READINGS IN MICROECONOMICS

Editors
EUGENE KOUASSI
MILTON .A. IYOHA
OKON T. EKANEM
SAM GUOBADIA
ALFRED BARIMAH

Readings in Microeconomics 2010

New Horizon Publishers
Arakan Printing Press,
P. O. Box 36, Koto Babi,
Accra, Ghana

ISBN 9988 - 8462 - 1 - 4

Designs: *Aggreh Meshack*

OUTLINE OF CHAPTERS

PART 5

GENERAL EQUILIBRIUM AND WELFARE ECONOMICS

CHAPTER NINE

9

Input Pricing

Okon T. Ekanem & Evans Osabuohien

INTRODUCTION

The optimizing behaviour of firms in the output market is crucial in the allocation of resources in a given economy. Hence, in this chapter effort is made to direct attention from product to resource market; that is from pricing and production of output to pricing and employment of resource inputs. There are basically two categories of productive resources, namely, human and non-human resources. Non-human resources, which can also be called material resources, are man-made goods, used in the production of further goods and services. They could be durable, long lasting inputs such as land, building, machinery and equipment, raw materials and so on. Human resources refer to the talents, skills and abilities of human beings that can be applied to the production of goods and services. To ensure uniformity and avoid possible ambiguity, resources and inputs are used almost synonymously in this chapter.

The daily operations of firms in productive activities involve the acquisition and usage of resource inputs. Changes in the rate of production or output can be necessitated by corresponding changes in input usage. Therefore, the firm's behaviour in the product market is obviously transmitted into the resource markets, which would influence both the quantity and prices of inputs used.

The main objective of this chapter is to analyse the input decision of firms: what determines the amount of *ith* resource (R_i) a firm is willing to employ at various input prices. The method adopted is to explain the theory of employment for resource inputs by examining resource input decisions making process of firms in three distinct cases, which include the following: when a firm:

- operates in a perfectly competitive output market and buys inputs from a perfectly competitive input market.
- operates in an imperfectly competitive output market and buys inputs from a perfectly competitive input market.
- is faced with imperfectly competitive conditions in both input and output markets.

Supply and Demand in the Input Market

The input or factor market is the market for factors of production- land, labour capital and managerial skills. In the input market, the individuals make up the suppliers' side of the market, while the firms constitute the demand or the buyers' side. Though factor inputs can be classified into four groups mentioned above, there are different kinds of each of these input classifications. For example, labour input may comprise the services of an accountant, a chemist, an engineer, a computer operator/analyst, a maintenance officer and so on. Thus, labour is not as homogenous as believed. Each type of labour service is characterized by different wage rates and market supply and demand conditions. Therefore, it would not be very appropriate to generally talk about a demand function for labour. Instead, labour market should be analysed with respect to the specified inputs such as the market for professional secretaries, electronic/electrical engineers or accountants. Moreover, each professional classification has its own association, which regulates entry requirements and general professional codes of conduct. This in turn influences supply as well as the market price. In Nigeria for example, we have Nigeria Bar Association (NBA), Nigeria Medical Association (NMA), Institute of Chartered Accountants of Nigeria (ICAN), Chartered Institute of Stockbrokers (CIS), Chartered Institute of Bankers of Nigeria (CIBN), among many others. However, the same general principle governs optimisation in each input classification. Thus, the discussions that follow will be of general applicability to all input classifications.

CASE 1: *Perfect Competition in both Output and Input Markets*

In a free market or capitalist economy, input prices respond to supply and demand conditions in the input market. When perfect competition exists in the input market, the price of each input (P_f) is determined by the equality of input supply (F_s) and input demand (F_d). When $F_s > F_d$, P_f will fall. Similarly, when $F_s < F_d$, P_f will rise. The application of the law of supply and demand in a competitive input market is graphically shown in Figure 1 below.

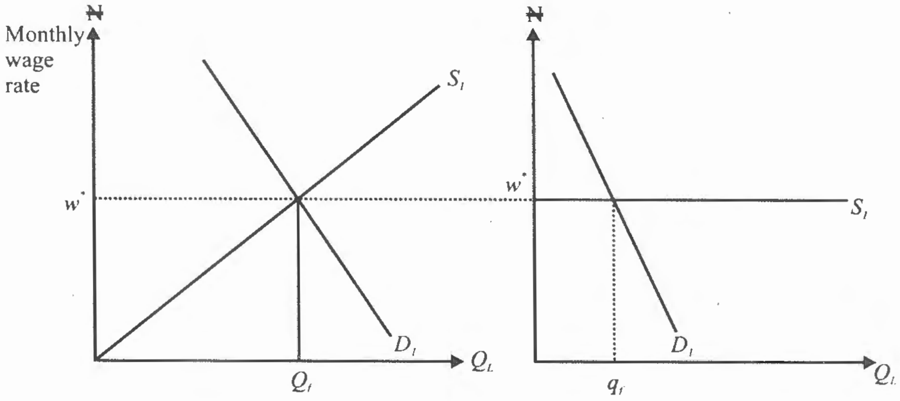

***Figure1**: Supply and Demand in a Competitive Input Market*

It is assumed in panel (a) as shown in Figure 18.1 above that the industry supply curve is upward sloping while the demand curve is downward slopping. The general explanation for this is that on the supply side, households are usually encouraged to supply more units of factor inputs such as land, labour or capital, when the related prices are higher. On the demand side, firms will always find it profitable to employ more units of an input when its price is low and less when its price is high. More so, it is their usual practice to substitute low priced factors for high priced factors whenever such substitutions are feasible.

The major argument in the marginal productivity theory is that the equilibrium price of an input should reflect the input's marginal contribution to the firm. Given the equilibrium wage rate in Figure 18.1, each firm employing labour input must pay the same wage rate w^*. The equilibrium employment for each firm is attained at the point where wage rate equals the value of marginal product of labour ($w = VMP_L$) i.e. extra cost of labour equals extra revenue from each additional unit of labour employed.

Assuming that the total industry demand for labour $= \sum D_i$; equilibrium wage rate = w^*; and the wage rate for the *ith* unit of labour = w_i. Then the following conditions exist:

$$\forall \;\; w < w^*, \;\text{ if } VMP_L > w_i, \text{ then } D_i \text{ will increase}$$
$$\forall \;\; w > w^*, \;\text{ if } VMP_L < w_i, \text{ then } D_i \text{ will decrease.}$$
$$\text{And at } w = w^*, \;\; VMP_L = w_i \text{ . It means } D_i \text{ is optimal.}$$

It follows from above that wage rate responds positively to changes in the VMP_L. The same principle is applicable for all other inputs.

Firms that employ inputs usually seek to minimise their costs in order to maximise their profits, while suppliers of the inputs desire employment of their resources in such a way that their net advantage is maximised, which may include pecuniary (monetary) and non-pecuniary (non-monetary) factors. The non-pecuniary factors could include favourable working conditions, location, and convenience, among others. Since these factors are relatively stable over time, the more variable pecuniary factors dominate in determining the factor supply schedule.

Given an input vector (*A, B,..., N*) required for the production of good *X*, the least cost combination for inputs can be given as:

$$MPP_a/P_a = MPP_b/P_b = ... = MPP_n/P_n = 1/MC_x \qquad (1)$$

where:

MPP_a = the marginal physical product of input *A*,

P_a = the unit price of input *A*, and

MC_x = the marginal cost of producing an extra unit of output *X*.

If competition exists in the input market then marginal factor cost (*MFC*) would be equal to factor price (P_a). Generally, if competition exists in factor market then,

$$MFC_a = P_a. \qquad (2)$$

The value of the marginal product of input *A* (VMP_a) can be expressed as:

$$VMPa = MPP_a * P_x \qquad (3)$$

where:

P = is the unit price of output *X*, and,

A = the only resource input,

For optimality:

$$VMP_a = MFC_a = P_a \quad (4)$$

In Equation 4 above, it could be observed that MFC_a is the same thing as the change in the firm's total cost (TC) resulting from a unit change in output.
Similarly,

$$MFC_a = {}^{\Delta TC}/_{\Delta Q_x} = MC_a$$
$$But \;\; \Delta TC = P_a \quad (5)$$
$$and \;\; \Delta Q_x = MPP_a$$

$$Therefore, {}^{\Delta TC}/_{\Delta Qx} = {}^{P_a}/_{MPP_a} = MC \quad (6)$$

(This is known as least cost combination of factors)

For n inputs, the optimum input mix is attained where:

$$P_a/MPP_a = P_b/MPP_b = \ldots = P_n/MPP_n = MC \quad (7)$$

This is means that all factors are equally efficient at the margin when the least cost combination rule is satisfied. That is when the marginal physical product per naira spent on each input is the same. This gives the optimum proportions for input usage. The absolute optimum must combine profit maximisation with the least cost rule as follows:

$$P_a/MPP_a = P_b/MPP_b = \ldots = P_n/MPP_n, \quad (8a)$$

(This is the least cost rule).

While $MR_x = MC_x$ (This is the π max. rule) *(8b)*

In sum, the marginal productivity theory fundamentally maintains that firms will minimise cost and maximise profits by hiring inputs at the rate, *ceteris paribus*, where:

$$MFC_a = P_x{*}MPP_a = VMP_a \qquad (9)$$

Given that perfect competition also exists in the product market, $P_x{*}MPP_a$ is also referred to as the marginal revenue product of A (MRP_a), which is equal to VMP_a. (This will be expounded later). Let the possible units of input A that can be used in producing various units of X range from $A = 1,\dots, n$.. Then the profit maximising combination of A based on the marginal productivity theory can be equally stated as follows:

$MRP_n = MFC_n$ (employment is optimal) *(10a)*

$MRP_{n-1} > MFC_{n-1}$ (it pays to employ more) *(l0b)*

$MRP_{n+1} < MFC_{n+1}$ (losses sustained by additional employment) *(10c)*

$TRP_n \geq TFC_n$ (optimum contribution to profit) *(10d)*

Factor Employment Equilibrium under Perfect Competition in the Input and Product Markets

When perfect competition exists in both factor and output markets this means that the firm is a price taker in both markets. The representative firm can buy all of its input requirements from the factor market at the prevailing prices. This also implies that it can sell all its products at the prevailing market price.

Given the optimisation decision rule in equation 10a, firm's profits are maximised whenever

$$MRP_n = MFC_n$$

On the cost side, optimality in factor employment under perfect competition in the input market implies that:

$$MFC_a = AFC_a \ (average\,factor\ cost) = P_a$$

On the output side,

$$Px = MRx = MRPa \qquad (11)$$

Since every additional unit of output can be sold at the prevailing price, it follows that:

$$MRP_a = VMP_a$$

Thus, the optimality in employment of factor inputs in this case stipulates that:

$$MFC_a = MRP_a = VMP_a$$

The numerical exposition of the equality above is shown in Table 1.

Observe the following definition:

i. VMP_a = Value of Marginal Product of input *A*
ii. *TRP* = Total Revenue Product
iii. *CTP* = Contribution to Profit, which is defined as *TRP-TVC*.

With competition in the product market,

$$P_x = MR_x.$$

$$\text{And } VMP_a = P_x * MPP_a$$

$$MRP_a = \Delta TR / \Delta A = MR_x * MPP_a$$

But under perfect competition:

$$P_x = MR_x$$

Therefore, $MRP_a = VMP_a$ (This equality is clearly shown in columns 6 and 7 in Table 1)

Table 1: *Profit Maximising Factor Employment for a Firm Operating in a Perfectly Competitive Output and Input Markets*

Units of Factor A (1)	Units of Output X (2)	MPP_a ΔX (3)	Price of Product $P_x = MR_x$ (4)	Total Revenue Product $X \cdot P_x$ (5)	MRP_a $\frac{\Delta TR}{\Delta A}$ (6)	VMP_a $P_x . MPP_a$ (7)	TVC $P_a . A$ (8)	GTP $TRP\text{-}TVC$ (9)
0	0	0	20	0	0	0	0	0
1	16	16	20	320	320	320	160	160
2	40	24	20	800	480	480	320	480
3	60	20	20	1200	400	400	480	720
4	75	15	20	1500	300	300	640	880
5	88	13	20	1760	260	260	800	960
6	98	10	20	1960	200	200	960	1000
7	107	9	20	2140	180	180	1120	1020
8	112	6	20	2240	120	120	1280	960
9	116	4	20	2320	80	80	1440	880
10	119	3	20	2380	60	60	1600	780

Note: Unit Price of input, P_a = N 160 =*MFC*

Source: Adapted from Ekanem and Iyoha (2000) with permission.

With perfect competition in the input market, every additional unit of input can be acquired at the same prevailing price. Thus:

$$MFC = P_a = MC_x \text{ (where } A \text{ is the only variable input)}$$

Then, equations 10a to 10d can be used to determine the profit maximising employment of input A in Table 1.

From equation 10, equilibrium employment is at the input usage rate of 7units. This means equation 10 is satisfied at input rate n= 7 units, where:

$$MFC_n = MRP_n, \quad (₦180 \text{ is closest to } ₦160) \text{ at the 7th unit.} \qquad (10a)$$

$$MFC_{n-1} < MRP_{n-1} \quad (₦160 < ₦200) \qquad (10b)$$

$$MFC_{n+1} > MRP_{n+1} \quad (₦160 > ₦120) \qquad (10c)$$

$$\text{And} \quad CTP = \text{Maximum.} \quad (₦1020) \qquad (10d)$$

Equation 10 satisfied at $n = 7$

Therefore, the optimal usage rate of input A is 7 units.

MATHEMATICAL EXPOSITION

Determination of Profit Maximising Input Rate under Conditions of Perfect Competition in the Product and Input markets

The central proposition in this regard is that a perfectly competitive firm will maximise profits by employing variable inputs up to the point where $MRP = MFC$. This can be demonstrated mathematically as follows:

Let the firm's production function be given as:

$$Q_k = f(A) \qquad (11)$$

where Q_k = Units of the *kth* product, A = Units of variable inputs.

$$MPP_a = \frac{dQ_k}{dA} = f'(A) \tag{12}$$

Firms profit function is given as

$$\pi = TR - TC \tag{13}$$

where $TR = P_k * Q_k$

and $TC = TFC + TVC$ and $TVC = P_a * A$.

The *TC* function can be re-written as

$$TC = TFC + P_a * A.$$

Substituting this into Eqn. (13), we have

$$\pi = P_k Q_k - [TFC + P_a * A] \tag{14}$$

Observe that $TR_k = P_k f(A)$, since $Q_k = f(A)$ and substituting this into Eqn. (4), we have

$$\pi = P_k Q_k - TFC - P_a * A \tag{15}$$

Taking a derivative of the profit function *wrt.* A and setting it equal to zero, we have

$$\frac{d\pi}{dA} = P_k f'(A) - P_a = 0 \tag{16}$$

$$P_k f'(A) - P_a = 0$$

But $f'(A) = MPP_a$

Thus $P_k MPP_a = MRP_a$ and $P_a = MFC_a$

The condition for attainment of $\pi\ max.$ emerges as $MRP_a = MFC_a$

CASE II: Factor Employment Equilibrium with Perfect Competition in the Factor Market and Imperfect Competition in the Product Market

A firm operating in an imperfectly competitive product market is usually faced with a downward slopping demand curve (*AR*). Column 4 of Table 2 is used to illustrate the extent of price reduction for the sale of each extra Unit of output of *X*. While Columns 5 and 6 show the behaviour of the TRP and the MRP, respectively for each additional unit of variable input, A, employed. The value of marginal product of input *A* (VMP_a) is given in column 7. Recall that under perfect competition in the product market, $VMP_a = MRP_a$ But with imperfect competition in the product market $MRP_a \neq VMP_a$. Equally recall that perfect competition in the factor market implies that $VMP_a = MFC = P_a$.

The cause and the relevance of this difference in the firm's decision making process is analysed as follows:

The *VMP* of a factor, by definition, refers to the gross addition to *TRP* resulting from the sale of one additional unit of output *X*. This is expressed algebraically in Equation 3 as:

$$\text{VM}P_a = P_x * MPP_a$$

$$MRP_a = MPP_a * MR_x = \frac{\Delta TR}{\Delta A}$$

But with imperfect competition in the output market, MR_x ? P_x

$$\therefore VMP_a \neq MRP_a$$

And VMP_a will equal MRP_a *iff* $P_x = MR_x$.

But with imperfect competition in the output market, $P_x > MR_x$, thus $P_x \neq MR_x$.

The relevant decision variable is the *MRP*. Note that for all units of input *A*. $VMP_a > MRP_a$. This means that *MRP* curve lies below the *VMP* curve and the *MRP* curve declines faster than the *VMP*. From Equation10, the optimum employment of input is attained at the point, where $MRP_n = MFC_n$.This occurs at the 5th unit of input in Table 2. Other conditions of Equation 10 are also met this output level as follows:

$MFC_n = MRP_n$ (least difference between MFC and MRP) *(10a)*

$MFC_{n-1} < MRP_{n-1}$ (₦160 < 255) *(l0b)*

$MFC_{n+1} > MRP_{n+1}$ (₦ 160 > 102) *(10c)*

$TCP = Maximum$ (₦ 960) *(l0d)*

Table 2: *Profit Maximising Factor Employment for a Firm Operating under Perfect Competition in the Input Market and Imperfect Competition in the Product Market*

Units of factor input A	Units of output X	Marginal Physical Product of A MPP_a	Price of Product X P_x	Total Revenue Product $TRP = X.P_x$	Marginal revenue product $MRP = \frac{\Delta TR}{A}$	Value marginal product VMP $P_x.MPP_a$	$TVC = P_a . A$	Marginal factor cost MFC	Contribution to profit CTP
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	0	0	25	0					
1	16	16	24	384	384	384	160	160	224
2	40	24	23	920	536	552	320	160	600
3	60	20	22	1320	400	440	480	160	840
4	75	15	21	1575	255	315	640	160	935
5	88	13	20	1760	185	260	800	160	960
6	98	10	19	1862	102	190	960	160	902
7	107	9	18	1926	64	162	1120	160	806
8	112	6	17	1904	-22	102	1280	160	624
9	116	4	16	1856	-48	64	1440	160	416
10	119	3	15	1785	-71	45	1600	160	185

Note: **Unit Price of input, P_a = N 160**

Source: *Adapted from Ekanem and Iyoha (2000) with permission.*

CASE III: Factor Employment Equilibrium for a Firm Operating in Imperfectly Competitive Product and Input Markets

Having looked at the first two cases above, next is to examine profit maximising employment of factor inputs with imperfect competition in both input and product markets. Imperfect competition in the product market and its impact on the firm's output decisions were examined in the last section. Using the same output data in Table 2, the unit input price variations to capture monopoly elements in the factor market is introduced. The distinctive feature of imperfectly competitive input market is the upward rising Average Factor Cost *(AFC)* and Marginal Factor Cost *(MFC)* curves. Note that with imperfect competition in the factor market, input price is no longer constant for the firm. Each additional unit of input is acquired at an additional cost. Thus, $P_a \neq MFC_a$ and average factor cost $AFC \neq MFC$. *MFC* is rising faster than *AFC*. The *AFC*, which is also the input supply curve, is upward rising. As in other cases, the relevant decision variables remain the MRP_a and the MFC_a. The graphical illustration of equilibrium factor employment under this market condition, is shown is Figure 2.

Table 3: *Factor Employment Equilibrium with Imperfect Competition in the Output and Input Markets*

Units of Factors A (1)	Units of Output X (2)	MPP_a (3)	Price of Output P_x (4)	TRP $P_x.X$ (5)	MRP (6)	Price of Factor P_a (7)	TVC $P_a.A$ (8)	Marginal Factor Cost (9)	TCP $TR\text{-}TVC$ (10)
0	0	0	25	0		140	0		
1	16	16	24	384	384	145	145	145	239
2	40	24	23	290	536	150	300	155	620
3	60	20	22	1320	400	155	465	165	855
4	75	15	21	1575	255	160	640	175	935
5	88	13	20	1760	185	165	825	185	935
6	98	10	19	1862	102	170	1020	195	842
7	107	9	18	1926	64	175	5225	205	701
8	112	6	17	1904	-22	180	1440	215	464
9	116	4	16	1856	-48	185	1665	225	191
10	119	3	15	1785	-71	190	1900	235	-114

Source: *Adapted from Ekanem and Iyoha (2000) with permission*

Applying the decision condition in Equation 10, at input rate $n = 5$

$$MRP_n = MFC_n \qquad (₦185) \qquad (10a)$$

$$MRP_{n-1} > MFC_{n-1} \qquad (225 > 175) \qquad (10b)$$

$$MRP_{n+1} < MFC_{n+1} \qquad (102 < 195) \qquad (10c)$$

$$TCP = maximum \qquad (₦935) \qquad (10d)$$

Thus, the equilibrium employment rate is 5 units.

A Situation when there are more than one Variable Inputs

The equilibrium-input usage under various market conditions has been so far examined. In all the three cases, the analyses were based on the assumption of a single variable input, A. The general optimisation procedure to cover cases with more than one variable factor inputs is treated in this section. Suppose the firm employs n factor inputs, then the optimum combination of inputs must satisfy the following criteria:

Under perfect competition in the factor market:

i. $MRP_a / P_a = MRP_b / P_b ... = MRP_n / P_n$

ii. With imperfect competition in the factor market, $P_f \neq MFC_f$ .The substitution of one input for another involves two things: (i) changes in the marginal product MP_f and (ii) changes in price of factor P_f .Since each additional unit of factor input must be purchased at a higher price, then it is the *MC* of inputs i.e. *MFC,* instead of price, that constitutes the relevant decision variable for the firm. For the *n* variable case profit is maximised when:

$$MRP_a/MFC_a = MRP_b/MFC_b = ... = MRP_n/MFC_n$$

if $MRP_a/MFC_a > MRP_b/MFC_b$,

then profit can be increased by substituting input A for B and vice versa. But since each input must be employed up to the point where $MRP_a = MFC_a$, then optimum input mix must be attained at the point where:

$$MRP_a/MFC_a = MRP_b/MFC_b = ... = MRP_n/MFC_n$$

Elasticity of Demand of Firms for Factor Inputs

Just as price elasticity of demand exists for consumers in the output market, it also exists in a similar manner in the factor market. There are three major factors that influence the firm's elasticity of demand for factor input, which are outlined below:

i. ***The behaviour of the marginal product function:*** Recall that under perfect competition in the factor market $MRP_f = MP_f * MR_x$ .The more rapid the decline in MP_f as additional units are used; the less elastic will be the firm's demand for the factor. On the other hand the more gradually MP_f declines, the greater the elasticity of demand for the factor.

i. ***Availability of substitutes:*** The fewer the number of acceptable substitutes, the more inelastic the demand for a factor. Similarly as the price of acceptable substitutes increase the less the possibilities of substitution and the more inelastic the demand for the factor.
ii. ***The relative magnitude of factor cost:*** The smaller the size of the factor cost relative to total cost, the smaller its elasticity of demand.

Monopoly, Monopsony and Bilateral Monopoly

Recall from Equation 10a that monopolists as well as competitive firms must employ input A for the production of output X at a point where $MRPa = MFCa$. But for a monopolist, $P_x > MRPa$ therefore $MRP_a < VMP_a$. Whereas $P_x = MR_x$, and $MRPa = VMP_a$ for competitive firms. In short, non-competitive firms pay factors an amount less than their VMP. But this does not mean that the monopolist will always pay each factor less than it can earn in a competitive firm. The issue here is that the monopolist is not employing sufficient factors that would bring about the equality between P_a and MRP_a.

Monopsony situation refers to the case of a single buyer – in the factor market. Similarly, oligopsony is used to describe the existence of just a few buyers in the factor market. The monopsonist faces an upward rising supply curve. This means that the monopsonist must pay a higher price for every additional unit of a factor employed. It should be noted that the higher wage must be paid not to the new employee alone, but to all the existing workers as well. Thus, for the monopsonist $MFC_a > P_a$ all the time. Suppose for example that a monopsonist is currently employing 20 people and paying them N10 each. To employ one additional worker he has to pay a higher wage of ₦11. Thus, the new price P_a =₦11. His total wage bill for 20 workers was N200 per hour. Since he must now pay each of the 21 workers ₦11, his new total wage bill is ₦231. And his MFC = N31. Thus, for the monopsonist, the MFC of employing any input is greater than the price of the input.

MATHEMATICAL EXPOSITION

Firm's Optimisation with Prefect Competition in the Output market and Mosopsony in the Input Market

Assuming the production and cost functions of the monopsonist are given as:

$$Q = f(L) \text{ and } C = wL, \text{ then :}$$

$$TR = PQ; \text{ and } TC = wL \qquad (11)$$

Because of monopsony in the labour market, w is an increasing function of the number of people employed.

$$\text{Thus, } \quad w = h(L) \qquad \text{where } \quad {}^{dw}\!/_{dL} > 0 \qquad (12)$$

$$MC_L = {}^{dC}\!/_{dL} = w + wh'(L) \qquad (13)$$

$$\text{Since } wh'(L) > 0, \qquad \text{then } MC_L > w. \; (\forall w > 0).$$

Monopsonist's π function, can be written as:

$$\pi = R\text{-}C = P_h\,(L)\text{-}wL \qquad (14)$$

The first order condition for optimality requires that:

$$^{d\pi}\!/_{dL} = ph'(L) - w - wh'(L) \qquad (15)$$

$$\text{Thus, } \quad ph'(L) = w + wh'(L) \qquad (16)$$

Equation (16) can be interpreted to mean that equilibrium employment by the monopsonist is achieved at the employment rate where the $VMPL^{(lhs)} = MCL^{(Rhs)}$.

$$d^2\pi / dL^2 = pf''(L) - 2h'(L) - Lh'' = 0$$

$$pf''(L) < 2h'(L) + Lh''(L)$$

This is interpreted to mean that the rate of change in MC_L be greater than the rate of change in VMP_L

Numerical Example

A monopsonists is production and cost functions are given as

$$Q = 40L^2 - .5L^3 \quad ; \quad C = 200 + 25L.$$

Assuming that the firm sells its output in a perfectly competitive market at ₦5 per unit, then his TR and TC functions will be

$$P.Q = 200L^2 - 2.5L^3 \quad ; \quad C = 200 + 25L.$$

$$MRP_L = \frac{\Delta TR}{\Delta L} = \frac{dTR}{dL} = 400L - 7.5L^2$$

$$MC_L = \frac{dC}{dl} = 25.$$

Invoking the MRP_L, MFC_L equality criterion for optimality, we have

$$400L - 7.5L^2 = 25.$$

This reduces to a standard quadratic equation

$$7.5L^2 - 400L + 25 = 0$$

This yields the solution, $L = 53.26$

The equilibrium solutions are

$$Q = 37{,}921 \text{ units}, \qquad L = 53 \text{ units.}$$

From Equation (10a) the monopsonist maximises his profit by employing at an input rate where $MRP_a = MFC_a$. This is illustrated in Figure 3. However, at the input rate L_1, $Pa < MRP_a$: This difference between the *MRP* of an input and the price of the input *P* is referred to as the monopsonistic exploitation of inputs in the factor market. Exploitation in the use of factors usually exists whenever any form of imperfection exists in the factor market. Although factor exploitation is always associated with the exploitation of labour, empirical observation shows that all factors, labour, capital are exploited, since their return may not always equal their *MRP*. The institution of minimum wage laws and the elimination of monopolistic tendencies in the factor market can remove exploitation of labour.

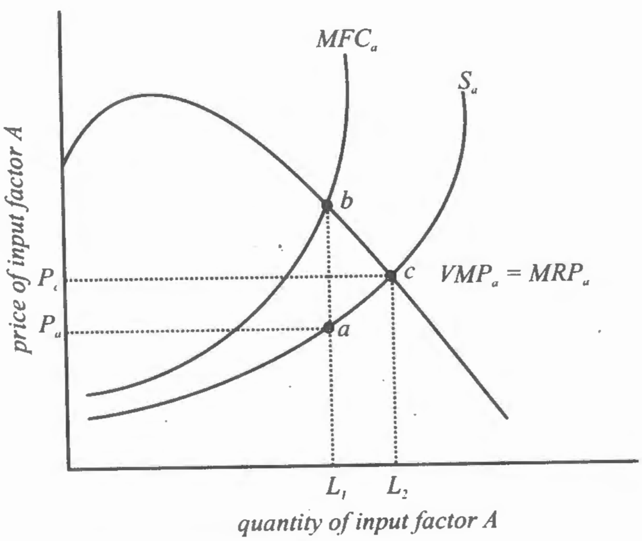

***Figure 3:** Monopsony Equilibrium and Monopsonistic Exploitation*

Bilateral Monopoly

A situation of bilateral monopoly is said to exist whenever there is a monopsonistic firm employing a factor supplied by a monopolistic firm. This may not be a common occurrence in the real world of business. A close approximation of this situation is the existence of a big employer and a big labour union. A close example is the relation between the Nigerian Railway Corporation and Rail Roads Workers Union. Another similar example is the case of the Government and Academic Staff Union of Universities. In this setting, the employer wants to behave like a monopsonist, and supplier wants to behave like a monopolist. A graphical illustration of equilibrium in this case is shown in Figure 4. The seller optimises his position by equating the *MR* with *MC* (*MR* = *MC*), supplying output L_1 and charging a price of P_1. The employer optimises his position guided by the *MRP* and *MFC* equality condition. Then the employer plans to employ L_2 units of labour and pay a price P_2. But there is an issue here to be clarified. The single buyer and the single seller have different optimal prices and quantities. The single seller optimises at quantity $L_1$and price P_1. The single buyer optimises at quantity L_2 and price P_2. Desired supply price exceeds desired purchase price ($P_1 > P_2$). The result would be a deadlock, and no economic forces exist to reconcile them. Equilibrium will depend upon the relative bargaining strengths of the monopolists, to determine whether price will move closer to P_2 or to P_1. External arbitration could operate to shift their positions closer to a competitive equilibrium at the equality of *MC* (short-run supply curve) and *MRP* (demand curve). At this point, employment level will be L^3 while the price will be P_3.

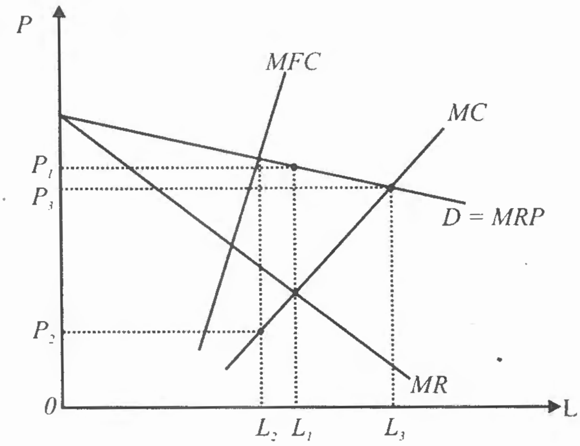

Figure 4: *Equilibrium under Bilateral Monopoly*

We can summarize thus,

The supplier	=	*monopolist;*
Employer	=	*monopsonist*
Monopolists optimising criterion:		$MR = MC$
Achieved at employment level L_1 and price P_1.		
Monopsonists optimising criterion:		$MRP_l = MFC_l$
Achieved at employment level L_2 and price P_2.		

External arbitration may force both parties to a market clearing equilibrium position at $MC = MRP$ establishing equilibrium price and quantity at P_3 and L_3, respectively.

Evaluating of the Marginal Productivity Theory

There exists some gaps between the theoretical possibilities that were illustrated in this chapter and what actually obtains in the empirical business world. The existence of this gap stems from numerous institutional interferences and market imperfections and other artificial restrictions, which exert remarkable impact on the self-functioning operation of the factor market. Over the years these interferences have thwarted the empirical realisation of the theoretical possibilities of the marginal productivity theory. They have in fact-introduced additional dimensions to the economic theories of input pricing and employment.

Some of these interferences include the establishment of minimum wage legislation, the growth of labour unions and their ability to extract wage increases in excess of productivity gains, the imposition of wage - price controls, the existence of wage differentials between certain occupations, the impact of monetary policy on interest rate and factor immobility among firms, industries and geographical locations. Federal character, ethnic quotas and other employment discriminatory practices are peculiar to Nigeria. Similar practices may be experienced in other developing countries of the world. All of these factors have tremendous effects on the prices and employment of factors. We now proceed to examine two of these factors namely minimum wage legislation and the impact of labour unionism.

The Impact of Minimum Wage Legislation on the Labour Market

It is within the powers of every prudent government to institute minimum wage laws in the economy. The major objectives of such laws include the following:

(i) To improve the standard of living of the masses at the lower echelon tail of the income strata.

(ii) To curtail the exploitation of the relatively immobile labour class. .

(iii) To increase the buying power of the low income families.

The economic effects of minimum wage legislation on labour market equilibrium are shown in Figure 5.

Let the equilibrium wage rate established by market forces be W^*. The establishment of minimum wage legislation pushes average wage to W_1 above the -market equilibrium that would otherwise have prevailed. This has the effect of suppressing employment from L^* to L_1 Meanwhile the increased wage has resulted in a rise in the supply of labour to L_2. This results in an excess supply of labour and the resultant unemployment rate is $L_1 L_2$.

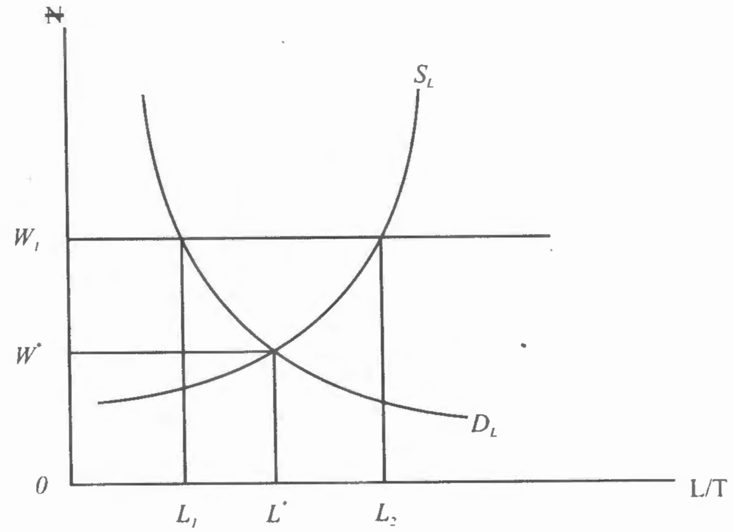

Figure 5: *The Impact of Minimum Wage Laws on Labour Markets.*

On the part of the firms, the higher wage bill caused by the institution of minimum wage laws imposes tremendous financial pressures on the marginal firms. In order to maintain the balance of their cost-price-profit relationship they may be constrained to adopt any of the following procedures:

i. Introduce new and more relevant technologies aimed at improving the efficiency and productivity of labour in accordance with the higher cost of labour.

ii. Upgrade management practices as well as the quality of supervision.

iii. Prune the work force by laying-off inefficient workers and the upgrading of employee selection standards and qualifications.

iv. Better working conditions may be introduced to reduce employee turnover, which could hamper the level of productivity and efficiency gains.

Generally, minimum wage legislation promises higher wages and improved conditions of services for workers, and increased productivity for the employers. Both parties gain, but only at the expense of the marginal workers and potential entrants who are denied access to the new order. Thus, in the short-run unemployment would definitely increase.

Economic Effects of Unions

In most free market economies trade unions exist and usually they exert great influence on the functioning of the labour market. For instance, in Nigeria, almost every trade or profession is unionised to a certain degree. Some of them include the following: Nigerian Medical Association (NMA), for the medical profession; Nigerian Bar Association (NBA) for the legal profession, Academic Staff Union of Universities (ASUU) for university lecturers, Nigerian Society of Engineers (NES), for the engineering profession, Institute of Chartered Accountants of Nigeria (ICAN) for the accounting profession, Chartered Institute of Bankers of Nigeria (CIBN) for banking profession, and a host of others. The nurses, electricians, fashion designers, music artists, teachers, automobile mechanics, barbers, and even the traders are unionised to some measures. Whenever wage-settlements are reached in any of the key industries, the pattern is quickly transmitted to other sectors of the economy. Thus, the existence of unions has great effect on the national labour market and the general price level.

Union activities usually have the effect of raising wage rate and reducing the number of people employed. Thus in studying the wage-employment behaviour in any economy the impact of collective bargaining must be taken into account for a more realistic approximation to be attained.

Strike, The Big Rod of Labour Unions

Through a system of collective bargaining, unionised workers have always succeeded in obtaining higher wages and better working conditions for their workers. The ultimate weapon of the union is the threat of a strike. In the words of Gwartney and Straub (1979), strike, the big stick of the union is an action of unionised employees whereby they simultaneously discontinue working and prevent other potential workers from offering their services to the employers. The purpose of strike is to impose economic costs on the employer of the union's services, which would pressurize him to accept the proposed terms of the union. When workers succeed in using a strike to disrupt the production process and also interfere with the employer's ability to market its goods and services to its clients, it is a very powerful weapon. Under such conditions the employer may submit to the wage demands of the union to avoid further costs. The Nigerian Union of Teachers (NUT) and Academic Staff Union of Universities (ASUU) have used this 'weapon' of strike a great deal in Nigeria (Osabuohien and Ogunrinola, 2007).

A strike can be costly both to union and management. When cost of living is high, prolonged strikes and the stoppage of pay cheques can mount up great pressures on union members to end the strike. This is because the baskets of products they can afford will reduce, which will dampen their total utility. On the part of the firm, the nature of the product, the nature of current demand and the ability of the firm to continue to meet customer demand during the strike all influence the effectiveness of the strike as a weapon. The more costly a work stoppage is, the greater the pressure on the union. Thus, the strike or the threat of it forces both management and the union to bargain painstakingly. The potential cost of a strike to both parties provides an incentive to settle without a work stoppage.

Not all unions are able to influence considerable increase in wage for their workers. Marshall (1920) maintains that the strength of a union to win substantial wage-gains for its workers is inversely related to the magnitude of the coefficient of elasticity of demand for labour in the industry. If labour demand is inelastic, union can win large wage gains without standing the risk of reduction in employment. On the contrary, if demand for labour is elastic, substantial wage gains will be at the expense of jobs.

In most of the free market economies, the problem of collective bargaining and the attendant strike actions it engenders have continued to pose disturbing questions in the minds of public relations operatives, public administrators and the general public. One of the most important questions is in respect of the usefulness of the strike in bringing about mutually acceptable and beneficial settlements to trade disputes. As earlier on stated, a strike can be costly both to management and employees as well as the public. As these actions continue the public becomes more and more irritated and frustrated by the inconveniences and hardships these strikes cause and by increases in prices and taxes which follow the settlement process (Rowan, 1972). Kennedy, (1970) has noted that as the frustration of the public intensifies over strikes, more voices are being raised in favour of some compulsory settlement of labour-management disputes. Freedom to strike comes at a price. But when the price is compared with the costs of compulsory settlement, it does not look nearly too high; free collective bargaining is essential for private enterprise system.

Economic Goals of Unions

Unions generally strive to accomplish some acceptable balance of increased incomes and improved conditions of service for their members. The state of affairs under which each union operates may differ markedly.

Accordingly, the emphasis and priorities of each union may differ as well. More generally, they do influence significantly, the wage-employment decisions of the organisations in which they operate. Whenever unions succeed in achieving higher wages for their members, it is always at the expense of employment. There could be two exceptions to this:

i. If the increased wage directly results in efficiency and productivity gains.

ii. When unions successfully counteract monopoly or monopsony powers.

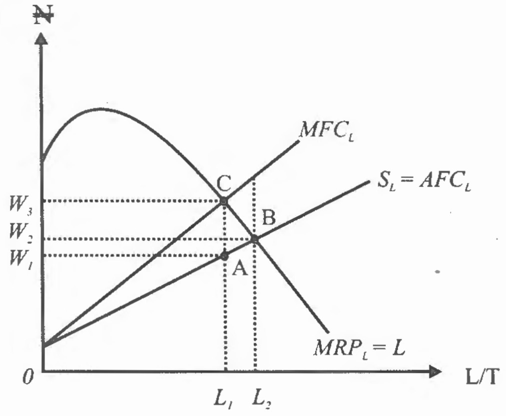

Figure 6: *The Effect of Unions in Counteracting Monopsony Power*

Under the above conditions the union usually accomplishes the two related goals of improved wage rate and employment. This is demonstrated in Figure 6. Let the initial employment equilibrium of the monopolist be OL_1 units of labour and wage rate OW_1. It is often argued that a union may initially have the effect of increasing both a firm's average wage rate and its employment rate, provided it does not insist upon pushing the wage rate up too far. If union wage contract is established at $0W_2$, then the firm will maximise its profits by employing $0L_2$ and paying $0W_2$. If the union is not interested in improved employment, then it could argue for $0W_3$ in wages and maintain employment at $0L_1$. But with the new equilibrium at point B, the union has succeeded in improving both wage rate from $0W_1$ to $0W_2$ and employment from $0L_1$ to $0L_2$.

Euler's Theorem

One way of linking factor demands to output levels is the production function. The production function has been previously defined as the quantitative relationship between the level of output and the inputs required for its production. Two important points in the analysis of production functions in this section are homogeneity and the applicability of Euler's Theorem. Recall that the economic theory of distribution anchors on the marginal productivity theory of resource employment developed by Alfred Marshall and others. According to this theory, factors of production must be paid the value of their marginal products. More explicitly, this theory states that factors would be hired until their contribution to the output of the equalled the cost of hiring one additional unit of that factor. Thus, given a firms production function:

$$q = f(x_i, x_2)$$

Let the price of inputs x_i be given as w_i and the price of final output be P. The marginal productivity theory states that the optimal employment of the *ith* input is achieved when:

$$pMP_i = pf_i = w_i$$

$$where \quad f_i = \frac{\delta f}{\delta x_i} = MPPx_i$$

But a theoretical question trailed the marginal productivity theory. The question was this: given that the firm employs a number of inputs, how could we be sure that the firm was capable of making these payments to all factors? Note that all factors payments must be derived from the output produced by the firm. Would enough output be produced (or perhaps too little) to be able to pay for each unit of each input the value of its marginal product. It was a Swiss mathematician Leonard Euler who developed a theorem to rescue economic theory from this problem in what came to be known as Euler's theorem. Euler explained that given a production function characterised by constant returns to scale, the sum of factor payments identically equals total output in the long run. In mathematical terms, let each factor xi be paid $w_i = pf_i$

Then total payment to all x_i will be $w_i x_i = pf_i x_i$ for all n factor inputs i.e. $(i = 1, 2, ..., n)$, total payment for the n factors is:

$$pf_1x_1 + pf_2x_2 + ... + pf_nx_n = p(f_1x_1 + f_2x_2 + ... + f_nx_n)$$

But with constant return to scale in the production function:

$$f_1x_1 + f_2x_2 + ... + f_nx_n = q = f(x_i).$$

Consequently we would have:

$$w_1x_1 + w_2x_2 + ... + w_nx_n \equiv p(f_ix_i + f_2x_2 + ... + f_nx_n) = pq$$

This implies that total cost (TC) equals total revenue (TR) and the product of the firm is exhausted in making payments to all factors.

Homogeneity and Constant Return to Scale

The feature of constant return to scale is a special case of a production function that is linearly homogenous. Putting this mathematically we say that a production function $q = f(x_1, x_2 ..., x_n)$ exhibits constant returns to scale if:

$$f(\lambda x_1, \lambda x_2, ..., \lambda x_n) \equiv \lambda f(x_i, x_2 ..., x_n) \qquad (17)$$

Equation (17) states that if all inputs are increased by a certain constant (λ), then output will rise by the same proportion. For example, if all inputs are doubled, then output will double. The equation is special case of a more general mathematical notion of homogeneity of functions.
By definition, a function:

$f(x_i, x_2, ..., x_n)$ is considered to be homogenous of degree k *iff*:

$$f(\lambda x_i, \lambda x_2, ..., \lambda x_n) \equiv \lambda^k f(x_i, x_2 ..., x_n) \qquad (18)$$

Equation (18) again states that changing all the decision variables in the equation by the same constant λ will result in a change in the value of the function by an amount λ^k. This identity holds at all points and for all λ, $x_1, x_2 ..., x_n$. We conclude by saying that constant return to scale is a special case of a production function that is homogenous to degree 1 and is said to be linearly homogenous.

Two most important cases of homogeneity are where k=0 and k = 1. When k = 0, doubling every input leaves output unchanged. That is

$$f(\lambda x) = \lambda^0 f(x) = f(x)$$

For linearly homogenous function, doubling input doubles output. i.e.

$$f(\lambda x) = \lambda^k f(x) \qquad \text{Where } k = 1.$$

Euler's Law
Given a differentiable function that is homogenous degree 1, then

$$f(x) = \sum_{i=1}^{n} \delta f(x) \big/ \delta x_i * x_i$$

By definition of linear homogeneity,

$$f(x) = \lambda f(x)$$

Differentiating this identity wrt λ, we have:

$$\sum_{i=1}^{n} \delta f(x) \Big/ \delta \lambda x_i * x_i = f(x)$$

We can set $\lambda = 1$ to obtain this result.

If $f(x_1, x_2, \ldots x_n)$ is homogenous of degree k, then the first partials, $f_1, f_2, \ldots, f_n$ are homogenous of degree $k-1$. i.e.:

$$\frac{\delta f(x)}{\delta x_i} x_i \text{ is homogenous of degree } k-1.$$

Proof

By definition, $f(\lambda x_i, \lambda x_2, \ldots, \lambda x_n) \equiv \lambda^k f(x_1, x_2 \ldots, x_n)$

Since this is an identity, we may differentiate both sides *wrt (xi)*. This yields:

$$\frac{\delta f}{\delta(\lambda x_i)} \frac{\delta(\lambda x_i)}{\delta x_i} \equiv \lambda^k \frac{\delta f}{\delta x_i}$$

But $$\frac{\delta \lambda x_i}{\delta x_i} = \lambda.$$

Thus dividing both sides of the identity by λ yields:

$$\frac{\delta f}{\delta(\lambda x_i)} \equiv \lambda^{k-1} \frac{\delta f}{\delta x_i}$$

This identity states that the function f_i evaluated at

$$(\lambda x_1, \lambda x_{2\ldots} \lambda x_n) = \lambda^{k-1} f_1(x_1, x_2, \ldots, x_n).$$

Thus the function f_i is homogenous of degree $k - 1$.

Summary and Conclusion

The concept of Euler's Theorem as discussed in this chapter set the central building block of the marginal productivity theory of distribution. The concept was applied logically to analyse the principles that underlie the optimising decisions of firms with respect to resource inputs and to explain the functioning of the markets for factors of production. In achieving this, 4 market cases were considered. They include a situation in which a firm:

- is experiencing perfect competition in both factor and product markets;
- experiences perfect competition in the resource market and imperfect competition in the product market;
- operates under imperfect competition in both resource and output markets; and
- is a monopsonist facing perfect competition in the output market.

In the four market models above, the optimising decisions of the firms are based on one principle – the marginal productivity theory. According to this theory the profit maximising condition is that a firm must adjust its factor employment to the level where the marginal revenue received from the last unit of factor employed equals the marginal cost associated with the employment of the factor. Thus, under this principle the firm's optimising position is attained wherever *MRP* is equal to *MFC*, regardless of the type of market circumstances.

More so, the marginal productivity theory has been found to be generally applicable to all types of market circumstances. But in real life, numerous types of imperfections and barriers could come into play to undermine the perfect application of the ideals of the marginal productivity theory. Such, situations include collective bargaining, the institution of minimum wage legislation by the government and so on. In sum, these conditions undermine the attainment of market determined equilibrium outcomes that may deviate significantly from the ideal condition of the Euler's marginal productivity theory of distribution.

REFERENCES

Ekanem,O.T and Iyoha, M.A (2000) *Microeconomic Theory,* Mareh Publishers, Benin City: Chapt. 13.

Eugene Silberberg and Wing Suen (2001): *The Structure of Economics: A Mathematical Analysis,* Irving McGraw-Hill Chapt.3

Gwartney James, D and Stroup, Richard (1980) *Microeconomics: Private and Public Choice*, New York: Academic Press.

Henderson, J.M. and Quandt, Richard, E. (1971) *Microeconomic Theory, A Mathematical Approach 3ed*, London: McGraw-Hill. Chapt. 8

Osabuohien, E.S.C and Ogunrinola, I.O (2007) "Causes and Effects of Industrial Crises in Nigeria: Some Empirical Clarifications", *Labour Law Review*, Vol. 1, No.4, Pp. 75-93.

Rowan, R.L (ed) (1976) Readings *in Labour Economics and Labour Relations* 3e*d* Homewood Illinois: Irwing Inc, Pp. 277-343.

Thompson, A A. (1973) *Economics of the Firm: Theory and Practice*, New Jersey: Prentice-Hall Inc. Chapt 14.