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Numerical Solution of Two-Point Boundary Value Problems via Differential Transform Method

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Abstracts

This paper considers the solution of two-point boundary value problems by the application of Differential Transform method. Two examples are solved to illustrate the technique and the results are compared with the exact solutions. The numerical results obtained show strong agreement with their corresponding exact solutions, and as such demonstrate reliability and great accuracy of the method.

Keywords: Two point boundary value problems, Numerical solution, Differential transform method.

Mathematics Subject Classification (2010): 30E25, 34A45, 65-XX

1. Introduction

Many linear and nonlinear boundary value problems of ordinary differential equations occur frequently in different areas of science and engineering. Various applications of these type of problems occur in fluid mechanics, quantum mechanics, optimal control, chemical reactor theory, aerodynamics, reaction-diffusion process, geophysics and other related fields of applied sciences.

Numerical-analytic methods of various types have been proposed by many researchers to solve boundary value problems (bvp). EL-Arabawy [1] used Picard iterative technique (PIT), Tatari *et al* [2], applied Adomian Decomposition method (ADM), Singh *et al* [3] also applied Adomian Decomposition method, Adesanya *et al* [4] presented Adomian Decomposition method for Bratu's problem, Mishra [5] employed He-Laplace method.

The method (Differential Transform method) adopted in this work was first proposed by Zhou (1998) to solve linear and nonlinear differential equations in electrical circuits analysis [6]. It has also been widely applied by many researchers in scientific literature to solve differential equations [7-10].

2. Analysis of the Method (DTM)

Taking an arbitrary one dimensional function $y = v(x)$ in Taylor's series about a point $x = x_\circ$ which can be written as:

$$V(k) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{d^k v}{dx^k}\right]_{x=x_0} \tag{1}$$

where $v(x)$ is the original function and $V(k)$ is the transformed function. The inverse differential transform of $V(k)$ is defined as:

$$v(x) = \sum_{k=0}^{\infty} V(k)(x - x_0)^k \tag{2}$$

Substituting (1) into (2) yields the following equation:

$$v(x) = \sum_{k=0}^{n} \frac{1}{k!}\left[\frac{d^k V(x)}{dx^k}\right]_{x=x_0} \left(x - x_0\right)^k \tag{3}$$

which gives the Taylor's series.

The following theorems can be deduced from equations (2) and (3):

Theorem 1: If $v(x) = \alpha q(x) \pm \beta r(x),\ then\ V(k) = \alpha Q(k) \pm \beta R(k)$, where α are β constants

Theorem 2: If $v(x) = x^n, then\ V(k) = \delta(k-n)$ where, $\delta(k-n) = \begin{cases} 1, & k = n \\ 0, & \mathrm{k} \neq n \end{cases}$

Theorem 3: If $v(x) = \dfrac{dh(x)}{dx}$, then $V(k) = (k+1)H(k+1)$

Theorem 4: If $v(x) = \dfrac{d^2h(x)}{dx^2}$, then $V(k) = (k+1)(k+2)H(k+2)$

Theorem 5: If $v(x) = \cos(px + \phi),\ then\ V(k) = \dfrac{P^k}{k!}\cos\left(\dfrac{k\pi}{2} + \phi\right)$, where p and ϕ are constants

3. Numerical Analysis and Applications

In this subsection we apply the differential transformation method to solve two examples of boundary value problems.

Problem 1: Consider the following bvp [12]

$$y'' - 4y = 0, y(0) = 0, y(1) = e \tag{4}$$

with a theoretical solution:

$$y(x) = \frac{(-1 + e^{4x})e^{3-2x}}{-1 + e^4} \tag{5}$$

The differential transform of the boundary value problem (4) yields the following recurrence relations:

$$(k+1)(k+2)Y(k+2) - 4Y(k) = 0 \tag{6}$$

And the differential transforms of the boundary conditions in (4) at $x = 0$ are:

$$Y(0)=0, \qquad \sum_{k=0}^{n} Y(k)=e \tag{7}$$

Using the recurrence relation in (6) and the boundary conditions (7) at $x=0$ (for $k \geq 0$) respectively, we obtain the following:

$$Y(2)=0,\ Y(3)=\frac{2A}{3},\ Y(4)=0,\ Y(5)=\frac{2A}{15},\ Y(6)=0,\ Y(7)=\frac{4A}{315}$$

Hence,

$$y(x)=\sum_{k=0}^{\infty} Y(k)x^k = Ax+\frac{2A}{3}x^3+\frac{2A}{15}x^5+\frac{4A}{315}x^7+\cdots \tag{8}$$

From (7) at $x=1$, using $A=y'(0)$, we obtain $A=1.499577$. Implementing the inverse transform rule in (2) yields the following series solution:

$$y(x)=1.499577x+0.99971836x^3+0.1999436x^5+0.019042247x^7+\cdots \tag{9}$$

Table I: *Analysis of Solutions-problem 1*

	EXACT	DTM	ABSOLUTE ERROR
0.00	0.000000	0.000000	0.000000
0.01	0.014991	0.014997	6.05E-06
0.02	0.029987	0.030000	1.21E-05
0.03	0.044996	0.045014	1.81E-05
0.04	0.060023	0.060047	2.42E-05
0.05	0.075074	0.075104	3.03E-05
0.06	0.090154	0.090191	3.64E-05
0.07	0.105271	0.105314	4.25E-05

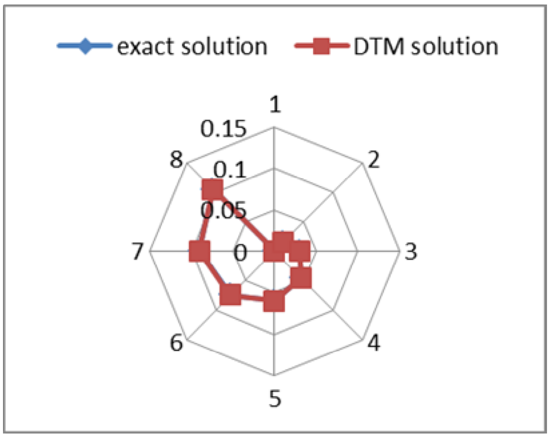

Fig 1: *Radar-view of problem 1 Solutions*

Problem 2: Consider the following bvp [5]:

$$y'' - y = \cos(x) \;\;,\; y(0) = 1,\; y(1) = 1 \tag{10}$$

with a theoretical solution:

$$\frac{\left(-3\cosh(1)+3\sinh(1)+\cos(1)+2\right)e^{x}}{4\sinh(1)} + \frac{\left(3\cosh(1)+3\sinh(1)-\cos(1)-2\right)e^{-x}}{4\sinh(1)} - \frac{\cos(x)}{2} \tag{11}$$

The differential transform of the bvp in (11) gives the following recurrence relation:

$$(k+1)(k+2)Y(k+2) = Y(k) + \frac{1}{k!}\cos\left(\frac{k\pi}{2}\right) \tag{12}$$

Also the differential transform of the boundary conditions at $x = 0$ are:

$$Y(0) = 0,\; \sum_{k=0}^{n} Y(k) = 1 \tag{13}$$

The recurrence relation in (11) together with the boundary conditions (13) at $x = 0$ (for $k \geq 0$) respectively, yield the following:

$$Y(2) = 1,\; Y(3) = \frac{A}{6},\; Y(4) = \frac{1}{24},\; Y(5) = \frac{A}{120},\; Y(6) = \frac{1}{360},\; Y(7) = \frac{A}{5040},$$

$$Y(8) = \frac{1}{40520},\; Y(9) = \frac{A}{362800},\; Y(10) = \frac{1}{181400},\; \cdots$$

$$\begin{aligned} y(x) &= \sum_{k=0}^{\infty} Y(k)x^{k} \\ &= 1 + Bx + x^{2} + \frac{B}{6}x^{3} + \frac{1}{24}x^{4} + \frac{B}{120}x^{5} + \frac{1}{360}x^{6} + \frac{B}{5040}x^{7} + \frac{1}{6}x^{8} + \frac{B}{362880}x^{9} + \cdots \end{aligned} \tag{14}$$

From (12) and (13) at $x = 1$, using $B = y'(0)$, we obtain $B = -0.888762526$

Applying the inverse transform rule in (2), yields the following series solution

$$\begin{aligned} y(x) &= 1 - 0.888762526x - x^{2} - 0.148127087x^{3} + 0.0416666x^{4} - 0.007406354x^{5} \\ &\quad - 0.002777778x^{6} + \cdots \end{aligned} \tag{15}$$

Table II: *Analysis of Solutions-problem 2*

	EXACT	DTM	ABSOLUTE ERROR
0.0	1.000000	1.000000	0.000000
0.2	0.861128	0.861127	1.03E-06
0.4	0.796019	0.796006	1.28E-05
0.6	0.799699	0.799571	0.000128
0.8	0.868488	0.867789	0.000699
1.0	1.000000	0.997371	0.002629

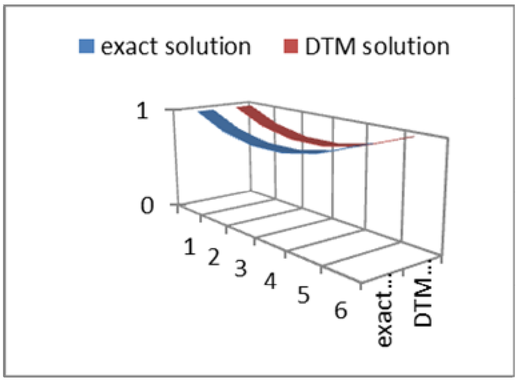

Fig 2: 3-D line-view of problem 2 Solutions

4. Conclusion

In this work, Differential Transform Method has been successfully applied to solve two-point boundary value problems. The two examples solved revealed that the method is fast, accurate and easy to apply. It converges faster to the exact solution. The results are presented in Tables I & II and Fig. 1 & 2. This method (DTM) is hereby recommended for all forms of differential equations due to the rapidity of its convergence and fewer computations.

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