Adebile, E, A, Anake, T. A., Adesanya, A. O. Ogbonyon, G. J are Lecturers at the Departments of Mathematics Federal Universities of Technology Akure Ondo and Covenant Sango Otta Ogun States Respectively

NUMEROV'S METHOD FOR THE SOLUTION OF SECOND ORDER INITIAL VALUE PROBLEMS OF ORDINARY DIFFERENTIAL EQUATION USING MODIFIED BLOCK METHOD

ADEBILE, E. A, ANAKE, T. A. ADESANYA, A. O., OGHONYON, G. J.

ABSTRACT

Collocation and interpolation method using power series approximate solution to generate a continuous Numerov's is considered in this paper. The existing block method was modified to accommodate the unknown parameter in the corrector. The modified block method generates the independent solution at the grid points. This method was found to be efficient when tested on second order ordinary differential equation. *Keywords: Collocation, interpolation, approximate solution, block method, grid point.* *Corresponding author: torlar10@yahoo.com.*

INTRODUCTION

Second order ordinary differential equation of the form

$$y'' = f(x,y,y'),\ y(a)=\eta_1,\ y'(a)=\eta_2 \tag{1.0}$$

is considered in this paper.

Setbacks of the conventional method of solving (1.0) which is the reduction to the system of first order ordinary differential equation are reported by Adesanya et.al [1], and Awoyemi [4]. Method of interpolation and collocation of power series approximate method to derived continuous linear multistep method for the solution of higher order ordinary differential equation was reported by Awoyemi [5], Adesanya et.al [2], Kayode [12], Olabode [1], Yahaya [16], Badmus et. al [7] to mention but few. The setbacks of this method are widely reported by Awpyemi et.al [6], Fatunla [8]. Though the construction of Numerov's method for the solution of second order ordinary differential equation is reported by many scholars like Yahaya [15], Adesanya, et.al [3]. In this paper, we proposed modified block method to accommodate general second order ordinary differential equation and compare our result with the existing block method. This method is self starting and does not require developing separate predictors to implement the method.

METHODOLOGY

Let consider an approximate solution in form of power series method given as

$$y(x) = \sum_{j=0}^{k} a_j \phi_j(x)$$

(2.1)

The second derivative of (2.1) is given as

$$y'' = \sum_{j=0}^{2k} j(j-)a_j \Psi_{j-2}(x) \qquad (2.2)$$

From (2.1) and (2.2) we have the differential system

$$\sum_{j=0}^{2k} j(j-1)a_j \Psi_{j-2}(x) = f(x, y, y') \qquad (2.3)$$

Collocating (2.3) at $x_{n+j}, j = 0(1)k$ and interpolating (2.1) at $x_{n+j}, j = 0(1)k-1$ yield a differential system of equation of the form

$$AX = B$$

(2.4)

$$\begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 \end{pmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+1} \\ f_n \\ f_{n+1} \\ f_{n+2} \end{bmatrix} \qquad (2.5)$$

Solving (2.5) for $a_j, j = 0(1)k$ using Gaussian elimination method and substituting the result in (2.1) gives a continuous linear multistep method of the form

$$y(x) = \sum_{j=0}^{1} \phi_j y_{n+j}(x) + \sum_{j=0}^{2} \beta_j f_{n+j}(x)$$

(2.6)

The co efficient of $\phi_j(x)$ and β_j are given as

$\phi_0 = -(t-1)$

$\phi_1 = t$

$\beta_0 = \frac{h^2}{24}(t^4 - 6t^3 + 12t^2 - 7t)$

$\beta_1 = \frac{h^2}{12}(-t^4 + 4t^3 - 3t)$

$\beta_2 = \frac{h^2}{24}(t^4 - 2t^3 + t)$

(2.7)

Where $t = \frac{x - x_n}{h}$

Substituting (2.8) in (2.2) and then evaluate at x $= x_{n+2}$ i.e. when $t = 2$ gives

$$y_{n+2} - 2y_{n+1} + y_n = \frac{h^2}{12}(f_{n+2} + 10f_{n+1} + f_n) \quad (2.8)$$

Evaluating the first derivative of (2.7) at t = 0, 1 and 2 respectively, give

$$hy'_n - y_{n+1} + y_n = \frac{h^2}{24}(f_{n+2} - 6f_{n+1} - 7f_n)$$

(2.9)

$$hy'_{n+1} - y_{n+1} + y_n = \frac{h^2}{24}(-f_{n+2} + 10f_{n+1} + 3f_n)$$

(2.10)

$$hy'_{n+2} - y_{n+1} + y_n = \frac{h^2}{24}(9f_{n+2} + 26f_{n+1} + f_n)$$

(2.11)

MODIFIED BLOCK METHOD

$$A^0 h^\lambda Y_m^{(n)} = h^\lambda \sum_{i=1}^{k} A^{(i)} Y_{m-i}^{(n)} + h^\mu \sum_{i=1}^{k} B^{(i)} F_{m-i} \quad (2.12)$$

n is the power of the derivative, μ is the order of the differential equation. A^0 is an identity $R \times R$. λ is the power relative to the derivative of the differential equation.

$A^{(i)}$ and A^0 are $R \times R$ matrix. $B^{(i)}$, may not necessarily be an r by r matrix. It must be noted that (2.20) is the modified block method.

$$h^{\lambda} Y_m^n = [y_{n+1}, y_{n+2}, ..., hy'_{n+1}, hy'_{n+2}, ...h^2 y''_{n+1}, h^2 y''_{n+2}, ...h^n y^n_{n+m}]^T \quad (2.13)$$

$$h^{\lambda} Y_{n-i}^n = [y_{n-1}, y_{n-2}, ...y_n, hy'_{n-1}, hy'_{n-2}, ...hy'_n, h^2 y''_{n-1}, h^2 y''_{n-2}, ..., h^2 y''_n, ..., h^m y_n^m]^T \quad (2.14)$$

$$F_{m-1} = [f_{n-1}, f_{n-2}, ...f_n, f_{n+1}, f_{n+2}, ...f_m]^T \quad (2.15)$$

Solving (2.8) - (2.11) for $y_{n+j}, y'_{n+j}, j = 1(1)k$, give

$$A^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A^i = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B^i = \frac{1}{24}\begin{bmatrix} 7 & 6 & -1 \\ 8 & 16 & 0 \\ 10 & 16 & -2 \\ 8 & 32 & 8 \end{bmatrix}$$

ANALYSIS OF THE SCHEME

CONVERGENCE

Theorem 3.1

According to Fatunla [9], the necessary and sufficient condition for a linear multistep method to be convergent is for it to be consistent and zero stable

Order of the scheme

Zero stability of the modified block

(2) If (2.12) be $R \times R$ matrix, then its zero stable if as $h^{\mu} \to 0$

$$\left| RA^0 - A^i \right| = R^{r-\mu}(R-1) = 0$$

For those roots with $|R_j| \leq 1$, the multiplicity must not exceed the order of the differential equation.

For our method

$$\lambda A^0 - A^i = \begin{bmatrix} \lambda & -1 & 0 & -1 \\ 0 & \lambda & 0 & -2 \\ 0 & 0 & \lambda & -1 \\ 0 & 0 & 0 & \lambda-1 \end{bmatrix} \qquad (2.20)$$

As $h \to 0$ in (2.12), (2.20) reduces to

Determinant of ( ) gives

$\lambda^4 - 2\lambda^3 + \lambda^2 = 0$

$\lambda^2(\lambda-1)^2 = 0$

Hence the modified block is zero stable

Numerical Example

Problem I

We considered a special second order initial value problem

$y'' = 2y' \quad y(1) = 1, y'(1) = -1, h = .1/40,$

Exact solution $y(x) = \dfrac{1}{x}$

Problem I1

We considered a general second order initial value problem

$y'' = y + e^{3x} \quad y(0) = \dfrac{-3}{32}, y'(0) = \dfrac{-5}{32}, h = \dfrac{.1}{40}$

Exact solution $y(x) = \dfrac{4x-3}{32\exp(-3x)}$

Problem III:

$y'' - x(y')^2 = 0$

$y(0) = 1, y'(0) = \dfrac{1}{2}, h = .1/40$

Exact solution $y(x) = 1 + \dfrac{1}{2}\ln\left(\dfrac{2+x}{2-x}\right)$

Problem IV

We consider a highly oscillatory test problem

$y'' + \lambda^2 y = 0$, we take $\lambda = 2$ with initial condition $y(0) = 1, y'(0) = 2$

Exact solution $\cos 2x + \sin 2x$

Our result was compared with Okunuga[11] who solved the same problem

Problem V

We considered a boundary value problem

$\varepsilon y'' = y'$ $y(0) = 1, y(1) = 0$ $x = [0,1]$ with $\varepsilon = 0.1$, h= 0.01

Exact solution $y(x) = \dfrac{1 - e^{-(\frac{1+x}{\varepsilon})}}{1 - e^{-\frac{1}{\varepsilon}}}$

Table 1 for problem I

x	Exact result	Calculated result	Error in [12]	Error
1.1	0.909090909090911	0.909090909434888	5.3731D-07	3.4397D-10
1.2	0.83333333333336	0.833333334129815	4.1426D-07	7.9647D-10
1.3	0.769230769230773	0.76230779543681	3.2609D-07	1.3129D-09
1.4	0.714285714285719	0.714285716180712	2.6126D-07	1.8949D-09
1.5	0.66666666666671	0.666666669213988	2.1254D-07	2.5473D-09
1.6	0.62500000000005	0.625000003275666	1.7522D-07	3.2756D-09
1.7	0.588235294117652	0.588235298203935	1.4615D-07	4.0862D-09
1.8	0.55555555555561	0.555555556054132	1.2317D-07	4.9855D-09
1.9	0.52631578947369	0.52315795453596	1.0476D-07	5.9799D-09
2.0	0.500000000000005	0.500000007075607	8.9856D-08	7.0756D-09

Table 2 for problem II

x	Exact result	Calculated result	Error in [12]	Error
0.1	-0.10967602811555	-0.109676028221597	9.056D-09	1.0604D-10
0.2	-0.125270667526847	-0.125270668013958	8.6408D-07	4.8711D-10
0.3	-0.138352675002578	-0.138352676278628	2.1677D-07	1.2760D-09
0.4	-0.145255115369724	-0.145255118038637	4.2779D-07	2.6689D-09
0.5	-0.140052783448064	-0.140052788398186	7.5992D-07	4.9501D-09
0.6	-0.112380072869013	-0.112380081509659	1.2878D-06	8.6406D-09
0.7	-0.051038561953551	-0.0510385759505469	2.0734S-06	1.3996D-08
0.8	0.0688948523790006	0.068894830185809	3.2509D-06	2.2193D-08
0.9	0.278994969841344	0.278994935523711	4.9987D-06	3.4317D-08
1.0	0.62767302884957	0.627672976775982	7.7571D-06	5.2073D-08

Table 3 for problem III

x	Exact result $y(x)$	Exact result $y_n(x)$	Errorin [12]	Error in [4]	Error
0.1	1.05004172927849	1.05004172923572	1.0532D-07	2.607D-10	4.2767D-11
0.2	1.10033534773108	1.10033534737803	2.1315D-07	1.9816D-09	3.5304D-10
0.3	1.151140404393647	1.15114043469945	3.2583D-07	6.5074D-09	1.2371D-09
0.4	1.202732554054408	1.2027325509718	4.4633D-07	1.5592D-08	3.0822D-09
0.5	1.256746590599891	1.256746584072	5.7812D-07	3.1504D-08	6.3116D-09
0.6	1.30951960420311	1.309519592193	7.2946D-07	5.6374D-08	1.2000D-08
0.7	1.34844976508925	1.348449747233	8.9879D-07	9.6164D-08	1.7855D-08
0.8	1.4236489301936	1.4236489301936	1.0973D-06	1.5686D-07	3.5150D-08
0.9	1.4847002785945	1.48470027859405	1.3336D-06	2.4869D-07	5.7283D-08
1.0	1.54930614433406	1.54930614433408	1.6307D-06	3.8798D-07	9.2040D-08

Table 4 for problem IV

x	Exact solution	Calculated solution	Okunuga[11]	Error $\|y(x)-y_n(x)\|$
0.01	1.01979867335991	1.0197986765946	-	3.234D-009
0.02	1.03918944084761	1.03918944824276	2.65-06	7.395D-009
0.03	1.05816454641465	1.05816450901434	3.98D-06	3.740D-008
0.04	1.07671640027179	1.0767163501034	5.30D-06	5.016D-008
0.05	1.09483758192485	1.09483751419107	6.62D-06	6.773D-008
0.06	1.11252084314279	1.11252075272287	7.94D-06	9.041D-008
0.07	1.12975911085687	1.1297589925526	9.25D-06	1.183D-007
0.08	1.14654548998987	1.14654533852972	1.06D-05	1.514D-007
0.09	1.16287326621395	1.16287307625733	1.19D-05	1.899D-007
0.1	1.1787359086363	1.17873567477786	1.32D-05	2.338D-007

Table 5 for problem V

x	Exact result	Calculated result	Error
0.01	0.999958918579681	0.999999892388082	4.097D-005
0.02	1.00000822998472	0.999999773561501	8.45D-006
0.03	1.00001176736881	0.99999642989839	1.212D-005
0.04	1.00001496812631	0.999999498809171	1.546D-005
0.05	1.00001786429145	0.99999933985998	1.852D-005
0.06	1.00002048485004	0.999999164629573	2.132D-005
0.07	1.00002285602951	0.999998797145019	2.388D-005
0.08	1.00002500156142	0.999998758483352	2.624D-005
0.09	1.00002694291897	0.999998523702194	2.841D-005
0.1	1.00002869953193	0.999998264872271	3.043D-005

REFERENCES

Adesanya, A O, Anake, T. A., Bishop S. A., Osilagun, J. A. Two steps block method for the solution of general second order initial value problems of ordinary differential equation (to appear).

Adesanya, A. O., Anake, T. A., Udoh, M. O. (2008): Improved continuous method for direct solution of general second order ordinary differential equation. Journal of the Nigerian Asso;iation of Mathematical Physics, 13, 59-62

Adesanya, A. O., Anake, T. A., Oghoyon, G. J. (2009): Continuous implicit method for the solution of general second order ordinary differential equation. Journal of Nigerian Association of Mathematical Physics (in press)

Awoyemi, D. O (2001): A new Sixth-order algorithm for general secondary order ordinary differential equation. International journal of computer mathematics 77, 117-124

Awoyemi, D. O. (1991): A class of continuous linear multistep method for general second order initial value problem in ordinary differential equation. Intern. J. Comp. Math. 72, 29-37

Awoyemi, D. O, Adesanya, A.O, Ogunyebi, S. N (2009): Construction of self Starting Numerov method for the solution of initial value problems of general second order ordinary differential equati:n. Int. J. Num. Math, University of Ado Ekiti, 4(2), 269-278