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ANALYSIS OF TORSIONAL RIGIDITY OF CIRCULAR BEAMS WITH DIFFERENT ENGINEERING MATERIALS SUBJECTED TO ST. VENANT TORSION

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ABSTRACT

Many engineering structures, such as airplane wings, beams and shafts are subjected to higher torsional forces today due to advancement in Structural Engineering, in terms of size and technology. In this paper, we analyzed the resistance of circular beams, of different engineering materials, to their corresponding twisting moments. We obtained the torsional rigidity for the different beams as the ratio of twisting moment to the angle of twist per unit length. It is observed that torsional rigidity of the beams is a function of their areas and the engineering material they are made up of. Specifically it is observed that the circular beam made up of brass engineering material has the greatest torsional rigidity among the twelve engineering materials considered.

KEYWORDS: Beams, Torsional Rigidity, Twisting Moment, St. Venant Torsion, Brass

INTRODUCTION

When a beam is transversely loaded in such a manner that the resultant force passes through the longitudinal shear central axis, the beam only bends and no torsion will occur. When the resultant force acts away from the shear central axis, then the beam will not only bend but also twist. [1, 2, 4]

Torsion is twisting about an axis produced by the action of two opposing couples acting in parallel planes [5]. Another name for couples is torque or twisting moment. Torsional rigidity of a beam is a ratio of moment to the angle of twist per unit length [6]. When torsion is applied to a structural member, its cross-section may warp in addition to twisting. If the member is allowed to warp freely, then the applied torque is resisted entirely by torsional shear stresses (called St. Venant's torsional shear stress). If the member is not allowed to warp freely, the applied torque is resisted by St. Venant's torsional shear stress and warping tension. This behavior is called non-uniform torsion [1, 2, 3, 4].

Beams of non-circular section tends to behave non-symmetrically when under torque and plane sections do not plane. Also the distribution of stress in a section is not necessarily linear[12].

St. Venant's theory is usually applied when the cross-section is non-deformable out of its plain or those deformations are very small [10].

Consider a circular beam with length l, with one of its bases fixed in the $xy-$plane, while the other base (in the plane $z=l$) is acted upon by a couple whose moment lies along $x-$ axis. The beam twists through an angle determined by the magnitude of applied couple and the modulus of rigidity of the beam. The amount of twist produced can thus be used to determine the applied force [5].

Saint-Venant (1885) was the first to produce the correct solution to the problem of torsion of bars subjected to moment couples at the ends

In material science, shear modulus or modulus of rigidity, denoted by μ or G, is defined as the ratio of shear stress to the shear strain [6].

FORMULATION OF THE PROBLEM

For a beam of constant circular cross-section subjected to torsion, the St. Venant's torsion is given by [1, 2, 3, and 4]

$$T_{SV} = I_r m \frac{df}{dz} \tag{1}$$

Where,

ϕ is the angle of twist (twist angle),

μ isthemodulus of rigidity,

T_{SV} is St. Venanttorsion,

I_ρ is the polar moment of inertia,

z isthedirectionalong axis of themember.

$\frac{df}{dz}$ = Twist rate

By symmetry, anysection of the beam perpendicular to $z-$axis remains perpendicular to this axis during deformation and the action of the couple will merelyrota teeach section through some angle ϕ, called the angle of twist [5]. Theamount of rotation wil lclearlydependonthedistance of the section from the base $z=0$, and since the deformations are small, the amount of rotation ϕ is proportional to the distance of the section from the fixed base.

Theangle of twist can then be written as

$$f = az , \tag{2}$$

Where a isthe twist per unitlength.

It is therelative angular displacement of a pair of crosssectionsthat are unitdistanceapart. Let w be the displacementalong z - axis.

$w = 0$, if the crosssection of the beam remainplaneafter deformation. Since w is independent of z, we write

$$w = a\Psi(x,y) \tag{3}$$

Where Ψ isthetorsión function

Consider a particle originalty at (x, y, z), since $f = az$, for the displacement of this particle

$u = -azy$, $v = azx$, $w = 0$.

The angle of twist f can also be written as [5]

$$f = \frac{Tl}{I_r m} \quad (4)$$

Where T is the applied torque and l is the length of the beam.

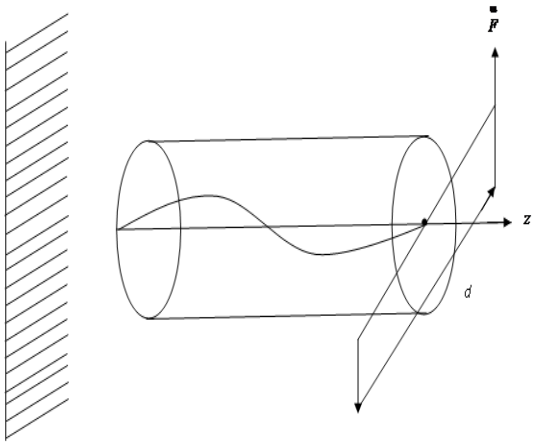

Figure 1: Twisting of Circular Section

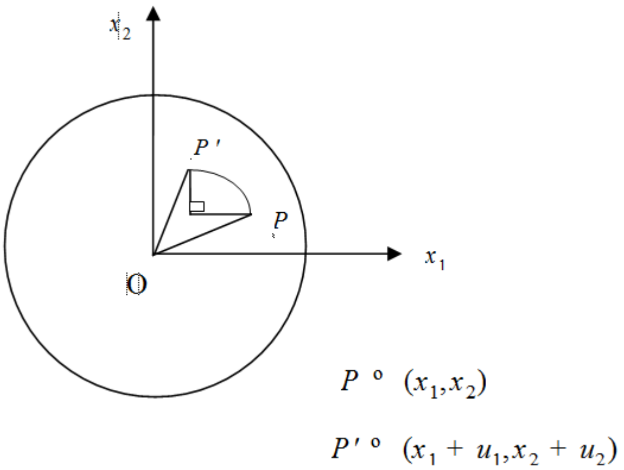

Figure 2: Showing the Displacement Along x_1 and x_2

Assumptions

- The bar is straight and of uniform cross section.
- The material of the bar has uniform properties.
- The only loading is the applied torque which is applied normal to the axis of the bar.
- The bar is stressed within its elastic limit.

The Eulerian strain is given in terms of displacement u , v , w by [5]

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{5}$$

Where $u_{i,j} = \frac{\partial u_i}{\partial x_j}$, $i = 1,2,3;\ j = 1,2,3$

$u_1 = u,\ u_2 = v$ and $u_3 = w$

$x_1 = x,\ x_2 = y$ and $x_3 = z$

$\Rightarrow\ u_1 = u = -azy = -ax_2x_3 = -f x_2$

$u_2 = v = -azx = ax_1x_3 = f x_1$

The representative strains, in matrix form, are as follows [5]

$$e_{ij} = \begin{pmatrix} 0 & 0 & \frac{f}{2}(-x_2 + \psi_{,1}) \\ 0 & 0 & \frac{f}{2}(x_1 + \psi_{,2}) \\ \frac{f}{2}(-x_1 + \psi_{,1}) & \frac{f}{2}(x_1 + \psi_{,2}) & 0 \end{pmatrix} \tag{6}$$

Where $\psi_{,1} = \frac{\partial \psi}{\partial x_1}$, $\psi_{,2} = \frac{\partial \psi}{\partial x_2}$, $\psi_{,11} = \frac{\partial \psi_{,1}}{\partial x_1}$, $\psi_{,22} = \frac{\partial \psi_{,2}}{\partial x_2}$

The stress-strain relation is given as [5]

$$d_{ij} = \frac{E}{1+u}\left(e_{ij} + \frac{u}{1-2u}e_{kk}d_{ij}\right) \tag{7}$$

Where d_{ij} is the stress tensor, E is the Young's modulus and u is the Poisson's ratio.

Sub stituting the respective strains in the stress-strain relation, we get

$$d_{ij} = \begin{pmatrix} 0 & 0 & xf(\psi_{,1} - x_2) \\ 0 & 0 & xf(\psi_{,2} + x_1) \\ xa(\psi_{,1} - x_1) & xf(\psi_{,2} + x_1) & 0 \end{pmatrix} \tag{8}$$

Where x is a lame constant, given as

$$x = \frac{E}{2(1 + u)} \tag{9}$$

Recall that $\tilde{\phi}$ is a function of x_1 and x_2 .

$\therefore$ for $i = 1$, $\tilde{\phi}_{,11} = 0$

for $i = 2$, $\tilde{\phi}_{,22} = 0$

$\Rightarrow \tilde{\phi}_{,11} + \tilde{\phi}_{,22} = 0$ (10)

This is the Laplace equation. Hence, $\tilde{\phi}$ is a harmonicfunction.

Since $\tilde{\phi}$ is a harmonicfunction in thesimplyconnected región R, representingthecross-section of thebeam, there exist sananalyticfunction $\tilde{\phi} + iy$ of the complex variable $x + iy$ where $y(x,y)$ is a harmonic conjúgate of $\tilde{\phi}$.

ThefunctionssatisfytheCauchy-Riemann equations, namely

$$\frac{\partial^2 \tilde{\phi}}{\partial x_1} = \frac{\partial^2 y}{\partial x_2} \quad \text{(i.e., } \tilde{\phi}_{,1} = y_{,2}\text{)} \tag{11}$$

And $$\frac{\partial^2 \tilde{\phi}}{\partial x_2} = -\frac{\partial^2 y}{\partial x_1} \quad \text{(i.e., } \tilde{\phi}_{,2} = -y_{,1}\text{)} \tag{12}$$

The harmonic conjugate y of $\tilde{\phi}$ isgiven as

$$\left.\begin{array}{l} y = \frac{1}{2}(x_1^2 + x_2^2) \text{ on C} \\ \text{and} \\ \nabla^2 y = 0 \text{ in R} \end{array}\right\} \tag{13}$$

This is a Dirichlet problem.

ANALYSIS

The torsional rigidity of a beam is defined as a ratio of moment to the angle of twist per unit length [5]. The torsional rigidity of a beam with circular cross-section is given as [3, 5]

$$D = \frac{M_3}{f} \tag{14}$$

Where M_3 is the resulting momento on the surface $x_3 = l$ and is given as [8]:

$$M_3 = \iint_R (x_1 d_{32} - x_2 d_{31})\, dA \tag{15}$$

where d_{32} and d_{31} are stress tensorswith $d_{31} = xf\left(\frac{\partial y}{\partial x_2} - x_2\right)$ and $d_{32} = xf\left(\frac{\partial y}{\partial x_1} + x_1\right)$.

On the surface $x_3 = l$, we have $x_j = (x_1, x_2, 0)$, hence $x_3 = 0$. Therefore, $M_1 = 0$ and $M_2 = 0$

$$\Rightarrow \quad M_3 = M = \iint_R (x_1 d_{32} - x_2 d_{31})\, dA \tag{16}$$

Letusconsidertheharmonicfunction [8]

$$y = c^2(x_1^2 - x_2^2) + k^2 \tag{17}$$

where c, k are constants.

$\Rightarrow \quad c^2(x_1^2 - x_2^2) + k^2 = \frac{1}{2}(x_1^2 + x_2^2)$ ontheboundary, or

$$\left(\frac{1}{2} - c^2\right)x_1^2 + \left(\frac{1}{2} + c^2\right)x_2^2 = k^2 \tag{18}$$

The curve defined by this equation is an ellipse

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1 \tag{19}$$

If we choose $c^2 < \frac{1}{2}$ and $a = \frac{k}{\sqrt{\frac{1}{2} - c^2}}$, $b = \frac{k}{\sqrt{\frac{1}{2} + c^2}}$, then

$$c^2 = \frac{1}{2}\frac{a^2 - b^2}{a^2 + b^2}, \quad k^2 = \frac{a^2 b^2}{a^2 + b^2}$$

$$\Rightarrow \quad \frac{1}{2}\frac{a^2 - b^2}{a^2 + b^2}(x_1^2 - x_2^2) + \frac{a^2 b^2}{a^2 + b^2}$$

So, d_{31} becomes $\frac{-2xf\, a^2 x_2}{a^2 + b^2}$

Similarly, δ_{32} becomes $\dfrac{2\xi\varphi\, b^2 x_1}{a^2 + b^2}$

From equation (16); the torsional momen to becomes

$$M = \frac{2\xi\varphi}{a^2 + b^2}\left(b^2 \iint_R x_1^2\, dA + a^2 \iint_R x_2^2\, dA \right) \tag{20}$$

$$= \frac{2\xi\varphi}{a^2 + b^2}\left(a^2 I_{x_1} + b^2 I_{x_2}\right) \tag{21}$$

where I_{x_1} and I_{x_2} are the moments of inertia of the elliptical section about the $x_1 -$ and $x_2 -$ axes.

But $I_{x_1} = \dfrac{\pi a b^2}{4}$ and $I_{x_2} = \dfrac{\pi a^3 b}{4}$, so we have

$$M = \frac{\pi\xi\varphi\, a^3 b^3}{a^2 + b^2} \tag{22}$$

For a beam with circular cross-section;

$$a = b = radius\,(r)$$

$$\therefore\ M = \frac{\pi\xi\varphi\, a^3 a^3}{a^2 + a^2} = \frac{\pi\xi\varphi\, a^4}{2} = \frac{\pi\xi\varphi\, r^4}{2}. \tag{23}$$

The torsional rigidity of the circular beam can be written as

$$D = \frac{M}{\varphi} = \frac{\pi\xi r^4}{2} \tag{24}$$

but $A = \pi r^2$

$$\Rightarrow\ \frac{A}{\pi} = r^2 \tag{25}$$

$$\frac{A^2}{\pi^2} = r^4 \tag{26}$$

Substiruting equation (26) into (24), wehave

$$D = \frac{\pi\xi A^2}{2\pi^2} \tag{27}$$

$$D = \frac{xA^2}{2p} \text{ (Torsional rigidity)} \quad (28)$$

$$Þ \frac{2pD}{A^2} = x = \frac{E}{2(1 + u)} \quad (29)$$

$$\backslash \ D = \frac{A^2E}{4p(1 - v)} \quad (30)$$

For a circular beam; the moment of inertia about the $x -$ axis is given as

$$I_{x_1} = \frac{pr^3}{4} \quad (31)$$

Also, the moment of inertia about the $y -$ axis isgiven as

$$I_{x_2} = \frac{pr^3}{4} \quad (32)$$

Thus, the polar moment of inertia about the $x_3 -$ axis is given as

$$I_p = \frac{pr^3}{4} + \frac{pr^3}{4} \quad (33)$$

$$I_p = \frac{pr^3}{2} \quad (34)$$

Hence, St. Venanttorsion becomes

$$T_{SV} = \frac{pr^3m}{2}\frac{dq}{dx_3} \quad (35)$$

With the boundary conditions

$$f(x_3) = 0 \text{ at } x_3 = 0, f(x_3) = f \text{ at } x_3 = l;$$

To find the twist angle f , we integratea long the length of thebeam as shownbelow

$$ò \frac{df}{dx_3}dx_3 = ò \frac{T_{SV}}{mI_p}dx_3 \quad (36)$$

If T_{SV} , m and I_p $A = \pi r^2$ constants along the beam.

$$f = \frac{T_{SV}}{mI_p} ò dx_3 \qquad (37)$$

$$\backslash \; f = \frac{T_{SV} x_3}{mI_p} \qquad (38)$$

If length of the beamalong $z -$ axis is l , then

$$f = \frac{T_{SV} l}{mI_p} \qquad (39)$$

From equation (3), the displacement is given as

$$x_3 = aT(x,y) = aT(x_1,x_2) \qquad (40)$$

Displacement along x_1 and x_2 axes are

$$u_1 = - a x_3 x_2 \text{ and } u_2 = a x_3 x_1 \qquad (41)$$

respectively , since displacement along $x_3 -$ iszero.

If the length of the beam along $x_3 -$ axis is l

$$u_1 = - f x_2 \text{ and } u_2 = f x_1 \text{ (since } f = az \text{)}$$

$$u_1 = - \frac{Tz}{mI_p} x_2 , \; u_2 = \frac{Tz}{mI_p} x_1 \qquad (42)$$

If the length of the beam along $x_3 -$ axis is l , then

$$u_1 = - \frac{Tl}{mI_p} x_2 , \; u_1 = \frac{Tl}{mI_p} x_1 \qquad (43)$$

From equation (13), the harmonic conjúgate y , of the torsión function which is a function of x_1 and x_2 , is considered for the different values of x_1 and x_2 and plotted on a graph as shown in Figure 2.

Table 1: The Following Chart Gives Typical Values for the Modulus of Rigidity, Young' S Modulus and Poisson Ratios for Different Engineering Materials [6, 9]

Engineering Materials	Modulus of Rigidity (Psi X 10^6)	Young's Modulus (Psi X 10^6)	Poisson Ratio (u)
Beryllium copper	6.7	17	0.285
Brass	5.8	102 – 125	0.331

Table 1: Contd.,			
Bronze	6.5	96 – 120	0.34
Copper	6.58	17	0.355
Iron (Malleable)	9.4	28.5	0.271
Magnesium	2.39	6.4	0.35
Molybdenum	17.16	40	0.307
Monel	9.57	26	0.315
Nickel silver	5.6	18.5	0.322
Nickel steel	10.8	29	0.291
Titanium	5.94	27	0.32
Zinc	6.1	12	0.331

For this paper, we consider a circular beam, for twelve different engineering materials of diameter 1.2m, angle of twist of 30^o, length of 10m and the Torque (T) as the St. Venant Torsion (T_{SV}). Table 2 shows the calculated values of polar moment (I_p), St. Venant Torsion (T_{SV}) and torsional rigidity (D).

i.e $r = 0.6m, f = 30^o, l = 10m, T = T_{SV}$

Table 2: Calculated Values of Polar Moment I_p, St. Venant Torsion T_{SV} and Torsional Rigidity D

Engineering Materials	m	I_p	T_{SV}	D
Beryllium copper	6700000	0.34	6834000	1.347
Brass	5800000	0.34	5916000	8.683
Bronze	6500000	0.34	6630000	8.207
Copper	6580000	0.34	6711600	1.278
Iron	9400000	0.34	9588000	2.283
Magnesium	2390000	0.34	2437800	0.483
Molybdenum	17160000	0.34	17503200	3.116
Monel	9570000	0.34	9761400	2.013
Nickel silver	5600000	0.34	5712000	1.425
Nickel steels	10800000	0.34	11016000	2.287
Titanium	5940000	0.34	6058800	4.757
Zinc	6100000	0.34	622000	0.918

Displacement along x_1 and x_2 for circular beam of different engineering materials, with $l = 10m$, $I_p = 0.34$

Table 3: Displacement Along x_1 and x_2 for Circular Beam with Different Engineering Materials

x_1	x_2	u_1	u_2
1	1	-30	30
2	2	-60	60
3	3	-90	90
4	4	-120	120
5	4	-150	150
6	6	-180	-180
7	7	-210	210
8	8	-240	240
9	9	-270	270
10	10	-300	300

The Relationship between μ , T_{SV} and D

When two variables x and y are related, they are said to be correlated [11]. In order to define the linear relationship and the amount of linear relationship between μ , T_{SV} and D, we adopt the concept of regression and correlation coefficient. We deduced the following:

$$\mu = 2970474 + 0.688458T_{SV} \qquad (44)$$

and

R = 0.878477 (45)

Similarly, we have

$$D = -4.9\times10^{-9} + 3.104177\mu \qquad (46)$$

And

R = -0.00659 (47)

Where r is the correlation coefficient, ($-1 \le r \le 1$).

Recall that

$D =$ Torsional rigidity,

I_p = Polar moment of inertial about x_3 - axis,

T_{SV} = St. Venant torsion,

$m =$ Modulus of rigidity

The Relationship between the Cross-Sectional Area (A) and Torsional Rigidity (D)

In this section we are interested in finding out if there is any relationship between the cross sectional area of the beams and torsional rigidity of the beams of different engineering materials. As shown in tables (4)-(8), where r is the radius , we considered five cases where the diameter of the cross sectional area is given values 1.2, 2.2, 3.2, 4.2 and 5.2 respectively.

Table 4: Torsional Rigidity of Beams with Different Materials When Cross-Sectionalareais 1.130973m^2

	r	A	E	v	D
BerylliumCopper	0.6	1.130973	17	0.285	1.346606406*10^6
Brass	0.6	1.130973	113.5	0.331	8.679859374*10^6
Bronze	0.6	1.130973	108	0.34	8.203776869*10^6
Copper	0.6	1.130973	17	0.355	1.277040024*10^6
Iron	0.6	1.130973	28.5	0.271	2.282412788*10^6
Magnessium	0.6	1.130973	6.4	0.35	4.825486315*10^5
Molybdenum	0.6	1.130973	40	0.307	3.115152316*10^6
Monel	0.6	1.130973	26	0.315	2.012530533*10^6
Nickel Silver	0.6	1.130973	18.5	0.322	1.426568663*10^6
Nickel Steels	0.6	1.130973	29	0.291	2.286475953*10^6

Table 4: Contd.,

Titanium	0.6	1.130973	27	0.32	2.082019130*10^6
Zinc	0.6	1.130973	12	0.331	9.176943822*10^5

Table 5: Torsional Rigidity of Beams with Different Materials When Cross-Sectionalareais 3.801327m^2

Engineering Material	R	A	E	v	D
BerylliumCopper	1.1	3.801327	17	0.285	1.521270401*10^7
Brass	1.1	3.801327	113.5	0.331	9.805696062*10^7
Bronze	1.1	3.801327	108	0.34	9.267862436*10^7
Copper	1.1	3.801327	17	0.355	1.442680787*10^7
Iron	1.1	3.801327	28.5	0.271	2.578457226*10^7
Magnessium	1.1	3.801327	6.4	0.35	5.451384652*10^6
Molybdenum	1.1	3.801327	40	0.307	3.519208725*10^7
Monel	1.1	3.801327	26	0.315	2.273569408*10^7
Nickel Silver	1.1	3.801327	18.5	0.322	1.609166175*10^7
Nickel Steels	1.1	3.801327	29	0.291	2.583047410*10^7
Titanium	1.1	3.801327	27	0.32	2.352071148*10^7
Zinc	1.1	3.801327	12	0.331	1.036725575*10^7

Table 6: Torsional Rigidity of Beams with Different Materials When Cross-Sectionalareais 8.042477m^2

Engineering Material	R	A	E	v	D
BerylliumCopper	1.6	8.042477	17	0.285	6.809505981*10^7
Brass	1.6	8.042477	113.5	0.331	4.389222715*10^8
Bronze	1.6	8.042477	108	0.34	4.148477786*10^8
Copper	1.6	8.042477	17	0.355	6.457723385*10^7
Iron	1.6	8.042477	28.5	0.271	1.154168245*10^8
Magnessium	1.6	8.042477	6.4	0.35	2.440147154*10^7
Molybdenum	1.6	8.042477	40	0.307	1.575267148*10^8
Monel	1.6	8.042477	26	0.315	1.017694453*10^8
Nickel Silver	1.6	8.042477	18.5	0.322	7.202944776*10^7
Nickel Steels	1.6	8.042477	29	0.291	1.156222903*10^8
Titanium	1.6	8.042477	27	0.32	1.052833377*10^8
Zinc	1.6	8.042477	12	0.331	4.640587891*10^7

Table 7: Torsional Rigidity of Beams with Different Materials When Cross-Sectional Area is 13.85442m^2

Engineering Material	R	A	E	V	D
BerylliumCopper	2.1	13.85442	17	0.285	2.020751239*10^8
Brass	2.1	13.85442	113.5	0.331	1.302521396*10^9
Bronze	2.1	13.85442	108	0.34	1.231079267*10^9
Copper	2.1	13.85442	17	0.355	1.916358186*10^8
Iron	2.1	13.85442	28.5	0.271	3.425045690*10^8
Magnessium	2.1	13.85442	6.4	0.35	7.241245398*10^7
Molybdenum	2.1	13.85442	40	0.307	4.674675445*10^8
Monel	2.1	13.85442	26	0.315	3.020053630*10^8
Nickel Silver	2.1	13.85442	18.5	0.322	2.137505955*10^8
Nickel Steels	2.1	13.85442	29	0.291	3.431142979*10^8
Titanium	2.1	13.85442	27	0.32	3.124329957*10^8
Zinc	2.1	13.85442	12	0.331	1.377115133*10^8

Table 8: Torsional Rigidity of Beams with Different Materials When Cross-Sectional Area is 21.23717m^2

Engineering Material	R	A	E	V	D
BerylliumCopper	2.6	21.23717	17	0.285	4.748200693*10^8
Brass	2.6	21.23717	113.5	0.331	3.060561278*10^9
Bronze	2.6	21.23717	108	0.34	2.892692239*10^9
Copper	2.6	21.23717	17	0.355	4.502906190*10^8
Iron	2.6	21.23717	28.5	0.271	8.047900209*10^8
Magnessium	2.6	21.23717	6.4	0.35	1.701490304*10^8
Molybdenum	2.6	21.23717	40	0.307	1.098418091*10^9
Monel	2.6	21.23717	26	0.315	7.096282046*10^8
Nickel Silver	2.6	21.23717	18.5	0.322	5.022541645*10^8
Nickel Steels	2.6	21.23717	29	0.291	8.062227127*10^8
Titanium	2.6	21.23717	27	0.32	7.341302272*10^8
Zinc	2.6	21.23717	12	0.331	3.235835711*10^8

RESULT DISCUSSIONS

The numerical calculations were carried out for a circular beam of length l, with one of its bases fixed in the xy-plane, while the other base (in the plane z= l) is acted upon by a couple whose moment lies along the z-axis (x_3-axis). As an illustration, the length l of the beam is taken to be 10m, the diameter,1.2m, the angle of twist, 30^o and the polar, 0.34. (i.e l=10m, r=0.6m, Æ=30^0, I_p = 0.34. Twelve different engineering materials with different values of E, $m$and υ were considered. The results are shown on the various tables and figures. It is observed from table 2, that circular beams of brass engineering material has the highest torsional rigidity under St. Venanttorsion, while circular beam made of magnesium engineering material has the lowest torsional rigidity. Clearly from the value of R in equation (45), which is approximately 0.9, it shows there is a strong positive correlation between the modulus of rigidity and St. Venant torsion, and equation (44) shows there is a linear relationship between modulus of rigidity and St. Venant torsion. Also, that the higher the modulus of rigidity of the engineering material the higher the St. Venant torsion. However, the value of R in equation (46), which is approximately -0.01, shows that apart from the fact that torsional rigidity and modulus of rigidity have a negative correlation, the correlation is very weak. This implies that the value of the modulus of rigidity has a little negative effect on the resistance of twist of a circular beam. The effect of the cross sectional area of the circular beam is shown in tables (4),(5),(6),(7) and (8).It can easily be seen that the cross-sectional area of the beams affect the torsional rigidity of the beams. The wider the cross-sectional area the higher the torsional rigidity.

CONCLUSIONS

This work deals with the analysis of torsional rigidity of circular beams with different engineering materials subjected to St. Venant torsion. The torsional rigidity of these beams were calculated as a ratio of twisting moment to the angle of twist per unit length. It is shown that the circular beam made of brass engineering material has high torsional rigidity, relatively, when subjected to St. Venant's torsion. Concerning the cross-sectional area of circular beams made of different materials; it was deduced that the wider the cross sectional area the higher the torsional rigidity and vice versa. This phenomenon is of great importance, especially in the field of civil and mechanical engineering.

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