Journal of Applied Sciences

ISSN 1812-5654

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Journal of Applied Sciences 15 (11): 1305-1311, 2015
ISSN 1812-5654

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RESEARCH ARTICLE OPEN ACCESS

DOI: 10.3923/jas.2015.1305.1311

The Weibull-Exponential Distribution: Its Properties and Applications

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ARTICLE INFO
Article History:
Received: July 24, 2015
Accepted: October 14, 2015

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ABSTRACT

A three parameter probability model, the so called Weibull-exponential distribution was proposed using the Weibull Generalized family of distributions. Some important models in the literature were found to be sub models of the new model. Explicit expressions for some of its basic mathematical properties like moments, moment generating function, reliability analysis, limiting behavior and order statistics were derived. The method of maximum likelihood estimation was proposed in estimating its parameters and real life applications were provided to illustrate its flexibility and potentiality over the exponential distribution.

Key words: Generalization, maximum likelihood estimation, order statistics, Weibull Generalized family of distributions

INTRODUCTION

A number of standard theoretical distributions have been found to be useful in the fields of insurance, engineering, medicine, economics and finance (among others). However, generalizing these standard distributions has produced several compound distributions that are more flexible compared to the base line distributions. To this end, several attempts have been made by notable authors to propose methods for generating new families of distributions, Bourguignon *et al*. (2014) for more details.

Exponential distribution is a well-known continuous probability model which has been identified as a life testing model among many other applications. Attempts to increase the flexibility of the exponential distribution gave rise to the beta exponential distribution (Nadarajah and Kotz, 2006), exponentiated exponential distribution (Gupta and Kundu, 2001), Generalized exponential distribution (Gupta and Kundu, 2007), Kumaraswamy exponential distribution (Cordeiro and Castro, 2011), inverse exponential distribution (Keller *et al*., 1982) and so on. This same exponential distribution will be considered as the baseline distribution in the rest of this article.

Out of the several families of distributions, of interest to us in this study is the Weibull Generalized family of distribution. The reason is that, about three different forms of such class of distribution have been observed (Alzaatreh *et al*., 2013; Bourguignon *et al*., 2014; Nasiru and Luguterah, 2015).

Let T denote a random variable that follows a Weibull distribution with parameters c and γ, the cumulative density function (pdf) of the Weibull-X family due to Alzaatreh *et al*. (2013) is obtained from:

$$F(x) = \int_0^{-\log(1-G(x))} r(t)\,dt \tag{1}$$

Therefore, in Eq. 2:

$$r(t) = \left(\frac{c}{\gamma}\right)\left(\frac{t}{\gamma}\right)^{c-1} e^{-\left(\frac{t}{\gamma}\right)^c};\ t \ge 0, c > 0, \gamma > 0 \tag{2}$$

where, c is the shape parameter and γ is the scale parameter.

The corresponding Probability Density Function (pdf) of the Weibull-X family due to Alzaatreh *et al*. (2013) is given by Eq. 3:

$$f(x) = \frac{c}{\gamma}\frac{g(x)}{1-G(x)}\left\{\frac{-\log(1-G(x))}{\gamma}\right\}^{c-1} \exp\left\{-\left(\frac{-\log(1-G(x))}{\gamma}\right)^c\right\} \tag{3}$$

For $x \ge 0$, $c<0$, $\gamma>0$.

where, g(x) is the pdf of the baseline distribution and G(x) is the cdf of the baseline distribution.

This form of Weibull Generalized family of distribution in Eq. 3 has been used to propose the Weibull Pareto Distribution (WPD) by Alzaatreh *et al*. (2013).

The cdf of another form of the Weibull generalized family of distribution due to Bourguignon *et al*. (2014) is given by Eq. 4:

$$F(x) = \int_0^{G(x)/1-G(x)} \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}} dt \quad (4)$$

Equation 4 was further solved to give Eq. 5:

$$F(x) = 1 - \exp\left\{-\alpha\left[\frac{G(x)}{1-G(x)}\right]^{\beta}\right\} \quad (5)$$

For x∈D⊆R; α>0, β>0.

The corresponding pdf of the Weibull-G family due to Bourguignon *et al*. (2014) is given by Eq. 6:

$$f(x) = \alpha\beta g(x)\frac{G(x)^{\beta-1}}{\left[1-G(x)\right]^{\beta+1}} \exp\left\{-\alpha\left[\frac{G(x)}{1-G(x)}\right]^{\beta}\right\} \quad (6)$$

Using the power series for the exponential function; Bourguignon *et al*. (2014) obtained Eq. 7:

$$f(x) = \alpha\beta g(x)\sum_{k=0}^{\infty}\frac{(-1)^k\alpha^k}{k!}\frac{G(x)^{\beta(k+1)-1}}{\left[1-F(x)\right]^{\beta(k+1)+1}} \quad (7)$$

This form of Weibull Generalized family of distribution in Eq. 6 has been used to define the pdf and cdf of Weibull-Uniform distribution, Weibull-Weibull distribution, Weibull-Burr XII distribution and Weibull-Normal distribution by Bourguignon *et al*. (2014).

Another form of Weibull-G family of distribution was used by Nasiru and Luguterah (2015) and the cdf is given by Eq. 8:

$$F(x) = \int_0^{1/1-G(x)} r(t)dt \quad (8)$$

where, r(t) is the pdf of the Weibull distribution.

The form of the Weibull-G family of distribution in Eq. 8 was used to define and study the New Weibull Pareto Distribution (NWPD) by Nasiru and Luguterah (2015).

With this understanding, this article seeks to increase the flexibility of the one parameter exponential distribution using the Weibull-G family of distributions due to Bourguignon *et al*. (2014). Our choice is based on the fact that this form of Weibull-G family is a new wider Weibull family of distributions as it extends several other widely known distributions such as the Uniform, Weibull and Burr XII distributions.

The rest of this article is organized as follows, Section 2 defines the pdf and the cdf of the proposed Weibull exponential distribution, followed by some basic mathematical properties of the proposed distribution including estimation of model parameters, Section 3 gives an illustration of the potentiality of the proposed model using real life applications followed by the discussion of results and a concluding remark.

MATERIALS AND METHODS

In this section, the methods used in deriving the proposed Weibull exponential distribution are clearly explained.

Weibull Exponential Distribution (WED): Consider the exponential distribution to be the parent (or baseline) distribution with a pdf and cdf respectively given by Eq. 9 and 10:

$$g(x) = \lambda e^{-\lambda x}; \quad x>0, \lambda>0 \quad (9)$$

$$G(x) = 1-e^{-\lambda x}; \quad x>0, \lambda>0 \quad (10)$$

where, λ is the scale parameter.

For some other details on the exponential distribution, readers can go through Oguntunde and Adejumo (2015a).

To derive the cdf of the Weibull exponential distribution, Eq. 10 is inserted into Eq. 5 and the resulting expression given in Eq. 11:

$$F(x) = 1-\exp\left\{-\alpha\left[\frac{1-e^{-\lambda x}}{e^{-\lambda x}}\right]^{\beta}\right\}; \quad x>0, \alpha, \beta, \lambda>0 \quad (11)$$

The expression in Eq. 11 can further be simplified to give Eq. 12:

$$F(x) = 1-\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]; \quad x>0, \alpha, \beta, \lambda>0 \quad (12)$$

The corresponding pdf is obtained as Eq. 13:

$$f(x) = \alpha\beta\left(\lambda e^{-\lambda x}\right)\left[\frac{\left(1-e^{-\lambda x}\right)^{\beta-1}}{\left(e^{-\lambda x}\right)^{\beta+1}}\right]\exp\left\{-\alpha\left[\frac{1-e^{-\lambda x}}{e^{-\lambda x}}\right]^{\beta}\right\} \quad (13)$$

For, x>0, α, β, λ>0.

The expression in Eq. 13 can further be expressed as Eq. 14:

$$f(x)=\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\};\ \ x>0,\alpha,\beta,\lambda>0 \tag{14}$$

Special cases:

- If β = 1 and α = θ/λ, for θ>0, the Weibull exponential distribution reduces to give the Gompertz distribution (Gompertz, 1825)
- If β = 1, the Weibull exponential distribution would reduce to give the exponential-exponential distribution (New)

Using the power series expansion in Eq. 7, the pdf of the Weibull exponential distribution can also be given by Eq. 15:

$$f(x)=\alpha\beta\lambda e^{-\lambda x}\sum_{k=0}^{\infty}\frac{(-1)^k\alpha^k}{k!}\frac{\left(1-e^{-\lambda x}\right)^{\beta(k+1)-1}}{\left[e^{-\lambda x}\right]^{\beta(k+1)+1}} \tag{15}$$

Using the generalized binomial theorem, the Eq. 16 will be as:

$$\left[e^{-\lambda x}\right]^{-(\beta(k+1)+1)}=\sum_{j=0}^{\infty}\frac{\Gamma(\beta(k+1)+j+1)}{j!\Gamma(\beta(k+1)+1)}\left(1-e^{-\lambda x}\right)^{j} \tag{16}$$

Therefore, Eq. 15 becomes as Eq. 17:

$$f(x)=\alpha\beta\lambda e^{-\lambda x}\sum_{k=0}^{\infty}\frac{(-1)^k\alpha^k}{k!}\left(1-e^{-\lambda x}\right)^{\beta(k+1)-1}\sum_{j=0}^{\infty}\frac{\Gamma(\beta(k+1)+j+1)}{j!\Gamma(\beta(k+1)+1)}\left(1-e^{-\lambda x}\right)^{j} \tag{17}$$

Possible plots for the pdf of the proposed model at some parameter values are shown in Fig. 1a and b.

It can be observed in Fig. 1a that the Weibull exponential distribution could be unimodal in shape while the plot in Fig. 1b shows that the shape could be reversed-J shape (depending on the value of the parameters).

A plot for the cdf of the proposed model at some selected parameter values is as shown in Fig. 1c.

Some basic properties of the Weibull exponential distribution: In this sub-section, expressions for some basic mathematical properties of the proposed Weibull exponential distribution are provided.

Reliability analysis: The reliability function is mathematically given by:

$$S(x) = 1-F(x)$$

In this case, F(x) is the cdf of the Weibull exponential distribution as defined in Eq. 12. Therefore, the reliability function of the proposed model is given by Eq. 18:

$$S(x)=\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right] \tag{18}$$

For, x>0, α, β, λ>0.

The corresponding plot for the reliability function of the Wei-exponential distribution is as shown in Fig. 1d.

For x>0, α, β, λ>0 and t>0, the probability that a system having age x units of time will survive up to x+t units of time is given by:

$$S(t\mid x)=\frac{S(x+t)}{S(x)}$$

Therefore, Eq. 19 will be as:

$$S(t\mid x)=\frac{\exp\left[-\alpha\left(e^{\lambda(x+t)}-1\right)^{\beta}\right]}{\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]} \tag{19}$$

For x>0, α, β, λ>0.

The failure rate is mathematically given by:

$$h(x)=\frac{f(x)}{1-F(x)}$$

Therefore, the corresponding failure rate for the Weibull exponential distribution is expressed as in Eq. 20:

$$h(x)=\frac{\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}}{\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]} \tag{20}$$

For x>0, α, β, λ>0.

Following (Bourguignon *et al*., 2014), the failure rate for the Weibull Generalized family of distributions is given by Eq. 21:

$$h(x)=\frac{\alpha\beta g(x)G(x)^{\beta-1}}{\left[1-G(x)\right]^{\beta+1}}=\frac{\alpha\beta G(x)^{\beta-1}}{\left[1-G(x)\right]^{\beta}}\left\{\frac{g(x)}{1-G(x)}\right\} \tag{21}$$

where, $\frac{g(x)}{1-G(x)}$ is the failure rate of the baseline distribution.

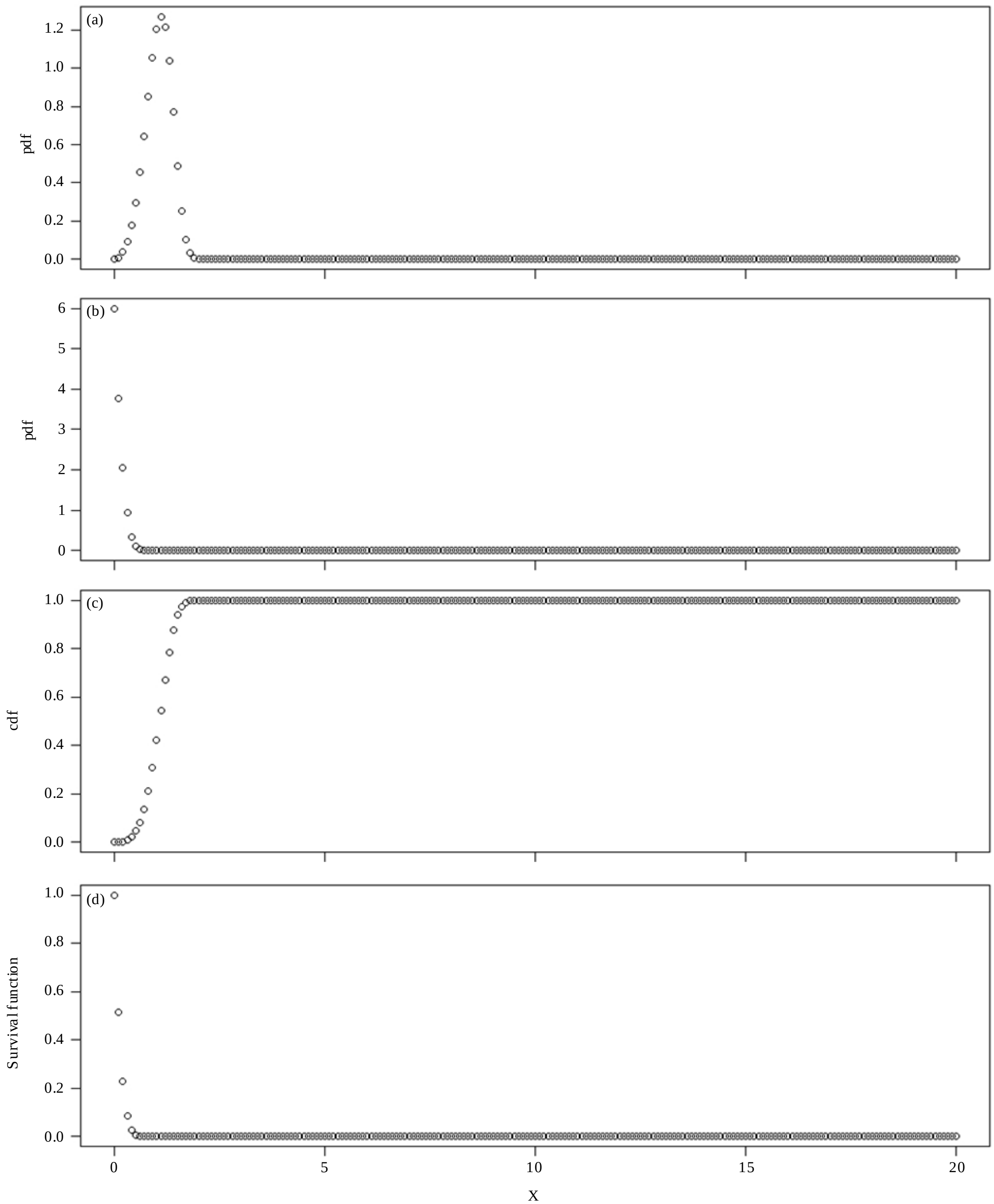

Fig. 1(a-d): (a-b) Pdf, (c) cdf, (d) survival function to Weibull exponential distribution at, (a) $\alpha = 2$, $\beta = 3$, $\lambda = 0.5$, (b) $\alpha = 3$, $\beta = 1$, $\lambda = 2$, (c) $\alpha = 2$, $\beta = 3$, $\lambda = 0.5$ and (d) $\alpha = 2$, $\beta = 3$, $\lambda = 0.5$

In this case, Exponential distribution is the baseline distribution and it has a constant failure rate which is expressed as λ.

With this understanding, the failure rate of the Weibull exponential distribution is further expressed as Eq. 22:

$$h(x) = \frac{\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}}{\left\{\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]\right\}^{\beta}} \qquad (22)$$

For $x>0$, α, β, $\lambda>0$.

Limiting behavior: The limit of the pdf of the Weibull exponential distribution as x→0 and as x→∞ is zero. In other words:

$$\lim_{x\to 0} f(x) = \lim_{x\to\infty} f(x) = 0$$

Proof:

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}\right]$$
$$= \alpha\beta\lambda(0)^{\beta-1}\times 1$$
$$= 0$$

Also:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left[\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}\right]$$
$$= \alpha\beta\lambda\times 1^{\beta-1}\times 0$$
$$= 0$$

This result shows that the proposed model has at least a unique mode.

Moments: Bourguignon *et al*. (2014) stated that some properties of the Weibull Generalized family of distributions can be directly obtained from those of the exponentiated Generalized family of distributions. Therefore, the rth moment of X is given by Eq. 23:

$$E\left[X^r\right] = \sum_{j,k=0}^{\infty} w_{j,k} E\left[Z_{j,k}^r\right] \tag{23}$$

Therefore, for Weibull exponential distribution, $Z_{j,k}$ is the exponentiated exponential distribution with power parameter β(k+1)+j-1.

Also, the Moment Generating Function (MGF) of X is given by:

$$M_X(t) = \sum_{j,k=0}^{\infty} w_{j,k} E\left(e^{tZ_{j,k}}\right) \tag{24}$$

where, $w_{j,k}$ as defined by Bourguignon *et al*. (2014) is given by:

$$w_{j,k} = \frac{(-1)^k \beta\alpha^{k+1}\Gamma\left(\beta(k+1)+j+1\right)}{k!\,j!\left[\beta(k+1)+j-1\right]\Gamma\left(\beta(k+1)+1\right)}$$

Order statistics: The pdf of the ith order statistic for a random sample $X_1,\ldots\ldots,X_n$ from a distribution function F(x) and an associated pdf f(x) is given by Eq. 25:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} f(x) F(x)^{i-1}\left[1-F(x)\right]^{n-i} \tag{25}$$

Now, take f(x) and F(x) in Eq. 25 to be the pdf and cdf of the Weibull exponential distribution. Therefore, the pdf of the ith order statistic for a random sample $X_1,\ldots\ldots,X_n$ from the Weibull exponential distribution is given as:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!}\left[\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}\right]\times$$
$$\left[1-\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]\right]^{i-1}\times\left[\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]\right]^{n-i} \tag{26}$$

Following Bourguignon *et al*. (2014), Eq. 26 can be re-written to give Eq. 27:

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!}\left[\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}\right]\times$$
$$\sum_{k=0}^{i-1}(-1)^k\binom{i-1}{k}\times\exp\left\{-\alpha\left(n+k-i\left[e^{\lambda x}-1\right]^{\beta}\right)\right\} \tag{27}$$

Hence, the pdf of the minimum order statistic $X_{(1)}$ and maximum order statistic $X_{(n)}$ of the Weibull exponential distribution are respectively given by Eq. 28:

$$f_{X_{(1)}}(x) = n\times\left[\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}\right]\times\left[\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]\right]^{n-1} \tag{28}$$

and:

$$f_{X_{(n)}}(x) = n\times\left[\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}\right]\times\left[1-\exp\left[-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right]\right]^{n-1} \tag{29}$$

Estimation: Let, $X_1, X_2,\ldots, X_n$ denote random samples drawn from the Weibull exponential distribution with parameters α, β and λ as defined in Eq. 14. By the method of Maximum Likelihood Estimation (MLE), the likelihood function is given by:

$$f\left(x_1, x_2,\ldots, x_n; \alpha, \beta, \lambda\right) =$$
$$\prod_{i=1}^{n}\left[\alpha\beta\lambda\left(1-e^{-\lambda x}\right)^{\beta-1}\exp\left\{\lambda\beta x-\alpha\left(e^{\lambda x}-1\right)^{\beta}\right\}\right]$$

The log-likelihood function, l is expressed as:

$$l = n\log\alpha + n\log\beta + n\log\lambda + (\beta-1)\sum_{i=1}^{n}\log\left[1-e^{-\lambda x_i}\right] + \sum_{i=1}^{n}\left\{\lambda\beta x_i - \alpha\left(e^{\lambda x_i}-1\right)^{\beta}\right\} \qquad (30)$$

The solution of the non-linear system of equations obtained by differentiating Eq. 30 with respect to α, β and λ gives the maximum likelihood estimates of the model parameters. The solution can also be obtained directly by using R software when data sets are available.

RESULTS

To assess the flexibility of the Weibull exponential distribution over the well-known exponential distribution, two real data sets are used and analyses performed with the aid of R software.

Data set I: The first data is on the breaking stress of carbon fibres of 50 mm length (GPa). The data has been previously used by Nichols and Padgett (2006), Cordeiro and Lemonte (2011) and Al-Aqtash *et al*. (2014). The data is as follows:

0.39	0.85	1.08	1.25	1.47	1.57	1.61	1.61	1.69	1.80	1.84
1.87	1.89	2.03	2.03	2.05	2.12	2.35	2.41	2.43	2.48	2.50
2.53	2.55	2.55	2.56	2.59	2.67	2.73	2.74	2.79	2.81	2.82
2.85	2.87	2.88	2.93	2.95	2.96	2.97	3.09	3.11	3.11	3.15
3.15	3.19	3.22	3.22	3.27	3.28	3.31	3.31	3.33	3.39	3.39
3.56	3.60	3.65	3.68	3.70	3.75	4.20	4.38	4.42	4.70	4.90

The summary of the data is given in Table 1.

Data set II: The second data set is on the strengths of 1.5 cm glass fibres. The data was originally obtained by workers at the UK National Physical Laboratory and it has been used by Smith and Naylor (1987) and Bourguignon *et al*. (2014). The data is as follows:

0.55	0.74	0.77	0.81	0.84	0.93	1.04	1.11	1.13	1.24	1.25	1.27
1.28	1.29	1.30	1.36	1.39	1.42	1.48	1.48	1.49	1.49	1.50	1.50
1.51	1.52	1.53	1.54	1.55	1.55	1.58	1.59	1.60	1.61	1.61	1.61
1.61	1.62	1.62	1.63	1.64	1.66	1.66	1.66	1.67	1.68	1.68	1.69
1.70	1.70	1.73	1.76	1.76	1.77	1.78	1.81	1.82	1.84	1.84	1.89
2.00	2.01	2.24									

The summary of the data is given in Table 3.

DISCUSSION

The model with the least Alkaike Information Criteria (AIC) or highest log-likelihood value is considered the best fit. Readers can also go through Aryal and Tsokos (2011), Bourguignon *et al*. (2014), Jafari *et al*. (2014), Oguntunde and Adejumo (2015b) and many other notable works where the same criterion was adopted. In Table 2, the AIC for the 2 models are the same; in this case, our conclusion is based on the value of the log-likelihood generated. In Table 4, the AIC corresponding to the Weibull exponential distribution is lower than that of the exponential distribution. Hence, the Weibull exponential distribution performed better than the exponential distribution.

This result supports the claim in section 1 of this article that "Generalizing standard (or baseline) distributions has produced several compound distributions that are more flexible compared to the baseline distributions". This can also be justified by the works of some other notable authors; For instance, Aryal and Tsokos (2011) where, Transmuted Weibull distribution performed better than the Weibull distribution, Jafari *et al*. (2014) where, Beta-Gompertz distribution and beta exponential distribution performed better than the Gompertz distribution and the exponential distribution, respectively.

Table 1: Data summary for breaking stress of carbon fibres

Parameters	Values
N	66
Minimum	0.390
Q1	2.178
Median	2.835
Q3	3.278
Mean	2.760
Maximum	4.900
Variance	0.7946875
Skewness	-0.128487
Kurtosis	3.223049

Table 2: Performance of the distributions (Standard errors in parentheses)

Distributions	Parameter estimates	Log-likelihood	AIC	Rank
Weibull exponential	$\hat{\alpha}$ = 5.25929 (7.54600) $\hat{\beta}$ = 2.80643 (0.31699) $\hat{\lambda}$ = 0.14236 (0.05404)	-85.88334	177.7667	1
Exponential	$\hat{\lambda}$ = 0.36238 (0.04461)	-132.9944	177.7667	2

AIC: Alkaike information criteria

Table 3: Data summary for strength of 1.5 cm glass fibres

Parameters	Values
N	63
Minimum	0.550
Q1	1.375
Median	1.590
Q3	1.685
Mean	1.507
Maximum	2.240
Variance	0.1050575
Skewness	-0.8785848
Kurtosis	3.923761

Table 4: Performance of the distributions (Standard errors in parentheses)

Distributions	Parameter estimates	Log-likelihood	AIC	Rank
Weibull exponential	$\hat{\alpha}$ = 0.0175 (0.05746) $\hat{\beta}$ = 2.87962 (1.94066) $\hat{\lambda}$ = 1.01779 (1.13950)	-14.40207	34.80415	1
Exponential	$\hat{\lambda}$ = 0.66365 (0.08361)	-88.83032	179.6606	2

AIC: Alkaike information criteria

It is also good to note that reverse might be the case in some situations, for instance, a generalization proposed in Oguntunde and Adejumo (2015b), the so called generalized inverted generalized exponential distribution did not perform better than a sub-model proposed by the same author inverted generalized exponential distribution based on the data set used.

CONCLUSION

A three-parameter Weibull exponential distribution has been successfully defined. The shape of model could be unimodal or decreasing (depending on the value of the parameters). Some basic mathematical properties of proposed model are rigorously discussed. The Weibull exponential distribution is useful as a life testing model. The model is applied to two real life data sets and it can be said that the Weibull exponential distribution is more flexible than the exponential distribution.

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