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Convergence and almost sure T-stability for a random iterative sequence generated by a generalized random operator

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Abstract

The aim of this paper is to introduce the concept of generalized ϕ-weakly contraction random operators and then to prove the convergence and almost sure T-stability of Mann and Ishikawa-type random iterative schemes. We also prove that a random fixed point of such operators is Bochner integrable. Our results generalize, extend and improve various results in the existing literature including the results in Berinde (Bul. Ştiinţ. - Univ. Baia Mare, Ser. B Fasc. Mat.-Inform. 18(1):7-14, 2002), Olatinwo (J. Adv. Math. Stud. 1(1):5-14, 2008), Rhoades (Trans. Am. Math. Soc. 196:161-176, 1974; Indian J. Pure Appl. Math. 21(1):1-9, 1990; Indian J. Pure Appl. Math. 24(11):691-703, 1993) and Zhang *et al.* (Appl. Math. Mech. 32(6):805-810, 2011).

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1 Introduction

Real world problems are embedded with uncertainties and ambiguities. To deal with probabilistic models, probabilistic functional analysis has emerged as one of the momentous mathematical disciplines and attracted the attention of several mathematicians over the years in view of its applications in diverse areas from pure mathematics to applied sciences. Random nonlinear analysis, an important branch of probabilistic functional analysis, deals with the solution of various classes of random operator equations and related problems. Of course, the development of random methods has revolutionized financial markets. Random fixed point theorems are stochastic generalizations of classical or deterministic fixed point theorems and are required for the theory of random equations, random matrices, random partial differential equations and various classes of random operators arising in physical systems (see [1, 2]). Random fixed point theory was initiated in 1950s by Prague school of probabilists. Spacek [3] and Hans [4] established a stochastic analogue of the Banach fixed point theorem in a separable complete metric space. Itoh [5] in 1979 generalized and extended Spacek and Han's theorem to a multivalued contraction random operator. The survey article by Bharucha-Reid [6] in 1976, where he studied sufficient conditions for a stochastic analogue of Schauder's fixed point theorem for random operators, gave wings to random fixed point theory. Now this area has become full fledged research area, and many interesting techniques to obtain the solution of nonlinear random system have appeared in the literature (see [1–3, 5, 7–16]).

Papageorgiou [13] established an existence of random fixed point of measurable closed and nonclosed valued multifunctions satisfying general continuity conditions and hence improved the results in [5, 17] and [18]. Xu [15] extended the results of Itoh to a nonself-random operator T, where T satisfies weakly inward or the Leray-Schauder condition. Shahzad and Latif [14] proved a general random fixed point theorem for continuous random operators. As applications, they derived a number of random fixed points theorems for various classes of 1-set and 1-ball contractive random operators. Arunchai and Plubtieng [7] obtained some random fixed point results for the sum of a weakly-strongly continuous random operator and a nonexpansive random operator in Banach spaces.

Mann [19] introduced an iterative scheme and employed it to approximate the solution of a fixed point problem defined by a nonexpansive mapping where the Picard iterative scheme fails to converge. Later, in 1974, Ishikawa [20] introduced an iterative scheme to obtain the convergence of a Lipschitzian pseudocontractive operator when a Mann iterative scheme is not applicable.

The study of convergence of different random iterative processes constructed for various random operators is a recent development (see [8–11] and references mentioned therein). Recently, Zhang *et al.* [16] studied the almost sure T-stability and convergence of Ishikawa-type and Mann-type random algorithms for certain ϕ-weakly contractive-type random operators in the setup of a separable Banach space. They also established the Bochner integrability of a random fixed point for such random operators.

In this paper, we introduce the notion of generalized ϕ-weakly contractive random operator and obtain the convergence and almost sure T-stability of Ishikawa-type random iterative scheme and Mann-type random iterative scheme for such operators. Our results extend, unify and generalize the comparable results in [21–25] and [16].

Our results improves and generalizes the deterministic fixed points results of Berinde [21], Olatinwo [22], Rhoades [23–25] in stochastic verse. Moreover, it extends and improves the results of Zhang *et al.* [16].

2 Preliminaries

Let (Ω, Σ, μ) be a complete probability measure space and $(E, B(E))$ be a measurable space, where E is a separable Banach space, $B(E)$ is Borel sigma algebra of E, (Ω, Σ) is a measurable space (Σ-sigma algebra) and μ is a probability measure on Σ, that is, a measure with total measure one. A mapping $\xi : \Omega \to E$ is called (a) E-valued random variable if ξ is $(\Sigma, B(E))$-measurable, (b) strongly μ-measurable if there exists a sequence $\{\xi_n\}$ of μ-simple functions converging to ξ μ-almost everywhere. Due to the separability of a Banach space E, the sum of two E-valued random variables is an E-valued random variable. A mapping $T : \Omega \times E \to E$ is called a random operator if for each fixed e in E, the mapping $T(\cdot, e) : \Omega \to E$ is measurable.

The following definitions and results will be needed in the sequel.

Definition 2.1 [1] Let (Ω, ξ, μ) be a complete probability measure space. A random variable $\xi : \Omega \to X$ is Bochner integrable if for each $\omega \in \Omega$,

$$\int_{\Omega} \|\xi(\omega)\| \, d\mu(\omega) < \infty, \tag{2.1}$$

where $\|\xi(\omega)\|$ is a nonnegative real-valued random variable.

The Bochner integral is a natural generalization of the familiar Lebesgue integral to the vector-valued setting.

Proposition 2.2 [1] *A random variable ξ is Bochner integrable if and only if there exists a sequence of random variables $\{\xi_n\}_{n=1}^{\infty}$ converging strongly to ξ almost surely such that*

$$\lim_{n\to\infty} \int_{\Omega} \|\xi_n(\omega) - \xi(\omega)\| \, d\mu(\omega) = 0. \tag{2.2}$$

Definition 2.3 [16] Let (Ω, ξ, μ) be a complete probability measure space, E be a nonempty subset of a separable Banach space X, and $T : \Omega \times E \to E$ be a random operator. Define $F(T) = \{\xi^* : \Omega \to E$ such that $T(\omega, \xi^*(\omega)) = \xi^*(\omega)$ for each $\omega \in \Omega\}$ (the random fixed point set of T).

Definition 2.4 [16] Let (Ω, Σ, μ) be a complete probability measure space and E be a nonempty subset of a separable Banach space X. A random operator $T : \Omega \times E \to E$ is called a ϕ-weakly contractive-type random operator if there exists a continuous and nondecreasing function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$ and $\phi(0) = 0$ such that for each $x, \varsigma \in E$, $\omega \in \Omega$, we have

$$\int_{\Omega} \|T(\omega, x) - T(\omega, \varsigma)\| \, d\mu(\omega) \le \int_{\Omega} \|x - \varsigma\| \, d\mu(\omega) - \phi\left(\int_{\Omega} \|x - \varsigma\| \, d\mu(\omega)\right). \tag{2.3}$$

Motivated by the above results, we hereby introduce the following contractive condition.

Definition 2.5 Let (Ω, ξ, μ) be a complete probability measure space and E be a nonempty subset of a separable Banach space X. A random operator $T : \Omega \times E \to E$ is of generalized ϕ-weakly contractive-type if there exists $L(\omega) \ge 0$ and a continuous and nondecreasing function $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ with $\phi(t) > 0$ for each $t \in (0, \infty)$ and $\phi(0) = 0$ such that for each $x, \varsigma \in E$, $\omega \in \Omega$,

$$\begin{aligned} &\int_{\Omega} \|T(\omega, x) - T(\omega, \varsigma)\| \, d\mu(\omega) \\ &\quad \le e^{L(\omega)\|x-\varsigma\|} \left[\int_{\Omega} \|x - \varsigma\| \, d\mu(\omega) - \phi\left(\int_{\Omega} \|x - \varsigma\| \, d\mu(\omega)\right)\right]. \end{aligned} \tag{2.4}$$

If $L(\omega) = 0$ for each $\omega \in \Omega$ in (2.4), then it reduces to condition (2.3).

The study of nonlinear operators have attracted the attention of several mathematicians (see, *e.g.* [26–31]). Several interesting fixed points results have emerged as a result of such study.

Let $T : \Omega \times E \to E$ be a random operator, where E is a nonempty convex subset of a separable Banach space X.

The random Ishikawa-type iterative scheme is a sequence of functions $\{\xi_n\}$ and $\{\eta_n\}$ defined by

$$\begin{cases} \xi_0(\omega) \in E, \\ \xi_{n+1}(\omega) = (1 - a_n)\xi_n(\omega) + a_n T(\omega, \eta_n(\omega)), \\ \eta_n(\omega) = (1 - c_n)\xi_n(\omega) + c_n T(\omega, \xi_n(\omega)). \end{cases} \tag{2.5}$$

The random Mann-type iterative scheme is a sequence of functions $\{\xi_n\}$ defined by

$$\begin{cases} \xi_0(\omega) \in E, \\ \xi_{n+1}(\omega) = (1 - a_n)\xi_n(\omega) + a_n T(\omega, \xi_n(\omega)), \end{cases} \tag{2.6}$$

where $0 \le a_n, c_n \le 1$ and $\xi_0 : \Omega \to E$ is an arbitrary measurable mapping.

For any given random variable $\xi_0 : \Omega \to E$, define a random iterative scheme with the help of functions $\{\xi_n\}_{n=0}^{\infty}$ as follows:

$$\xi_{n+1}(\omega) = f\big(T; \xi_n(\omega)\big), \quad n = 0, 1, 2, \ldots, \tag{2.7}$$

where f is some function measurable in the second variable.

Definition 2.6 [16] Let ξ^* be a random fixed point of a random operator T and Bochner integrable with respect to $\{\xi_n\}_{n=0}^{\infty}$. Let $\{\zeta_n\}_{n=0}^{\infty}$ be an arbitrary sequence of random variables. Set

$$\epsilon_n(\omega) = \big\| \zeta_{n+1}(\omega) - f\big(T; \zeta_n(\omega)\big) \big\|, \tag{2.8}$$

and assume that $\|\epsilon_n(\omega)\| \in L^1(\Omega(\xi, \mu))$ $(n = 0, 1, \ldots)$. The iterative scheme (2.7) is almost surely T-stable (or the iterative scheme (2.7) is almost surely stable with respect to T) if and only if

$$\lim_{n\to\infty} \int_{\Omega} \big\| \epsilon_n(\omega) \big\| \, d\mu(\omega) = 0 \tag{2.9}$$

implies that $\xi^*(\omega)$ is Bochner integrable with respect to $\{\zeta_n(\omega)\}_{n=0}^{\infty}$.

The following lemma will be needed in the sequel.

Lemma 2.7 [32] *Let $\{\gamma_n\}$ and $\{\lambda_n\}$ be two sequences of nonnegative real numbers, $\{\sigma_n\}$ be a sequence of positive numbers satisfying the conditions: $\sum_{n=1}^{\infty} \sigma_n = \infty$ and $\lim_{n\to\infty} \frac{\gamma_n}{\sigma_n} = 0$. If $\lambda_{n+1} \le \lambda_n - \sigma_n \phi(\lambda_n) + \gamma_n$ hold for each $n \ge 1$, where $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous and strictly increasing function with $\phi(0) = 0$, then $\{\lambda_n\}$ converges to 0 as $n \to \infty$.*

3 Main results

We start with the following result.

Theorem 3.1 *Let $(E, \|\cdot\|)$ be a separable Banach space, $T : \Omega \times E \to E$ be a generalized ϕ-weakly contractive-type random operator with $F(T) \neq \emptyset$, and $\{\xi_n\}$ be a random iterative sequence as defined in (2.5) where $\{a_n\}$ and $\{c_n\}$ are real sequences in $(0, 1)$ such that $\sum_{n=1}^{\infty} a_n c_n = \infty$. Then the random fixed point ξ^* of T is Bochner integrable.*

Proof It suffices to show that $\lim_{n\to\infty}\int_\Omega \|\xi_n(\omega)-\xi^*(\omega)\|\,d\mu(\omega)=0$. By (2.4) and (2.5), we have

$$\begin{aligned}
&\int_\Omega \bigl\|\xi_{n+1}(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\quad\le (1-a_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)+a_n\int_\Omega \bigl\|T\bigl(\omega,\eta_n(\omega)-\xi^*(\omega)\bigr)\bigr\|\,d\mu(\omega)\\
&\quad\le (1-a_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\qquad + a_n\biggl\{e^{L(\omega)\|\eta_n(\omega)-\xi^*(\omega)\|}\biggl[\int_\Omega \bigl\|\eta_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\qquad - \phi\biggl(\int_\Omega \bigl\|\eta_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\biggr)\biggr]\biggr\}\\
&\quad\le (1-a_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\qquad + a_n e^{L(\omega)\|\eta_n(\omega)-\xi^*(\omega)\|}\int_\Omega \bigl\|\eta_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega).
\end{aligned}\tag{3.1}$$

Now we compute the following estimate:

$$\begin{aligned}
&\int_\Omega \bigl\|\eta_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\quad\le (1-c_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)+c_n\int_\Omega \bigl\|T\bigl(\omega,\xi_n(\omega)\bigr)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\quad\le (1-c_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\qquad + c_n\biggl\{e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|}\biggl[\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\qquad - \phi\biggl(\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\biggr)\biggr]\biggr\}\\
&\quad\le (1-c_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)+c_n e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|}\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\qquad - c_n e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|}\phi\biggl(\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\biggr).
\end{aligned}\tag{3.2}$$

Using (3.2) in (3.1), we obtain

$$\begin{aligned}
&\int_\Omega \bigl\|\xi_{n+1}(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\quad\le (1-a_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)+a_n e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|}\biggl\{(1-c_n)\\
&\qquad \times\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)+c_n e^{(L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|)}\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\\
&\qquad - c_n e^{(L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|)}\phi\biggl(\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)\biggr)\biggr\}\\
&\quad= (1-a_n)\int_\Omega \bigl\|\xi_n(\omega)-\xi^*(\omega)\bigr\|\,d\mu(\omega)+a_n e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|}(1-c_n)
\end{aligned}$$

$$\begin{aligned}
&\times \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega) + a_n c_n e^{(2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|)} \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega) \\
&- a_n c_n e^{(2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|)} \phi\left(\int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega)\right) \\
\leq{}& \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega) + a_n e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|} \\
&\times \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega) + a_n c_n e^{2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|} \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega) \\
&- a_n c_n e^{2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|} \phi\left(\int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega)\right) \\
={}& \left(1 + a_n e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|} + a_n c_n e^{2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|}\right) \\
&\times \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega) - a_n c_n e^{2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|} \\
&\times \phi\left(\int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega)\right). \qquad (3.3)
\end{aligned}$$

Set

$$\begin{aligned}
&\lambda_n = \left(1 + a_n c_n e^{2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|}\right) \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega), \\
&\sigma_n = a_n c_n e^{2L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|} \quad \text{and} \\
&\gamma_n = a_n e^{L(\omega)\|\xi_n(\omega)-\xi^*(\omega)\|} \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega)
\end{aligned}$$

in Lemma 2.7, it follows that conditions of Lemma 2.7 are satisfied. Hence

$$\lim_{n\to\infty} \int_\Omega \|\xi_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega) = 0. \qquad (3.4)$$

□

Remark 3.2 Theorem 3.1 improves and generalizes the results of Akewe and Okeke [33], Akewe *et al.* [34], Berinde [21, 35], Olatinwo [22], Zhang *et al.* [16] and Rhoades [23–25].

Now we obtain the following theorem as a special case of Theorem 3.1.

Theorem 3.3 *Let $(E, \|\cdot\|)$ be a separable Banach space, $T : \Omega \times E \to E$ be a generalized ϕ-weakly contractive-type random operator with $F(T) \neq \emptyset$, and $\{\xi_n\}$ be a random iterative sequence as defined in* (2.6) *where $\{a_n\}$ is a real sequence in* $(0,1)$ *such that* $\sum_{n=1}^{\infty} a_n = \infty$. *Then the random fixed point ξ^* of T is Bochner integrable.*

Theorem 3.4 *Let $(E, \|\cdot\|)$ be a separable Banach space, $T : \Omega \times E \to E$ be a generalized ϕ-weakly contractive-type random operator with $F(T) \neq \emptyset$, and $\{\xi_n\}$ be a random iterative sequence as defined in* (2.5) *converging strongly to the random fixed ξ^* of T almost surely, where $\{a_n\}$ and $\{c_n\}$ are real sequences in* $(0,1)$ *such that* $0 < a \leq a_n$ *and* $0 < c \leq c_n$ $(n \geq 1)$. *Then $\{\xi_n\}_{n=0}^{\infty}$ is almost surely T-stable.*

Proof Let $\{\varsigma_n\}_{n=0}^{\infty}$ be any sequence of random variables and

$$\|\epsilon_n(\omega)\| = \|\varsigma_{n+1}(\omega) - (1 - a_n)\varsigma_n(\omega) - a_n T(\omega, k_n(\omega))\|, \quad n = 0, 1, \ldots, \qquad (3.5)$$

where $k_n(\omega) = (1-c_n)\varsigma_n(\omega) + c_n T(\omega, \varsigma_n(\omega))$ and $\lim_{n\to\infty}\int_\Omega \|\epsilon_n(\omega)\|\,d\mu(\omega) = 0$. Next, we show that $\xi^*(\omega)$ is Bochner integrable with respect to the sequence $\{\varsigma_n(\omega)\}_{n=0}^\infty$. From (3.5), we obtain

$$\begin{aligned}\int_\Omega \|\varsigma_{n+1}(\omega) - \xi^*(\omega)\|\,d\mu(\omega) &\le \int_\Omega \|\varsigma_{n+1}(\omega) - (1-a_n)\varsigma_n(\omega) - a_n T(\omega, k_n(\omega))\|\,d\mu(\omega) \\ &\quad + (1-a_n)\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\quad + a_n \int_\Omega \|T(\omega, k_n(\omega)) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\le \int_\Omega \|\epsilon_n(\omega)\|\,d\mu(\omega) + \int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\quad + a_n \int_\Omega \|T(\omega, k_n(\omega)) - \xi^*(\omega)\|\,d\mu(\omega). \end{aligned} \tag{3.6}$$

Using (2.4), we have

$$\begin{aligned}&\int_\Omega \|T(\omega, k_n(\omega)) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\quad \le e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}\biggl[\int_\Omega \|k_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) - \phi\biggl(\int_\Omega \|k_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega)\biggr)\biggr] \\ &\quad \le e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}\int_\Omega \|k_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\quad \le e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}\biggl\{(1-c_n)\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\qquad + c_n \int_\Omega \|T(\omega, \varsigma_n(\omega)) - \xi^*(\omega)\|\,d\mu(\omega)\biggr\} \\ &\quad \le e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}(1-c_n)\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\qquad + c_n e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}\biggl[e^{L(\omega)\|\varsigma_n(\omega)-\xi^*(\omega)\|}\biggl\{\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\qquad - \phi\biggl(\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega)\biggr)\biggr\}\biggr] \\ &\quad \le e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\qquad + c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) - c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|} \\ &\qquad \times \phi\biggl(\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega)\biggr). \end{aligned} \tag{3.7}$$

By (3.7) in (3.6), we obtain that

$$\begin{aligned}\int_\Omega \|\varsigma_{n+1}(\omega) - \xi^*(\omega)\|\,d\mu(\omega) &\le \int_\Omega \|\epsilon_n(\omega)\|\,d\mu(\omega) + \int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega) \\ &\quad + a_n e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}\int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\|\,d\mu(\omega)\end{aligned}$$

$$
\begin{aligned}
&+ a_n c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|} \int_\Omega \big\| \varsigma_n(\omega) - \xi^*(\omega) \big\| \, d\mu(\omega) \\
&- a_n c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|} \phi\biggl(\int_\Omega \big\| \varsigma_n(\omega) - \xi^*(\omega) \big\| \, d\mu(\omega) \biggr) \\
= {} & \int_\Omega \big\| \epsilon_n(\omega) \big\| \, d\mu(\omega) + \bigl(1 + a_n e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|} \\
&+ a_n c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|} \bigr) \int_\Omega \big\| \varsigma_n(\omega) - \xi^*(\omega) \big\| \, d\mu(\omega) \\
&- \bigl(a_n c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|} \bigr) \\
&\times \phi\biggl(\int_\Omega \big\| \varsigma_n(\omega) - \xi^*(\omega) \big\| \, d\mu(\omega) \biggr).
\end{aligned}
\tag{3.8}
$$

Using the conditions that $\lim_{n\to\infty} \int_\Omega \|\epsilon_n(\omega)\| \, d\mu(\omega) = 0$, $0 < a \leq a_n$ and $0 < c \leq c_n$ $(n \geq 1)$, we have

$$
\lim_{n\to\infty} \frac{\int_\Omega \|\epsilon_n(\omega)\| \, d\mu(\omega)}{a_n c_n} \leq \lim_{n\to\infty} \frac{\int_\Omega \|\epsilon_n(\omega)\| \, d\mu(\omega)}{ac} = 0.
\tag{3.9}
$$

Take $\lambda_n = (1 + a_n e^{L(\omega)\|k_n(\omega)-\xi^*(\omega)\|} + a_n c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}) \int_\Omega \|\varsigma_n(\omega) - \xi^*(\omega)\| \, d\mu(\omega)$, $\sigma_n = a_n c_n e^{2L(\omega)\|k_n(\omega)-\xi^*(\omega)\|}$, $\gamma_n = \int_\Omega \|\epsilon_n(\omega)\| \, d\mu(\omega)$. Note that all the conditions in Lemma 2.7 are satisfied. Therefore, we obtain

$$
\lim_{n\to\infty} \int_\Omega \big\| \varsigma_n(\omega) - \xi^*(\omega) \big\| \, d\mu(\omega) = 0.
\tag{3.10}
$$

Conversely, if $\xi^*(\omega)$ is Bochner integrable with respect to the sequence $\{\varsigma_n(\omega)\}_{n=1}^\infty$, we have

$$
\begin{aligned}
\int_\Omega \big\| \epsilon_n(\omega) \big\| \, d\mu(\omega) &= \int_\Omega \big\| \varsigma_{n+1}(\omega) - (1-a_n)\varsigma_n(\omega) - a_n T\bigl(\omega, k_n(\omega)\bigr) \big\| \, d\mu(\omega) \\
&\leq \int_\Omega \big\| \varsigma_{n+1}(\omega) - \xi^*(\omega) \big\| \, d\mu(\omega) \\
&\quad + (1-a_n) \int_\Omega \big\| \xi^*(\omega) - \varsigma_n(\omega) \big\| \, d\mu(\omega) \\
&\quad + a_n \int_\Omega \big\| \xi^*(\omega) - T\bigl(\omega, k_n(\omega)\bigr) \big\| \, d\mu(\omega).
\end{aligned}
\tag{3.11}
$$

Using (2.4), we have

$$
\begin{aligned}
&\int_\Omega \big\| \xi^*(\omega) - T\bigl(\omega, k_n(\omega)\bigr) \big\| \, d\mu(\omega) \\
&\quad = \int_\Omega \big\| T\bigl(\omega, \xi^*(\omega)\bigr) - T\bigl(\omega, k_n(\omega)\bigr) \big\| \, d\mu(\omega) \\
&\quad \leq e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \biggl[\int_\Omega \big\| \xi^*(\omega) - k_n(\omega) \big\| \, d\mu(\omega) - \phi\biggl(\int_\Omega \big\| \xi^*(\omega) - k_n(\omega) \big\| \, d\mu(\omega) \biggr) \biggr] \\
&\quad \leq e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \int_\Omega \big\| \xi^*(\omega) - k_n(\omega) \big\| \, d\mu(\omega) \\
&\quad \leq e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \biggl[(1-c_n) \int_\Omega \big\| \xi^*(\omega) - \varsigma_n(\omega) \big\| \, d\mu(\omega)
\end{aligned}
$$

$$
\begin{aligned}
&+ c_n \int_\Omega \bigl\| T\bigl(\omega, \xi^*(\omega)\bigr) - T\bigl(\omega, \varsigma_n(\omega)\bigr) \bigr\| \, d\mu(\omega) \biggr] \\
&\le e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \biggl[(1-c_n) \int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \\
&+ c_n \biggl\{ e^{L(\omega)\|\xi^*(\omega)-\varsigma_n(\omega)\|} \biggl[\int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \\
&- \phi\biggl(\int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \biggr) \biggr] \biggr\} \biggr] \\
&\le (1-c_n) e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \\
&+ c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \\
&- c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \phi\biggl(\int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \biggr) \\
&\le \bigl(e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} + c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \bigr) \\
&\times \int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) - c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \\
&\times \phi\biggl(\int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \biggr).
\end{aligned}
\tag{3.12}
$$

Using (3.12) in (3.11), we have

$$
\begin{aligned}
\int_\Omega \bigl\| \epsilon_n(\omega) \bigr\| \, d\mu(\omega) &\le \int_\Omega \bigl\| \varsigma_{n+1}(\omega) - \xi^*(\omega) \bigr\| \, d\mu(\omega) + \int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \\
&\quad + a_n \bigl(e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} + c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \bigr) \\
&\quad \times \int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) - a_n c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \\
&\quad \times \phi\biggl(\int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \biggr) \\
&= \int_\Omega \bigl\| \varsigma_{n+1}(\omega) - \xi^*(\omega) \bigr\| \, d\mu(\omega) + \bigl(1 + a_n e^{L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \\
&\quad + a_n c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \bigr) \int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \\
&\quad - a_n c_n e^{2L(\omega)\|\xi^*(\omega)-k_n(\omega)\|} \phi\biggl(\int_\Omega \bigl\| \xi^*(\omega) - \varsigma_n(\omega) \bigr\| \, d\mu(\omega) \biggr).
\end{aligned}
\tag{3.13}
$$

Hence

$$
\lim_{n\to\infty} \int_\Omega \bigl\| \epsilon_n(\omega) \bigr\| \, d\mu(\omega) = 0. \tag{3.14}
$$

□

Remark 3.5 Theorem 3.4 generalizes and improves the results of Zhang *et al.* [16], Berinde [21, 35], Olatinwo [22], Rhoades [23–25] and several others in the literature.

We now state the following theorem as a special case of Theorem 3.4.

Theorem 3.6 *Let $(E, \|\cdot\|)$ be a separable Banach space, $T : \Omega \times E \to E$ be a generalized ϕ-weakly contractive-type random operator with $F(T) \neq \emptyset$, and $\{\xi_n\}$ be a random iterative sequence as defined in* (2.6) *converging strongly to the random fixed ξ^* of T almost surely, where $\{a_n\}$ is a real sequence in $(0,1)$ such that $0 < a \leq a_n$ $(n \geq 1)$. Then $\{\xi_n\}_{n=0}^{\infty}$ is almost surely T-stable.*

4 Example

Example 4.1 Consider the following nonlinear stochastic integral equation:

$$\begin{aligned}(\xi;\omega) &= \int_0^\infty \frac{(1-t^2)e^{|\xi(t;\omega)-\varsigma(t;\omega)|}}{8(1+|\xi(s;\omega)|)}\,ds \\ &\leq e^{|\xi(t;\omega)-\varsigma(t;\omega)|}\left[\int_0^\infty \frac{1}{8(1+|\xi(s;\omega)|)}\,ds - t^2\int_0^\infty \frac{1}{8(1+|\xi(s;\omega)|)}\,ds\right]. \end{aligned} \tag{4.1}$$

From (4.1), we see that (2.4) is satisfied with $\phi(t) = t^2$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally to this work. Both authors read and approved the final manuscript.

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