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Design and Analysis of Two FTRNN Models with Application to Time-Varying Sylvester Equation

Jie Jin, Lin Xiao, Ming Lu, and Jichun Li

***Abstract*—In this work, to accelerate the convergence speed of Zhang neural network (ZNN), two finite-time recurrent neural networks (FTRNNs) are presented via devising two novel design formulas. For verifying the advantages of the proposed FTRNN models, a solution application to time-varying Sylvester equation (TVSE) is given. Compared with the conventional ZNN model, the presented new FTRNN models in this work are theoretically proved to have better convergence performance, and they are more effective for online solving TVSE within finite time. At last, superiority and effectiveness of the new FTRNN models for solving TVSE are verified by numerical simulations.**

***Index Terms*—Recurrent neural network (RNN), Finite time, ZNN, Time-varying Sylvester equation.**

I. INTRODUCTION

Sylvester equations are frequently encountered in science and engineering fields, and they are widely used in image processing [1], automatic control theory [2], eigenvalue assignment [3] and state estimation [4]. Solving Sylvester equations has been deeply studied in the past few decades. However, most of the reported works mainly focused on the solution of static Sylvester equations [5-9], and these methods are difficult to solve the TVSEs effectively.

Because of its superiority and effectiveness, solving time-varying Sylvester matrix equations using ZNN models is deeply investigated in recent years. Instead of using Frobenius norm in the gradient-based RNN, the ZNN uses the lagging error as performance indicator. The linear activation error function of the ZNN exponentially converges to zero, and ZNN models are suitable for solving TVSEs [10-11].

Several novel nonlinear activation functions are reported to further decrease the convergence time of the ZNN models [12-15]. However, the ZNN models with all the activation functions mentioned above could not converge in finite time. For enhancing the convergence property of ZNN, the sign-bi-power activation function is provided, and the resultant ZNN models have the finite time convergence ability [16-17].

Rather than improving activation functions to make the ZNN converge in finite time, this paper focuses on improving the ZNN model to accelerate its convergence speed. Based on the conventional ZNN model, two finite-time recurrent neural networks (FTRNNs) are designed and investigated for online solving TVSE. Compared with the conventional exponential convergence ZNN model, the outputs of the new FTRNN models could converge to the theoretical solution of the TVSE faster.

II. PROBLEM DESCRIPTION AND ZNN MODEL

The following TVSE is considered:

$$A(t)X(t) - X(t)B(t) = C(t) \in \mathbb{R}^{n\times n} \quad (1)$$

where the matrices A(t), B(t) and C(t) $\in \mathbb{R}^{n\times n}$ are time-varying coefficient, and $X(t) \in \mathbb{R}^{n\times n}$ is the time-varying matrix to be solved. The main work of this article is to quickly find the solution $X(t) \in \mathbb{R}^{n\times n}$ in finite time by using the new FTRNN models.

The ZNN is an effective tool to find the solution of the time-varying equations, and the steps to build a ZNN model for solving TVSE are as follows [18-19]:

First, we define an error matrix:

$$E(t) = A(t)X(t) - X(t)B(t) - C(t) \in \mathbb{R}^{n\times n} \quad (2)$$

Unlike the gradient-based RNNs, there are no special requirements for the the error matrix E(t).

Then, selecting the following formula for the error matrix E(t) exponentially converges to zero:

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$$\frac{dE(t)}{dt} = -\beta E(t) \tag{3}$$

where β > 0 is a coefficient for adjusting the convergence speed.

Substituting equation (2) into (3), the ZNN model is realized as:

$$A(t)\dot{X}(t) - \dot{X}(t)B(t) = -\beta\big(A(t)X(t) - X(t)B(t) - C(t)\big) + X(t)\dot{B}(t) + \dot{C}(t) - \dot{A}(t)X(t) \tag{4}$$

III. Two FTRNN Models

As we know the convergence performance of a RNN can be improved significantly by selecting proper activation function, and several novel activation functions are presented in recent year [20-21]. Unlike the thought of developing activation functions, this paper focuses on improving the ZNN model to accelerate its convergence performance.

A. The first FTRNN

Based on the existing original ZNN model, design steps of the first FTRNN model can be expressed as follows.

We also define an error matrix:

$$E(t) = A(t)X(t) - X(t)B(t) - C(t) \in \mathrm{R}^{n\times n} \tag{5}$$

Then, an improved design method for the error matrix E(t) is presented as:

$$\frac{dE(t)}{dt} = -\beta\left(k\left(E(t)\right)^{1/\alpha}\right) \tag{6}$$

where the design parameters in (6) are α >1, β > 0 and k > 0.

On basis of the proposed design formula for the error matrix E(t) in (6), the following theorem will demonstrate the advantages of the improved formula in (6) over the original formula in (3).

Theorem 1. The improved formula for the error matrix E(t) in (6) converges to zero within finite time t_f:

$$t_f = \frac{\alpha\left(E(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)} \tag{7}$$

where E(0) is the initial state of the error matrix E(t).

Proof. The improved formula in (6) can be rewritten as:

$$dt = -\frac{1}{\beta k}\left(E(t)\right)^{-\frac{1}{\alpha}} dE(t) \tag{8}$$

Solving the simple differential equation in (8),

$$t = \frac{\alpha}{\beta k(\alpha-1)}\left(\left(E(t)\right)^{\frac{\alpha-1}{\alpha}} - \left(E(0)\right)^{\frac{\alpha-1}{\alpha}}\right) \tag{9}$$

According to the known conditions, E(t) decreases to 0 at time t_f, i.e., E(t_f) = 0. Hence, when $t = t_f$, equation (9) can be rewritten as:

$$t_f = \frac{\alpha\left(E(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)} \tag{10}$$

From the above analysis, we can conclude that compared with the original design formula in (3), the new improved design formula in (6) converges to zero within finite time, and the proof is completed. ■

Based on the proposed design formula for error matrix E(t) in (6), the first FTRNN model for solving TVSE can be represented as:

$$A(t)\dot{X}(t) - \dot{X}(t)B(t) = -\beta k\big(A(t)X(t) - X(t)B(t) - C(t)\big)^{\frac{1}{\alpha}} + X(t)\dot{B}(t) + \dot{C}(t) - \dot{A}(t)X(t) \tag{11}$$

The main theoretical results of the FTRNN in (11) can be summarized in theorem 2.

Theorem 2. Given smoothly time-varying coefficient matrices A(t)∈$\mathbb{R}^{n\times n}$, B(t) ∈$\mathbb{R}^{n\times n}$, the state output X(t) of the new FTRNN model in (11) is globally convergence to the theoretical solutions of the TVSE in (1) in finite time t_x:

$$t_x \le \max\left\{\frac{\alpha\left(\varepsilon^+(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)}, \frac{\alpha\left(\varepsilon^-(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)}\right\} \tag{12}$$

where $\varepsilon^+(0)$ and $\varepsilon^-(0)$ present the largest and smallest initial values in error matrix E(0), respectively.

Proof. Let us define $\varepsilon^+(0)$ = max{E(0)}, $\varepsilon^-(0)$ = min{E(0)}, $\varepsilon_{ij}(t)$ is the *ij*th element in E(t). Then we can have $\varepsilon^-(0) \le \varepsilon_{ij}(t) \le \varepsilon^+(0)$. If both $\varepsilon^-(0)$ and $\varepsilon^+(0)$ decrease to zero, and $\varepsilon_{ij}(t)$ will converge to zero.

Let t_x^+ and t_x^- present the time of $\varepsilon^+(0)$ and $\varepsilon^-(0)$ converge to zero, and t_x presents the time of $\varepsilon_{ij}(t)$ decrease to zero. From the above analysis, it is clear $t_x \le \max\{t_{\varepsilon+(0)}, t_{\varepsilon-(0)}\}$, and the upper bound of t_x can be obtained by calculating t_x^+, t_x^- with $\varepsilon^+(0)$ and $\varepsilon^-(0)$.

According to theorem 1, the time for $\varepsilon^+(0)$ convergence to zero is:

$$t_{\varepsilon^+(0)} = \frac{\alpha\left(\varepsilon^+(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)} \tag{13}$$

The time for $\varepsilon^-(0)$ convergence to zero is:

$$t_{\varepsilon^-(0)} = \frac{\alpha\left(\varepsilon^-(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)} \tag{14}$$

The time for $\varepsilon_{ij}(t)$ convergence to zero satisfies $t_x \le \max\{t_{\varepsilon+(0)}, t_{\varepsilon-(0)}\}$, and we can conclude that convergence time of new FTRNN model in (11) is bounded by

$$t_x \le \max\left\{\frac{\alpha\left(\varepsilon^+(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)}, \frac{\alpha\left(\varepsilon^-(0)\right)^{\frac{\alpha-1}{\alpha}}}{\beta k(\alpha-1)}\right\} \tag{15}$$

From the above analysis, the first FTRNN model in (11) has a finite time convergence property, and the proof is completed.■

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B. The second FTRNN

The second FTRNN model is depicted as follows.

An error matrix is also defined as:

$$E(t) = A(t)X(t) - X(t)B(t) - C(t) \in \mathrm{R}^{n\times n} \quad (16)$$

Then, another improved different design formula for error matrix E(t) is presented as:

$$\frac{dE(t)}{dt} = -\beta\left(k_1(E(t))^{1/\alpha} + k_2 E(t)\right) \quad (17)$$

where the design parameters α > 1, β > 0 and k > 0.

The theoretical analysis of the improved formula in (17) is demonstrated as below.

Theorem 3. The improved formula for the error matrix E(t) in (17) converges to zero in finite time t_f:

$$t_f = \frac{\alpha}{\eta_2(1-\alpha)}\ln\left(\frac{\eta_2 I}{\eta_2 I + \eta_1(E(0))^{1-\frac{1}{\alpha}}}\right) \quad (18)$$

where $\eta_1 = k\beta_1$, $\eta_2 = k\beta_2$, E(0) is the initial state of the error matrix E(t).

Proof. The new formula in (17) can be rewritten as:

$$(E(t))^{-\frac{1}{\alpha}}\frac{dE(t)}{dt} = -\eta_2(E(t))^{1-\frac{1}{\alpha}} - \eta_1 I \quad (19)$$

where $\eta_1 = k\beta_1$, $\eta_2 = k\beta_2$, and I is a n × n identity matrix.

Let $Z(t) = (E(t))^{1-\frac{1}{\alpha}}$, $\frac{dZ(t)}{dt} = \left(1-\frac{1}{\alpha}\right)(E(t))^{\frac{-1}{\alpha}}\frac{d(E(t))}{dt}$, and equation (19) can be simplified as:

$$\frac{dZ(t)}{dt} + \frac{(\alpha-1)\eta_2}{\alpha}Z(t) + \frac{(\alpha-1)\eta_1}{\alpha}I = 0 \quad (20)$$

Solving the first-order differential equation in (20), the expression of Z(t) is:

$$Z(t) = \left(\frac{\eta_2}{\eta_1} + Z(0)\right)\exp\left(-\left(\frac{\alpha-1}{\alpha}\right)\eta_2 t\right) - \frac{\eta_2}{\eta_1}I \quad (21)$$

Because t_f presents the time of E(t) convergence to zero, and $E(t_f) = 0$, and $Z(t_f) = 0$.Then equation (15) can be simplified as:

$$\left(\frac{\eta_2}{\eta_1} + Z(0)\right)\exp\left(-\left(\frac{\alpha-1}{\alpha}\right)\eta_2 t_f\right) - \frac{\eta_2}{\eta_1}I = 0 \quad (22)$$

The convergence time t_f is:

$$t_f = \frac{\alpha}{\eta_2(\alpha-1)}\ln\left(\frac{\eta_2 I + \eta_1(E(0))^{1-\frac{1}{\alpha}}}{\eta_2 I}\right) \quad (23)$$

From above analysis, we can conclude that the new improved design formula in (17) converges to 0 in finite time. The proof is completed. ■

Similarly, based on the new improved formula for E(t) in (17), the second FTRNN for solving TVSE can be presented as:

$$\begin{aligned} A(t)\dot{X}(t) - \dot{X}(t)B(t) = &-\beta k_1(A(t)X(t) - X(t)B(t) - C(t))^{\frac{1}{\alpha}} \\ &-\beta k_2(A(t)X(t) - X(t)B(t) - C(t)) \\ &+ X(t)\dot{B}(t) + \dot{C}(t) - \dot{A}(t)X(t) \end{aligned} \quad (24)$$

The main theoretical solution of the second FTRNN in (24) can be presented in theorem 4.

Theorem 4. Given smoothly time-varying coefficient matrices A(t)∈ℝ n×n, B(t) ∈ℝ n×n, the state matrix X(t) of the second FTRNN in (24) is globally convergence to the theoretical solutions of the TVSE (1) within finite time t_x:

$$t_x \le \max\left\{\frac{\alpha}{\eta_2(\alpha-1)}\ln\left(\frac{\eta_2 + \eta_1(\varepsilon^+(0))^{1-\frac{1}{\alpha}}}{\eta_2}\right), \frac{\alpha}{\eta_2(\alpha-1)}\ln\left(\frac{\eta_2 + \eta_1(\varepsilon^-(0))^{1-\frac{1}{\alpha}}}{\eta_2}\right)\right\} \quad (25)$$

$\varepsilon^+(0)$ and $\varepsilon^-(0)$ are the largest and smallest initial values in the error matrix E(0).

Proof. Let us define $\varepsilon^+(0) = \max\{E(0)\}$, $\varepsilon^-(0) = \min\{E(0)\}$, $\varepsilon_{ij}(t)$ is the *ij*th element in E(t), and $\varepsilon^-(0) \le \varepsilon_{ij}(t) \le \varepsilon^+(0)$. If both $\varepsilon^-(0)$ and $\varepsilon^+(0)$ decrease to zero, and $\varepsilon_{ij}(t)$ will converge to zero.

Let t_x^+ and t_x^- present the time of $\varepsilon^+(0)$ and $\varepsilon^-(0)$ converge to zero, and t_x presents the time of $\varepsilon_{ij}(t)$ decrease to zero. From the above analysis, it is clear $t_x \le \max\{t_{\varepsilon+(0)}, t_{\varepsilon-(0)}\}$, and the upper bound of t_x can be obtained by calculating t_x^+, t_x^- with $\varepsilon^+(0)$ and $\varepsilon^-(0)$.

According to theorem 3, the time for $\varepsilon^+(0)$ convergence to zero is:

$$t_{\varepsilon^+(0)} = \frac{\alpha}{\eta_2(\alpha-1)}\ln\left(\frac{\eta_2 + \eta_1(\varepsilon^+(0))^{1-\frac{1}{\alpha}}}{\eta_2}\right) \quad (26)$$

The time for $\varepsilon^-(0)$ convergence to zero is:

$$t_{\varepsilon^-(0)} = \frac{\alpha}{\eta_2(\alpha-1)}\ln\left(\frac{\eta_2 + \eta_1(\varepsilon^-(0))^{1-\frac{1}{\alpha}}}{\eta_2}\right) \quad (27)$$

The time for $\varepsilon_{ij}(t)$ convergence to zero satisfies $t_x \le \max\{t_{\varepsilon+(0)}, t_{\varepsilon-(0)}\}$, we can conclude that convergence time of new FTRNN model in (24) is bounded by

$$t_x \le \max\left\{\frac{\alpha}{\eta_2(\alpha-1)}\ln\left(\frac{\eta_2 + \eta_1(\varepsilon^+(0))^{1-\frac{1}{\alpha}}}{\eta_2}\right), \frac{\alpha}{\eta_2(\alpha-1)}\ln\left(\frac{\eta_2 + \eta_1(\varepsilon^-(0))^{1-\frac{1}{\alpha}}}{\eta_2}\right)\right\} \quad (28)$$

From the above analysis, the second FTRNN model in (24) has a finite time convergence property, and the proof is completed.■

IV. Numerical Simulation Results

For purpose of further verifying the finite time convergence of the two FTRNN models in (11) and (24) for solving TVSE, the computer numerical simulation results with Matlab software are presented in this part. Moreover, simulation results of the conventional ZNN in (4) for solving TVSE are also provided to compare with the new models.

The time-varying coefficient matrices of the TVSE in (1) are provided as follows:

$$A=\begin{bmatrix}\sin 3t & \cos 3t\\ -\cos 3t & \sin 3t\end{bmatrix} B=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix} C=\begin{bmatrix}\sin 4t & \cos 4t\\ -\cos 4t & \sin 4t\end{bmatrix}$$

Fig.1-Fig.3 are the simulation results of solving TVSE in (1) with different models. Solid blue curves are neural state solutions (generated by the models in (4), (11) and (24)), and red dotted curves are theoretical solutions. From Fig.1-Fig.3, the neural state solutions X(t) of all the three models converge to the theoretical solutions of the TVSE.

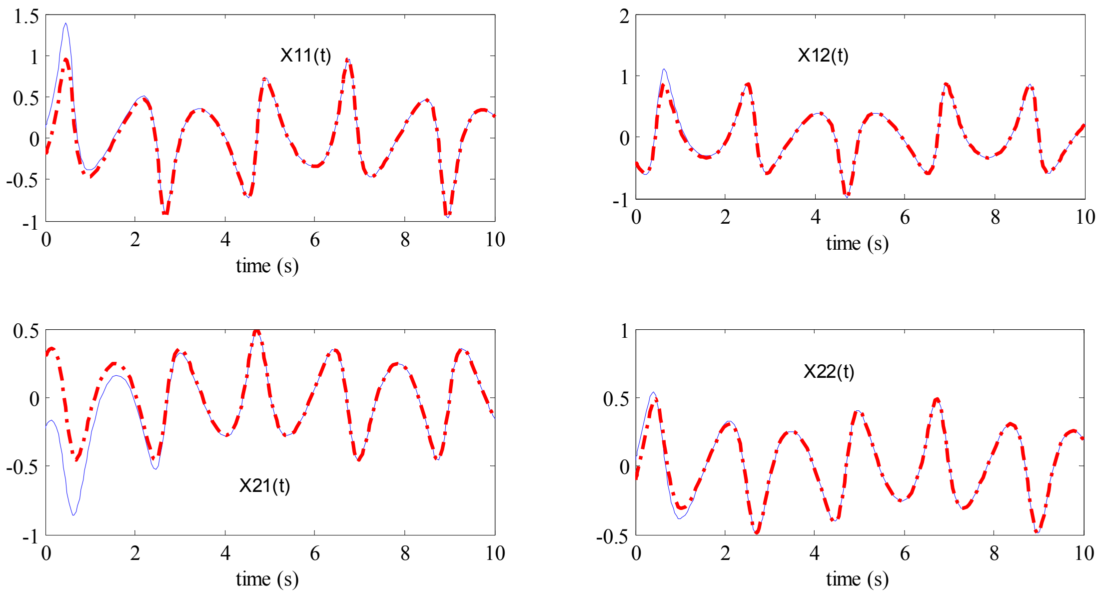

Fig.1 Neural state solution X(t) generated by the conventional ZNN model in (4) for TVSE with β = 1 (solid blue curves are neural state solutions; Red dotted curves are theoretical solutions)

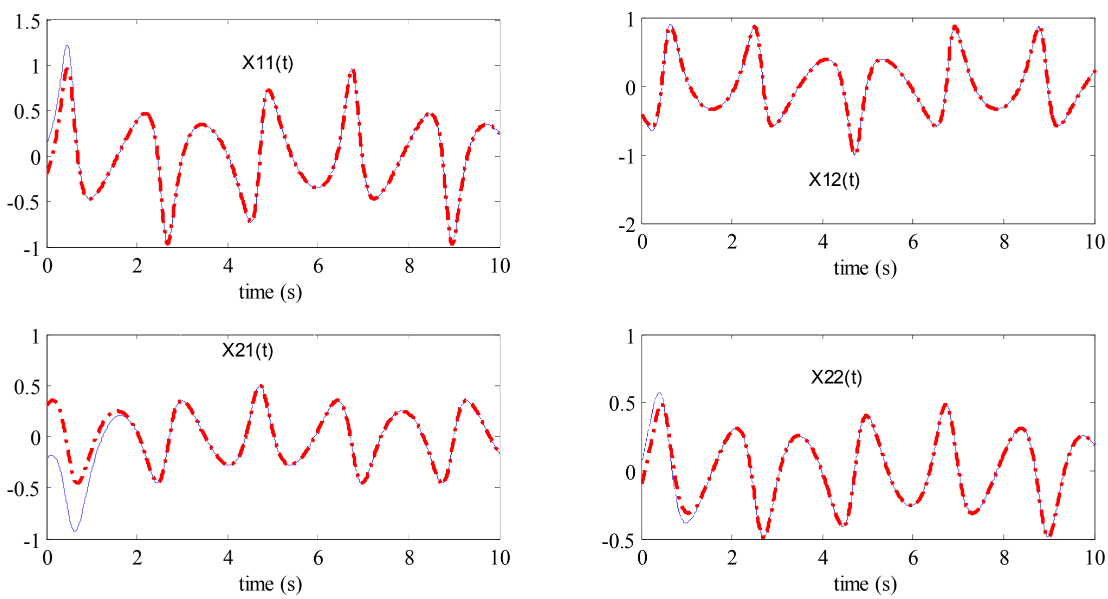

Fig.2 Neural state solution X(t) generated by the conventional ZNN model in (11) for TVSE with β = 1 (solid blue curves are neural state solutions; Red dotted curves are theoretical solutions)

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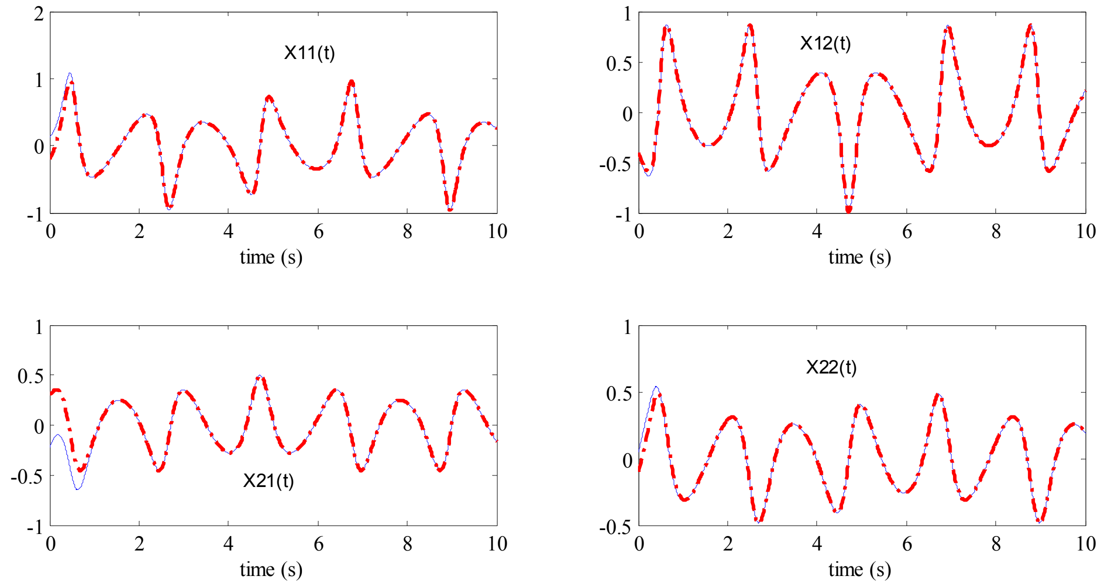

Fig.3 Neural state solution X(t) generated by the finite time RNN (24) for TVSE with β = 1 (solid blue curves are neural state solutions; Red dotted curves are theoretical solutions)

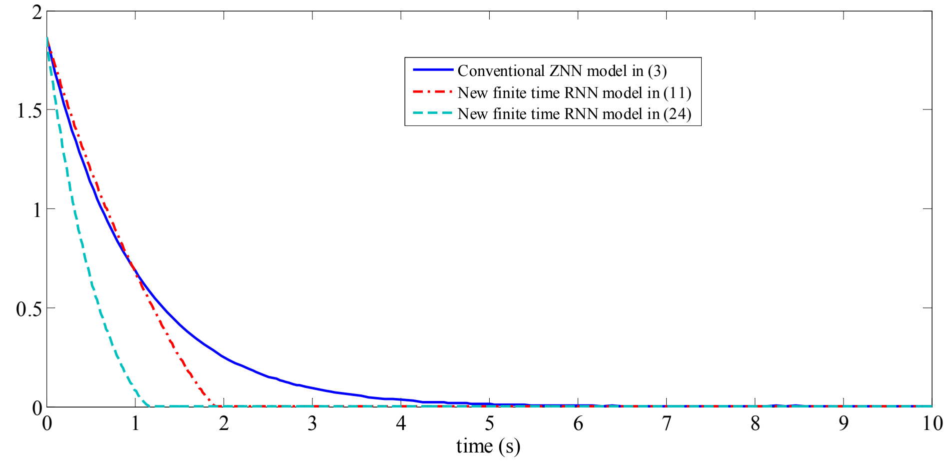

Fig.4 Convergence behaviors of $\|A(t)X(t) - X(t)B(t) - C(t)\|_F$ generated by the conventional ZNN model in (3), finite time RNN model in (11) and finite time RNN model in (24) for TVSE with β = 1

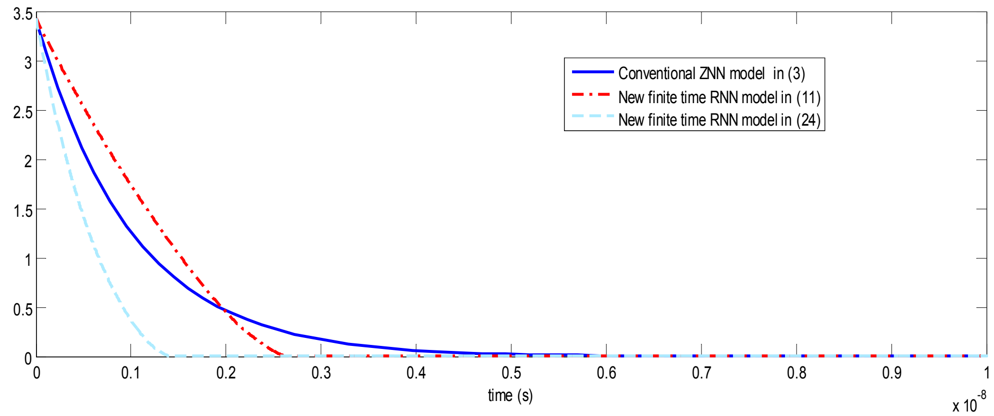

Fig.5 Convergence behaviors of $\|A(t)X(t) - X(t)B(t) - C(t)\|_F$ generated by the conventional ZNN model in (3), finite time RNN model in (11) and finite time RNN model in (24) for TVSE with β = 10^9

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The residual error $\|A(t)X(t) - X(t)B(t) - C(t)\|_F$ in Fig.4 and Fig.5 are used to further investigate the convergence characteristics of all the three models. It can be observed from two figures that the conventional ZNN model in (4) exponentially converges to the theoretical solutions of the TVSE, while the new models in (11) and (24) converge to the theoretical solutions within finite time. The new FTRNN models in (11) and (24) have better convergence property than conventional ZNN model in (4).

Moreover, the convergence performances of all the three models also have important relationship with the parameter β. Fig.4 presents the convergence characteristics of the residual errors of the models in (4), (11) and (24) with β=1, and Fig.5 presents the convergence characteristics of the residual errors of the models in (4), (11) and (24) with $\beta=10^9$. It is clear that the convergence time of all the three models could be further reduced by choosing a large value of β.

Remark 1. In general, the parameters in the novel design formulas have close relationship with the convergence speed of the two FTRNN models. Therefore, the values of these parameters cannot be set arbitrarily. Specifically, the values of these parameters can be set according to specific practical requirements to ensure a timely convergence. Specific guidance for choosing these parameters can be seen in Theorems 1-4 and related works [22-28].

V. Conclusions

In this paper, illustrated via solving TVSE, two new FTRNN models are presented and investigated by devising two new formulas, which both possess finite time performance. The theoretical analysis and numerical simulation results are conducted to verify the effectiveness for solving TVSE. Compared with the conventional ZNN model, two new FTRNN models has remarkable improvements in convergence performance, and the future directions may conclude its circuit implementations and some practical applications to engineering fields.

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