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Heuristic Optimization of Consumer Electricity Costs Using a Generic Cost Model

Chris Ogwumike *, Michael Short * and Fathi Abugchem

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School of Science and Engineering, Teesside University, Middlesbrough TS1 3BA, UK; F.Abugchem@tees.ac.uk
* Correspondence: C.Ogwumike@tees.ac.uk (C.O.); M.Short@tees.ac.uk (M.S.); Tel.: +44-1642-73-8099 (C.O.); +44-1642-34-2528 (M.S.)

Abstract: Many new demand response strategies are emerging for energy management in smart grids. Real-Time Energy Pricing (RTP) is one important aspect of consumer Demand Side Management (DSM), which encourages consumers to participate in load scheduling. This can help reduce peak demand and improve power system efficiency. The use of Intelligent Decision Support Systems (IDSSs) for load scheduling has become necessary in order to enable consumers to respond to the changing economic value of energy across different hours of the day. The type of scheduling problem encountered by a consumer IDSS is typically NP-hard, which warrants the search for good heuristics with efficient computational performance and ease of implementation. This paper presents an extensive evaluation of a heuristic scheduling algorithm for use in a consumer IDSS. A generic cost model for hourly pricing is utilized, which can be configured for traditional on/off peak pricing, RTP, Time of Use Pricing (TOUP), Two-Tier Pricing (2TP) and combinations thereof. The heuristic greedily schedules controllable appliances to minimize smart appliance energy costs and has a polynomial worst-case computation time. Extensive computational experiments demonstrate the effectiveness of the algorithm and the obtained results indicate the gaps between the optimal achievable costs are negligible.

Keywords: demand side management; smart grid; decision support system; heuristic algorithm; load scheduling

1. Introduction

Smart grids are modern electricity infrastructure networks. They cost-effectively integrate the actions and behaviors of all the connected users in order to ensure safe, sustainable and reliable electricity supply [1]. The emerging smart grid, by use of an Advanced Metering Infrastructure (AMI)—a two way communication infrastructure—can deliver real-time prices of electricity to consumers and simultaneously send back their consumption data to the utility service companies for billing and other purposes [2]. This enables consumers to manage energy distribution efficiently by modifying their consumption behavior in line with the pricing signals. Currently, most household consumers buy electricity on flat rate tariff and have no demand response incentives to encourage shifting energy consumption from peak to off peak period. Smart pricing mechanisms such as RTP, critical time pricing (CPP), and TOUP could lead to cost-reflective consumption, driven by aspects of the entire supply chain involved in delivering electricity during a certain period of time in a given quantity at a specific location [3]. However, the major difficulties in utilizing the pricing incentives are the current lack of automated decision support system, coupled with most users not being knowledgeable enough (or having enough spare time) to respond to the time varying electricity prices. Hence, an automated energy management system or Intelligent Decision Support

System (IDSS) for load scheduling is highly desirable. Even disregarding the technical challenges and complexities of connecting an IDSS to both an AMI and controllable home appliances, smart home load scheduling using variable price signals remains a difficult problem to solve computationally. In most cases the problem is NP-hard and is also affected by uncertainties such as variations in appliance power profiles. Moreover, an IDSS is ideally also required to be responsive to unexpected or emergency events, such as specific DSM requests relayed through the AMI following unexpected events affecting the wider grid. Therefore, we consider a rolling-horizon framework such that regular re-optimization with updated information regarding the current system state and energy cost updates provided by the electricity supplier can be implemented. To be of practical use, the optimization carried out by the IDSS must be able to deliver results of reasonable quality in a short space of time. In this paper, we present a low-overhead heuristic scheduling algorithm for use in a consumer IDSS for minimizing smart appliance energy costs.

In the wider context, effective distributed energy generation based upon renewable resources is a major goal of the smart grid. Such generation can provide clean and sustainable energy and (potentially) enhance power system capacity and security. In addition to reducing consumer energy costs, the enhanced DSM support that can potentially be delivered by consumer IDSSs should be able to help manage the integration of renewable resources, since a large proportion of energy generation in smart grids is expected to come from non-dispatchable renewable resources such as wind, solar and wave energy [4]. These renewables are intermittent in nature and it remains an important challenging factor to manage their output generation with demand fluctuations. However, the potential coordination of distributed energy generation, energy storage systems and smart home loads will lead to more robust optimization and corresponding energy cost savings. Utilizing price signals that reflect the forecasted value of energy during a particular hour—and also its uncertainty—may help to enable this optimization and coordination. In this paper, we consider a generic and flexible cost function for hourly energy pricing in our optimizer. This model can be configured for traditional on/off peak pricing, RTP, Time of Use Pricing (TOUP), Two-Tier Pricing (2TP) and various combinations thereof. The heuristic we propose greedily schedules controllable appliances to minimize this cost function, and has a polynomial worst-case computation time. Extensive computational experiments demonstrate the effectiveness of the algorithm, and the obtained results indicate the gaps between the optimal achievable costs are negligible; although some differences in solution structure are evident in certain cases. The remainder of this paper is structured as follows: Section 2 presents a review of related work and highlights the contribution of the current paper. Section 3 describes the models employed, while Section 4 describes the optimization procedure we propose. Sections 5 and 6 describe the simulation studies that have been carried out to investigate the efficiency and performance of the heuristic algorithm. Section 7 presents our conclusions and outlines areas for future work.

2. Related Work and Contribution

Extensive demand side management strategies using techniques such as Mixed Integer Linear Programming (MILP) [5–7], Direct Load Control (DLC) [8], branch and bound algorithms [9,10], *etc.*, have been presented in the literature as potentially effective solutions for the consumer load scheduling problem. The fact remains that more work has to be done in practice, as most existing methods are not readily applicable for scheduling large numbers of appliances and for real-time implementation in households. Additionally, metaheuristic search algorithms have also been proposed in the literature over the last two decades for scheduling residential and commercial loads. Most of the existing metaheuristic such as Particle Swarm Optimization (PSO) [11,12], Ant Colony Optimization ACO [13], Simulated Annealing (SA) [14], Genetic Algorithm (GA) [15–17], *etc.*, are inspired by natural phenomenon. These studies explore alternative means of scheduling and optimizing a power profile at any hour of the day since an optimal deterministic technique is unrealistic to most customers.

A significant focus of recent research has also been on heuristic algorithms applicable to residential and industrial scheduling problems. Heuristic approaches can be efficient in achieving faster solutions which could be implemented on an embedded system or computer for the purposes of a consumer decision support system. On the other hand, a "good", but not necessarily optimal solution to the optimization problem can only be found, but it will be found in a reasonable time. In [18], an intelligent Home Energy Management (HEM) algorithm is presented for managing high power consumption household loads according to a preset priority. Reference [19] proposed a heuristic algorithm to determine price update interval and step size required for limiting deviation of power load from a desired load. An aggregator-based residential DR approach for scheduling residential assets was proposed in [20]. They further designed a heuristic framework to perform optimization on the profit of the aggregator.

In previous work by the current authors [21,22], an efficient heuristic for scheduling residential appliances in the presence of RTP was proposed and partially evaluated. The heuristic is based upon greedy list-processing of smart appliances into a feasible energy schedule, with the aim of minimizing costs subject to a set of constraints over the appliances. The heuristic, although it does not guarantee the lowest possible costs, is simple and efficient enough to ensure that regular re-optimization with updated state information can take place. Initial investigations indicated that the heuristic was in fact highly competitive against an exact algorithm in terms of achieved cost reductions in a simulation-based evaluation.

Our contribution in the current paper is first to extend the cost model employed in this heuristic algorithm, and secondly to further explore its properties under realistic simulation conditions using a wider range of pricing signals. In particular, for the latter we consider cost-based scheduling of the smart home appliances in response to RTP, TOUP and 2TP. To the best of the author's knowledge, no results related to the performance of such a heuristic in the presence of such cost models have previously been published in the literature.

3. Optimization Model

This section provides the mathematical formulation of the residential load scheduling problem and the generic cost model employed. Our focus is to optimize the power (and hence energy) profile at any given timeslot to minimize costs subject to the given constraints. Figure 1 shows the block diagram of a demand side management system, which comprises the data and power flow between a smart meter, decision support system and the smart home appliances. The smart meter receives external signals (e.g., spot prices of electricity) from the utility service providers. This information is used by the optimizer to determine a cost effective scheduling of controllable smart appliances. Residential users can visualize the appliance scheduling and recommendations provided by the energy management decision support to enable informed decisions on their energy consumption pattern/usage.

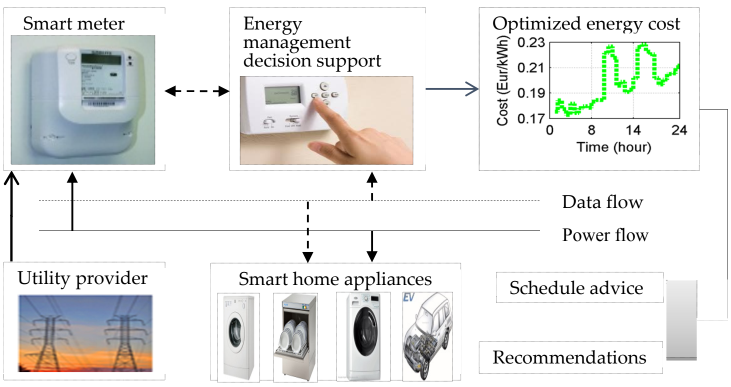


Figure 1. Block diagram of Residential energy management system.

3.1. Optimization Overview

We assume that the scheduling/planning horizon is divided into $H > 0$ uniform time slots; each time slot is of length $T > 0$ h. Typically, each slot will be of length $T = 1$ h, although this does not necessarily have to be so in the general case. Let the number of appliances be denoted as N, and the number of stages of appliance i be denoted as $n_i > 0$. The power consumption during stage j of appliance i is denoted by $P_{i,j}$, $i \in [1, N]$, $j \in [1, n_i]$. Let the starting time of appliance i be denoted by the integer variable $s_i \in [1, H]$. Then the power consumed by appliance i with start time s_i during timeslot h is given by:

$$x_i(h) = \begin{cases} P_{i,h-s_i+1} & : \text{If } 0 < (h - s_i + 1) \leqslant n_i \\ 0 & : \text{Otherwise} \end{cases} \tag{1}$$

Let the cost of consuming $x_h \geqslant 0$ units of energy during a particular hour h be represented by the cost function $C_h(x_h) \geqslant 0$. The optimization problem objective function J can then be formulated as the sum of the energy costs across each slot in the horizon as follows:

$$J = \sum_{h=1}^{H} C_h \left(T \cdot \sum_{i=1}^{N} x_i(h) \right) \tag{2}$$

The basic form of the optimization problem can then be formulated as follows:

$$\begin{aligned} &\min(J) \\ &\text{with respect to:} \\ &s_j : 1 \leqslant j \leqslant N; \end{aligned}$$

$$\begin{aligned} &\text{subject to:} \\ &s_i^{Min} \leqslant s_i \leqslant s_i^{Max},\ s_i \in I : 1 \leqslant i \leqslant N; \end{aligned} \tag{3}$$

$$\sum_{i=1}^{N} x_i(h) \leqslant X_h^{Max} : 1 \leqslant h \leqslant H; \tag{4}$$

Constraints (3) are the user start time preferences which ensure that each appliance does not operate outside of the set time preference interval given by ${s_i}^{Min}$ and ${s_i}^{Max}$. Constraints (4) ensure the maximum power consumption for all the appliances at any time slot h does not exceed the power threshold, where ${X_h}^{Max}$ is the threshold at slot h. Typically this will be set by the household to suit its own specific constraints, such as the maximum power rating of the incoming supply or consumer unit. In addition, appliance specific constraints can be applied to ensure certain appliances start or finish before each other. An example is the case of washing machine and dryer where the latter must not start until the former has completed all of its operation stages. For certain types of interruptible appliances, it may also be possible to schedule a bounded amount of time-delay between two consecutive operation stages (e.g., a delay between a rinse cycle and the next wash cycle in a washing machine). In such cases, the model may be extended by appropriate splitting of the main appliance into a number of sub-appliances, each with a separately considered start-time; appropriate constraints relating the start times of each sub-appliance will then model the required behavior. By appropriate choice of T and H, the model may be configured to a given level of temporal fidelity and future planning horizon length. In the remainder of the paper we assume that $T = 1$ and $H = 24$, *i.e.*, hourly slots are considered over a planning horizon of one day. In Appendix A, we shown that the decision version of the problem described above is NP-Complete, and is hence intractable for large problem sizes unless P = NP. The optimization version of the problem is therefore NP-hard.

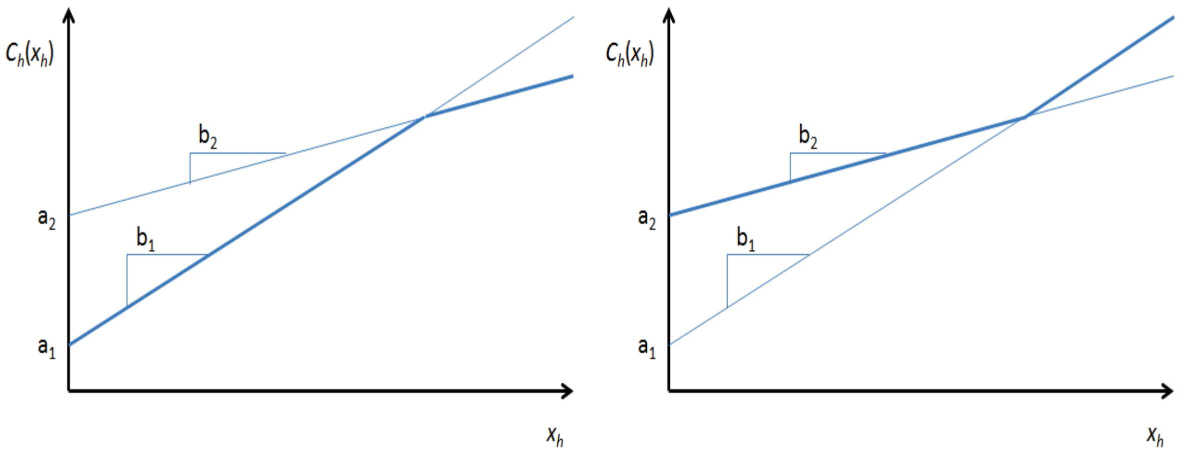
3.2. Generic Cost Model

We assume that the cost of energy during a particular slot h is a generic function $C_h(x_h)$ of the amount of energy consumed, which is x_h units. Typically, the form of C_h will depend heavily upon pricing of electricity in a day-ahead (spot) market and also any specific DSM initiatives advertised to the subscribed residents by the supply/distribution company via the smart meter/AMI. The source of the energy supply is assumed to be a hybrid generation comprising the conventional forms of generation (gas, coal *etc.*) plus distributed renewables (solar, wind, biomass *etc.*); hence the nature and form of C_h can also depend upon the availability of these latter renewables, and have components linked to balancing (real-time) energy market prices. Two particular cases seem to be of most interest at the present time for representing costs in the presence of fluctuating costs and DSM signals; in these cases, costs are represented by a concave/convex combination of two piecewise affine functions:

$$C_h(x_h) = \max\{a_1 + b_1 x_h, a_2 + b_2 x_h\}, b_1 \le b_2 \tag{5}$$

$$C_h(x_h) = \min\{a_1 + b_1 x_h, a_2 + b_2 x_h\}, b_2 \le b_1 \tag{6}$$

In particular, Equation (5) represents a case in which a cover charge (a_1) plus a base price (b_1 €/kWh) is incurred for energy used up to a certain limit $(a_1 - a_2)/(b_2 - b_1)$ kWh, beyond which a higher price (b_2 €/kWh) is incurred for each extra unit consumed. This represents a pricing model in which increased production costs are reflected as increased consumer costs for increased consumption, and with the prices and low consumption limit linked to external market conditions. Equation (6) represents a similar situation except a reduction in cost is incurred for consumption above the limit, reflecting an economy of scale. Models (5) and (6) can be used to reflect specific cost incentives encouraging consumers to shift their consumption from peak to off-peak times, with both base and high consumption prices that can be linked to an underlying pricing plan. The cost functions Equations (5) and (6) are shown graphically in Figure 2 below.



(**a**) (**b**)

Figure 2. Illustration of the piecewise affine price model. (**a**) Concave (min) configuration; (**b**) Convex (max) configuration. Note: In most cases, either the parameter a_1 or a_2 will be equal to zero.

By appropriate choice of the parameters a_1, a_2, b_1 and b_2 for each hour, such a cost model is flexible enough to capture the salient features of RTP, TOUP, 2TP and various combinations in addition to specific DSM incentives. Unlike RTP, TOUPs are more customer friendly due to the predictable nature of the pricing signals. Adopting TOUP scheme has an effect on load shifting, which in turns helps to achieve demand response [23]. TOUP mainly consist of two or more tier rates namely peak, off-peak and in some cases mid-peak prices depending on customers need and load profile pattern which varies across different countries and locations. However, the more the tiers, the more difficult the model would be for customers to participate. Hence, in this paper we will consider the more difficult the model would be for customers to participate. Hence, in this paper we will consider the two tiers pricing to reduce complexity since mid-peak rates only examines the average costs between the peak and off-peak periods. 2TP is organized such that the rate of tariff paid below a certain power threshold is lower than the rate paid above it; this to penalize high consumption in any one hour and encourage even load distribution. However, the effect on demand response of combining 2TP with RTP—in which a customer may pay a basic unit rate until the threshold is exceeded, at which time a price linked to the spot price is incurred—has not been investigated fully in the presence of load scheduling. The simple functions we propose in Equations (5) and (6) allow such an investigation to be carried out.

Under the assumption that the cost functions $C_h(x_h)$ are linear, or piecewise linear and convex, the optimization problem above can be solved using mixed integer linear programming (MILP) software such as the IBM ILOG CPLEX and the YALMIP interface to Matlab [24]. Nevertheless, solving such MILPs efficiently can only be done for relatively small instances of appliances [25]. Algorithms such as cutting plane methods and the branch and bound method [26] can also be used to reduce the average execution time complexity. In the case that the costs may be arbitrary non-linear functions—or combinations of even simple convex and concave functions at different hours over the horizon—then a large number of additional binary variables may need to be introduced to solve the problem. This may result in unacceptable overhead, even for relatively small numbers of appliances; in addition, the use of specialized solvers will be impractical and should be avoided on small devices such as smart meters and an IDSS computer. Therefore, instead we seek to find *good*—not necessarily optimal—solutions to this problem, in a reasonable time without undue computational overheads. The heuristic we propose is described in the next Section.

4. Scheduling Algorithms

In this section, we improve the scheduling algorithms (exact and heuristic) that were proposed in [22] with the addition of the cost models described in the previous Section. The algorithms use appliance start times s_i as the decision variables and search over the future time horizon (window) H for the start times which minimize the expected electricity cost J subject to the given constraints. Parameters such as the number of appliances N, length of timeslot T, hourly timeslot cost functions ($C_h(x_h)$), constraints *etc.* are assumed given and define the problem instance. In the sequel, the performance of the proposed heuristic algorithm will be evaluated and compared against the proposed exact method in simulation studies.

4.1. Exact Method

In principle, exact methods can guarantee an optimal solution to this NP-hard optimization problem. This can be achieved by searching the timeslots within the set time window exhaustively. In our proposed exact method—shown in pseudocode below—the algorithm exhaustively searches appliance start times for the best possible combination of starting times to obtain the minimum costs which satisfy the constraints. The exact algorithm iterates through each possible combination of start times in the specified user intervals in turn. In the worst case, each of these intervals will be of length H timeslots, giving an exponential run-time complexity of $O(H^N)$ for the algorithm. During the search iteration, the exact algorithm updates the best solution whenever a feasible cheaper cost solution is found. The algorithm could clearly be improved by adding features such as back-tracking of partial solutions that cannot improve upon the best solution found so far; however, its use in this paper was principally to obtain optimal solutions for comparative purposes.

Algorithm 1. Exact Method.

```
Initialization: Set and initialize the N appliances, constraints and cost functions;
for i = 1 to N do
    s_i := s_i^Min;
end for;
C_B := INF;
S := [];
Done := FALSE;
while Done == FALSE do
if Constraints Satisfied do
    J := Evaluate Full Schedule Cost;
    if J < C_B do
        C_B := J;
        S := [s_1, s_2, … s_N];
    end if;
end if;
for i = 1 to N do
s_i := s_i + 1;
    if s_i > s_i^Max do
        s_i = s_i^Min;
        if i == N do
            Done=TRUE;
        end if;
    else
        break;
    end if;
end for;
end while;
return [C_B, S];
```

4.2. *Heuristic Method*

In the proposed heuristic algorithm, appliances are scheduled sequentially based on a greedy strategy without back-tracking. Appliance start times are scheduled one-by-one, and the cost is evaluated for each feasible start time and considers only the current appliance and those which have already been scheduled and their start times fixed. Once the minimum cost for the current appliance is determined, its start time is fixed and is not subsequently changed once scheduling continues to the next un-scheduled appliance. All appliances are scheduled in this way. A single loop over N appliances, considering the start times of each appliance within its specified user interval is performed. In the worst case, each of these intervals will be of length H timeslots, giving a polynomial run-time complexity of $O(HN)$ for the heuristic algorithm.

Given the similarity of the heuristic algorithm to the "List Processing" algorithm for multiprocessor scheduling, and the similarity of the considered appliance scheduling to multiprocessor scheduling (as demonstrated in the Appendix A), it follows that the heuristic we propose may inherit some of the known good performance bounds of the "List Processing" algorithm. Indeed, if appliances are all single-stage and are sorted in non-increasing order of power requirements, then our heuristic would achieve a cost not greater than a factor of $4/3 - 1/(3H)$ away from the optimal cost [27]. For a typical configuration with $H = 24$, the heuristic cost would never be larger than 32% more than the optimal cost. In order to investigate the heuristic properties in more depth, detailed computational experiments now follow.

Algorithm 2. Heuristic Method.

Initialization: Set and initialize the N appliances, constraints and cost functions;
$C_B := 0$;
$S := []$;
for $i = 1$ **to** N **do**
    $C_B := C_B + \text{INF}$;
    for $s_i = s_i^{Min}$ **to** s_i^{Max} **do**
        $J :=$ Evaluate Partial Schedule Cost;
        if Constraints Satisfied
            if $J < C_B$
                $C_B := J$;
                $s_B := s_i$;
            end if;
        end if;
    end for;
    $s_i := s_B$;
end for;
$S := [s_1, s_2, \ldots s_N]$;
return $[C_B, S]$;

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5. Simulation Studies

To demonstrate the effectiveness of the proposed algorithms, we present several different computational experiments. First, to evaluate and compare the minimum cost of appliance schedule for exact and heuristic algorithms based on RTP; second to evaluate the test result based on TOUP, combined with 2TP (TOUP/2TP), and to compare the results (exact and heuristic) with the RTP/2TP test results and the corresponding power distribution for RTP and 2TP across different hours, days, months and respective seasons of the year. In these first sets of experiments, we consider a single household with 4 controllable home appliances. We assume two instances where the household user is subscribed to receive the advertised RTP, 24 h in advance; and a case where the household buys electricity based on TOUP. The power maximum limit (safety threshold) is assumed to be 5500 w throughout. Details of the appliance technical specifications are as given below:

5.1. Minimum Cost Evaluation Based on Real-Time Energy Pricing (RTP)

In this experiment, we aim to determine the differences in the cost of scheduling appliances with the exact and heuristic algorithms respectively. RTP was used for optimization which is carried out once every 24 h for one simulated year duration, considering the period from 1 December 2013 to 30 November 2014. The scheduling consists of four controllable appliances namely washing machine, dishwasher, tumble dryer and Electric Vehicle (EV) as indicated in Table 1. In the scheduling, appliance operation constraints are applied such that the washing machines stages must finish before the tumble dryer phase starts. The hourly pricing data for the RTP was taken from the Scandinavian electricity market Nordpoolspot [28] and samples of these prices are shown in Figure 3 below. Note that the raw (wholesale) costs for electricity were employed; in reality, consumer costs would also include per-unit taxation and distribution charges which actually form a large proportion of the final price, and are typically over 50% in the EU. Nevertheless, price variations with the inclusion of these extra charges are still primarily as a result of wholesale price fluctuations, and our experiments still give a realistic indication of algorithm behavior.

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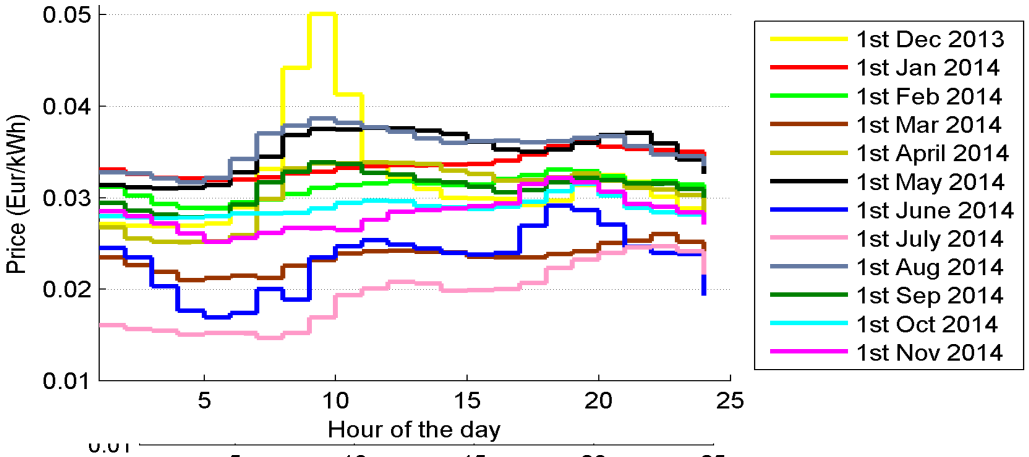


Figure 3. Example of the hourly pricing of electricity used in the simulation, showing the plot for 1st day of every month from December 2013 to November 2014 [28].

Table 1. Data specification of the appliance scheduling.

Devices	Power Consumption (Watts)	User Time Preference
Washing machine	2100	10:00–20:00
Tumble dryer	1200	10:00–22:00
Dish washer	1900	17:00–23:00
Electric vehicle	1000	1:00–5:00

The simulation results of the total consumption costs for the exact and heuristic algorithms across the different months of the year used are plotted in Figure 4.

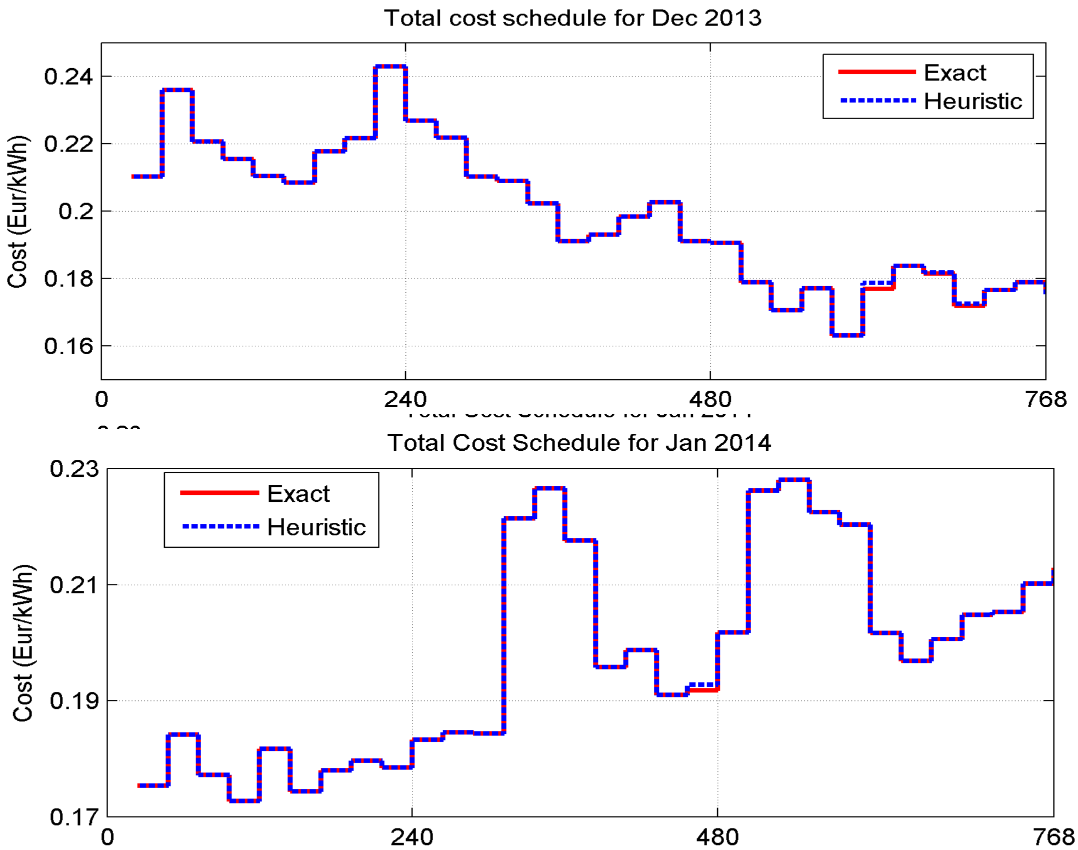
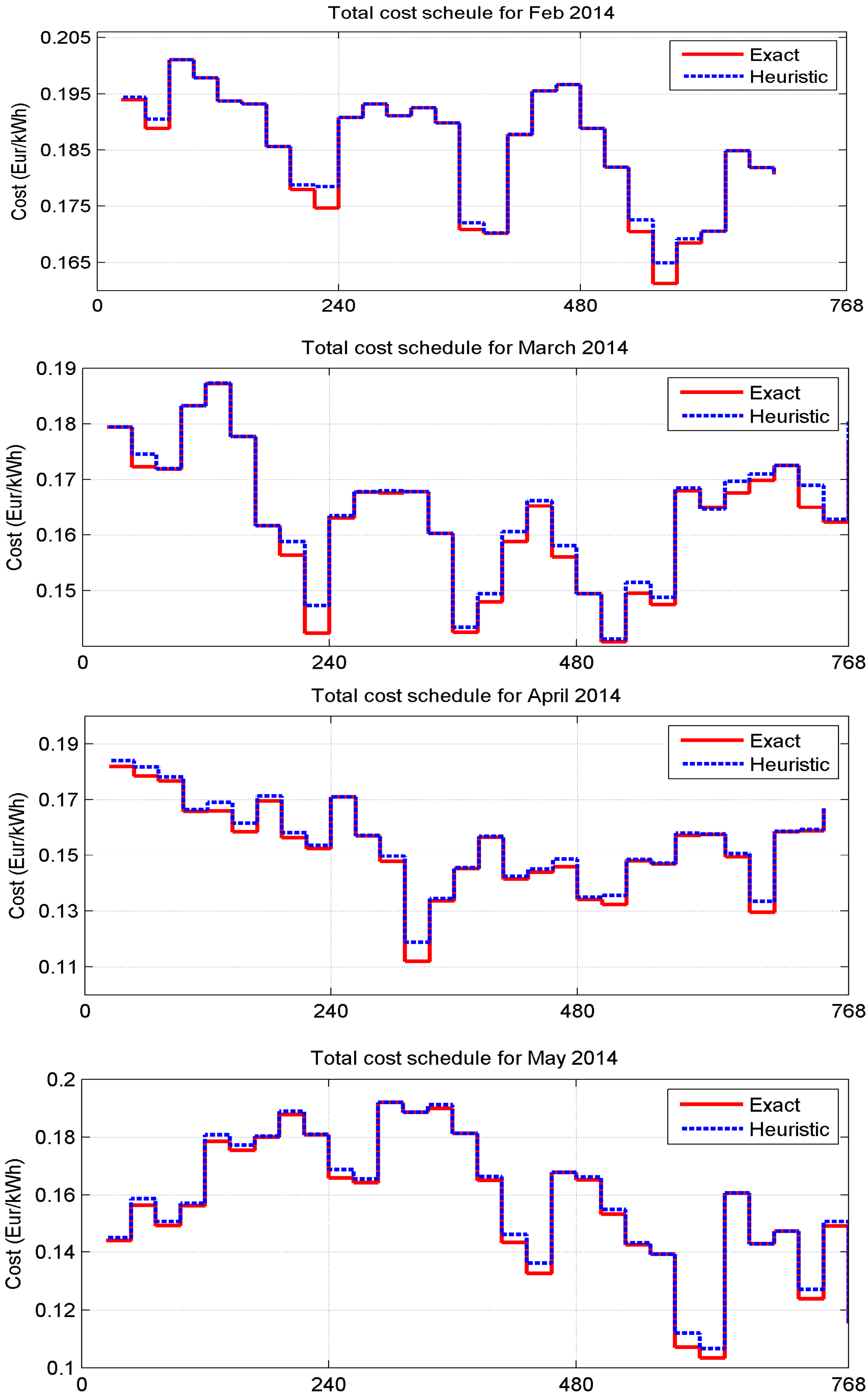


Figure 4. *Cont.*



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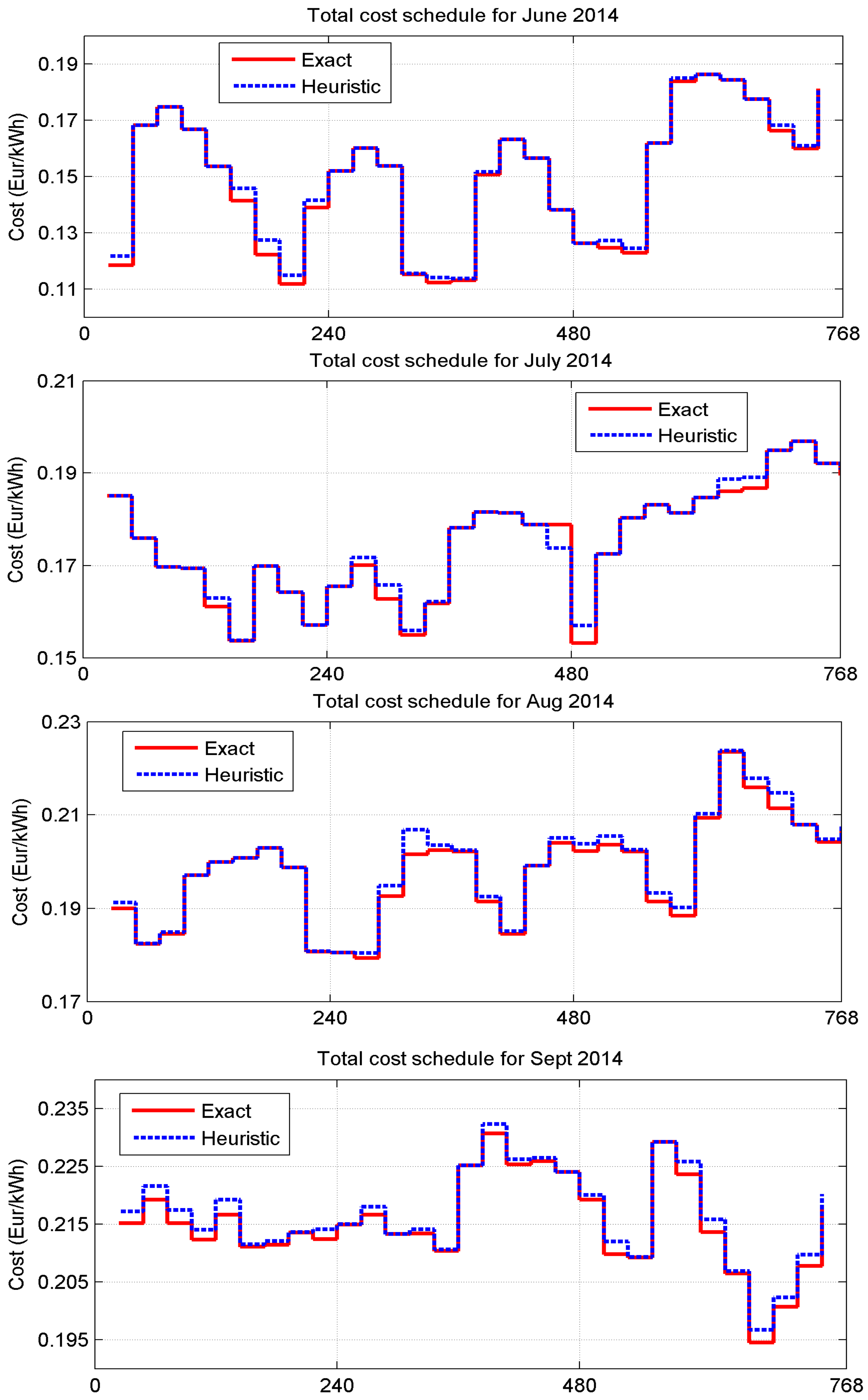

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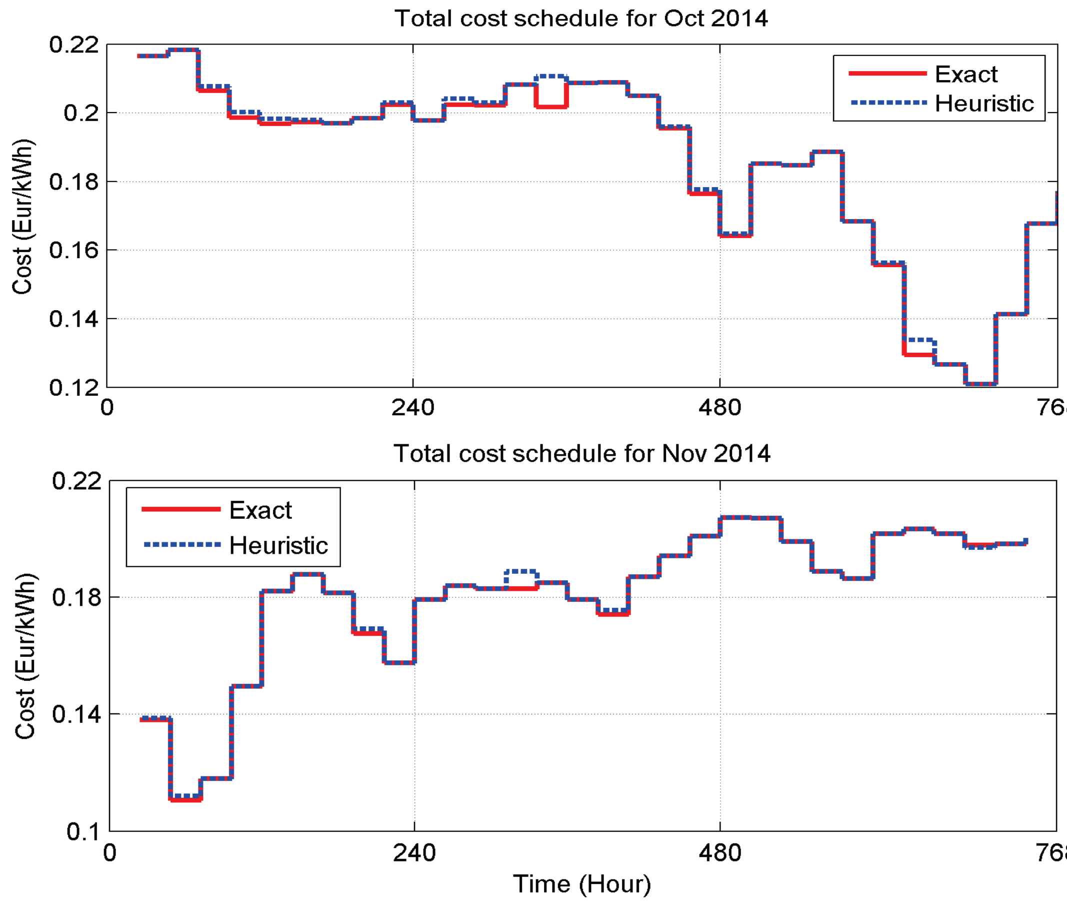

Figure 4. Total consumption cost solutions obtained with the exact and heuristic algorithms across 12 months for the simulation period from December 2013 to November 2014.

From the results, we can see that the heuristic achieves a near optimal solution across the course of the whole year. Average monthly consumption costs of €2.1674 were incurred as compared to €2.1582 obtained by the exact algorithm. Percentage cost difference in these figures confirms that the proposed heuristic achieves up to (2.1674 − 2.1582)/2.1674 = 0.0042% of the optimal solution obtained by the exact algorithm. However, both algorithms schedule the same amount of energy in the household, but the heuristic takes a significantly smaller amount of computation time (0.000704 s) when compared to the proposed exact algorithm (0.00246 s) which is approximately 71% difference in the solving time (see [21] for a detailed comparison of CPU execution times for typical configurations).

5.2. Cost Evaluation Based on Two-Tier Pricing (2TP)

This experiment studies the impact of using a 2TP model in conjunction with an RTP model on both the residential electricity consumption cost and energy consumption profile. In a basic 2TP, the amount of energy consumed above a given power threshold is set as the tier-two price, and a tier-one is charged for consumption below this threshold. This allows more even balancing of the electricity used during the overall billing period [29]. Both the tier-one and tier-two prices could be fixed, follow typical on/off peak periods, or even change hourly. In our study, the 2TP is modelled such that the basic rate charged follows the hourly RTP (wholesale price as in Section 5.1), and the high rate—charged for consumption over a fixed threshold—is a multiple (>1) of this base price. In this simulation, we set the higher price to be 150% of the base price for consumption exceeding 1500 Wh. This configuration was motivated by the configuration of the British Columbia hydro two-tier pricing system as shown in Figure 5. We investigate whether the heuristic algorithm would be as effective at enabling residential energy consumers to respond to the 2TP/RTP charges by shifting peak consumption to off-peak period as with the response to the RTP-only charges reported above.

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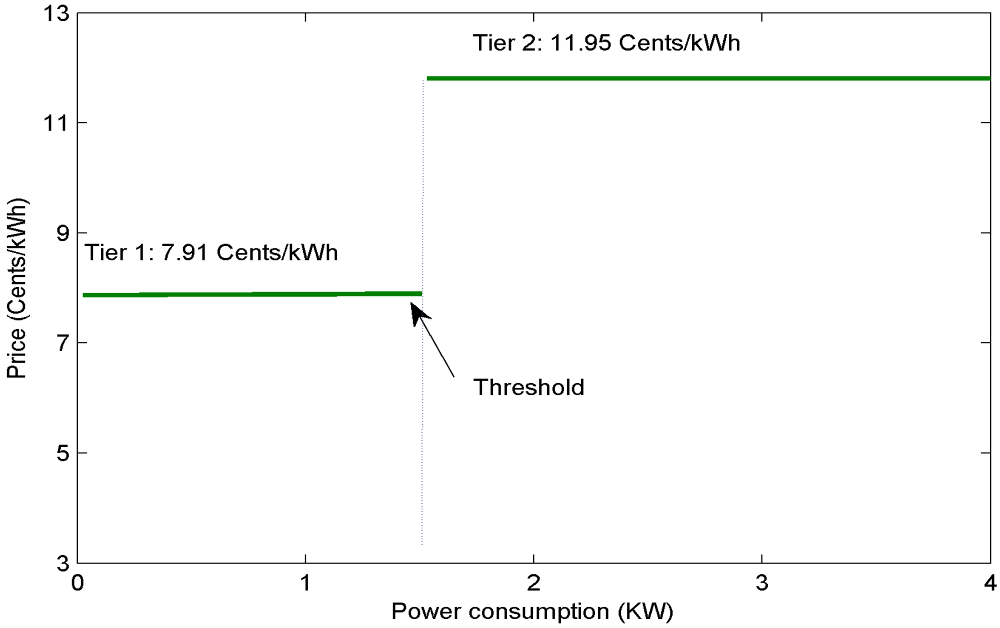

Figure 5. Example of 2TP model used by the British Columbia Hydro Residential usage charge updated in 1st April 2015 [29].

In this experiment, the simulation was carried out across four months in 2014—January, April, July and October, representing samples of the four seasons of the year (winter, spring, summer and autumn) respectively. Again, the heuristic achieves almost the same result—with respect to costs—compared with the exact algorithm as seen in Table 2 below. Comparing these results with those obtained for RTP alone in the previous section, the relative cost between the RTP and RTP/2TP is approximately a 20% increase for the latter under both the heuristic and exact algorithm solutions. RTP/2TP being the more expensive of the two schemes is to be expected, however, given the nature of the cost models.

Table 2. Simulation Result of 2TP/RTP Model across representative seasons of the year.

Months of the Year (2014)	Heuristic Algorithm Average Total Cost (Eur/kWh)	Exact Algorithm Average Total Cost (Eur/kWh)	Relative Difference in Average Total Cost (%)
January	0.46451	0.46221	0.00495
April	0.36140	0.35502	0.01765
July	0.40928	0.40461	0.00467
October	0.43770	0.43247	0.01195

Furthermore, additional experiments were carried out in which the RTP and 2TP/RTP were evaluated against a basic TOUP cost model and also a 2TP/TOUP with the same appliance characteristics. The TOUP model used in this experiment was derived as follows: the highest and lowest daily rates of the hourly RTP (wholesale price as in previous Section 5.1) were taken as the fixed prices charged for peak and off-peak periods across the simulation period. The peak period was defined as the hours between 06:00–08:00 and 17:00–21:00, and the off-peak the remaining hours of the day. In the 2TP/TOUP cost model, the same procedure was used to set the 2TP base price during the on-peak and off-peak times, with the higher price again set to be 150% of the base price for consumption exceeding 1500 Wh. The resulting solutions obtained with both the heuristic and exact algorithms for TOUP and 2TP/TOUP are shown in Tables 3 and 4, while the differences in the average total cost consumption are plotted in Figures 6 and 7 below.

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exact algorithm for STOUP and 2TP/TOUP model are shown in Tables 3 and 4 while the differences in the average total cost consumption are plotted in Figures 6 and 7 below.

Table 3. Simulation Result of TOUP Model across representative seasons of the year.

Months of the Year (2014)	Heuristic Algorithm Average Total Cost (Eur/kWh)	Exact Algorithm Average Total Cost (Eur/kWh)	Relative Difference in Average Total Cost (%)
January	0.18635	0.18635	0.00000
April	0.16816	0.16814	0.00011
July	0.16049	0.16049	0.00000
October	0.16028	0.16028	0.00000

Table 4. Simulation Result of 2TP/TOUP Model across representative seasons of the year.

Months of the Year (2014)	Heuristic Algorithm Average Total Cost (Eur/kWh)	Exact Algorithm Average Total Cost (Eur/kWh)	Relative Difference in Average Total Cost (%)	Difference with 2TP/RTP (%)
January	0.43131	0.43130	0.0000023	0.004948
April	0.38822	0.38807	0.0000390	0.17611
July	0.37157	0.37157	0.0000000	0.00467
October	0.37098	0.37098	0.0000000	0.01195

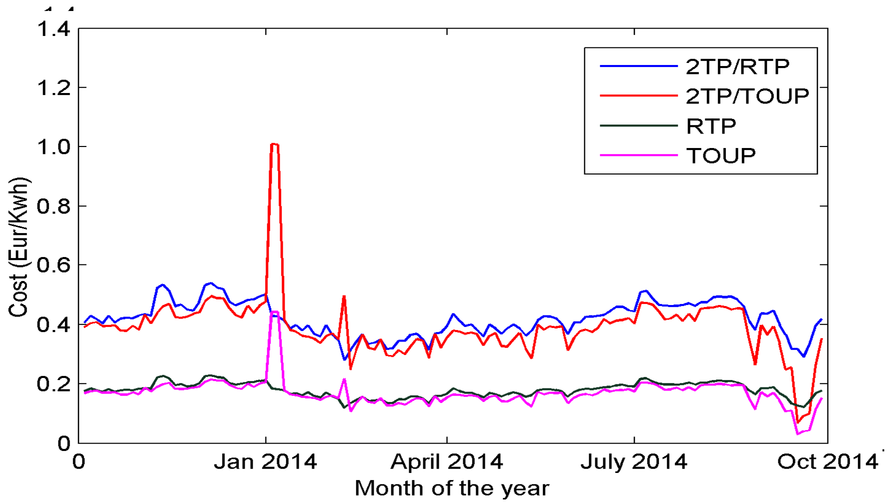

Figure 6. RTP and RTP/2TP *vs.* TOUP and 2TP/TOUP cost scheduling solution for heuristic algorithm.

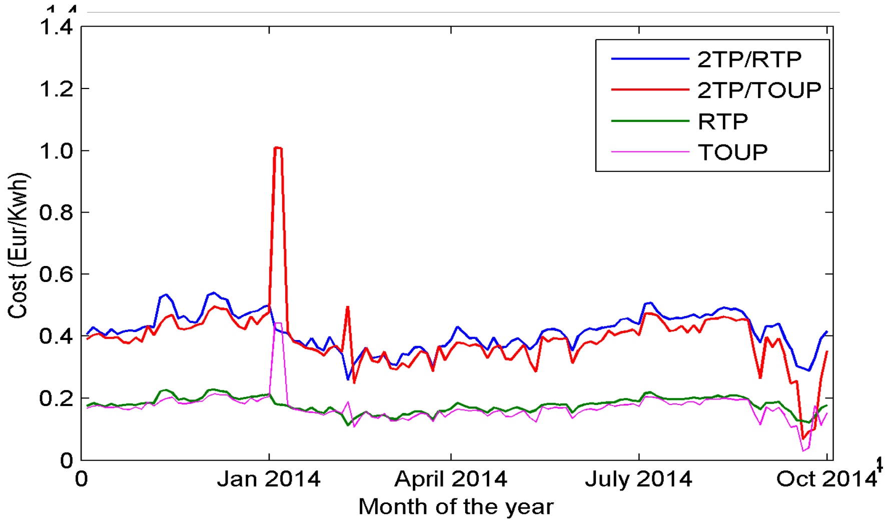

Figure 7. RTP and RTP/2TP *vs.* TOUP and 2TP/TOUP cost scheduling solution for exact algorithm.

Considering Figures 6 and 7, one may observe that the heuristic algorithm achieves almost identical costs when compared to the exact algorithm over the course of the simulated months.

Considering Figures 6 and 7 one may observe that the heuristic algorithm achieves almost identical costs when compared to the exact algorithm over the course of the simulated months. In terms of consumer costs, better results (in terms of slightly lower billing) are achieved with 2TP/TOUP when compared to the 2TP/RTP model. In summary, the results that have been presented in these Sections suggest that the proposed heuristic algorithm is very effective across different types of pricing model when compared to the exact algorithm, in terms of the end consumer costs. In the next Section, it is evaluated in terms of the achieved power consumption profile.

5.3. Power Consumption for Real-Time Energy Pricing (RTP) and Two-Tier Pricing (2TP) with Heuristic and Exact Algorithms

The obtained power consumption for the two pricing models is tested with both the exact and heuristic algorithms to verify the energy distribution across different hours, days, months and respective seasons of the year. This is displayed in Figures 8–11 below. As can be seen in these Figures, under the RTP-only model there are several hourly timeslots in which the power consumption is significantly different between the heuristic and exact algorithm. In particular, there are 13 situations in which the heuristic consumes over 1.5 kW while the exact algorithm remains below this level. Whilst this has an almost negligible impact upon cost—as detailed in the previous Section—it indicates that problem solution is quite sensitive near the optimal cost. Examining the results obtained for the RTP/2TP pricing model, it can be observed that, although some differences exist, they are less pronounced under the 2TP extension. In particular, there are now no situations in which the heuristic consumes over the 1.5 kW thresholds while the exact algorithm remains below it.

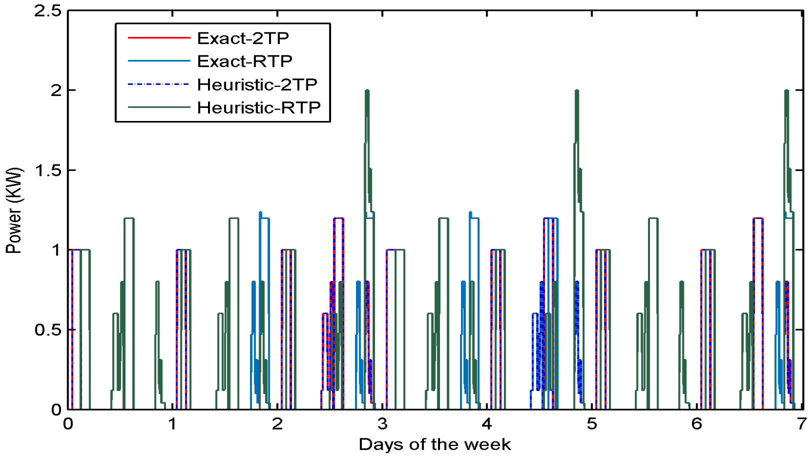

Figure 8. Power distribution across the first week of January 2014, representing winter period.

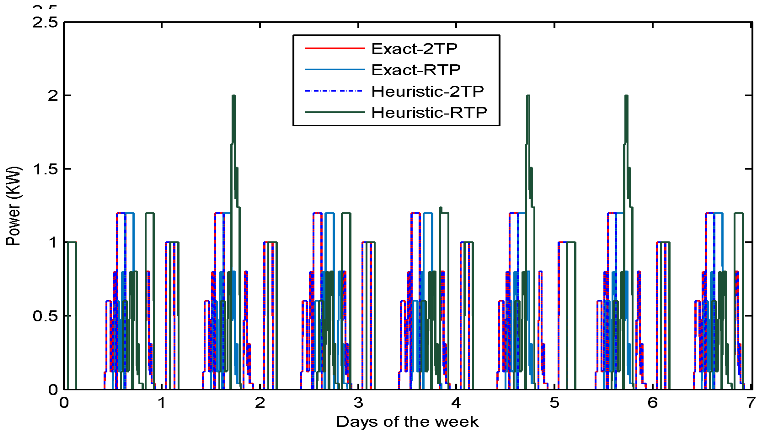

Figure 9. Power distribution across the first week of April 2014, representing spring period.

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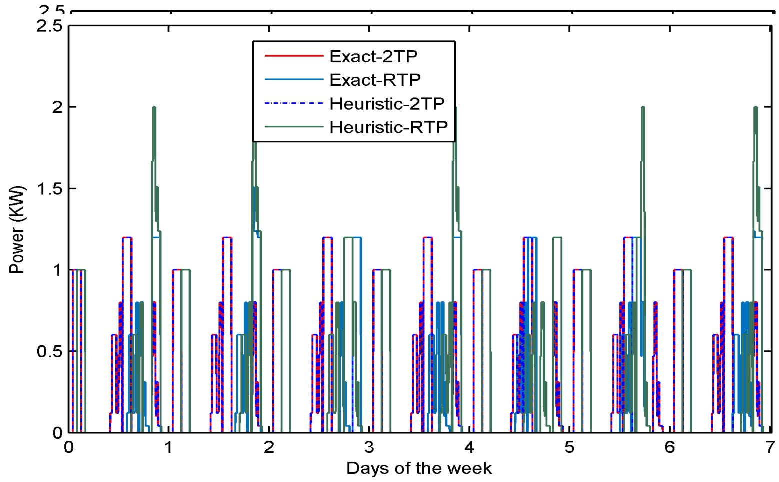

Figure 10. Power distribution across the first week of July 2014, representing summer period.

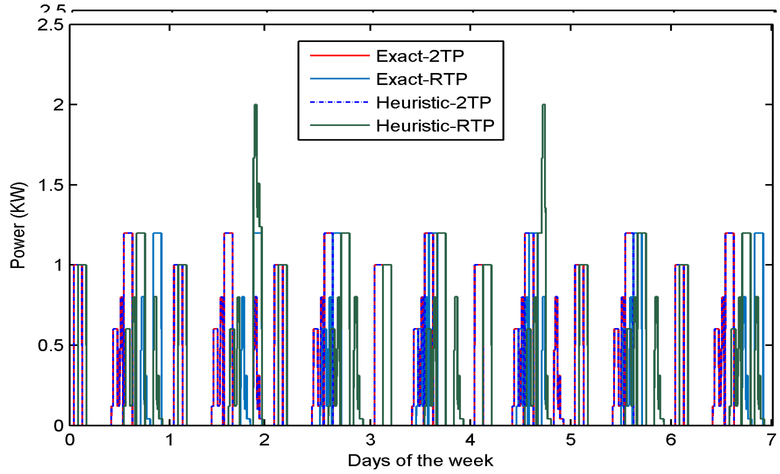

Figure 11. Power distribution across the first week of October 2014, representing autumn period.

This is indicative that the 2TP extension may be more effective at peak power reduction and load balancing than the basic RTP approach when households employ approximate, near-optimal scheduling of appliances. The aspects of multiple households with different appliances and configuration will be investigated in the next section.

6. Cost Evaluation Based on Multiple Household Configurations

In this experiment, we consider multiple households with various appliance configurations and pricing mechanisms. Parameters such as Time preference Range (H), Length of timeslot (T) and Total power range ($P_{i,j}$) are the varying inputs that can be set by different household users based on the consumption pattern, desired comfort level and appliance manufacturer's constraints. We assume that each household has configuration (C1–C8), with a particular pricing model (e.g., RTP, RTP/2TP, TOUP, and TOUP/2TP) and its input parameters (e.g., different start times, timeslot length operation as well as appliance power rating and assignment). These inputs are selected randomly in comparison to the recorded data of household appliance technical specification used in [30], and will determine the changes in energy cost with exact and heuristic algorithms across different households. Please see Appendix B and Tables A1 and A2 for the details of the household configurations used in these experiments. The pricing data used in this experiment is the same with the previous set of experiments reported in Section 5. For comparison purposes with the exact algorithm, given that the problem is NP-hard it is very difficult to obtain extensive exact results for large problem instances, so we conducted experiments with five and six appliances, each with four different configurations and price model. The average yearly simulation results for the eight different configurations were as found in Table 5.

Table 5. Simulation result for multiple households with five & six appliances with different configurations and pricing model.

Average Yearly Total Cost (Eur/kWh)	Five Appliances with Configurations (C1~C4)				Six Appliances with Configurations (C5~C8)			
	C1 RTP	C2 RTP/2TP	C3 TOUP	C4 TOUP/2TP	C5 RTP	C6 RTP/2TP	C7 TOUP	C8 TOUP/2TP
Heuristic algorithm	0.2071	0.4763	0.3043	0.5123	0.2276	0.4829	0.2117	0.4781
Exact algorithm	0.2068	0.4722	0.3037	0.5111	0.2273	0.4790	0.2087	0.4758
% Difference	0.0014	0.0086	0.0019	0.0023	0.0013	0.0080	0.0142	0.0048

The simulation results indicate that our heuristic algorithm with the proposed generic cost model seems to be effective with different appliance and user preference configurations, and has managed to bring the final consumption cost close to the optimal results (within 0.15%) across all pricing models and configurations.

7. Conclusions

This paper has presented details of an extensive study into a heuristic scheduling algorithm for use in a consumer IDSS for minimizing smart appliance energy costs. A generic and flexible cost model for hourly pricing has been utilized in the model, which captures the salient characteristics of traditional on/off peak pricing, RTP, Time of Use Pricing (TOUP), Two-Tier Pricing (2TP) and combinations thereof. In comparisons with an exact (optimal) scheduling algorithm, the effectiveness of the algorithm has been evaluated in extensive simulations and computational experiments. The obtained results indicate that, although the worst-case performance of the algorithm could be closer to 32%, in representative simulations the gaps between the heuristic cost solutions and the optimal achievable costs have been found to be much lower and almost negligible. Although the costs differences observed were negligible, some differences were however observed in the power consumption profile between the algorithms, especially in the presence of the RTP policy; this indicates that underlying the appliance scheduling problem is potentially sensitive to small changes in the decision variables around the optimal achievable costs. In comparison, a combination of RTP and RTP/2TP was found to be less sensitive than RTP alone, and gave a better distribution of the power consumption. These issues will be investigated in more depth in our future work.

Author Contributions: The corresponding authors (Chris Ogwumike and Michael Short) contributed equally to the paper and were responsible for the manuscript preparation, development of the generic cost model, design of the experiments, evaluation & analysis of the experimental results and review of the manuscript before submission. The co-author (Fathi Abugchem) contributed to the development of the generic cost model and algorithm designs.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Proof of NP-Completeness of the IDSS energy scheduling problem considered in this paper. Consider the decision version of the optimization model presented in Section 3:

IDSS PROBLEM INSTANCE: An integer $H > 0$ representing the number of considered time slots, an integer $T > 0$ representing the length of each slot, an integer $N > 0$ representing the number of appliances, integers $n_i > 0$ representing the number of appliance stages, and real-valued power consumption values for each stage denoted by $P_{i,j} > 0$, $i \in [1, N]$, $j \in [1, n_i]$, cost function $C_h(x_h) \geqslant 0$ and maximum power consumption thresholds $X_h{}^{Max} > 0$ for each hour of the day, plus user start time preferences $0 \leqslant s_i{}^{Min} \leqslant s_i{}^{Max} \leqslant H$ for each appliance, and a real-valued cost budget $B \geqslant 0$.

QUESTION: Is there a set of appliance start times s_i such that Constraints (3) and (4) are satisfied, and the cost calculated using Equations (1) and (2) satisfies $J \leqslant B$?

MULTIPROCESSOR SCHEDULING PROBLEM INSTANCE: Set Γ of tasks with cardinality L, number $M > 1$ of uniform processors, real-valued length $l_i > 0$ for each task, real-values deadline $D > 0$.

QUESTION: Does a non-preemptive M-processor schedule for Γ exist, *i.e.*, a function $f(j) \in [1, \ldots, M]$ mapping all L tasks $j \in \Gamma$ to a processor (without overlap), such that the finish time for the schedule F:

$$F = \max_{1 \leqslant i \leqslant M} \sum_{\substack{f(j) = i \\ j \in \Gamma}} l_j$$

Satisfies the constraint that it is less than the deadline, *i.e.*, $F \leqslant D$?

The multiprocessor scheduling problem above is known to be NP-Complete [27], and is in fact NP-Complete in the strong sense when $M \geqslant 2$. NP-Completeness of the IDSS problem is now shown by transformation from MULTIPROCESSOR SCHEDULING.

*Theorem*1 : IDSS is NP-Complete.

Proof : Transformation from the MULTIPROCESSOR SCHEDULING PROBLEM. Given an instance of the MULTIPROCESSOR SCHEDULING problem, we configure the following instance of an IDSS problem:

$$\begin{aligned}
&H = M; \\
&T = 1; \\
&N = L; \\
&s_i^{MIN} = 1;\ 1 \leqslant i \leqslant N; \\
&s_i^{Max} = M;\ 1 \leqslant i \leqslant N; \\
&n_i = 1;\ 1 \leqslant i \leqslant N; \\
&P_{i,1} = l_i,\ 1 \leqslant i \leqslant N; \\
&X_i^{Max} = D;\ 1 \leqslant i \leqslant H; \\
&C_i(x) = x,\ 1 \leqslant i \leqslant H; \\
&B = \sum_{i=1}^{L} l_i;
\end{aligned}$$

Observe that M timeslots have been created in IDSS, each with unit length, and that L appliances have been constructed each with a single stage having power requirement l_i. By the choice of ${s_i}^{Min}$ and ${s_i}^{Max}$, each appliance is free to be started in any of the M available timeslots and incurs an economic cost l_i regardless of which slot it is assigned to. Given the choice of the budget B, any assignment of start times satisfies the budget constraint eliminating it from the IDSS problem. It is clear from this construction, however, that assigning an appliance start time $s_i = j$ incurs a power cost of l_i units in timeslot j. The claim is that a feasible schedule to this instance of the IDSS problem exists if and only if a feasible schedule exists for this instance of the MULTIPROCESSOR SCHEDULING problem. This is proven by taking the assignment of $s_i = j$ as equivalent to the assignment of task i on processor j, and equivalently it must hold that:

$$\forall h, 1 \leqslant h \leqslant H : \\ \sum_{i=1}^{N} x_i(h) = \sum_{\substack{f(i) = h \\ i \in \Gamma}} l_i$$

From which it is easy to see that the finish time of the schedule F is equivalent to the maximum power assigned to any of the $H = M$ timeslots, and since the maximum power constraints are constructed as ${X_i}^{Max} = D$ for each timeslot a feasible schedule to MULTIPROCESSOR SCHEDULING exists if and only if there is a feasible solution to IDSS, proving the claim.□

Appendix B

Table A1. Configuration for five appliance scheduling with dynamic pricing [30].

Devices	Input Parameters	Household Configuration			
		C1 RTP	C2 RTP/2TP	C3 TOUP	C4 TOUP/2TP
Washing Machine	Start time Range	10~20	10~20	10~20	10~20
	Timeslot Lenght	136	161	130	154
	Power	2249.96	2249.96	2249.96	2149.96
Dish washer	Start time Range	9~23	9~23	9~23	9~23
	Timeslot Lenght	82	134	78	87
	Power	1739.96	1880.96	1740.96	1840.96
Tumble dryer	Start time Range	13~23	13~23	13~23	13~23
	Timeslot Lenght	90	120	105	70
	Power	1200	1200	1500	1200
Electric vehicle	Start time Range	1~6	1~6	1~6	1~6
	Timeslot Lenght	120	110	150	120
	Power	1100	1000	2500	2000
Water heater	Start time Range	5~20	5~20	5~20	5~20
	Timeslot Lenght	105	60	90	60
	Power	950	900	700	1000

Table A2. Configuration for six appliance scheduling with dynamic pricing [30].

Devices	Input Parameters	Household Configuration			
		C5 RTP	C6 RTP/2TP	C7 TOUP	C8 TOUP/2TP
Washing Machine	Start time Range	10~20	10~20	10~20	10~20
	Timeslot Lenght	135	135	155	135
	Power	1939.96	1899.96	2249.96	1899.96
Dish washer	Start time Range	9~23	9~23	9~23	9~23
	Timeslot Lenght	89	88	132	108
	Power	1720.96	1700	1960.96	1700
Tumble dryer	Start time Range	13~23	13~23	13~23	13~23
	Timeslot Lenght	90	90	90	90
	Power	1100	1000	1100	1000
Electric vehicle	Start time Range	1~6	1~6	1~6	1~6
	Timeslot Lenght	120	120	120	110
	Power	1500	1200	1000	1300
Water heater	Start time Range	5~20	5~20	5~20	5~20
	Timeslot Lenght	90	90	90	90
	Power	900	900	900	900
Electric cooker	Start time Range	6~22	6~22	6~22	6~22
	Timeslot Lenght	75	75	75	75
	Power	600	600	600	600

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