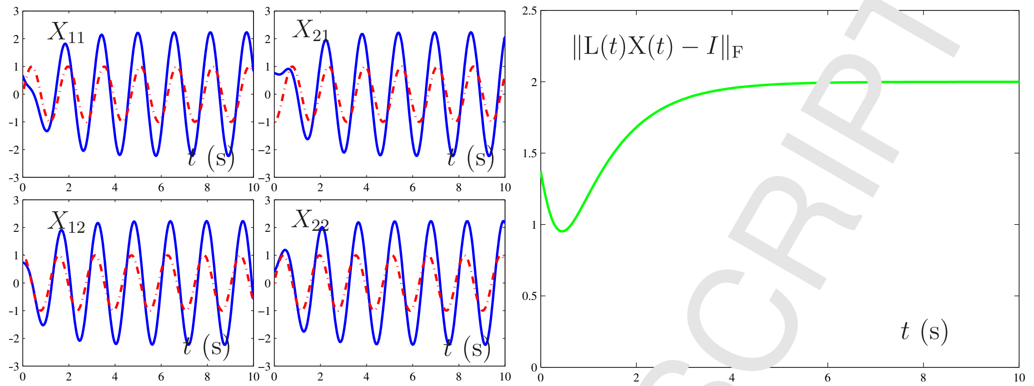
Let us first consider the following time-dependent invertible matrix to authenticate the efficacy of NNTZNN model (5):

$$\mathrm{L}(t) = \begin{bmatrix} \sin(4t) & \cos(4t) \\ -\cos(4t) & \sin(4t) \end{bmatrix} \in \mathrm{R}^{2\times 2}. \tag{9}$$

Through mathematical calculations, one can calculate the theoretical solution $\mathrm{L}^*(t)$ of (9) as

$$\mathrm{L}^*(t) = \begin{bmatrix} \sin(4t) & -\cos(4t) \\ \cos(4t) & \sin(4t) \end{bmatrix}. \tag{10}$$

Without loss of generality, one can set $\lambda = a_1 = a_2 = a_3 = a_4 = 1, \eta = 0.2, w = 5$, and it is easy to calculate the predefined finite time as $t_{\mathrm{c}} \leq 1.5$ s. First, NNTZNN model (5) is employed to find the time-dependent matrix inversion of (9) without noise. The simulation consequences are shown in Fig. 1, where the red dotted line in the figure represents the theoretical solution of the time-dependent matrix inversion problem (9), and the solid blue line represents the state solution $X(t)$ from the randomly generated initial state $X(0)$. From Fig. 1, one can see the blue solid lines in the four subgraphs can quickly coincide with the red solid line in a very short time (i.e., approximately 0.5 s that is less than $t_{\mathrm{c}} = 1.5$ s), which means that when using NNTZNN model (5) to find the time-dependent matrix inversion (9)

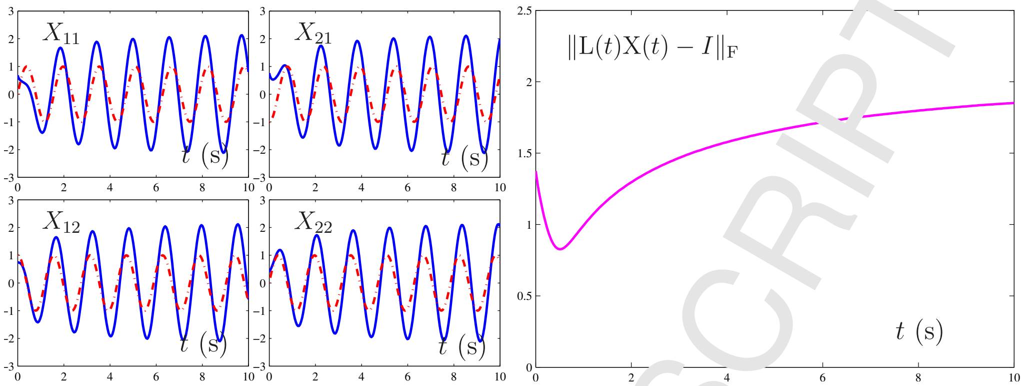


(a) state solutions (b) residual error

Figure 2: Transient behavior of the ZNN model activated by LAF for solving time-dependent matrix inversion (9) with noise $\mathrm{Y}(t) = 1$.

without noise, the state solution $X(t)$ from any randomly generated initial state $X(0)$ can quickly converge to the theoretical solution $\mathrm{L}^*(t)$. That is, NNTZNN model (5) is effective when applied to finding the time-dependent matrix inversion of (9).

When noise is considered [we first set the noise $\mathrm{Y}(t) = 1$], the NNTZNN model using the VAF and several other ZNN models using LAF, PSAF, and SBPAF are hired to solve the same problem (9). The corresponding simulation consequences are shown in Figs. 2-5. When the ZNN model using LAF was hired to solve the problem (9), the corresponding simulation consequences are shown in Fig. 2. From Fig. 2(a), one can see that there is always a certain distance between the blue solid line and the red dotted line, and there is no coincidence all the time. From Fig. 2(b), one can see that when the time reaches 10 s, the resultant residual remains stable at approximately 2 instead of 0. That's to say, the ZNN model using LAF cannot solve problem (9) accurately when there is a noise $\mathrm{Y}(t) = 1$. When the ZNN models activated by PSAF and SBPAF are employed to solve problem (9) with noise $\mathrm{Y}(t) = 1$, the corresponding state solution trajectories are shown in Fig. 3(a) and Fig. 4(a) respectively. One can see that all the state solutions of this two subfigures are not convergent to the theoretical solution as time going to infinity. From Fig. 3(b) and Fig. 4(b), one can see that the transient behavior of the residual error does not converge to 0 when the time reaches 10 s. Specifically, the residual error of the ZNN model activated by PSAF converges to approximately 1.8, and the residual error of the ZNN

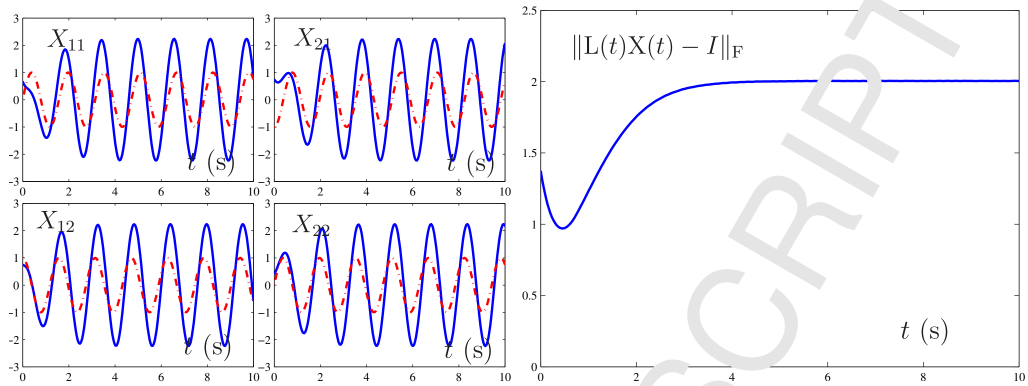


(a) state solutions (b) residual error

Figure 3: Transient behavior of the ZNN model activated by PSAF for solving time-dependent matrix inversion (9) with noise $Y(t) = 1$.

model activated by SBPAF converges to approximately 2. In addition, the simulation consequences of the NNTZNN model employed to solve problem (9) are shown in Fig. 5. Fig. 5(a) shows that the state solution $X(t)$ from the randomly generated initial state $X(0)$ converges to the theoretical solution in a very short period of time. Fig. 5(b) shows that the transient behavior of the synthesized residual error converges to 0 rapidly in a very short time (i.e., approximately 0.6 s that is less than $t_c \leq 1.5$ s). By analyzing Figs. 2-5, one can see that when the noise $Y(t) = 1$ is considered, only the NNTZNN model (6) activated by VAF can be employed to solve problem (9) accurately, while other ZNN models (activated by LAF, PSAF, and SBPAF) are employed to solve problem (9), there will be a larger error.

To further verify the superior predefined-time convergence and the denoising capability of the NNTZNN model (6) in solving problem (9), some more simulation results synthesized by NNTZNN model (6) activated by VAF and other ZNN models activated by LAF, PSAF, and SBPAF under different noise environments are shown in Fig. 6. Fig. 6(a) shows the transient behavior of the residual errors $\|L(t)X(t) - I\|_F$ synthesized by NNTZNN model (6) activated by VAF and other ZNN models activated by LAF, PSAF, and SBPAF can converge to 0 when noise $Y(t) = 0$, noting that the time required for the residual error synthesized by NNTZNN model (6) to converge to 0 is the shortest (i.e., only approximately 0.5 s), while other ZNN models take longer time to converge to 0 (i.e., LAF takes approximately 6 s , PSAF takes approximately 3 s, and SBPAF takes approximately 2.5 s). Fig. 6(b)

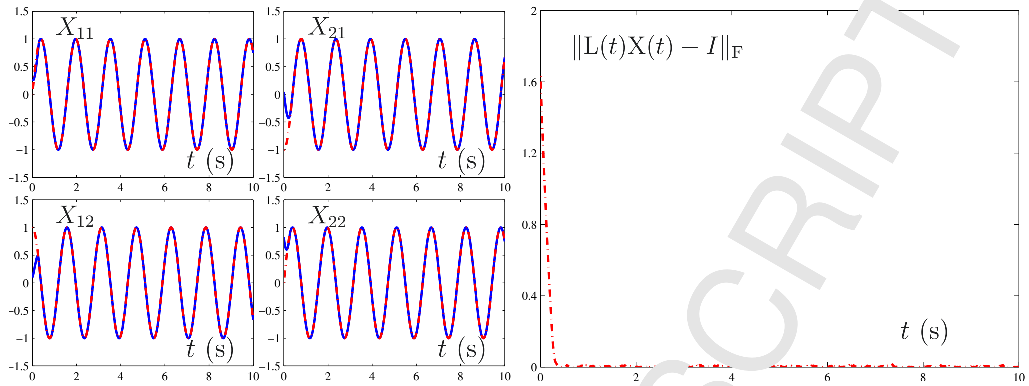


(a) state solutions (b) residual error

Figure 4: Transient behavior of the ZNN model activated by SBPAF for solving time-dependent matrix inversion (9) with noise $Y(t) = 1$.

considers a gradually disappearing noise disturbance. One can see that all the models can solve problem (9) effectively, but the NNTZNN model is the fastest compared with other ZNN models. Fig. 6(c) considers an unbounded time-dependent noise $Y(t) = 0.1\exp(0.2t)$, from which we can see that only the residual error of NNTZNN model (6) activated by VAF can converge to 0 within approximately 0.5 s, while the residual errors of the ZNN models activated by other activation functions gradually diverge over time rather than converge to 0. The fact validates the accuracy of NNTZNN model (5) for problem (9) not only in a steady noise disturbance, but also in time-dependent noise interference. From Fig. 6(d), one can see that when there is time-dependent bounded noise interference, the residual error of NNTZNN model (6) for solving problem (9) can still converge to 0 in approximately 0.5 s, while the residual errors of the ZNN models activated by LAF, PSAF, and SBPAF for solving problem (9) are always fluctuating over time.

At last, let us consider different values of parameters for NNTZNN model (6) and other ZNN models to verify its effectiveness and advantage further. Specifically, we set $\lambda = 10$ and $\lambda = 20$ for these models under the injection of different external disturbances. First, let us consider a time-dependent unbounded noise $Y(t) = 2t$. As seen from Fig. 7(a), in this simulation, when one increases the value of design parameter $\lambda = 1$ to $\lambda = 10$, the residual error of NNTZNN model (6) converges to 0 at approximately 0.05 s, while the residual errors of other ZNN models cannot converge to 0 over time. Then, let us consider a large noise $Y(t) = 20$. In this situation, one can set



(a) state solutions (b) residual error

Figure 5: Transient behavior of NNTZNN model (6) activated by VAF for solving time-dependent matrix inversion (9) with noise $\mathrm{Y}(t) = 1$.

the design parameter $\lambda = 20$. As observed in Fig. 7(b), NNTZNN model (6) only takes approximately 0.05 s to accurately solve the problem (9), while other ZNN models cannot converge to 0.

5.2. Example 2: Six-Dimensional Coefficient

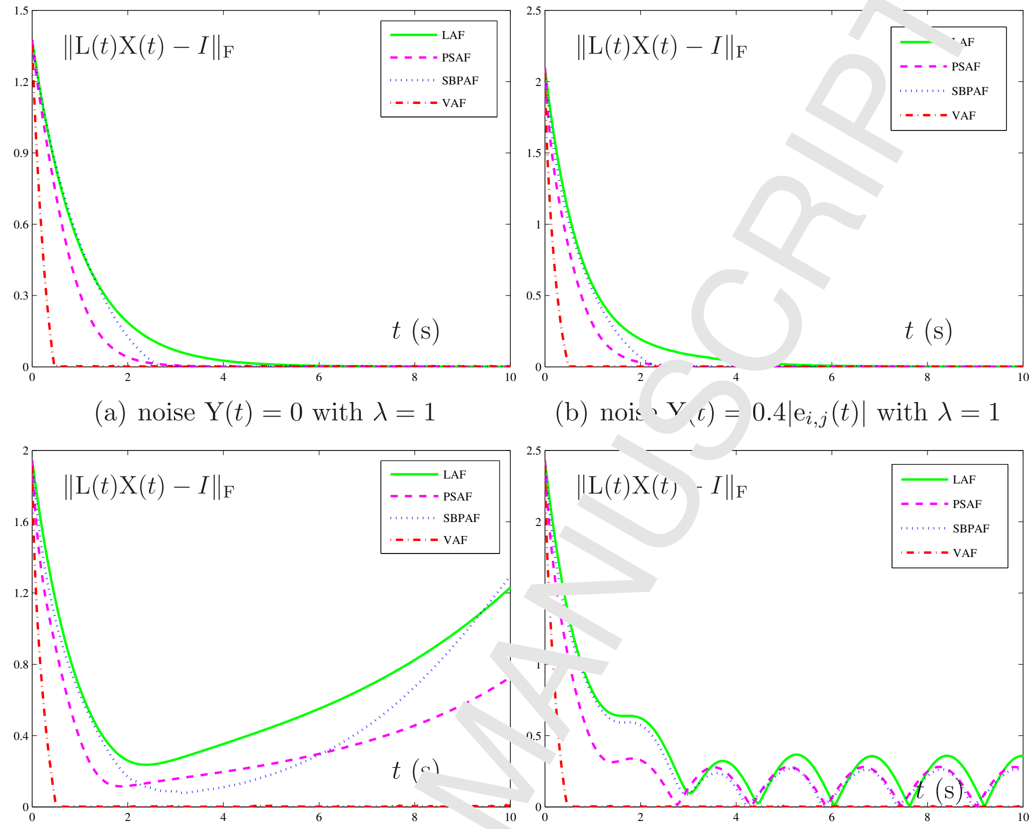
To further verify the efficacy and generalization of the NNTZNN model (6), a 6-dimensional time-dependent Toeplitz matrix is considered as

$$\mathrm{L}(t) = \begin{bmatrix} l_{11}(t) & l_{12}(t) & l_{12}(t) & \cdots & l_{1n}(t) \\ l_{21}(t) & l_{22}(t) & l_{13}(t) & \cdots & l_{2n}(t) \\ l_{31}(t) & l_{32}(t) & l_{33}(t) & \cdots & l_{3n}(t) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ l_{n1}(t) & l_{n2}(t) & l_{n3}(t) & \cdots & l_{nn}(t) \end{bmatrix} \in \mathrm{R}^{n\times n}, \tag{11}$$

with $l_{ij}(t)$ denoted

$$l_{ij}(t) = \begin{cases} 6 + \sin(2t), & i = j, \\ \cos(2t)/(i-j), & i > j, \\ \sin(2t)/(j-i), & i < j, \end{cases}$$

For comparison purposes, GNN and EIZNN models [19, 20, 31, 32] are also used to solve the above-mentioned time-dependent Toeplitz matrix under the same conditions. All design parameters are kept identical, and simulation results are shown in Fig. 8.

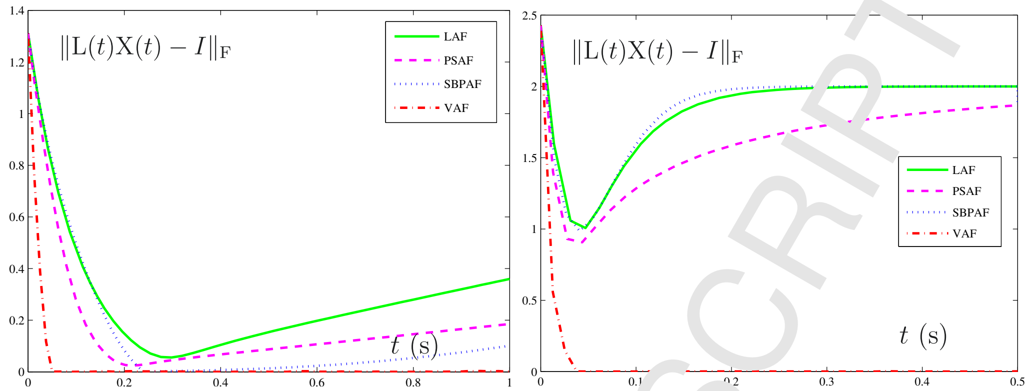


(a) noise $Y(t) = 0$ with $\lambda = 1$ (b) noise $Y(t) = 0.4|e_{i,j}(t)|$ with $\lambda = 1$

(c) noise $Y(t) = 0.1\exp(0.2t)$ with $\lambda = 1$ (d) noise $Y(t) = 0.4\cos(2t)$ with $\lambda = 1$

Figure 6: Transient behavior of residual errors $\|L(t)X(t) - I\|_F$ synthesized by NNTZNN model (6) activated by VAF and other ZNN models activated by different activation functions in different noise environments for solving time-dependent matrix inversion (9).

As seen from Fig. 8(a), when there is constant noise $Y(t) = 1$, the residual error of NNTZNN model (6) activated by VAF from the randomly generated initial state can converge to zero rapidly (approximately 1 s). In contrast, residual errors synthesized by the GNN model and other ZNN models activated by LAF, PSAF, and SBPAF cannot converge to 0 over time. Note that the residual error of the IEZNN model can slowly converge to 0.1 when the time is $t = 10$ seconds. As seen from Fig. 8(b), when there is time-dependent bounded noise interference, the residual error of NNTZNN model (6) for solving problem (11) can still converge to 0 in approximately 0.5 s while the residual errors of the GNN model, IEZNN model, and ZN-



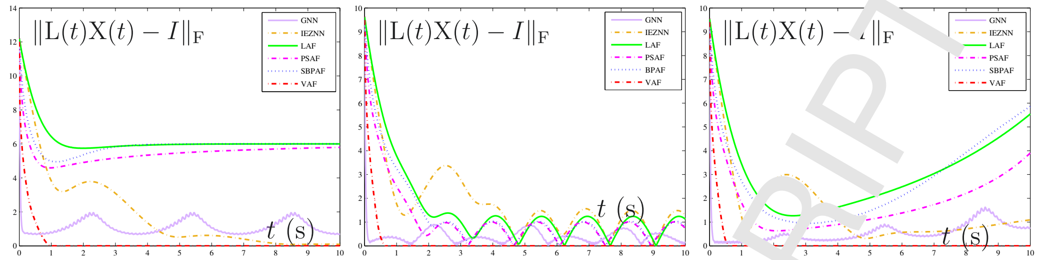
(a) noise $Y(t) = 2t$ with $\lambda = 10$ (b) noise $Y(t) = 20$ with $\lambda = 20$

Figure 7: Transient behavior of residual errors $\|L(t)X(t) - I\|_F$ synthesized by NNTZNN model (6) activated by VAF and other ZNN models activated by different activation functions in different noise environments for solving time-dependent matrix inversion (9) with different values of parameter λ.

N models activated by LAF, PSAF, and SBPAF for solving problem (11) are always fluctuating over time. Fig. 6(c) considers an unbounded time-dependent noise $Y(t) = 0.15\exp(0.2t)$, from which one can see that only the residual error of NNTZNN model (6) activated by VAF can converge to 0 within approximately 0.5 s, while the residual errors of the other neural models gradually diverge over time rather than converge to 0. The observation validates the accuracy of NNTZNN model (5) for problem (11) not only in a steady noise disturbance, but also in time-dependent noise interference.

From the results of this example, one would have the remark to discuss how NNTZNN model (5) respond and converge for more training data.

Remark 1: Different from the BP neural network, given the input [i.e., the initial value of neural state matrix $X(0)$], the proposed NNTZNN model (5) will fall into a dynamic process, repeatedly compute, and finally reach a steady state (i.e., the error function converges to 0). In this sense, the training procedure of the proposed NNTZNN model (5) can be viewed as global convergence of the error function. In addition, it is worth pointing out that the proposed NNTZNN model (5) can be solved by Matlab routines, such as "ode45", "ode15s" and so on. For more data [e.g., a larger matrix $X(t)$ shown in this example], the solving process and convergence for the proposed NNTZNN model will be acted just like before. Generally speaking, for a specific matrix, the weights of the corresponding NNTZNN model are



(a) $Y(t) = 1$ (b) $Y(t) = 0.5\sin(2.2t)$ (c) $Y(t) = 0.15\exp(0.2t)$

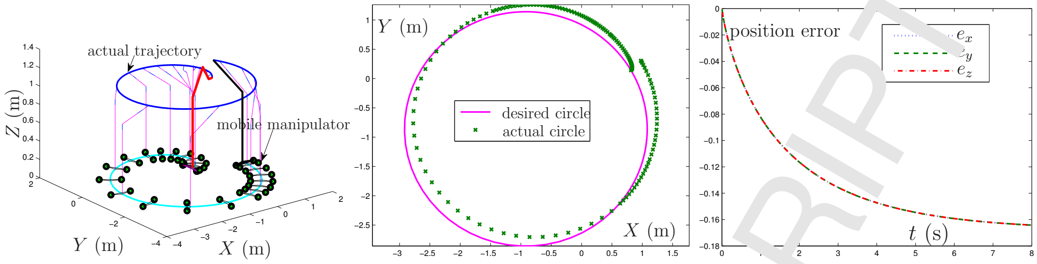
Figure 8: Transient behavior of residual errors $\|L(t)X(t) - I\|_F$ synthesized by NNTZNN model (6) activated by VAF, GNN model, IEZNN model and other ZNN models activated by LAF, PSAF, and SBPAF under different noise environments for solving time-dependent matrix inversion (11).

fixed. Therefore, as long as a random initial state is given, the corresponding NNTZNN model will output an accurate solution. That is to say, no matter how many times the NNTZNN model executes, it always outputs an accurate solution (i.e., the execution is repeated). In this work, simulation results show the value of a single run calculation. In addition, as the NNTZNN model is globally convergent, the results generated by the NNTZNN model will reach the consensus whatever the initial value is.

As for the computational complexity, as shown in [14], the original ZNN model with LAF contains $4n$ addition operations, $3n^2 + n$ multiplication operations, and n integrator operations, and thus is of $O(n^2)$ operations. The complexity of a RNN is mainly dictated by its architecture. Therefore, as for the NNTZNN model, only the activation function $\Phi(\cdot)$ is different and increases its computational complexity, as compared with that of the original ZNN model with the linear activation function. If the VAF is regarded as a whole and realized by co-processors, it only increases one multiplication operation. Therefore, the computational complexity of the NNTZNN model is of $O(n^2)$ operations to some degree. Although the NNTZNN model possesses more operations on calculating, the computational complexity is on the same order of $O(n^2)$. In addition, the presented model will finally be implemented in hardware with a parallel processing nature, so the ability for real-time computation can be guaranteed.

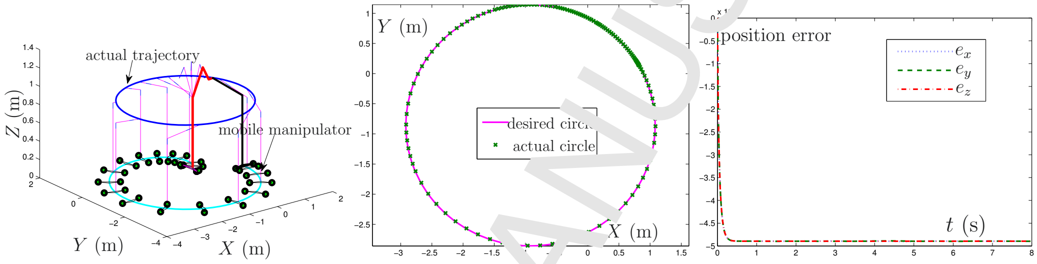
5.3. Example 3: Application to Mobile Manipulator

In this example, the proposed NNTZNN model (6) would be applied to path tracking of a mobile manipulator [46] by solving inverse kinematical



(a) whole tracking process (b) task comparison (c) position error

Figure 9: Circular task tracking synthesized by the original ZNN model activated by the SBP activation function in the presence of additive noise Y(t) = 0.35.



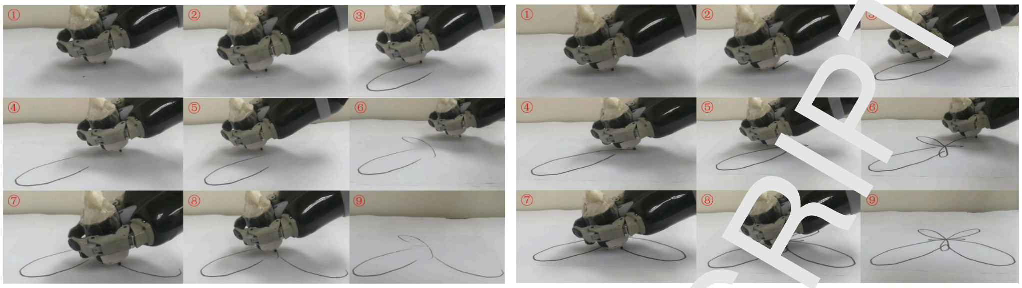
(a) whole tracking process (b) task comparison (c) position error

Figure 10: Circular task tracking synthesized by NNTZNN model (6) in the presence of additive noise Y(t) = 0.35.

equation in a noisy environment. For comparison, the ZNN model activated by SBPAF is also explored under the same condition. The desired path is set as a circle with the radius being 2 m, and the external additive noise $Y(t) = 0.35$. Simulative comparison results are described in Figs. 9 and 10. As seen from these two figures, it follows that the ZNN model activated by SBPAF does not complete the desired path tracking, while the proposed NNTZNN model (6) successfully fulfill the desired circular tracking task. These robotic application results further validate the efficacy and superior robustness of the proposed NNTZNN model (6).

5.4. Example 4: Physical Comparative Experiments

To further validate the real applicability of NNTZNN model (6), the Kinova JACO^2 manipulator is adopted as a test, and its detailed information can be referred to [47, 48, 49, 50]. In this example, two physical comparative experiments generated respectively by the SBPAF activated ZNN model and



(a) Failure by SBPAF activated ZNN. (b) Success by NNTZNN.

Figure 11: Physical comparative experiments of a butterfly-path tracking task generated by different ZNN models and performed on the Kinova JACO^2 robot manipulator when disturbed by external noise.

NNTZNN model (6) will be presented for comparison purposes in front of external noise via tracking a butterfly path. Without loss of generality, the external noise is set as $Y(t) = [1, 2, 3; 4, 5, 6; 7, 8, 9]$, $\lambda = 100$, the parameters of a butterfly path is the same as those of [47, 48, 49]. The whole process of comparative physical experiments are snapshoted and integrated in Fig. 11. Observed from Fig. 11(a) produced by the SBPAF activated ZNN model, when the external noise disturbance is present, the end-effector of the robot deviates from the desktop occasionally when tracking the given butterfly path. Therefore, the tracking experiment in this situation is fail, which can be verified by the last snapshot of Fig. 11(a). In contrast, observed from Fig. 11(b) produced by NNTZNN model (6), under the same conditions, the Kinova JACO^2 manipulator successfully completes the given butterfly-path tracking task.

Through the above comparative consequences, one can conclude that N-NTZNN model (6) and other ZNN models can be employed to find time-independent matrix inversion accurately in the absence of noise, but the NNTZNN model (6) has the largest convergence rate (i.e., the predefined time convergence) when problem (9) is solved. When noise is considered, only NNTZNN model (6) can be hired to solve problem (9) accurately in a predefined time, while other ZNN models are not suitable for solving problem (9) due to excessive errors. The robotic application examples also validates this conclusion.

6. Conclusions

In this current work, a new noise-tolerant zeroing neural network (NNTZNN) with a versatile activation function (VAF) was proposed and researched for finding time-dependent matrix inversion. Unlike the traditional zeroing neural network (ZNN) models that can only be used to find time-dependent matrix inversion in a disturbance-free environment, the proposed NNTZNN model can still be used to find time-dependent matrix inversion under various external noises. In addition, the convergence time of the NNTZNN model for time-dependent matrix inversion problem can be calculated in advance (i.e., the upper bound of the predefined convergence time is known). In addition to this property, the excellent robustness of the NNTZNN model was also analyzed in detail under the injection of different external disturbances. The main reason for these excellent properties is to add a new sign function, which makes the NNTZNN model satisfy the requirement of the predefined time convergence. At last, two numerical simulations with different dimensions and two practical applications were used as test examples in a noisy environment, of which the final consequences further substantiated the effectiveness, excellence, and applicability of the NNTZNN model, as compared with the traditional ZNN models using existing activation functions (such as LAF, PSAF, and SBPAF). The future direction of this work may include its model discretization and circuit implementations.

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