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Solutions of the Schrödinger Equation with Inversely Quadratic Hellmann Plus Inversely Quadratic Potential Using Nikiforov-Uvarov Method

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Abstract. By using the Nikiforov-Uvarov (NU) method, the Schrödinger equation has been solved for the interaction of inversely quadratic Hellmann (IQHP) and inversely quadratic potential (IQP) for any angular momentum quantum number, *l*. The energy eigenvalues and their corresponding eigenfunctions have been obtained in terms of Laguerre polynomials. Special cases of the sum of these potentials have been considered and their energy eigenvalues also obtained.

Keywords: Schrödinger equation; inversely quadratic Hellmann potential; inversely quadratic potential; Nikiforov-Uvarov method; Energy eigenvalues; Eigenfunctions; Laguerre polynomial
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INTRODUCTION

The Schrödinger equation (SE) is a total energy equation where the Hamiltonian operator acts on a suitable wave function of the system to give us the energy eigenvalues of the system. With the experimental verification of the Schrodinger equation, many physicists, mathematicians and chemists have devoted much interest now than before in solving the Schrodinger equation to obtain bound state solutions for some potentials of physical interest [1-5]. The use of some special potentials to obtain exact or approximate solutions of the Schrödinger equation has also been reported in the literature [6-10]. Some of these potentials are known to play very important roles in many fields of Physics such as Molecular Physics, Solid State and Chemical Physics [8].

The purpose of the present work is to present the solution of the Schrodinger equation with the inversely quadratic Hellmann potential [11] and inversely quadratic potential [12] of the form $V(z) = -\frac{a}{r} + \frac{b}{r^2} e^{\delta r}$ and $V(z) = \frac{g}{r^2}$, respectively. Where g is the inverse quadratic potential strength.

The sum of these potentials (IQHIQP) can be written as

$$V(r) = b\delta^2 - \frac{1}{r}(a + b\delta) + \frac{1}{r^2}(b + g), \tag{1}$$

where r represents the internuclear distance, a and b are the strengths of the coulomb and Yukawa potentials, respectively, and δ is the screening parameter. Equation (1) is then amenable to Nikiforov-Uvarov method. Researches involving the Hellman potential in the Schrodinger and Dirac formalisms have already been reported [13-15]. However, the solutions of radial Schrodinger equation for any angular momentum quantum number, l, with IQHIQP using Nikiforov-Uvarov method which is the aim of this paper, has not yet been reported.

Overview of the Nikiforov-Uvarov method

The overview of Nikiforov-Uvarov (NU) method has already been reported [16-17].

The Schrödinger equation

In spherical coordinate, Schrödinger equation with the potential $V(r)$ is given as [18-19]

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$$-\frac{\hbar^2}{2\mu}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2 sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi(r,\theta,\phi)+V(r)\psi(r,\theta,\phi)$$

$$= E\psi(r,\theta,\phi). \quad (2)$$

Using the common ansatz for the wave function:

$$\psi(r,\theta,\phi)=\frac{R(r)}{r}Y_{lm}(\theta,\phi) \quad (3)$$

in equation (2) we get the following set of equations:

$$\frac{d^2R_{nl}(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[E-V(r)-\frac{\lambda\hbar^2}{2\mu r^2}\right]R_{nl}(r)=0, \quad (4)$$

$$\frac{d^2\Theta_{ml}(\theta)}{d\theta^2}+cot\theta\frac{d\Theta_{ml}(\theta)}{d\theta}\left(\lambda-\frac{m^2}{sin^2\theta}\right)\Theta_{ml}(\theta)=0, \quad (5)$$

$$\frac{d^2\Phi_m(\phi)}{d\phi^2}+m^2\Phi_m(\phi)=0, \quad (6)$$

where $\lambda=l(l+1)$ and m^2 are the separation constants. $Y_{lm}(\theta,\phi)=\Theta_{ml}(\theta)\Phi_m(\phi)$ is the solution of equations (5) and (6) and their solutions are well known as spherical harmonic functions [18].

Solutions to the radial equation

Equation (4) is the radial part of the Schrodinger equation which we are interested in solving. Equation (4) together with the potential in equation (1) and with the transformation $z=r^2$ yields the following equation [19]:

$$\frac{d^2R(z)}{dz^2}+\frac{1}{2z}\frac{dR(z)}{dz}+\frac{1}{4z^2}(-\alpha z^2+\beta z-\gamma)R(z) = 0, \quad (7)$$

where the radial wave function is $R(z)$ and

$$\alpha=-\frac{2\mu(E-b\delta^2)}{\hbar^2},\beta=\frac{2\mu(a+b\delta)}{\hbar^2},\gamma=\frac{2\mu(b+g)}{\hbar^2}+l(l+1). \quad (8)$$

Following the Nikiforov-Uvarov method as reported in our previous paper [19] the energy eigenvalue equation with the IQHIQP is obtained as

$$E=b\delta^2-\frac{\mu(a+b\delta)^2/2\hbar^2}{\left(n+\frac{1}{2}+\sqrt{\frac{2\mu(b+g)}{\hbar^2}+\left(l+\frac{1}{2}\right)^2}\right)^2} \quad (9)$$

and the wave function $R(z)$ is obtained in terms of the generalized Laguerre polynomials as

$$R(z)=N_n z^{(-1+\sqrt{1+4\gamma})/2}e^{-\sqrt{\alpha}z}L_n^{\sqrt{1+4\gamma}}(2\sqrt{\alpha}z). \quad (10)$$

N_n is the normalization constant.

RESULT AND DISCUSSION

The energy eigenvalues and the corresponding un-normalized eigenfunctions have been obtained using the NU method for the Schrodinger equation with the inversely quadratic Hellmann plus inversely quadratic potential (IQHIQP). Special cases of the potential are considered:

Case 1: If we set the parameters, $b = g = 0, a = ze^2$, it is easy to show that equation (9) reduces to the bound state energy spectrum of a particle in the Coulomb potential, *i.e.*, $E_{np} = -Z^2\mu e^4/2\hbar^2 n_p^2$, where $n_p = n + l + 1$, is the principal quantum number.

Case 2: Similarly, if we set $g = 0, a, b \neq 0$ equation (9) results in the bound state energy spectrum of a vibrating-rotating diatomic molecule subject to the inversely quadratic Hellmann potential as follows:

$$E = b\delta^2 - \frac{\mu(a+b\delta)^2/2\hbar^2}{\left(n+\frac{1}{2}+\sqrt{\frac{2\mu b}{\hbar^2}+\left(l+\frac{1}{2}\right)^2}\right)^2} \ . \tag{10}$$

It is interesting to note that similar equation was obtained by Ita [11] when he carried out calculations on inversely quadratic Hellmann potential for the Schrodinger equation. The two deductions from equation (9) reveal that our calculations are exact.

CONCLUSION

The bound state solutions of the Schrodinger equation have been obtained for the inversely quadratic Hellmann plus inversely quadratic potential. Special cases of the potential are also considered and their energy eigen values obtained.

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