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## INVESTIGATING THE SELECTION OF A SUITABLE SLACK BUS: A CASE STUDY OF THE MULTI-GENERATING STATIONS OF THE NIGERIAN 330-KV POWER SYSTEM NETWORK.

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**ABSTRACT:** *Slack bus is a bus with generating unit and used to balance the real power ( $P$ ) and reactive power ( $Q$ ) in the power system while performing load flow studies. This study therefore, investigated the best slack bus suitable to be used in the load flow study of the Nigerian 330-kV transmission network with nine (9) generating stations. The method involve the load flow analysis of the existing network with Egbin, Shiroro and Kanji generating stations chosen as a slack bus at different instances and simulated using Newton-Rapson method and Gauss Seidel. This study revealed that the use of Egbin power station as a slack bus brought about the lowest power mismatch in the network. The result also indicated violation of voltages in some of the network and high reactive power loss.*

**Keywords:** Load flow, Slack bus, Grid, Matlab tool, Reactive power

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### INTRODUCTION

The planning, design and operation of power system network requires extensive system analysis in order to determine the power system network performance and to evaluate proposed network expansion plans, stability or reliability of the power system network (PSN). Load flow analysis is used to carry out this study on the power system network. [1] To obtain the complete description of the power system, therefore, it is necessary to know the voltage at node points or buses and the power flowing through the elements of the system. Due to the growing demand of power and the unbundling of the network, the power system network has become very complex. Hence, the load flow study is a very significant tool for the power system operator.

The study would be carried out by performing the load flow analysis method of Newton-Rapson iterative algorithms to provide real time result of a power flow simulation using Matlab software package.

To solve the problem of load flow on the Nigerian 330-kV transmission grid, the total generated power is required to match the total demand plus transmission losses. Since the losses in the system cannot be determined in advance, it is necessary to have at least one bus called the slack bus or reference bus where the voltage magnitude and angle are known and whose real power ( $P$ )

generation can be replaced to supply the difference between the total system load plus losses and the sum of specified active powers at generator busses [2]. The rest of the paper is organized as follows: section 2 presents the formulation of the load flow equations and bus classifications. Section 3 gives the description of the existing Nigeria power system network and section 4 will explain the discussion and evaluation of result. Section 5 will conclude this paper.

## FORMULATION OF LOAD FLOW EQUATIONS

For an easy and convenient analysis and formulation of load flow equations, a balanced 3-phase network is assumed. The analysis of an electrical power system starts with the formulation of a referenced nodal system and it describes the relationship between the electrical variables (voltages and currents) as is stated by the second Kirchhoff's law or nodal law [3]

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus} * \mathbf{V}_{bus} \quad (1)$$

Where

$\mathbf{I}_{bus}$  is an  $n \times 1$  vector whose components are the electrical net current injections in the  $n$  network nodes

$\mathbf{V}_{bus}$  is an  $n \times 1$  vector with the nodal voltages measured with respect to the referenced node and

$\mathbf{Y}_{bus}$  is the  $n \times n$  nodal admittance matrix of the electrical network.

$$S_k = V_k \cdot I_k^* = V_k \left( \sum_{m=1}^n Y_{k,m} V_m \right) \quad (2)$$

Where  $Y_{k,m}$  is the element ( $k, m$ ) of  $\mathbf{Y}_{bus}$  matrix

$S_k$  can also be represented for its real and imaginary components such

$$S_k = P_k + jQ_k, \text{ for } k = 1, 2, \dots, n \quad (3)$$

Where  $P_k$  and  $Q_k$  are the net active and reactive power injected at node  $k$  of the system.

They are defined as:

$$P_k = P_k^{\text{gen}} - P_k^{\text{load}} \quad (4)$$

$$Q_k = Q_k^{\text{gen}} - Q_k^{\text{load}} \quad (5)$$

Where  $P_k$  represents the active power and  $Q_k$  represents reactive powers

$$S_k = S_k^{\text{gen}} - S_k^{\text{load}} \quad (6)$$

$$P_k - jQ_k = (P_k^{\text{gen}} - P_k^{\text{load}}) + j(Q_k^{\text{gen}} - Q_k^{\text{load}}) \quad (7)$$

$$S_k = P_k - jQ_k = V_k I_k^* \quad (8)$$

$$I = \frac{S_k}{V_k^*} = \frac{P_k - jQ_k}{V_k^*} \quad (9)$$

$$\frac{P_k - jQ_k}{V_k^*} = \sum Y_{k,m} V_k \quad (10)$$

Knowing that

$$I_i = \sum_{j=1}^n Y_{ij} V_j \quad (11)$$

In polar form it would be

$$I_i = \sum_{j=1}^n |Y_{ij}| |V_j| \angle \theta_{ij} + \delta_j \quad (12)$$

Complex power at bus  $i$  is

$$P_i - jQ_i = V_i I_i^* \quad (13)$$

$$P_i - jQ_i = |V_i| \angle -\delta_i \sum_{j=1}^n |Y_{ij}| |V_j| \angle \theta_{ij} + \delta_j \quad (14)$$

Separating the real and the imaginary parts we have

$$P_i = \sum_{j=1}^n |Y_{ij}| |V_i| |V_j| \cos(\theta_{ij} - \delta_i + \delta_j) \quad (15)$$

$$Q_i = - \sum_{j=1}^n |Y_{ij}| |V_i| |V_j| \sin(\theta_{ij} - \delta_i + \delta_j) \quad (16)$$

All Expanding equations (2.20) and (2.21) in Taylor's series results in a set of linear equations called the Jacobian matrix. This matrix gives the linear relationship between small changes in voltage angle with small changes in real power and small changes in magnitude voltage with small changes in reactive power.

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2}^{(k)} & \dots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \left| \frac{\partial P_2}{\partial |V_2|}^{(k)} \right| & \dots & \left| \frac{\partial P_2}{\partial |V_n|}^{(k)} \right| \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2}^{(k)} & \dots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \left| \frac{\partial P_n}{\partial |V_2|}^{(k)} \right| & \dots & \left| \frac{\partial P_n}{\partial |V_n|}^{(k)} \right| \\ \hline \frac{\partial Q_2}{\partial \delta_2}^{(k)} & \dots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \left| \frac{\partial Q_2}{\partial |V_2|}^{(k)} \right| & \dots & \left| \frac{\partial Q_2}{\partial |V_n|}^{(k)} \right| \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_2}^{(k)} & \dots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \left| \frac{\partial Q_n}{\partial |V_2|}^{(k)} \right| & \dots & \left| \frac{\partial Q_n}{\partial |V_n|}^{(k)} \right| \end{bmatrix} \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}$$

This can be rewritten in short form as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J1 & J2 \\ J3 & J4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix}$$

$\Delta P$  and  $\Delta Q$  represents the difference between specified value and calculated value respectively.  $\Delta V$  and  $\Delta \delta$  represents magnitude voltage and voltage angle respectively in an incremental form.

### **Buses in Power System Network:**

Busses are classified into three (3) types in power system network for the purpose of load flow study. They are

**1 The Voltage-Controlled (PV) Bus:** These buses are also called generator buses. In these buses the active and reactive power load demand are known. Thus the active power generation and voltage magnitude are specified. The voltage magnitude is made constant by adjusting the reactive power [6]. The maximum and the minimum limits on the value of the reactive power are also specified and the voltage magnitude and its phase angle are to be determined in these buses.

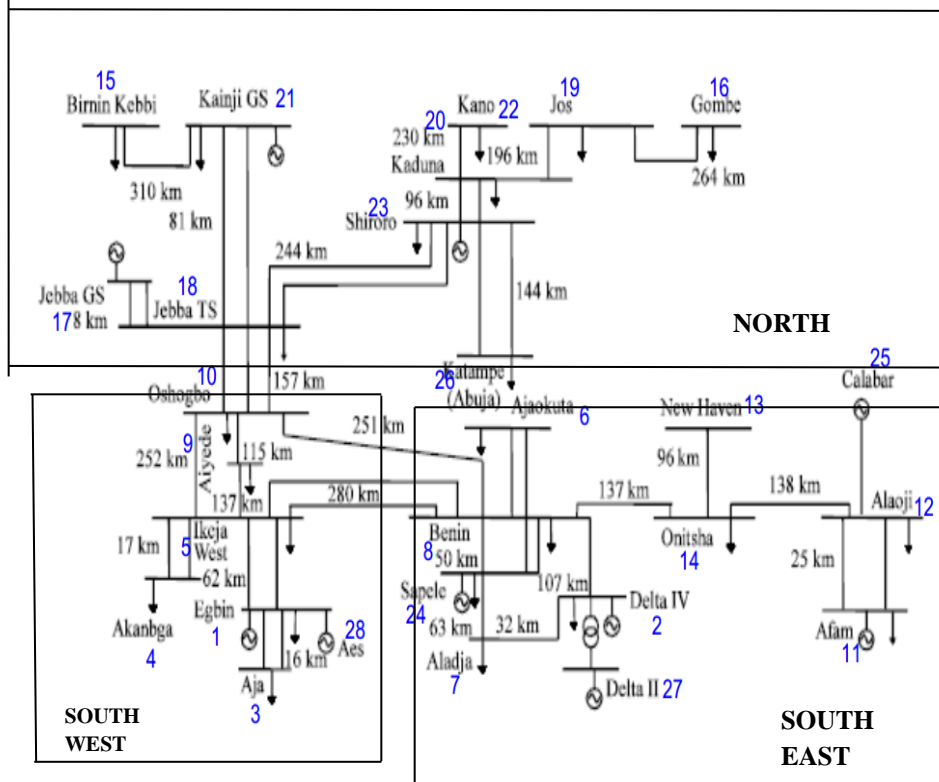
**2. The PQ Bus:** The total injected complex power is specified at this bus and no generator is connected to it. That is, the active (P) and the reactive (Q) power of the load demands are known and the voltage magnitude and phase angle of the bus are unknown. It is also known as load bus.

**3. The Slack Bus:** This bus is also called the reference bus. In this bus the voltage magnitude and its phase angle are specified. The specified phase angle acts as the standard against which the phase angles of other buses are measured [7]. Computation is not needed for this bus. In order to solve the load flow problem in the country's power system network, it is required that the total generated power matches the total demand plus transmission losses. Since transmission losses cannot be predetermined, it is therefore required to have at least one slack bus selected whose real power generation can be rescheduled to supply the difference between total systems load plus losses and the sum of active powers specified at generation buses [8]. Slack bus should therefore be properly selected in order to minimize the system power in balance in load flow studies.

### **DESCRIPTION OF THE EXISTING NIGERIA POWER SYSTEM NETWORK**

The Nigerian grid is made up of interconnected network of 5650km of 330kV, 6687km of 132kV transmission lines, 60 numbers of 330kV circuits, 153 circuits of 132kV, 28 numbers of 330kV substations and 132 substations [4]. The single line diagram of the Nigerian 330kV transmission network as shown in fig 1 is divided into three major sections: the North, South-East and the South-West. The North is connected to the South through the one-triple circuit lines between Jebba and Oshogbo while the West is linked to the East through one transmission line from

Oshogbo to Benin and one double circuit line from Ikeja to Benin [5]. The network is made up of 9 voltage controlled buses (slack bus included) and 19 load buses.



**Fig 1: 28-bus Nigeria 330kv transmission network indicating the 3 regions**

The bus data and transmission line data of the 28-bus 330kV Nigerian transmission system with 9 generator buses and 19 load buses as obtained from Transmission Company of Nigeria (TCN) and were used in the simulations.

**EVALUATION OF RESULT**

Using the Matlab R2012a tool for the load flow simulation the following results were gotten

1. Load flow using the Newton-Raphson and Gauss Seidel iterative method (Egbin as slack bus)
2. Effects of change of slack bus in the power system network.

The result of the base case simulation using Egbin power station as the slack bus is as shown in table 1, while the result of the simulation using Shiroro power station as the slack bus is shown in table 2 and the result of the simulation using Kanji power station as slack bus is as tabulated in table 3.

**Table 1: Base case result of the 28 bus power system network with Egbin selected as the slack bus**

Bus No	Voltage Magnitude	Angle Degree	Load		Generation		Injected Mvar
			MW	Mvar	MW	Mvar	
1	1.050	0.000	68.927	51.7	251.443	641.327	0
2	1.050	15.427	0	0	670	82.632	0
3	1.040	-0.570	274.4	205.8	0	0	0
4	0.970	0.483	344.7	258.5	0	0	0
5	0.986	1.409	633.2	474.9	0	0	0
6	1.026	8.742	13.8	10.3	0	0	0
7	1.046	14.042	96.5	72.4	0	0	0
8	1.011	9.309	385.4	287.5	0	0	0
9	0.95	2.337	275.8	206.8	0	0	0
10	0.966	8.644	201.2	150.9	0	0	0
11	1.050	13.277	52.5	39.4	431	590.558	0
12	1.007	12.06	427	320.2	0	0	0
13	0.905	3.325	177.9	133.4	0	0	0
14	0.949	6.271	184.6	138.4	0	0	0
15	1.010	26.304	114.5	85.9	0	0	0
16	0.844	4.911	130.6	97.9	0	0	0
17	1.046	25.527	11	8.2	0	0	0
18	1.050	26.027	0	0	495	159.253	0
19	0.930	12.907	70.3	52.7	0	0	0
20	0.951	8.798	193	144.7	0	0	0
21	1.050	31.824	7.5	5.2	624.7	-65.317	0
22	0.818	-1.554	220.6	142.9	0	0	0
23	1.050	13.486	70.3	36.1	388.9	507.985	0
24	1.050	12.017	20.6	15.4	190.3	283.414	0
25	0.9511	21.708	110	89	0	0	0
26	1.000	9.251	290	145	0	0	0
27	1.050	46.874	0	0	750	193.09	0
28	1.050	5.872	0	0	750	488.148	0
<b>TOTAL</b>			4372.3	3173.2	4551.343	2881.091	0

**Table 2: Base case result of the 28 bus power system network with Shiroro selected as the slack bus**

Bus No	Voltage Magnitude	Angle Degree	Load		Generation		Injected Mvar
			MW	Mvar	MW	Mvar	
1	1.050	15.996	68.927	51.7	792	554.095	0
2	1.050	18.989	0	0	670	71.845	0
3	1.040	15.427	274.4	205.8	0	0	0
4	0.975	11.126	344.7	258.5	0	0	0
5	0.990	12.297	633.2	474.9	0	0	0
6	1.030	12.297	13.8	10.3	0	0	0
7	1.046	17.607	96.5	72.4	0	0	0
8	1.014	12.861	385.4	287.5	0	0	0
9	0.95	8.020	275.8	206.8	0	0	0
10	0.990	9.908	201.2	150.9	0	0	0
11	1.050	11.808	52.5	39.4	431	584.040	0
12	1.008	10.589	427	320.2	0	0	0
13	0.907	4.550	177.9	133.4	0	0	0
14	0.951	6.271	184.6	138.4	0	0	0
15	1.010	9.835	114.5	85.9	0	0	0
16	0.817	-3.318	130.6	97.9	0	0	0
17	1.049	14.323	11	8.2	0	0	0
18	1.050	14.836	0	0	495	21.997	0
19	0.909	5.119	70.3	52.7	0	0	0
20	0.942	-3.121	193	144.7	0	0	0
21	1.050	15.355	7.5	5.2	206	-47.727	0
22	0.806	-13.732	220.6	142.9	0	0	0
23	1.050	0	70.3	36.1	238.837	565.606	0
24	1.050	15.584	20.6	15.4	190.3	260.177	0
25	0.941	17.133	110	89	0	0	0
26	1.000	-4.237	290	145	0	0	0
27	1.050	42.489	0	0	750	211.354	0
28	1.050	16.522	0	0	750	445.566	0
<b>TOTAL</b>			<b>4372.3</b>	<b>3173.2</b>	<b>4551.343</b>	<b>2881.091</b>	<b>0</b>

In comparing the base case result gotten from using the Newton-Raphson iterative with the result from the Gauss-Seidel iterative method, the results are the same but the iterations for convergence are different. The Newton-Raphson converged at 5 iterations while the Gauss-Seidel converged after 174 iterations. This therefore proves that the Newton-Raphson method has faster convergence, which results in lower use of computer memory hence the best method for use in carrying out load flow analysis on large power systems.

### The Effects of Change of Slack Bus in the Power System Network

The slack bus being the reference bus helps in the balancing the real and reactive power transmission in the system. In order to investigate the effect of the choice of slack bus on the PSN, the Shiroro and Kainji power stations (buses) were used as slack bus at different instances and simulations were carried out, producing the results in table 3 and 4. The results were compared with the result of simulations from Egbin as slack bus and comparisons were made in their base case results.

**Table 3: Base case result of the 28 bus power system network with kainji selected as the slack bus**

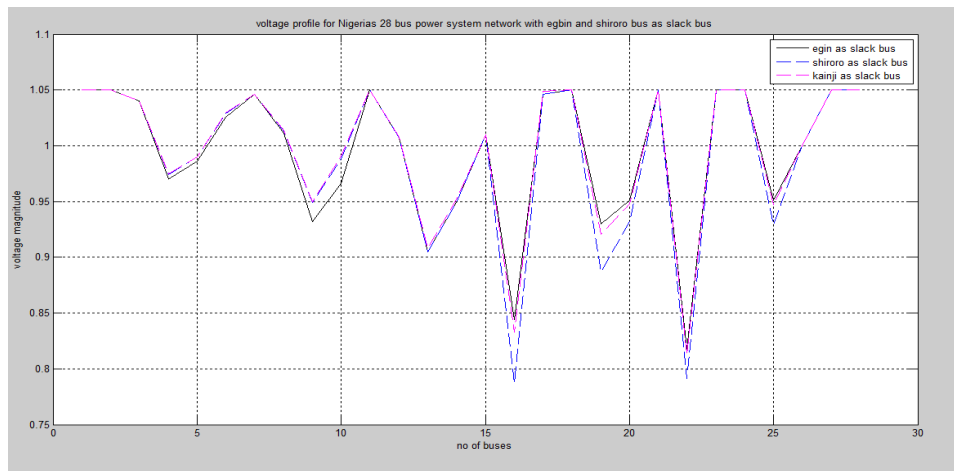
Bus No	Voltage Magnitude	Angle Degree	Load		Generation		Injected Mvar
			MW	Mvar	MW	Mvar	
1	1.050	6.364	68.927	51.7	792	554.095	0
2	1.050	10.202	0	0	670	71.845	0
3	1.040	5.794	274.4	205.8	0	0	0
4	0.975	1.494	344.7	258.5	0	0	0
5	0.990	2.411	633.2	474.9	0	0	0
6	1.030	3.509	13.8	10.3	0	0	0
7	1.046	8.819	96.5	72.4	0	0	0
8	1.014	4.073	385.4	287.5	0	0	0
9	0.951	-1.827	275.8	206.8	0	0	0
10	0.991	-0.121	201.2	150.9	0	0	0
11	1.050	5.430	52.5	39.4	431	584.040	0
12	1.008	4.210	427	320.2	0	0	0
13	0.908	-3.100	177.9	133.4	0	0	0
14	0.952	-0.172	184.6	138.4	0	0	0
15	1.010	-5.520	114.5	85.9	0	0	0
16	0.833	-6.407	130.6	97.9	0	0	0
17	1.049	3.088	11	8.2	0	0	0
18	1.050	3.604	0	0	495	21.997	0
19	0.921	1.769	70.3	52.7	0	0	0
20	0.948	-4.471	193	144.7	0	0	0
21	1.050	0	7.5	5.2	-119.062	-47.727	0
22	0.813	-14.924	220.6	142.9	0	0	0
23	1.050	-0.615	70.3	36.1	550	565.606	0
24	1.050	6.797	20.6	15.4	190.3	260.177	0
25	0.947	12.206	110	89	0	0	0
26	1.000	-4.852	290	145	0	0	0
27	1.050	37.444	0	0	750	211.354	0
28	1.050	6.890	0	0	750	445.566	0
<b>TOTAL</b>			<b>4372.3</b>	<b>3173.2</b>	<b>4551.343</b>	<b>2881.091</b>	<b>0</b>



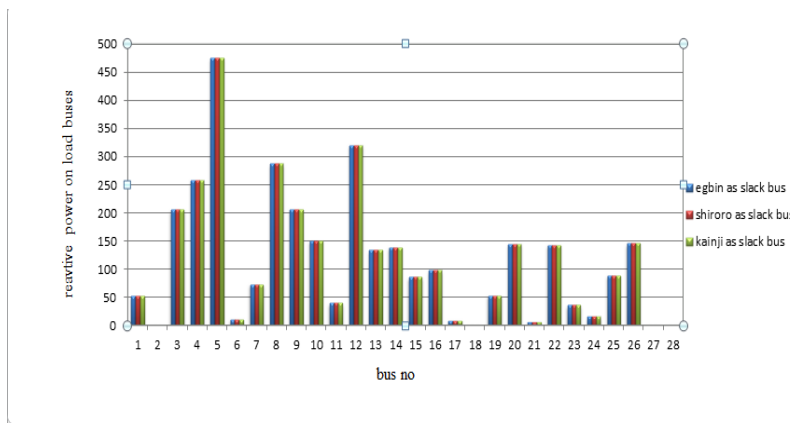
**Table 4: Base case result of the 28 bus power system network with Shiroro selected as the slack bus.**

Bus No	Voltage Mag.	Angle Degree	Load		Generation		Injected Mvar
			MW	Mvar	MW	Mvar	
1	1.050	15.996	68.927	51.7	792	554.09	0
2	1.050	18.989	0	0	670	71.845	0
3	1.040	15.427	274.4	205.8	0	0	0
4	0.975	11.126	344.7	258.5	0	0	0
5	0.990	12.297	633.2	474.9	0	0	0
6	1.030	12.297	13.8	10.3	0	0	0
7	1.046	17.607	96.5	72.4	0	0	0
8	1.014	12.861	385.4	287.5	0	0	0
9	0.95	8.020	275.8	206.8	0	0	0
10	0.990	9.908	201.2	150.9	0	0	0
11	1.050	11.808	52.5	39.4	431	584.04	0
12	1.008	10.589	427	320.2	0	0	0
13	0.907	4.550	177.9	133.4	0	0	0
14	0.951	6.271	184.6	138.4	0	0	0
15	1.010	9.835	114.5	85.9	0	0	0
16	0.817	-3.318	130.6	97.9	0	0	0
17	1.049	14.323	11	8.2	0	0	0
18	1.050	14.836	0	0	495	21.997	0
19	0.909	5.119	70.3	52.7	0	0	0
20	0.942	-3.121	193	144.7	0	0	0
21	1.050	15.355	7.5	5.2	206	-47.72	0
22	0.806	-13.732	220.6	142.9	0	0	0
23	1.050	0	70.3	36.1	238.837	565.60	0
24	1.050	15.584	20.6	15.4	190.3	260.17	0
25	0.941	17.133	110	89	0	0	0
26	1.000	-4.237	290	145	0	0	0
27	1.050	42.489	0	0	750	211.35	0
28	1.050	16.522	0	0	750	445.56	0
TOTAL			4372.3	3173.2	4551.343	2881.0	0

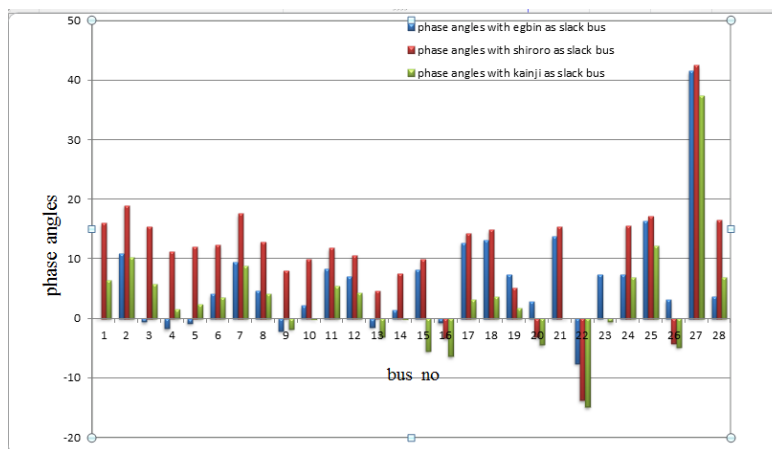
From figures 2, 3 and 4 below, it could be seen en that a change in slack bus causes little or no change in the voltage profile as well as the reactive power on the load buses but causes a major change in the phase angles of the buses and the reactive power at the generator buses therefore leading to a change in the total real and reactive power loss on the system network.



**Figure 2:** Merge of voltage profiles when Egbin, Shiroro and Kainji buses are selected as the slack bus.



**Figure3:** Showing the reactive power on the load buses when Egbin, Shiroro and Kainji buses were selected as the slack bus.



**Figure 4:** Show the voltage angles of the buses when Egbin, Shiroro and kainji were selected as the slack bus.

Using the maximum power mismatch on the base case results as standard of comparison, the table 6 below shows that the use of Egbin bus as a slack bus brought about the lowest power mismatch in the network, hence, the generator with the largest power should be used as slack bus.

**Table 5: showing the maximum power mismatch in the base case result when Egbin, Shiroro and kainji buses were selected as slack bus respectively**

Slack bus	Maximum power mismatch
Egbin	0.00915
Shiroro	0.012
Kainji	0.0161

## DISCUSSION AND OBSERVATIONS

Using the standard voltage limit of  $\pm 5\%$ , it was observed that buses 13 (New Haven), 14 (Onitsha), 16 (Gombe), 19 (Jos) and 22 (Kano) did not meet the voltage limit. Using the transmission line power limit of 760MVA as specified by the power holding company of Nigeria (PHCN) [10], it was observed that the line from bus 5 to 28 and bus 27 to 25 are overloaded.

## CONCLUSION:

In power system network design, care has to be taken in order to select the right slack bus as it is used to balance the active and reactive power in the system. It serves as a reserve for the unaccounted active and reactive power which constitutes the system losses. A change in the slack bus selection thus has no effect on the voltage magnitude of the buses, the total real and reactive power at the load and generating nodes but has an effect in the phase angles which in turn affects the total real and reactive power losses on the line. After much comparison, analysis and observations it is therefore it is best to select the largest generator bus as the slack bus.

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