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Sequences of numbers obtained by digit and iterative digit sums of Sophie Germain primes and its variants

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Abstract

Sequences were generated by the digit and iterative digit sums of Sophie Germain and Safe primes and their variants. The results of the digit and iterative digit sum of Sophie Germain and Safe primes were almost the same. The same applied to the square and cube of the respective primes. Also, the results of the digit and iterative digit sum of primes that are not Sophie Germain are the same with the primes that are notSafe. The results of the digit and iterative digit sum of prime that are either Sophie Germain or Safe are like the combination of the results of the respective primes when considered separately.

Keywords: Sophie Germain prime, Safe prime, digit sum, iterative digit sum.

INTRODUCTION

In number theory, many types of primes exist; one of such is the Sophie Germain prime. A prime number p is said to be a Sophie Germain prime if $2p+1$is also prime. A prime number $q=2p+1$ is known as a safe prime. Sophie Germain prime were named after the famous French mathematician, physicist and philosopher Sophie Germain (1776-1831). She used the primes up to 100 to solve the Fermat Last Theorem [1]. See [2-4] for some theories and recent developments onsome conjectures on the Sophie Germain primes.

The details of Sophie Germain primes can be found in OEIS (A005384) [5] while the safe prime is OEIS (A005385) [6]. Safe primes have some unique conditions for computation [7] and [8] devised some polynomial time based methods of generating them. Research has shown that there are infinitely Sophie Germain primes [9] while

[10] showed that the set of all Sophie Germain prime numbers are an infinite set. Large Sophie Germain primes have been discovered by researchers such as [11, 12]. Relationships exist between Sophie Germain primes and other primes numbers [13], for example the equal-sum-and-product problem [14]. Sophie Germain primes can be applied in testing for primality [15, 16], pseudorandom number generation [17], cryptography [18, 19].

Digit and iterative digit sum approach was used in this paper to create sequences which revealed some observed properties of Sophie Germain and safe primes. Digit sum of a given integer is the sum of its digit while the iterative digit sum or digital root of a non-negative integer is obtained after its successive iterative sum of digit until a single digit is obtained. Also the digit and iterative digit was also used to provide another way of relating the two primes. The relationships were extended to their squares and cubes.

The objectives are used to achieve the aforementioned aims. That is, to examine some sequences of numbers generated by the digit sum and iterative digit sum of the following:

a). Sophie Germain prime p
b). Safe prime $q = 2p + 1$
c). The square of the Sophie Germain primes
d). The square of the safe primes
e). The cube of the Sophie Germain primes. f) The cube of the safe primes
g). The primes that are not Sophie Germain primes
h). Theprime numbers that are not Safe primes.
i). The primes that are neither Sophie Germain nor Safe primes.
j). The primes that are either Sophie Germain or Safe primes.

The motivation of the research or the reason of the use of digit and iterative digit sum approach was that these primes are integers and some authors have studied the behavior of the sum of digitof some sequences of integers [20], sum of digit of squared positive integers [21], sum of digit of cubed positive integers [22], sum of digit of palindromes [23] and others. The digit and iterative digit sum of these integers resulted to some unique sequences.

Digit and iterative digit sums of Sophie Germain primes.

Sophie Germain prime number sequence containing the first few primes is given as:

2, 3, 5, 11, 23, 29, ... (A)

Sequence (B) of integers can be obtained from the sum of digit of Sophie Germain primes.

2, 3, 5, 8, 11, 14, 17, 20, 23, ... (B)

With the exception of the second term, the nth term of sequence (B) is given as:

$$B_n = 3n - 1 \quad (1)$$

This follows from arithmetic progression and with the exception of 3, the second term, the nth term is $B_n = a + (n-1)d$, $B_n = 2 + (n-1) \times 3 = 3n - 1$ The sum of digit increases as the primes increases as shown in **figure 1**. The iterative sum of the digit of Sophie Germain primes produced only 4 numbers namely 2, 3, 5 and 8. The number 3 appeared only once which is the iterative digit sum of the second term of sequence (A).

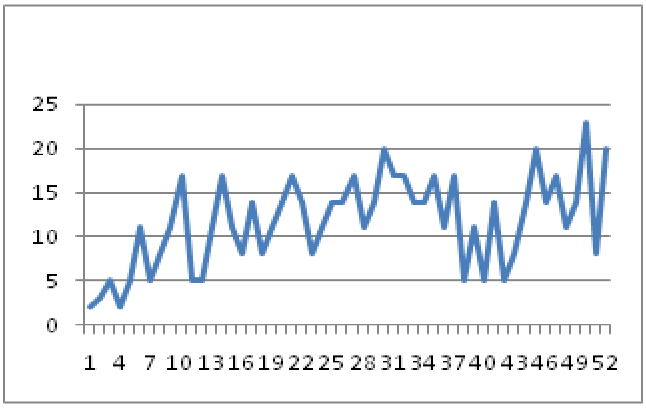

Figure 1: The sum of Digit of the first few Sophie Germain primes. The x axis is the primes and the y axis is the values of the sum of digit.

Digit and iterative digit sums of safe primes

Safe primes sequence containing the first few primes is

5, 7, 11, 23, 47, 59, 83, ... (C)

Sequence (D) of integers can be obtained from the sum of digit of Safe primes.

2, 5, 7, 8, 11, 14, 17, 20, ... (D)

With the exception of the third term, the nth term of sequence (D) is the same as equation1. The sum of digit increases as the primes increases as shown in **figure 2**. The iterative sum of the digit of Safe primes produced only 4 numbers namely 2, 5, 7 and 8. The number 7 appeared only once which is the iterative digit sum of the second term of sequence (C).

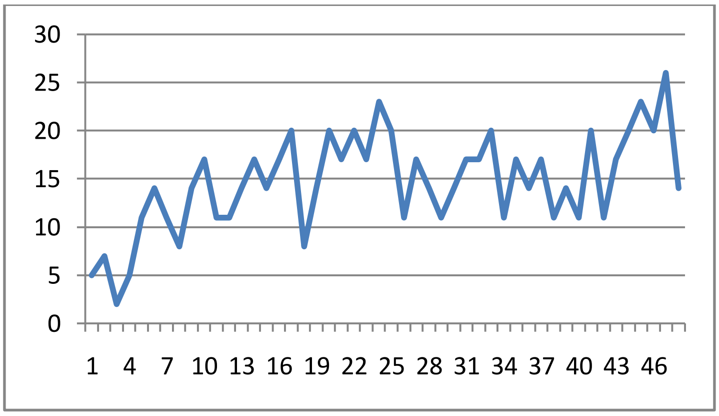

Figure 2: The sum of Digit of the first few Safe primes. The x axis is the primes and the y axis is the values of the sum of digit.

Digit and iterative digit sums of squared Sophie Germain primes.
The squared Sophie Germain primes sequence containing the first few numbers is
4, 9, 25, 121, 529, ... (E)
The Square of the Sophie Germain primes are not closed under that operation. That is numbers that are not Sophie Germain can be obtained. Emphasis was placed on the behavior and comparison with the results of section 2. 1.
Sequence (F) of integers can be obtained from the sum of digit of squared Sophie Germain primes.
4, 7, 9, 10, 13, 16, 19, 22, ... (F)
With the exception of the third term, the nth term of sequence (F) is given as:
$$F_n = 3n + 1 \qquad (2)$$
This follows from arithmetic progression and with the exception of 9, the third term, the nth term is
$$F_n = a + (n-1)d\ ,\ F_n = 4 + (n-1) \times 3 = 3n + 1$$
The iterative sum of the digit of squared Sophie Germain primesproduced only 4 numbers namely 1, 4, 7 and 9. The number 9 appeared only once which is the iterative digit sum of the second term of sequence (E).

Digit and iterative digit sums of squared Safe primes
These are obtained by the square of each element of the sequence of Safe primes. The squared Safe primes sequence containing the first few numbers is
25, 49, 121, 529, ... (G)
The square of the Safe prime is not closed under that operation. That is numbers that are not Safe primes can be obtained from the square. Emphasis was however, placed on the behavior and comparison with the results of section 2. 2.
Sequence (H) of integers can be obtained from the sum of digit of squared Safe primes.
4, 7, 10, 13, 16, 19, 22, ... (H)
The nth term of sequence (H) is the same as equation 2.
The iterative sum of the digit of squared Safe primes produced only 3 n umbers namely 1, 4 and 7.

Digit and iterative digit sums of cubed Sophie Germain primes.
These are obtained by the cube of each element of the sequence of Sophie Germain primesThe cubed Sophie Germain primes sequence containing the first few numbers is
8, 27, 125, 1331, ... (I)
The cube of the Sophie Germain prime is not closed under that operation. Emphasis was placed on the behavior and comparison with the results of section 2. 1 and 2. 3.
Sequence (J) of integers can be obtained from the sum of digit of cubed Sophie Germain primes.
8, 9, 17, 26, 35, 44, 53, ... (J)
With the exception of the second term, the nth term of sequence (J) is given as:

$J_n = 9n - 1$ (3)

This follows from arithmetic progression and with the exception of 9, the second term, the nth term is

$J_n = a + (n-1)d$, $J_n = 8 + (n-1) \times 9 = 9n - 1$

The iterative sum of the digit of cubed Sophie Germain primes produced only 2 numbers namely 8 and 9. The number 9 appeared only once which is the iterative digit sum of the second term of sequence (I). With the exception of iterative digit sum of the second term, all the result implied primality and 9 connote divisibility which is the opposite of primes. Note that divisibility is for odd and even numbers and not primes.

Digit and iterative digit sums of cubed Safe primes

The cubed Safe primes sequence containing the first few numbers is

125, 343, 1331, 12167, ... (K)

The cube of the Safe prime is not closed under that operation. Emphasis is placed on the behavior and comparison with the results of section 2. 2 and 2. 4.

Sequence (L) of integers can be obtained from the sum of digit of cubed Safe primes.

8, 10, 17, 26, 35, 44, 53, ... (L)

With the exception of the second term, the nth term of sequence (L) is the same as equation 3. The iterative sum of the digit of cubed Safe primes will produce only 2 numbers namely 1 and 8. The number 1 appeared only once which is the iterative sum of digit of the second term of sequence (K). The iterative sum of digit of cubed positive integers yielded only 3 numbers; namely 1, 8 and 9. Primality is violated if 9 were obtained from the iterative digit sum of cubed safe primes.

Digit and iterative digit sums of primes that are not Sophie Germain primes

These are the prime numbers that are not Sophie Germainprimes. The prime numbers that are not Sophie Germain primes forms the sequence:

7, 13, 17, 19, 31, 37, ... (M)

Sequence (N) is obtained by the sum of the digit of prime numbers that are not Sophie Germain primes.

2, 4, 5, 7, 8, 10, 11, 13, 14, ... (N)

This is a sequence of numbers without 1 and multiples of 3.

The iterative sum of the digit of the prime numbers that are not Sophie Germain primes produced 6 numbers namely 1, 2, 4, 5, 7 and 8. The numbers also excluded the multiples of 3.

Digit and iterative digit sums of primes that are not Safe primes

This is the sequence of prime numbers with the exclusion of the safe primes.

The prime numbers that are not Safe primes forms the sequence:

2, 3, 13, 17, 19, 19, 29, 31, ... (O)

Sequence (P) is obtained by the sum of the digit of prime numbers that are not Safe primes.

2, 3, 4, 5, 7, 8, 10, 11, 13, 14, ... (P)

Only the second term is the multiple of 3 in the sequence.

The iterative sum of the digit of the prime numbers that are not Safe primes produced 7 numbers namely 1, 2, 3, 4, 5, 7 and 8. The numbers also exclude the other multiples of 3. The number 3 appear only once which is the iterative digit sum of the second term of sequence (O).

Digit and iterative digit sums of primes that are neither Sophie Germain nor Safe primes

The prime numbers that are neither Sophie Germain primes nor Safe primes forms the sequence:

13, 17, 19, 31, 37, 43, ... (Q)

The sequence is very similar to sequence (M). The sum of digitand the iterative sums of digitof neither primes that are Sophie Germain nor Safe primes are the same with the results of the primes that are not Sophie Germain.

Digit and iterative digit sums of primes that are either Sophie Germain or Safe primes

The prime numbers that are either Sophie Germain primes or Safe primes forms the sequence:

2, 3, 5, 7, 11, 23, 29, ... (R)

Sequence (S) is obtained by the sum of the digit of prime numbers that are either Sophie Germain or Safe primes. .

2, 3, 5, 7, 8, 10, 11, 14, 17, 20, ... (S)

The iterative sum of the digit of the prime numbers that are either Sophie Germain or Safe primes produced 5 numbers namely, 2, 3, 5, 7 and 8. The numbers 3 and 7 appeared only once which is the iterative digit sums of the second and fourth terms of sequence (R).

CONCLUSION

Some sequences of numbers were generated and comparison was made on the different nth terms of the sequences. The results of the digit and iterative digit sum of Sophie Germain and Safe primes are almost the same. The same applied to the square and cube of the respective primes. The results obtained from the digit sum of the square and cube of Sophie Gernain and Safe primes are some of the results of [21] and [22]. Also, the results of the digit and iterative digit sum of primes that are not Sophie Germain are the same with the primes that are not Safe. The results of the digit and iterative digit sum of prime that are either Sophie Germain or Safe are like the combination of the results of the respective primes when considered separately. The results of Sophie Germain and Safe primes are different from their squares and cubes. This is because their squares and cubes are not primes. The digit and iterative digit sum approach has helped to establish some relationships between the Sophie Germain and Safe primes in different variants.

The result of this paper was limited to the observed data and sequences generated by the digit and iterative digit sums. The detailed mathematical analysis is a subject of further research. See [24] and [25] for similar research results. The result of the digit and iterative digit of primes are equivalent to that of Sophie Germain and Safe primes, however, this also requires a further research.

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