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Portfolio Selection Problem Using Generalized Differential Evolution 3

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Abstract

This Portfolio selection Problem (PSP) remains an intractable research problem in finance and economics and often regarded as NP-hard problem in optimization and computational intelligence. This paper solved the extended Markowitz mean-variance portfolio selection model with an efficient Metaheuristics method of Generalized Differential Evolution 3 (GDE3). The extended Markowitz mean-variance portfolio selection model consists of four constraints: bounds on holdings, cardinality, minimum transaction lots, and expert opinion. There is no research in literature that had ever engaged the set of four constraints with GDE3 to solve PSP. This paper is the first to conduct the study in this direction. The first three sets of constraints have been presented in other researches in literatures. This paper introduced expert opinion constraint to existing portfolio selection models and solved with GDE3. The computational results obtained in this research study show improved performance when compared with other Metaheuristics methods of Genetic algorithm (GA), Simulated Annealing (SA), Tabu Search (TS) and Particle Swarm Optimization (PSO).

Keywords: portfolio selection, generalized differential evolution 3; expert opinion; Metaheuristics method

1. Introduction

The ability of financial practitioners, individuals and corporate investors to select appropriate assets to build a portfolio in order to minimize risk and maximize expected returns remained a difficult task to perform over the years. This has drawn the interest of many researchers especially in the domain of finance and economics to propound a solution [1, 2, 3]. Many models have been formulated to tackle this problem with several variable definitions, objective functions, constraint sets, benchmarks and heuristic techniques [4]. The work of the Markowitz portfolio selection model remains a foundational framework which other researchers had built upon [5]. Over the years, the Markowitz model has been extended with the introduction of one constraint or the other to make the model realistic in a real-life scenario. Among the few works in literature that introduced one constraint or the other to Markowitz mean-variance portfolio selection model are as follows: In the research study of [6, 7] they used cardinality and bounding constraints with efficient metaheuristics method. Minimum transaction lots constraint was engaged by [8] while [9] used probability and upper and lower constraints and [10] used a set of three constraints namely, minimum transaction lot, cardinality and sector capitalization. The work of [3] used four sets of constraints, bounds on holding, cardinality, minimum transaction lots and sector capitalization in the extending Markowitz mean-variance model. Others research works that has introduced varieties of practicable constraints including cardinality constraint to the Markowitz portfolio model are [11, 12, 13, 14, 15, 16, 17] to mention a few.

Sequel to additional constraints being added to the portfolio selection model realistically to satisfy a typical real-life situation increases the complexity of the problem. Thus, there are increasing attempts to develop efficient heuristics that will find an optimum solution within minimal computational time. Many Metaheuristics methods have been developed to provide solutions to the extended Markotwitz mean-variance portfolio selection model in particularly Genetic Algorithm (GA) have been explored widely as reported in the literatures. Among them are the works of [8, 9, 10, 12, 15, 16]. However, the work of [10] demonstrated that the results obtained by GA outperform some other methods of heuristics such as Simulated Annealing (SA) and Tabu Search (TS). However, in recent times, swarm intelligence (SI) has proven to be an alternative promising approach to solve PSP model. The work of [18] was the first to proposed and used particle swarm optimization (PSO) to provide solutions to the standard Markowitz model. The results obtained with PSO in [2, 3] showed improved performance when compared with GA. Other works that used PSO for PSP are [19, 20]. Few related works that engaged GDE for portfolio selection problem are as follows [21, 22,].

This paper contrast significantly from other researches in literatures being that it is the first ever to use the set of constraints of bounds on holdings, cardinality, minimum transaction lots, and expert opinion with efficient Metaheuristics method GDE3 to find a solution to the extended Markowitz portfolio selection problem. The computational results obtained in this research study are compared with existing studies.

The rest of the paper is organized as follows. Section 2 presents the portfolio selection problem and the new model evolved. The methodology used to address the research problem is explained in section 3. Selection 4 contained the computational results obtained in this work and the paper concluded in section 5.

2. Portfolio Selection Problem

The extended Markowitz model as formulated in the work of [2] upon which our proposed model was built on is as follows:

$$\min \quad \sigma_{Rp}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j COV(\bar{R}_i, \bar{R}_j) \tag{1}$$

where

$$w_i = \frac{x_i c_i z_i}{\sum_{j=1}^{N} x_j c_j z_j}, i = 1,\dots,N \tag{2}$$

where

$$\sum_{i=1}^{N} z_i = M \le N; \quad M, N \in \mathrm{N}, \forall_i = 1,\dots,N, z_i \varepsilon \{0,1\} \tag{3}$$

subject to

$$\sum_{i=1}^{N} x_i c_i z_i \bar{R}_i \ge BR \tag{4}$$

$$\sum_{i=1}^{N} x_i c_i z_i \le B \tag{5}$$

$$0 \le B_{low_i} \le x_i c_i \le B_{up_i} \le B, i = 1,\dots,N \tag{6}$$

$$\sum_{i_s} W_{i_s} \ge \sum_{i_s} W_{i_{s^1}} \; ; \tag{7}$$

$$\forall y_s, y_{s^1} \ne 0, s, s^1 \varepsilon (1,\dots,S), s \prec s^1 \tag{8}$$

where

$$y_s = \begin{cases} 1 & if \quad \sum z_i < 0 \\ 0 & if \quad \sum z_i = 0 \end{cases} \tag{9}$$

$$i_s, i_{s^1} \varepsilon (1,...,N)$$

where

N is the number of available assets;

M is the number of assets to be selected from N available assets

$\bar{R}_i$ is the mean return of asset i

$\bar{R}_j$ is the mean return of asset j

$COV(\bar{R}_i, \bar{R}_j)$ is the covariance of returns of asset i and j;

R is the investor's expected rate of return and

B_{low_i} is the minimum amount of budget that can be invested in asset i

B_{up_i} is the maximum amount of budget that can be invested in asset i

c_i is the minimum transaction lots for asset i

x_i is the number of c_i 's that is purchased

z_i is a binary variable $\{0,1\}$ if 1 asset i is in the portfolio and otherwise 0

σ^2_{Rp} is the return variance of the portfolio.

w_i is the decision variable that represents the weight of the budget to be invested in asset i.

w_j is the decision variable that represents the weight of budget to be invested in asset j;

s sector in which asset i belong to;

y_s is equal to 1 if sector s has at least one selected asset, and 0 otherwise.

i_s is the set of asset indices which belong to sector s

The following constraints such as bounds on holdings, cardinality, and minimum transaction lots and sector capitalization are particularly important in making significant investment decision in real-life financial market. The bounds on holding constraint, ensures that the amount invested in each asset lie between predetermined upper and lower bounds. The carnality constraint ensures that the total number of assets selected in the portfolio is equal to the predefined number, the minimum transaction lots constraint requires that each asset can only be purchased in batch with a given number of units while sector capitalization constraint ensure that asset with highest sector capitalization should be selected in the portfolio. The four aforementioned constraints have been well researched in portfolio selection problem [3, 10, 16, 24].

In order to make the model realistic and attaining the goal set in reducing investment risk, an important constraint known as expert opinion is added. The importance of expert opinion in portfolio selection cannot be over-emphasized due to fact that the expert is well informed and can do a thorough analysis of each security before selection of an asset to be part of the portfolio. There are other factors known to the expert beyond sector capitalization that can enhance selection of an asset. This research differs significantly from the previous studies on the portfolio selection problem by the introduction of new four set of constraints which are bounds on holdings, cardinality, and minimum transaction lots and expert opinion to the portfolio selection problem.

2.1 Proposed Model

This section describes the proposed model. The proposed model is an extension of Markowitz's mean variance portfolio selection model in the work of [2]. The Markowitz's model lack real market situation scenario. To explain the proposed model the definition of following variables are of importance. Therefore:

M is the number of assets to be selected from N available assets
B is the total available budget
R is the investor's expected rate of return
σ_p^2 is the return variance of the portfolio.
σ_{ij} is the covariance of returns of asset i and j;
B_{lower_i} is the minimum amount of budget that can be invested in asset i
B_{upper_i} is the maximum amount of budget that can be invested in asset i
c_i is the minimum transaction lots for asset i
x_i is the number of c_i's that is purchased
w_i is the decision variable that represents the weight of the budget to be invested in asset i.
w_j is the decision variable that represents the weight of budget to be invested in asset j;
z_i is a binary variable $\{0,1\}$ if 1 asset i is in the portfolio and otherwise 0
e_i is the expert opinion, a random variable of equal or greater than 0.5 if the asset i is selected and otherwise 0
i is the index of securities

Investors always desire to minimize risk of investment and maximize possible return. The extended Markowitz model for the portfolio selection problem proposed in this paper is, thus, formulated as follows:

$$\min \quad \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{10}$$

where

$$w_j = \frac{x_i c_i z_i}{\sum_{i=1}^{N} x_i c_i z_i}, \; j = 1,\ldots,N \tag{11}$$

and

$$\sum_{i=1}^{N} z_i = M \leq N; \quad M, N \in \mathrm{N} \tag{12}$$

subject to

$$\sum_{i=1}^{N} x_i c_i z_i e_i r_i \geq BR \tag{13}$$

$$\sum_{i=1}^{N} x_i c_i z_i e_i \leq B \tag{14}$$

$$0 \leq B_{lower_i} \leq x_i c_i \leq B_{upper_i} \leq B, i = 1,\ldots,N \tag{15}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{16}$$

$$w_i \geq 0, \quad \forall_i \in \{1,2,\ldots,N\} \tag{17}$$

$$e_i \in \{0,1\} \tag{18}$$

where

$$z_i = \begin{cases} 1 & if \quad e_i \geq 0.5 \\ 0 & if \quad e_i < 0.5 \end{cases} \tag{19}$$

$x_i c_1$ represents the number of units of asset i in the selected portfolio. z_i is the decision variable in which it is equal to 1 if the asset i is upheld in the portfolio and otherwise 0. The inequality in equation (12) denotes cardinality constraint while the inequality in equation (13) is the same as equation (4). Equation (14) represents the budget constraint. Equation (15) indicates the bounds on holdings constraint. The equations (16) and (17) ensure that the total budgets are invested in the portfolio. The equations (18) and (19) represent the expert opinion constraint. The expert opinion constraint is a practicable and useful constraint in a real life scenario of portfolio selection because the expert has detailed information about sector capitalization where each asset i to be selected in the portfolio belong in order to minimize investment risk. Beyond sector capitalization the expert or financial analyst can access other information regarding each asset i to be selected in the portfolio such as price/annual earning, management calibre, dividend rate, book value and so on. An in-depth analysis of these information can guide the expert upon which an opinion is formed whether asset i should be included in the portfolio or not. This paper is the first to introduce these set of important constraints in the portfolio selection problem.

This extended model requires efficient Metaheuristics to find the solution because it is classified as a quadratic mixed integer programming model. In the next section which contained the methodology used in this work, Generalized Differential Evolution 3 (GDE3) is reviewed and used to solve the proposed extended Markowitz model as formulated above.

3. Methodology

This section describes briefly the concept expert opinion and Metaheuristics used in this work in particular the Generalized Differential Evolution 3 (GDE3). The data used and experimental details are also discussed.

3.1 Expert Opinion

An expert is defined by [25] as "professional who have acquired knowledge and skill through study and practice over the years in a particular field or subject, to the extent that is his/her opinion may be helpful in fact finding, problem solving, or understanding of a situation". Similarly, [26] defined skilled expert as individual who have acquired extensive knowledge and experience that affects how they perceive systems and how they are able to organize and interpret information. The work of [27] advocated that "it is very important for experts or decision makers to use their experience or knowledge to predict the performance of each stock that a make stock portfolio". They proposed that linguistic variables are suitable to express expert opinions for the performance evaluation of each stock to be selected. Since expert opinions are considered vital in solving problem, perceive systems and situations and interpreting information. There is no doubt of it potential to enhance selection of assets to make a portfolio. Other works in literature has used expert judgement in portfolio selection as follows [28, 29, 30, 31].

3.2 Generalized Differential Evolution 3

Several extensions of differential evolution [32] exist for solving constrained and non-constrained multi-objective optimization problem [33, 34, 35]. In comparison to the extension of differential evolution, GDE3 makes differential evolution a suitable algorithm for multi-objective optimization as well as constrained optimization with little changes to the basic differential evolution algorithm. GDE3 extends DE/rand/1/bin strategy which exhibit slow convergence rate and strong exploration properties. GDE3 is the third version of generalized differential evolution modifying the selection process of the basic differential evolution algorithm [36]. The selection process in GDE3 is guided by these three rules:

Rule 1: Feasible vector is selected in a situation where both feasible and infeasible vectors are generated.

Rule 2: In a scenario where both the old vector and trial vector are infeasible, the old vector is selected if it dominates the trial vector, but if the trial vector weakly dominates the old vector, then the trial vector is selected.

Rule 3: In a scenario where both the old vector and trial vector are feasible, the old vector is selected if it dominates the trial vector, but if the trial vector weakly dominates the old vector, then the trial vector is selected.

GDE3 performs the sorting of the vector by calculating the crowding distance of the vector. The selection process based on crowding distance gives GDE3 an advantage over NSGAII. In the case of comparing feasible, incomparable and non-dominating solutions, both offspring and parent vectors are saved for the population of the next generation [37]. As a result, this procedure reduces the computational costs of the Metaheuristics and improves its efficiency. Readers interested in GDE3 should refer to the texts by [38, 39, 40].

3.3 Data Used and Experimental Setting

The proposed extended Markowitz model developed in this work was implemented with efficient Metaheuristics method of GDE3 with each set of data of 31 and 85 stocks from the stock markets of Hong Kong Hang Seng and the German DAX 100 respectively. The data were obtained from test data from OR-Library [41]. Each data set contains the number of assets (N). The mean return and standard deviation of return for each asset i and correlation between asset i and j for all possible pairs of assets. In order to evaluate the performance of the algorithm on the proposed portfolio model. It was run on a PC with Intel Pentium 4.3 GHz with 2GB RAM. The parameter settings for each of the data set is as follows: expert opinion was set to greater than 0.5 if the asset is selected in the portfolio, the value of the budget was set to 2800, expected rate of returns was set to 0.004, 0.005, 0.006, 0.07, 0.08 and 0.009 respectively. A predetermined upper and lower bound was set for each of the selected assets. The size of portfolio was set to 15, 20, 25 for each of the data set

Four criteria were used to compare the performance of the results obtained by the GDE3 algorithm used for the proposed portfolio model. The criteria are as follows:

- Mean variance; the average of the objective function found by the algorithm.
- Worst variance, depicts the highest risk from algorithm runs, showing the worst solution.
- Standard deviation of variance, depicts how close the solution found by the algorithms are close to each other and,
- Mean execution time, depicts the amount of time needed to arrive to a solution.

4. Computational Result and Discussion

The results of GDE3 algorithms for data set of 31 stocks are tabulated in table 1. Similarly, the results obtained for data set of 85 stocks with GDE3 are contained in table 2 accordingly. The table 3 consists of the results reported in the work of [42] and the results obtained with the proposed portfolio model implemented with GDE3.

Table 3 are the results of other Metaheuristics methods namely genetic algorithm (GA), Simulated Annealing (SA), Tabu Search (TS) and Particle Swarm Optimization (PSO) used to compare with the proposed portfolio model solved with efficient heuristics of GDE3 developed in this work. The results of the GA, SA, TS, and PSO are from [42]. From the results obtained in table 3. When the size of asset is 31. The proposed model shows improved performance over the other heuristics commonly used in literatures.

Similarly, to further evaluate the performance of the improved extended portfolio model in a complex scenario of larger dataset of 85 stocks. Table 2 shows the results obtained with 85 stock data set and comparison with other heuristics also in table 3. The performance of efficient Metaheuristics of GDE3 to the portfolio model shows superior performance over other Metaheuristics with less computation time.

Table 1: Results of GDE3 algorithm of Hang Seng 31 stocks data set across 50 independent executions

Size of portfolio		Expected rate of return	0.007	0.008	0.009	0.010	Average
15	Variance	Mean	0.563994883	0.583538754	0.569434822	0.614140926	0.582777
		Worst	0.965689627	1.098968215	0.967734479	0.957364336	0.997439
		Std. Dev.	0.14488222	0.201706052	0.152108253	0.154142507	0.16321
		Mean exe. time (s)	26.0242	23.40956	21.87144	22.30456	24.15244
20		Mean	0.803687702	0.780372357	0.820243883	0.782411362	0.796679
		Worst	1.14091346	1.149916668	1.276348554	1.248753492	1.203983
		Std. Dev.	0.167143278	0.175205807	0.213475292	0.204415099	0.19006
		Mean exe. time (s)	21.90408	20.91218	20.94464	21.10034	21.21531
25		Mean	0.87681794	0.871821765	0.922272678	0.861066813	0.882995
		Worst	1.33100497	1.350028382	1.587300034	1.261058188	1.382348
		Std. Dev.	0.22507781	0.178424065	0.221563296	0.200820474	0.206471
		Mean exe. time (s)	23.0121	21.04312	23.68118	23.25478	22.7478

Table 2: Results of GDE3 algorithm of Dax100 of 85 stocks data set across 50 independent executions

Size of portfolio		Expected rate of return	0.007	0.008	0.009	0.010	Average
15	ria nc	Mean	1.153426633	1.092303731	1.201436643	1.154792333	1.15049
		Worst	2.182757287	1.957339709	1.911125224	1.892663134	1.985971

Table 2: (Continued): Results of GDE3 algorithm of Dax100 of 85 stocks data set across 50 independent executions

		Std. Dev.	0.316993858	0.333457876	0.313750357	0.310681004	0.318721
		Mean exe. time (s)	26.8754	36.4358	28.81348	27.98884	30.02838
20		Mean	1.557621718	1.538650809	1.574681603	1.52479037	1.548936
		Worst	2.45157812	2.555971016	2.164951505	2.235240523	2.351935
		Std. Dev.	0.331674621	0.406520562	0.308754781	0.333222188	0.345043
		Mean exe. time (s)	27.35952	25.77718	28.3102	26.96552	27.10311
25		Mean	1.712905247	1.66820084	1.852586721	1.756736944	1.747607
		Worst	2.602102722	2.686838775	2.649699903	2.50261784	2.610315
		Std. Dev.	0.43247937	0.431014481	0.396969188	0.391768473	0.413058
		Mean exe. time (s)	31.77896	30.72344	28.0944	26.87462	29.36786

Table 3: Comparison of the proposed GDE3 Portfolio Model with other Heuristics

Stock Data	Asset (N)	GA	SA	TS	PSO	**Proposed Model with GDE3**
Hang Seng	31	1.0974	1.0957	1.1217	1.0953	**0.75415**
Dax100	85	2.5424	2.9297	3.3049	2.5417	**1.482344**

5. Conclusion

The model developed in this paper is an improvement of the extended Markowitz portfolio model. This new proposed extended Markowitz portfolio model consists of four constraints namely: bounds on holdings, cardinality, minimum transaction lots, and expert opinion. The proposed extended portfolio model developed was implemented with efficient Metaheuristics of GDE3 algorithm. There is no study in the literature that has ever used these sets of constraints and solved with GDE3. The results obtained were compared with GA, SA, TS, and PSO. The performance of the new portfolio model shows improved performance. Further studies are to engage comparative study of other swarm intelligence techniques to the new extended portfolio model developed in this paper.

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