Wave-Based Control of Under-Actuated Flexible Structures with Strong External Disturbing Forces

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Abstract

Wave-based control of under-actuated, flexible systems has many advantages over other methods. It considers actuator motion as launching a mechanical wave into the flexible system which it absorbs on its return to the actuator. The launching and absorbing proceed simultaneously. This simple, intuitive idea leads to robust, generic, highly efficient, precise, adaptable controllers, allowing rapid and almost vibrationless re-positioning of the system, using only sensors collocated at the actuator-system interface. It has been very successfully applied to simple systems such as mass-spring strings, systems of Euler-Bernoulli beams, planar mass-spring arrays, and flexible 3-D space structures undergoing slewing motion. In common with most other approaches, this work also assumed that, during a change of position, the forces from the environment were negligible in comparison with internal forces and torques. This assumption is not always valid. Strong external forces considerably complicate the flexible control problem, especially when unknown, unexpected or un-modelled. The current work extends the wave-based strategy to systems experiencing significant external disturbing forces, whether enduring or transient. The work also provides further robustness to sensor errors. The strategy has the controller learn about the disturbances and compensate for them, yet without needing new sensors, measurements or models beyond those of standard wave-based control.

Keywords

Flexible system control, Robust control, Wave-based control, Impact, Disturbance

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1 Introduction

There is an enormous literature, for over five decades now, on the control of flexible mechanical systems. Several textbooks have been published (e.g. [1-4]) as well as encyclopaedic review papers (e.g. [5-8]). These books and papers refer in turn to many hundreds of papers on the topic. Much of the literature on flexible system control assumes that external disturbing forces are negligible in comparison with forces internal to the system. This is quite reasonable, for example, in many robotics and gantry crane applications, especially when the devices are indoors. But clearly flexible systems are always disturbed by their environment to some extent, and the interaction can sometimes be strong. This paper considers coping with such strong external interactions.

The sources and nature of external forces are diverse. They can have very different time profiles, ranging from impulsive forces occurring any time during or after a manoeuvre (e.g. from impacts), to more slowly varying forces, either transient (e.g. from friction or viscous damping) or enduring (e.g. from on-going contact reaction or changed gravitational strain). Often the forces arrive unexpectedly, so that the level of the forces, where they act on the system and their timing will not be known beforehand. Even where they are known or can be measured, the effects of such forces on the flexible system and its control system will often be poorly modelled. The effect of an impact, for example, can be very different depending on the impacting body's dynamics, its collision path (obliqueness), its inertia, rigidity, hardness, and surface texture. Similar variations will arise on the side of the flexible system undergoing the impact. For all these reasons, strong external disturbing forces pose a major challenge to the approaches to controlling flexible systems published to date. By their nature they are difficult to model and to deal with in a generic way.

The topic of this paper is adapting the wave-based control (*WBC*) technique, for under-actuated flexible systems, to enable it to cope with such strong external forces. The term "under-actuated" means that there are fewer controllers than degrees of freedom, and frequently just one actuator is controlling a flexible system with arbitrarily many degrees of freedom. Transient and enduring disturbing forces have different effects on the *WBC* technique and so, when both are potentially present, the control law needs to be modified in two ways to deal with the two effects simultaneously and comprehensively. But before considering the two modifications, a brief review of the unmodified *WBC* will be presented, to set a context, to remind readers and to make the paper more self-contained. There is, however, no attempt to be comprehensive. More about the nature of *WBC*, its merits, and its advantages over other methods are available in many other papers [9-21].

The ideas presented here apply to a wide range of under-actuated flexible systems: lumped, distributed or mixed; uniform or not; translating, rotating or both simultaneously; moving in a line, plane or in three dimensions; with wide ranges in the level of internal damping; with linear or non-linear elasticity; with single or multiple actuators, in series or in parallel; with actuators having far from ideal behaviour; with number of degrees of freedom ranging from one to infinity; and so on [9-21]. To restrict the paper length and to minimise confusion, however, the paper will restrict its focus to two specific systems, both with single actuators.

The first is a rectilinear, system of a string of masses and springs controlled by a linear actuator, as in Fig.1. This system is a test-bed for the basic strategies. Its control system also gives the building blocks for controlling systems with more complex motions. The second is a mass-spring array, here in a beam-like arrangement (long and narrow), undergoing translation and rotation in the plane, controlled by an actuator which can translate and rotate, with three degrees of freedom. This arrangement, with its 10 spring masses moving in the plane, could also be regarded as a 20 DOF planar mechanism. See Fig.3. In this second case the flexible system undergoes bending, shearing, extension, and compression, in a complex motion. It might be remarked in passing that, although this system is relatively simple in concept, it is still of much higher order and complexity than most of those appearing in the literature to date on control of flexible systems, and, for example, automatically models centrifugal and Coriolis effects.

Such lumped models are of interest in their own right, in modelling systems having inherently lumped characteristics, such as robot arms and some space structures. They can also be considered as finite-order approximations of distributed systems. For example, by choosing suitable spring and mass values for the array, the lumped array could, for example, simulate Euler-Bernouilli or Timoshenko beam-like behaviour, with specified density and elastic moduli, whether uniform or non-uniform throughout.

These modelling considerations, however, are not relevant to the message of the paper. For present purposes the array is a generic, representative flexible system, with no damping, having many degrees of freedom (actually 23 DOF in total, including 3 for the actuator) and complex internal dynamics. It is a system which is difficult to move rapidly from rest in one position to rest in another, while controlling its vibrations, all to be achieved by a single actuator, using measurements at the actuator interface. Such dynamic complexities are intended to test the controller rather than to model any given system more or less precisely. Because the proposed control system is nonspecific, not model-dependent, and robust to system changes, the flexible

system which it is controlling could be modelled by any preferred technique, of arbitrary complexity. Of course, the controller can also be directly applied to, and tested on, hardware real physical system, which is the main purpose of the work, thereby dispensing with any need for a model, even for testing.

In this paper, this section, Section 1, has been introductory. Section 2 reviews the standard implementation of *WBC* which has previously been shown to function well, in many applications, in the absence of external, disturbing forces. Section 3 then considers how various external forces can affect the wave-based control performance. Section 4 shows how standard *WBC* can be adapted to deal with these external disturbing forces, both enduring and transient. The adaptation for enduring forces also prevents possible position drifting under *WBC* due to steady-state zero errors in force sensors. Comparative results are presented in Section 5 while Section 6 has comments and conclusions.

2 Review of standard *WBC*

To review the basic idea, consider first a rectilinear mass-spring system, moving in one dimension, controlled by a single actuator, with no external forces, an example of which is shown in Fig.1 for a 3-mass case, which could be uniform or not. As explained, a directly-controlled actuator is indirectly controlling the attached, lumped flexible system. To move the system from rest to rest through a target displacement, the requested motion input to the actuator, $c(t)$, is set to be the sum of a "launch" displacement $a(t)$ of half the reference displacement, $\frac{1}{2}r(t)$, and a measured "return" displacement, $b(t)$. The returning motion component $b(t)$ provides active vibration damping. Furthermore, it can be proven [20] that, in the absence of external disturbances, the additional net displacement caused by adding $b(t)$ equals the second half of the target displacement, $\frac{1}{2}r(\infty)$. Thus the wave leaves behind an associated net displacement, on passing through the system, from actuator to tip and back to the actuator, leaving the system at rest at the new target position.

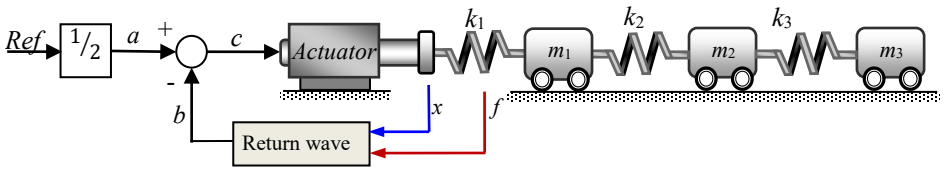

Fig. 1: WBC of rectilinear system without external disturbances: $x = c = a+b = \frac{1}{2}r + b$

Thus *WBC* seamlessly combines position control and active vibration damping in a single actuator motion. As required, the position control and active vibration damping reach their completion at the same time, as otherwise further movement of the actuator to achieve the one would perturb the other. This simple, intuitive wave-based idea leads to robust, generic, highly efficient, precise, adaptable controllers, allowing rapid and almost vibrationless re-positioning of the system, using only sensors collocated at the actuator-system interface. The reference displacement, $r(t)$, can have any desired shape, including step, ramp, or s-shaped (double parabola), provided it settles at the target, rest displacement, $r(\infty)$.

The returning wave $b(t)$ can be determined from two independent interface measurements. The two variables used in the *WBC* implementation of this paper are the actuator position, $x(t)$, and the force, $f(t)$, which the actuator applies to the flexible system. Measuring $b(t)$ provides what could be described as real-time system identification. It gives the sub-system directly controlling the actuator the information about the system dynamics it needs to achieve its goal. As system dynamics change (e.g. due to change of payload, change in system configuration, or even physical damage) the returning wave changes, while the control strategy and control law remain unchanged. This is partly why the control system is so robust. The apparent delay between the launch and return wave becomes larger and easier to observe as the system becomes longer or more flexible, whereas for shorter, stiffer systems, the absorbing action begins almost immediately and strongly overlaps the launch action. Again the one control law copes with all cases and needs no adjusting.

In the literature [9- 21] there are various ways to define $b(t)$ and the simplest version is used here. In the following control law it is based on a force integral (3rd line):

$$\begin{aligned} x(t) &= a(t) + b(t); \\ a(t) &= \tfrac{1}{2} r(t); \\ b(t) &= \tfrac{1}{2} \left[x(t) - Y \int f(t)\, dt \right]; \\ x(t) &= c(t). \end{aligned} \tag{1}$$

The Y parameter is a mechanical admittance term and here it is assumed to be constant. The force $f(t)$ is taken as positive when the first spring is in compression.

By way of motivation and justification for this control law, two important points can be noted. Firstly, by differentiating $c(t) = a(t) + b(t)$ with respect to time, it can be seen that, certainly when $a(t)$ becomes constant, the $b(t)$ component of the actuator motion is providing a viscous damping effect for the returning motion, that is, a velocity proportional to the force associated with the returning wave, with a damping coefficient equal to $1/Y$. In other words, it

causes the actuator to provide active vibration damping with an appropriate damping coefficient, or a terminating impedance to returning waves, which minimises their reflection on arrival back to the actuator.

Secondly, for rest-to-rest manoeuvres, when the initial and final momenta are zero, the force integral in $b(t)$ will return to zero, so the final position of $x(t)$ must equal $r(\infty)$, the final value of $r(t)$. Note that this effect is independent of the value of Y. For a lumped system, Y can be set to $1/\sqrt{(k_1 m_1)}$, that is, the reciprocal of the square root of the product of the spring stiffness at the interface, k_1, by the first lumped mass, m_1. But responses and zero steady-state errors are assured for a wide range of values of Y, say up to ±50% of the suggested, nominal value. Such variations have only secondary effects, seen only in the transient parts of the response. The Y value can be used as a fine-tuning parameter, for example to achieve some desired trade-off between rise-time, overshoot and settling time.

The control strategies presented here still function when non-ideal actuator behaviour is taken into account, including saturation, loading effects and real dynamic responses [21]. The only requirement is that the final position be correct, that is $x(\infty)=c(\infty)$, which is easily achieved in practice. For simplicity, however, we here assume ideal actuator behaviour, implying $x(t) = c(t)$, as assumed in Eq.(1).

The discussion above has been in terms of launching a wave of $a(t) = ½r(t)$, and absorbing the measured returning $b(t)$ wave. But once conceived in this way, the controller can then be put in a very simple form, namely

$$c(t) = r(t) - Y \int f(t)dt, \qquad (2)$$

as shown in Fig. 2. This control law, Eq.(2), is simply a rearrangement of Eq.(1). The equivalence of the two is exact only when $x(t) = c(t)$, but when this requirement is relaxed, and realistic actuator dynamics are used, the simple control law remains very effective. It implies only one feedback measurement, namely the force $f(t)$. The actuator sub-controller will still need to measure $x(t)$ to ensure it follows $c(t)$ as closely as possible, but the *WBC* requires only $f(t)$.

The simplicity of Eq.(2) tends to hide aspects of how it works. In Fig.2, if the system starts from rest, it can be shown that the value of the feedback variable, $d(t)$, is initially close to $½r(t)$, so $c(t)$ is about $½r$(t), which is the launch wave, as before. Later the force integral returns to zero, and $c(t)$ rises to $r(t)$, thereby absorbing the returning wave and vibrations, as before. This is equivalent to adding $b(t)$ to $a(t)$ as in Fig.1. If required, the return wave, $b(t)$ of Fig.1, can be determined from the variables in Fig.2 as $½(x(t) - d(t))$.

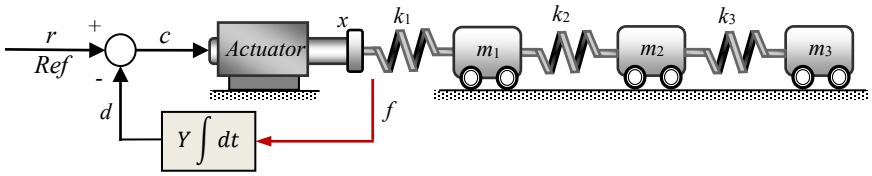

Fig. 2: A simplification of Fig.1 giving similar performance (identical if x=c)

Figures 1 and 2 show systems undergoing translation only. For systems whose motion is rotational rather than translational, a similar control strategy works well [14, 21]. The variables $r(t)$ and $x(t)$ will then correspond to the angular displacements, and $f(t)$ to a torque (moment). The axis for taking moments can be the actuator axis, but only if the rotational actuator undergoes fixed-axis rotation. If the rotary actuator also undergoes translation, then moments of forces should be taken either about a fixed point in space (such as the initial actuator axis position) or about the system mass centre (which would then need to be calculated and updated continually). This is to ensure that angular momentum about that point will be conserved between the start and end of the motion.

This control strategy has been thoroughly tested on flexible systems of many kinds, sizes and flexibilities, lumped and distributed, undergoing different kinds of motion, including translation, rotation, and simultaneous translation and rotation in the plane and in 3-D space [13-19]. This testing has been carried out in numerical simulation and experimentally [13].

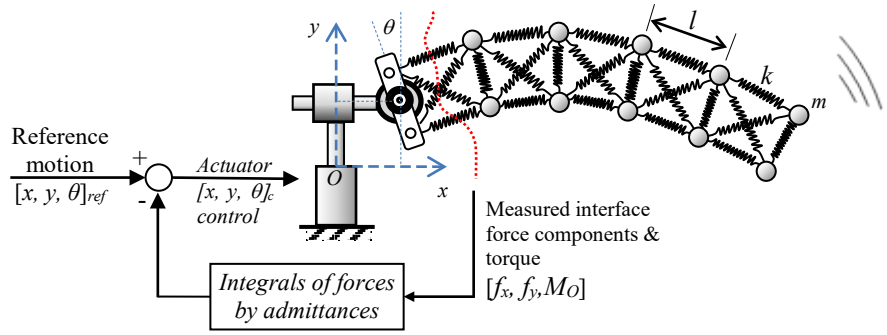

Fig. 3: Planar mass-spring array controlled by actuator which translates and rotates.

More challenging than the rectilinear case above is the mass-springs array of Fig.3, with a directly controlled actuator which can translate and rotate in the plane. In other words, the actuator has its own sub-controllers which can give controlled $x(t)$ and $y(t)$ translation and $\theta(t)$ rotation in the plane. For the present the plane is horizontal, so that gravity effects are constant and ignorable. The *WBC* implementation for this system has *three* controllers (each as in Fig.2

or Eq.(2)) working in parallel, independently of each other. So each line in the control arrangement of Fig.3 has three signals. The three inputs to the actuator sub-controllers are each made up of half the corresponding reference variable minus the feedback term based on an admittance times the time integrals of f_x, f_y and M_O. Thus there is one *WBC* algorithm for each actuator translation component, $x(t)$ and $y(t)$, and one for rotation, $\theta(t)$.

In the absence of external disturbances or external forces which change during the manoeuvre, these control systems give very rapid, rest-to-rest motion to target, with no steady-state error, for rectilinear, planar and (although not considered here) even for 3-D flexible systems undergoing both translation and rotation in space [20]. As a measure of its rapidity, the rest-to-rest time is typically no more than about 15% longer than the theoretical minimum time when an ideal, acceleration-limited actuator is assumed, and, by suitable choice of reference input, it can even achieve that minimum time exactly [15].

WBC can be viewed as standing between two broad classes of strategies for controlling under-actuated flexible devices, combining the benefits of both while avoiding their respective drawbacks. One class embraces essentially open loop techniques (e.g. time-optimal, input shaping, time-delay filtering, sliding mode and posicast), even if sometimes they are subsequently refined with a degree of feedback. See for example, [22-28]. These methods can give excellent performance under specified conditions, but they usually depend on having accurate system models and so they are not robust to system changes, modelling errors or external disturbances. Some also assume actuator performances which are far from realistic.

The second class of flexible control systems uses generic techniques (e.g. classical feedback such as PID control, passivity-based control, LQR, energy, and Lyapunov-based methods) which are blind to some characterising features of flexible system dynamics. Some of these techniques wait for an error to arise and then move to correct it, and for this reason cannot be even close to optimal. Others rely on having a very good system model, or require special tuning for each case within each application. ([1-3]).

By contrast, *WBC* is generic and robust to modelling, actuation and implementation errors. It anticipates the error and moves the actuator to absorb it before it becomes established. It first identifies, then measures and finally exploits the propagation delay effects inherent in flexible systems, but without relying on a full system model, achieving close to time-optimal performance. It has the benefits of feedback yet without requiring multiple sensors. It has the stability associated with the collocation of sensor and actuator, despite the physical distance between the actuator and the controlled tip ([28]).

Because no system model is required, the strategy is inherently robust to system changes and unknown and un-modelled system dynamics. References [15] and [29], for example, consider the robustness aspects of WBC. Furthermore, because the wave measurements are made after the actuator, and so are based on motion actually achieved by a given actuator, *WBC* automatically deals with non-ideal actuator behaviour including actuator saturation and bandwidth limitations. This can be understood as follows. If, due to poor dynamics, the actuator absorbs, say, only 90% of the returning wave, the 10% is reflected into the system. This 10% is thus re-launched and soon returns to the actuator, where 90% of the 10% is then absorbed. Thus the system still quickly settles, with just a minor extension of the settling time.

The merits of model-free control have previously been identified, for example in [8], which considers two model-free approaches, one energy-based and the other using neural adaptive control. Of the two, the energy-based approach is closest to *WBC* in its philosophy and in its focus on work-energy at the actuator-system interface. It is, however, less generic and less optimised. Unlike *WBC* it does not deliberately, and explicitly, manage and exploit the launch-and-absorb, two-way momentum-and-energy transfer, between actuators and flexible structures, as they pass through the dynamics of the interfaces.

3 External disturbances: effects and categories

The interface force (or torque) in the control systems above (in the absence of external disturbances), such as $f(t)$ in Fig.1 or 2, is the dynamic force experienced by the actuator, due to the interaction between its own motion (which it has initiated) and the resulting motion of the flexible system to which it is attached and which it is attempting to control. It is strongly dominated by the system inertia and by the internal flexible dynamics. For rest-to-rest motion, this internal, dynamic force starts and ends at zero, and its integral over time must also be zero (since there is no net momentum change).

As noted above, frequently, in comparison with these internal forces, external forces are negligible. This is the case, for example, for a robot moving through air and in many crane systems (if the usual swinging of the load under gravity is considered to be analogous to an internal, elastic force). But not always. Where external forces are significant relative to the internal, and particularly when they are not constant or are un-modelled, they pose a challenge to any control scheme for flexible systems. In the case of *WBC*, their effects manifest themselves as a change in the return wave, $b(t)$, which in turn leads to errors in the final position, if not properly handled.

This paper considers how to adjust the control law to cope with external forces of different kinds. Sometimes such forces are predictable or quantifiable beforehand; sometimes they are entirely unpredictable in size, direction or timing. Likewise auxiliary sensor information may, or may not, be available. In keeping with the *WBC* philosophy to date, however, it is here assumed that no such foreknowledge or extra measurements are available, and that all measurements, control and system identification should be done at the actuator interface. As before, the only measured variable is the interface force (assuming the actuator sub-controller is already measuring the interface motion). This restriction arguably increases the challenge, but meeting this challenge results in exceptional robustness in the control law to both external and internal changes and disturbances.

4 WBC modifications to cope with various disturbances

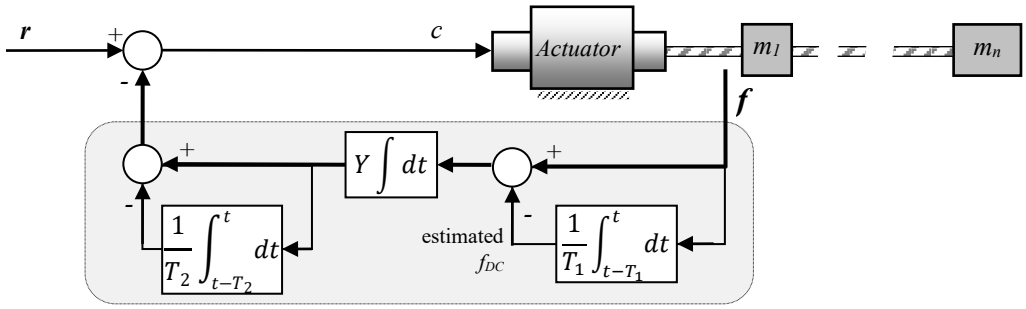

Fig. 4: New controller to cope with all types of external forces acting on the flexible system.

Figure 4 shows modifications to the control system to deal with all kinds of external disturbances while retaining the many benefits of standard *WBC*. It shows an implementation for the rectilinear system (as in Fig.2), which then easily extends to systems translating and rotating in the plane (or indeed moving in 3-D). The various aspects of the controller will now be explained.

As before, the input to the overall system is a reference signal, $r(t)$, corresponding to the desired displacement over time of the flexible system. To achieve rest-to-rest motion this reference changes to a target displacement, perhaps as a ramp, or (more gently) as an s-shaped, double parabola curve, or (less gently) as a step. The essential requirement is that it should settle at the target position, $r(\infty)$.

The main feedback line in the box (the heavy line), as before, takes the time integral of the force multiplied by the admittance *Y*. When this is fed into the actuator it again causes the actuator to act as a viscous damper to the interface force, thereby acting as an active vibration damper, rapidly absorbing vibrational energy from the system for as long as vibrations persist.

Again this would be fine if there were no disturbances and the oscillating force returned to zero. However any continuing (or DC) component present in the measured force $f(t)$ will, when integrated, cause a drift in the feedback signal which in turn causes actuator drift. The solution is to subtract from f an estimate of this f_{DC}, obtained as a running average of f over a suitable time interval T_1, and calculated as

$$f_{DC} = {}^{1}/_{T_1} \int_{t-T_1}^{t} f(t)\, dt. \tag{3}$$

Here the averaging time, T_1, should be long enough to smooth over residual oscillations in f and short enough to track any longer-term variations in f_{DC}. In practice a value slightly longer than the lowest periodic time of the flexible system works well. This ensures that only the oscillatory components of $f(t)$ contribute to the feedback to the actuator, with an average value which quickly approaches zero. In the time domain, the control law for the actuator is now

$$c(t) = r(t) - Y \int_0^t \left(f(t) - {}^{1}/_{T_1} \int_{t-T_1}^{t} f(t)dt \right) dt \tag{4}$$

An added bonus is that this control law will also automatically take care of a secondary problem frequently noticed in practical implementations of the scheme. There will generally be some zero-errors in the force measurement, for example due to strain gauge zero voltage offsets. The subtraction of the DC component of the measured $f(t)$ in Eq.(4) also removes such zero-offset errors, which, no matter how small, would otherwise cause the actuator to drift from the reference position as their effects are integrated over time.

But this is not enough. Certainly if, at steady state, the combination $(f - f_{DC})$ is exactly zero, and all the oscillations have been absorbed, the system will appear to settle. But if the system has undergone impulsive external forces, it may take a long time to settle at the target position, $r(\infty)$. Impulsive (transient) forces become negligible after they act. But while acting they impart a (potentially large) change to the system momentum, and they make an extra contribution to the external force integral. Most of this contribution can endure long after vibrations have ceased and the integrand, $(f - f_{DC})$, has become zero. It causes an apparent settling in the wrong place, with a very slow crawl to the correct final value. Mathematically it is like an integration constant, being carried by the integral. It could also be compared to integration wind-up, which takes a long time to clear the system (even though its source and nature are quite different from the integration wind-up found in PID control systems, for example).

To deal with these effects a further adaptation of the control law proves very beneficial. A time averaged (or a filtered and delayed) version of the entire last term in Eq.(4) is used to cancel this term over a relatively short time. This ensures that, as things settle, the feedback

quickly returns to zero, leaving the actuator at target, despite any accumulated values in the force integral. This is achieved, again, without interfering with the active vibration effects associated with the vibratory components of *f*, which ensures settling. This control arrangement is shown in Fig.4. The equivalent actuator control law in the time domain is

$$c = r - Y\int_0^t \left(f - {}^1\!/_{T_1}\int_{t-T_1}^t f dt\right)dt + {}^1\!/_{T_2}\int_{t-T_2}^t \left[Y\int_0^t \left(f - {}^1\!/_{T_1}\int_{t-T_1}^t f dt\right)dt\right]dt \quad (5)$$

where, for brevity, the time dependence of variables is not explicitly shown. The second averaging time, T_2, can be similar or identical to T_1. In any case its value is not critical. Except for the additional, fourth term, Eq.(5) is identical to Eq.(4). At steady state the sum of the last two terms will be zero, so $c(\infty) = r(\infty)$. In fact, each of the last two terms individually also approach zero, but very slowly, while the final value of ${}^1\!/_{T_1}\int_{t-T_1}^t f(t)\,dt$ approaches the steady state force at the actuator.

These corrections are designed to remove unwanted components but without interfering with the active vibration damping associated with any vibratory components in *f*, so vibrations are continuously absorbed throughout the manoeuvre, with rapid settling at target. The same applies if there are further disturbances after reaching steady state in a new position.

The same modifications easily extend to the *WBC* of the planar mass-spring array. In this case, as already explained, the control system comprises three *WBC* systems acting in parallel, similar to the above, one for the *x*-motion, one for the *y*-motion, and one for the rotation, θ. The spring forces acting at the interface are resolved into horizontal and vertical forces, to give f_x and f_y, and the moment of these forces are taken about a fixed origin (taken as O in Fig.3). The admittance values for each of the component WBC systems can be used as tuning parameters to achieve a preferred trade-off between classical performance metrics such as rise time, settling time and overshoot (all of which are already small in any case).

5 Sample Results

Representative results are now presented for both the rectilinear and planar systems with no external disturbances, and then under significant external disturbances of different kinds. For the rectilinear model the sample system is uniform with spring stiffness k=400 *N/m* and mass m=1 kg, with three masses. The parameters for the planar model are given in Table 1. The mesh is uniform, with k_x, k_y and k_d the spring stiffness in horizontal, vertical and diagonal directions respectively, m the mass values and l the (unstretched) spacing between masses throughout the

mesh. These values are arbitrary in the sense that similar results are obtained when different values are chosen, the system size or shape are varied, or the number of masses and springs is increased or reduced.

Table 1: Parameter values for the model of Fig. 2

k_x (kN/m)	k_y (kN/m)	k_d (kN/m)	m (kg)	l (m)
8	7	10	0.1	0.25

First consider the effects of enduring, smooth, non-impact, external forces. Perhaps the most obvious example of such a force is when, during a manoeuvre, gravity becomes active or the orientation of a system changes with respect to gravity. Changing gravity effects could arise in the "rectilinear" system of Fig.1 as suggested by Fig.5. The control system works very well for this system. Clearly it is no longer "rectilinear". Arguably it could be called "quasi-rectilinear" but it really is planar. So, for this case, the illustrative results are taken for the planar array of Fig.3, but now with the plane of motion vertical (with, say, the x-axis horizontal and y-axis vertical). Gravity, acting in the negative y-direction, now plays a strong role, particularly under rotations. To increase the challenge, it is assumed that the controller has no information initially about the weight of the system, perhaps because it is fully supported before the manoeuvre begins, resting on a platform of some kind.

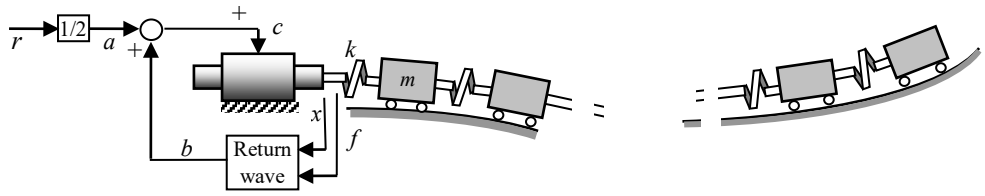

Fig. 5: Quasi rectilinear system experiencing changing external forces from gravity requiring a modified WBC

In the first test then, the reference input is in the vertical direction, with a ramped displacement up to one meter, with and without gravity. Although apparently simple, this motion involves a strongly slewing action of a very flexible system having many degrees of freedom. Figure 6 shows how the actuator responds to this input under different control strategies.

In the absence of gravity, standard *WBC*, described in Eqs. (1) or (2), gives a good response. The system quickly reaches the target displacement while absorbing the flexible system vibrations. When gravity effects are incorporated, however, the same controller fails badly. Following an initial inadequate attempt to follow the reference, there is then a dramatic falling

away, as the control system interprets the weight of the system as a returning wave, which it tries to absorb, leading to an ever-increasing error.

The third curve shows the response under gravity when the *WBC* system is modified as in Fig.4 or Eq.(4) or (5). The system travels quickly to target, with no overshoot, and settles rapidly. This implies that the controller is managing to detect and separate the additional force at the actuator due to the system weight under gravity (applied suddenly and unknown initially), and to compensate for this, automatically, without impairing the active vibration damping action associated with the vibratory dynamics of the system. Thus, at the cost of a modest increase in the settling time, the revised strategy achieves a combination of position control and active vibration damping, in a robust way, using only forces measured at the interface.

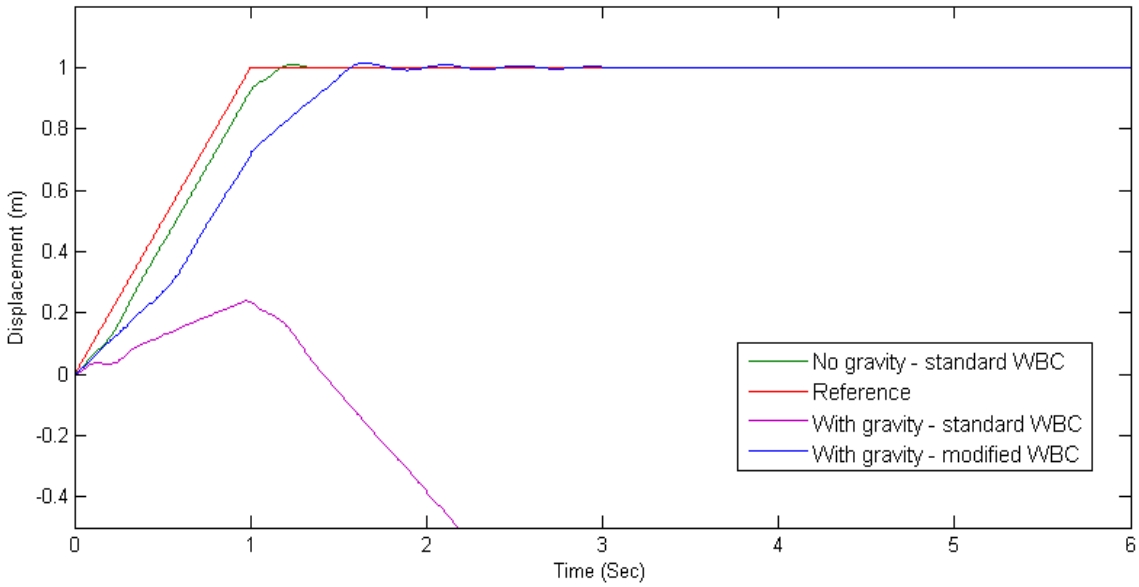

Fig. 6: *WBC* response to slewing reference motion, with and without gravity, using standard *WBC*, Eq.(2), and modified, Eq.(4).

Figure 7 shows a controlled rotation through 1 radian, in the vertical plane (under gravity), of the planar array system of Fig.3. The controller actively absorbs the vibrations and brings the system to rest as required. Note that there is an apparent steady-state error, but only because the response shows the angle at the tip, whereas the input is the angle at the actuator. Due to the gravitational strain these are not identical, indicating a final curvature along the structure. This difference can be considered more a question of statics than of dynamics, and if required can easily be compensated for in various ways. For example, if the reference, target input (1 radian in this case) is intended for the tip rather than for the actuator, then the actuator input can easily be offset to achieve this, either using a pre-calculated offset, experience, or an adjustment determined by observation of the strain.

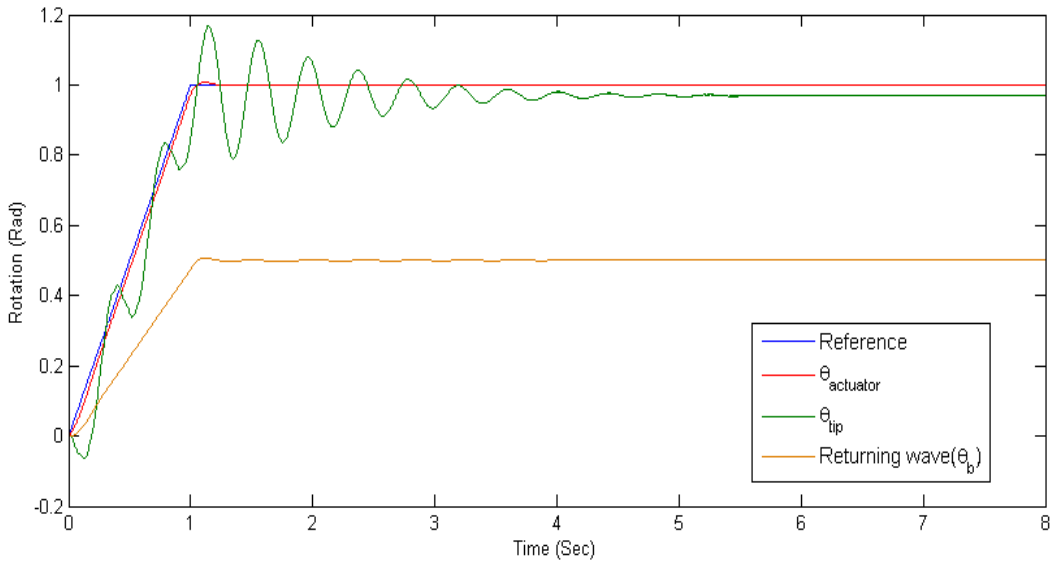

Fig.7: A rotational maneuver under gravity using *WBC* and the control system of Fig.4.

Although not shown, the controller was found to work well when the system was simultaneously translated in x and y directions while also being rotated. The system was also tested for the effects of sensor offset errors and slowly drifting errors. Under the original *WBC* law of Eq. (1) or (2) when such an error was introduced into the models a slow drift was observed in the final position, but under the revised control strategy the system stayed precisely at the target position.

Now consider impulsive forces. The first example is with the three-mass rectilinear system, undergoing a strong impact at the tip during a controlled rightwards displacement. See Fig. 8. The impulse is about 8 *N.s*, with a force of 100 N acting over 0.08 seconds. It acts to oppose the system motion, which has a target displacement of 1 m to the right. The system is uniform with spring stiffness k=400 *N/m* and mass m=1 kg.

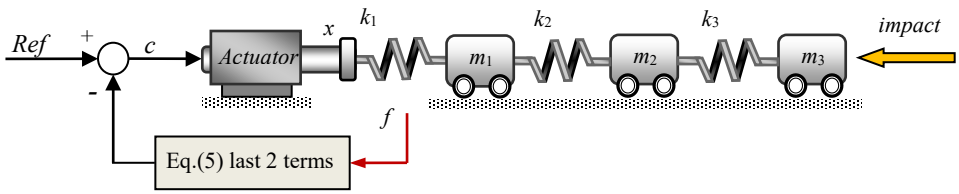

Fig. 8: Three-mass rectilinear flexible system undergoes an opposing impulsive force

Figure 9 shows the response of the third mass under the unmodified *WBC* of Eq.(2) and then under the modified control system of Fig.4. In all cases the system is hit at the same time, about t=4s, while at the settling stage shortly after reaching the target. Under standard *WBC*, Eq.(2), the system settles very well, but in the wrong position with a large steady-state error. What has happened is that the controller has absorbed the returning wave as before, but it now has the

added leftwards momentum from the external impact. This pushes the system back, causing it to come to rest well short of target (reference) input of 1m.

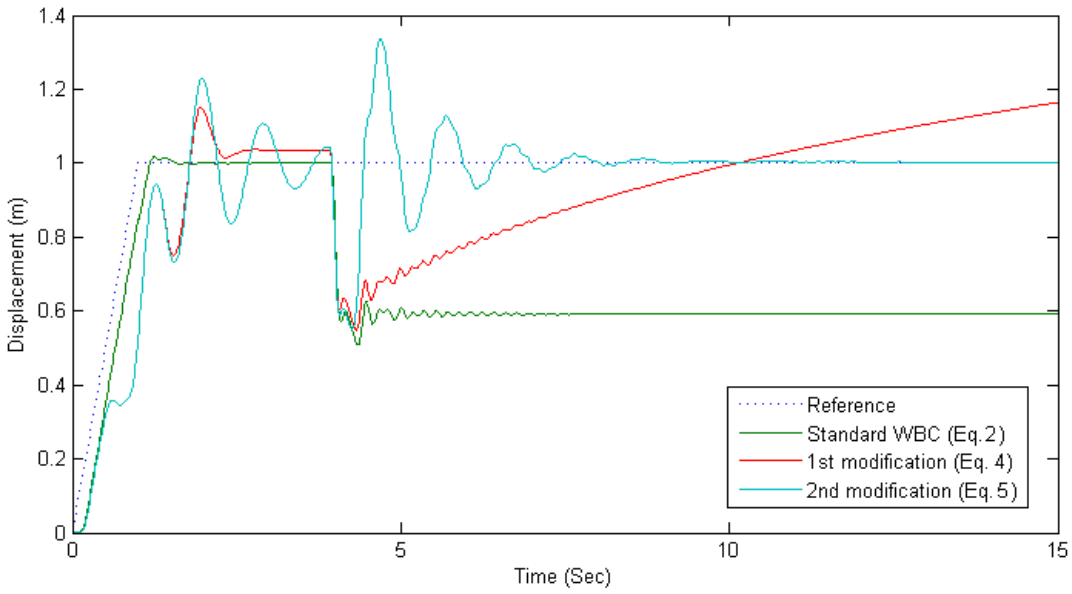

Fig. 9: Tip response of three-mass rectilinear system subject to strong impact under standard and modified *WBC* laws

The first modification, Eq.(4), designed to remove errors due to steady-state forces, reacts to the position error to try to remove it, but it does so poorly. Given enough time it would eventually correct the error, but the delay would be far too long. As can be seen the control law of Eq.(5) or Fig.4 reacts quickly to absorb the vibration and again settle the system exactly at target.

The second illustration, Fig.10, uses the planar array system of Fig.3 undergoing a complex translational manoeuvre under gravity. The *x* and *y* reference motions have different slopes and different final values. In addition to the gravity effects, there are two impacts on the tip of the system, in the *y*-direction at about t= 3s and in the *x*-direction around t= 4.5s. The magnitudes and timing of these impacts are unknown to the controller. The control law is Eq.(5) or Fig.4. The following can be observed.

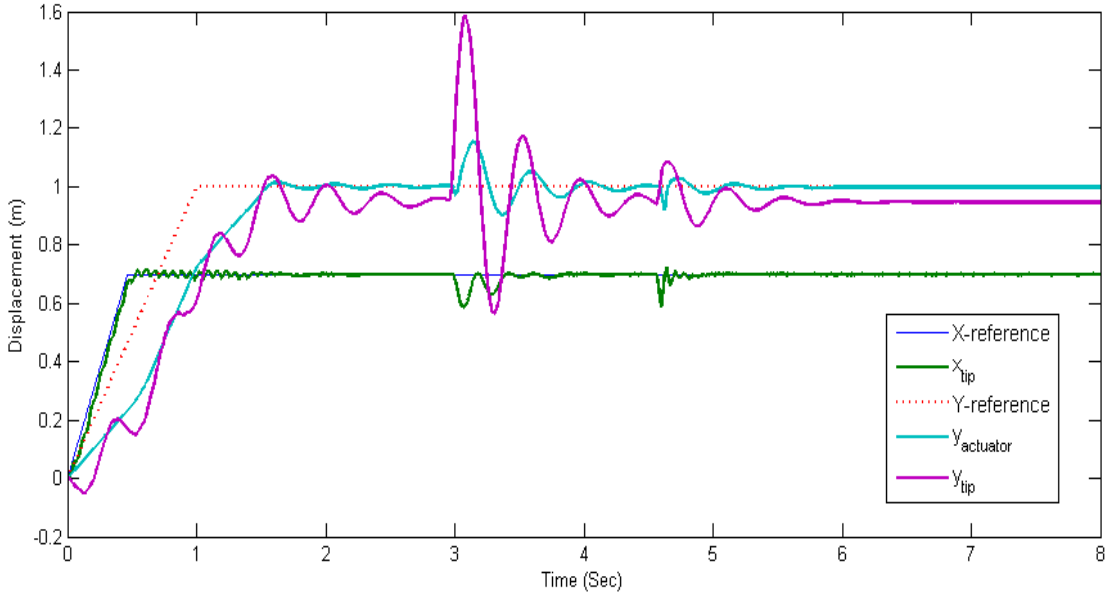

Fig.10: Response of planar system under gravity with two impacts, under the modified *WBC*.

The initial *y* oscillations of the tip are caused by the vertical translation and the initial effects of gravity, which the control system works to overcome while positioning the system to target. The system is beginning to settle at target when the impacts occur. There is some cross-coupling between the *x* and *y* impact effects. These are unequal because the stiffness along the longitudinal beam axis (*x*-direction) is considerably greater than in the transverse, bending *y*-direction. The reference inputs refer to the actuator displacements, whereas the outputs are those of the tip. The apparent error in the final settling position of the tip is simply the deflection due to the weight of system. If the target position is intended to apply to the final position of the tip, the actuator control reference should be adjusted to allow for this deflection.

As a final example, *viscous* forces were added to all the masses in the model. Each mass was given an external damping force proportional to its absolute velocity, which therefore changes magnitude and direction with the system motion and oscillations. Unlike impact forces, viscous forces continue for as long as the system is in motion. With standard *WBC* the external viscous dampers will absorb some of the motion which would otherwise return to the actuator and be absorbed. This results in a final settling of the system short of the target displacement, as seen in Fig.11. If the viscous damping coefficients are known, it is possible to predict the net shortfall, and so compensate for it [30]. But the application of the control law of Eq.(5) also works, as can be seen in Fig.11. An advantage of this strategy is that it does not need any information about the viscosity coefficients.

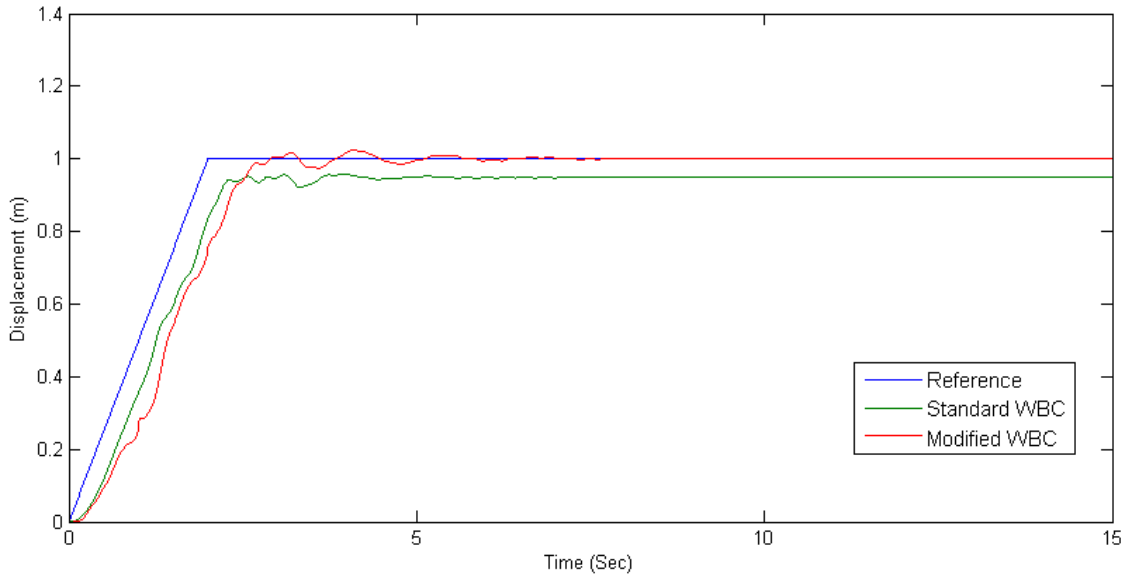

Fig.11: System experiencing external viscous forces, under standard and modified *WBC*

Note that if the damping forces are all internal, for example with viscous dampers connected between masses (rather than to ground), then the unmodified *WBC* system works perfectly. In this case, although there is certainly a loss of mechanical energy, there is no momentum lost to the environment during a manoeuvre. This is the decisive point, because despite the dampers, all the forces are still internal, and the external forces are still negligible.

5 Concluding remarks

Complete motion control of a complex, under-actuated flexible system, undergoing unquantified external disturbances, has been achieved using simple measurements taken only at the actuator. This complete, robust control, by a single actuator, has been achieved without a detailed system model. The controller needs no measurements from other parts of the system or details of the system properties (such as inertias, stiffnesses, dimensions, vibration modes, or damping). Neither does it need to know the size, location and timing of the external forces. The modifications to *WBC* here presented also retain the many advantages of standard *WBC*, including strong robustness to system variations, robustness to actuator dynamic limitations, precision, speed of response and ease of implementation.

No detailed system model is needed. It is true that the controller requires two or three parameters, namely Y, T_1 and T_2, but a) these are easily estimated, and b) none of them is critical. It is found, for example, that changing their values by ±50% or more from the suggested nominal values still produces good control responses, with no steady-state error. The main effects are variations in the transient behaviour. If desired, they can be tuned to achieve a classical trade-off between rise time, overshoot (small in any case) and settling time.

If desired, further information can be extracted from the measure interface force and from the waveform of the measured return wave $b(t)$. Such information could be useful in some applications for system identification, system monitoring, or environment monitoring. For example, the modified returning wave and measured f_{DC} can be used to say whether or not an external force acted on the system during the manoeuvre; whether or not it continues at the end; the associated momentum and the energy added to, or removed from, the system; whether the waveform was impulsive (impact-like) or spread over time; and approximately when the interaction happened.

The relative magnitudes of the external and internal forces can vary widely, depending on both the system dynamics (stiffness and inertia values) and on the nature of the external forces. The proposed control systems will cope with the entire range of possibilities, and the extremes where one or other set of forces dominates.

The *WBC* literature has an alternative way to resolve interface motion into outgoing and returning waves, especially for lumped systems, based on "wave transfer functions" [14, 21]. All the techniques and results here presented can also be obtained using these techniques. They would involve a way of evaluating the content of the boxes in the diagrams for calculating the returning waves different from the control laws given above. They give a slight improvement in the control performance at the cost of a small increase in computational complexity. It was decided in this paper, however, to do the wave analysis exclusively using the simple force integral and interface admittance approach.

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