Dynamic Response of Two Viscoelastically Connected Rayleigh Beams Subjected to Concentrated Moving Load.

Jacob A. Gbadeyan, Ph.D.[1]; Moses S. Dada, Ph.D.[1]; and Olasunmbo O. Agboola, M.Sc.[*2]

[1]Department of Mathematics, University of Ilorin, Ilorin, Nigeria.
[2]Department of Mathematics, Covenant University, Canaan Land, Ota, Nigeria.

E-mail: dimejiagbo@yahoo.com[*]

ABSTRACT

A theory concerning the dynamic response of two identical simply supported Rayleigh beams viscoelastically connected together by a flexible core and traversed by a concentrated moving load is developed in this paper. The solution technique employed is based on finite Fourier and Laplace integral transformations. It is observed that the maximum amplitude of the deflection of the upper beam increases with an increase in the value of the rotatory inertia while the maximum amplitude of deflection of the lower beam decreases with increasing values of rotatory inertia.

(Keywords: viscoelastically connected Rayleigh beams, dynamic response, moving load)

INTRODUCTION

The behavior of elastic structures such as beams under moving loads is of great theoretical and practical importance. The problem of analyzing the behavior of a single beam under the influence of a moving load has been studied in various fields of engineering, applied mathematics, as well as applied physics. For over three decades ago, this moving load problem has attracted much attention of a large number of investigators. [3 – 13]. As a matter of fact, there are many designs involving moving loads in one form or the other. An extensive review on moving load problems has been reported by Fryba [3] in his excellent monograph. However, work on the dynamic response of two elastically connected beams under moving load is scanty. This is perhaps due to the fact that, unlike the case of a single beam, the governing equation is made up of two coupled partial differential equations. Nonetheless, the vibration of problem of double beams is of practical importance. For instance, the dynamic behavior of composite material has been studied by modeling the latter by elastically connected beams [14 – 16].

Furthermore, Gbolagade et al. [9] carried out a study on the response of two Euler-Bernoulli beams elastically connected together by a flexible core with an attached dash-pot and traversed by a concentrated force moving with a constant velocity. In their work, the inertia effects of the beams and the moving load were neglected. Two years later, Gbadeyan et al. [5] investigated the problem of the dynamic response of two identical thin beams which are viscoelastically connected and subjected to uniform partially distributed moving force. The work paid special attention on the effects of uniform partially distributed moving forces on viscoelastically connected Bernoulli-Euler beams. It is found that, for various values of the speed of the moving force considered, the amplitude of the transverse deflection of the primary beam increases with the speed.

Oniszczuk [12] presented exact theoretical general solutions of undamped forced vibrations for a simply supported Euler-Bernoulli double-beam system. Several cases of particularly interesting excitation loadings, the moving concentrated force in particular, are investigated. The steady-state dynamic response of an embedded railway track to a moving train is investigated in [13]. The model of the railway is made up of a flexible plate performing vertical vibrations, two beams that are connected to the plate by continuous viscoelastic elements and an elastic foundation which supports the plate. The dynamic behavior of a double-beam system traversed by a constant moving load is also studied in [1]. The system is composed of two simply supported Euler-Bernoulli beams connected continuously by a viscoelastic layer.

This paper is concerned with the development of dynamic analysis of a double Rayleigh beam system traversed by a concentrated moving load. The system consists of two identical, uniform, elastic homogeneous isotropic simply-supported beams. The beams are (i) arranged horizontally, one upon the other and (ii) continuously and visco-elastically connected together by a distributed spring-damper layer. In essence, the main focus of this study is to assess the effect of rotatory inertia, which is neglected in Euler-Bernoulli beams, on the dynamic response of the said Rayleigh beams.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider two identical finite Rayleigh beams (with small deflections) attached together by a flexible core as shown Figure 1.

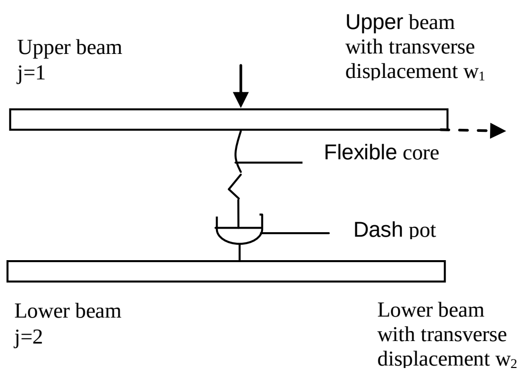

Figure 1: Two Viscoelastically Connected Rayleigh Beams.

The x-coordinate is along the upper beam and the y-coordinate is normal to x-axis. Thus, we consider a one–dimensional elastic system where bending moment and rotatory inertia effects are not neglected. The index j is attached to the variables associated with the two beams such that with the upper beam, j = 1 and lower beam, j =2.

The displacement (transverse deflection) of the beam in the y–direction denotes and measures the deflection of the middle plane of the beam. If the core medium is represented by a viscoelastic (Kelvin) model with stiffness k and damping coefficient ε_0, the restoring forces from the core are expressed as:

$$(k+\varepsilon_0 \frac{\partial}{\partial t})(w_2 - w_1)$$

and

$$(k+\varepsilon_0 \frac{\partial}{\partial t})(w_1 - w_2)$$

for the upper and lower beams, respectively.

In the formulation of the governing equation of the two viscoelastically connected beams, the following assumptions are adopted. Firstly, the beams are assumed to be of constant cross-section with uniform mass distribution. Secondly, the effects of rotatory inertia and the gravitational effect of load are taken into account. Thirdly, the load moves with a uniform velocity and is guided in such a way that it keeps contact with the beam at all times. Fourthly, the beams are assumed to be simply supported and consequently the computations are performed for simply supported end conditions.

Under the above assumptions, the problem of interest is described by the partial differential equation of the form:

$$\frac{EI\partial^4 w_j}{\partial x^4}(x,t) + \mu \frac{\partial^2 w_j}{\partial t^2}(x,t) + 2\mu\omega_b \frac{\partial w_j}{\partial t}(x,t)$$
$$-\frac{r^2\partial^4 w_j}{\partial x^2 \partial t^2}(x,t) = \rho_i(x,t) - \left(k + \varepsilon_0 \frac{\partial}{\partial t}\right)(w_1 - w_2)(-1)^j ,$$

for j =1, 2 **(1)**

where, EI =flexural rigidity of the beam (Nm^2)

$w_j(x,t)$= jth beam deflection at point x and time t, measured form the equilibrium position when beam is loaded with its own weight (s)

x = length (axial) co-ordinate with the origin at the left-hand end of the beam

t = time coordinate with the origin at the instant of the load arriving on the beam

μ = constant mass per unit length of the beam (kgm^{-1})

ω_b = circular frequency of the damping of the beam

r =radius of gyration of beam cross-section (m)

$\rho_i(x,t)$ is the applied force defined by,

$$\rho_i(x,t) = \begin{cases} P\delta(x-vt), \ j=1 \\ 0, \qquad \text{otherwise} \end{cases}$$

such that,

$\delta(x-vt)$ = Dirac delta function (or unit impulse function) at point $x = vt$

v = constant velocity of the load's motion (ms^{-1})

P=concentrated moving force of constant magnitude (N).

The boundary conditions associated with the set of equations (1) are:

$$w_j(0,t) = 0 = w_j(l,t), \quad j = 1,2$$
$$\frac{\partial^2 w_j}{\partial x^2}(0,t) = 0 = \frac{\partial^2 w_j(l,t)}{\partial x^2}, \quad j = 1,2 \qquad \textbf{(2)}$$

where, l =length of the beam (m)
(since the displacement and the bending moment vanish at a simply supported end) and the initial conditions are:

$$w_j(x,0) = 0 = \frac{\partial w_j}{\partial t}(x,0)\ , \ j = 1,2 \qquad \textbf{(3)}$$

SOLUTION OF THE MATHEMATICAL PROBLEM

In this section, we proceed to solve the initial boundary value problem (IBVP) described by Equations (1) – (3). We remark that the integral transform techniques have been proved suitable and effectively applicable to solving moving load problems such as the one under investigation [3]. Therefore, this method is adopted in the solution of the IBVP. Specifically, the Fourier transformation for the length coordinates and the Laplace transformation for the time coordinate with boundary and initial conditions are used in this work.

Finite Fourier Transformed Governing Equations

The set of Governing Equations (1) for the deflections of the two beams may be rewritten as:

$$EI\frac{\partial^4 w_1(x,t)}{\partial x^4} + \mu\frac{\partial^2 w_1(x,t)}{\partial t^2} + 2\mu\omega_b\frac{\partial w_1(x,t)}{\partial t}$$
$$-r^2\frac{\partial^4 w_1(x,t)}{\partial x^2 \partial t^2} - \left(k + \varepsilon_0\frac{\partial}{\partial t}\right)(w_1 - w_2)$$
$$= P\delta(x - vt) \qquad \textbf{(4)}$$

and

$$EI\frac{\partial^4 w_2(x,t)}{\partial x^4} + \mu\frac{\partial^2 w_2(x,t)}{\partial t^2} + 2\mu\omega_b\frac{\partial w_2(x,t)}{\partial t}$$
$$-r^2\frac{\partial^4 w_2(x,t)}{\partial x^2 \partial t^2} + \left(k + \varepsilon_0\frac{\partial}{\partial t}\right)(w_1 - w_2) = 0 \qquad \textbf{(5)}$$

respectively. Equations (4) and (5) are fourth order partial differential equations with respect to the variables x and t.

Next, Fourier transform of each of the governing partial differential Equations (4) and (5) is taken. We find, however, that the boundary conditions in Equation (2) may be accommodated only by using a finite Fourier sine transform, so we shall apply the Fourier finite sine integral transformation for the length coordinate and this is defined as:

$$\overline{w}_j(n,t) = \int_0^l w_j(x,t)\sin\frac{n\pi x}{l}dx, n = 1,2,3, \qquad \textbf{(6)}$$

with the inverse transform defined as:

$$w_j(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} w_j(n,t)\sin\frac{n\pi x}{l}dx \qquad \textbf{(7)}$$

Thus, by invoking Equation (6) on Equations (4) and (5), we have:

$$\ddot{\overline{w}}_1(n,t)+\left(\frac{l^2(2\mu\omega_b-\varepsilon_0)}{\mu l^2+r^2n^2\pi^2}\right)\dot{\overline{w}}_1(n,t)$$
$$+\left(\frac{EIn^4\pi^4-kl^4}{\mu l^4+r^2n^2\pi^2l^2}\right)\overline{w}_1(n,t)$$
$$+\left(\frac{\varepsilon_0\ell^2}{\mu l^2+r^2n^2\pi^2}\right)\dot{\overline{w}}_2(n,t)$$
$$+\left(\frac{kl^2}{\mu l^2+r^2n^2\pi^2}\right)\overline{w}_2(n,t)$$
$$=\frac{Pl^2}{\mu l^2+r^2n^2\pi^2}\ Sin\frac{n\pi vt}{l} \qquad \textbf{(8)}$$

and

$$\ddot{\overline{w}}_2(n,t)+\frac{l^2(2\mu\omega_b-\varepsilon_0)}{\mu l^2+r^2n^2\pi^2}\dot{\overline{w}}_2(n,t)$$
$$+\left(\frac{EIn^4\pi^4-kl^4}{\mu l^4+r^2n^2\pi^2\ell^2}\right)\overline{w}_1(n,t)$$
$$+\left(\frac{\varepsilon_0l^2}{\mu l^2+r^2n^2\pi^2}\right)\dot{\overline{w}}_1(n,t) \qquad \textbf{(9)}$$
$$+\left(\frac{kl^2}{\mu l^2+r^2n^2\pi^2}\right)\overline{w}_1(n,t)=0$$

respectively. We can conveniently write Equations (8) and (9) respectively as:

$$\ddot{\overline{w}}_1(n,t)+b_1\dot{\overline{w}}_1(n,t)+b_2\overline{w}_1(n,t)+b_3\dot{\overline{w}}_2(n,t)$$
$$+b_4\overline{w}_2(n,t)=b_5\sin a_8t \qquad \textbf{(10)}$$

$$\ddot{\overline{w}}_2(n,t)+b_1\dot{\overline{w}}_2(n,t)+b_2\overline{w}_2(n,t)+b_3\dot{\overline{w}}_1(n,t)$$
$$+b_4\overline{w}_1(n,t)=0 \qquad \textbf{(11)}$$

where,

$$a_1=\mu l^2+r^2n^2\pi^2,\ \ a_2=\mu l^4+r^2n^2\pi^2l^2,$$
$$a_3=l^2(2\mu\omega_b-\varepsilon_0),\ a_4=EIn^4\pi^4-kl^4,$$
$$a_5=\varepsilon_0l^2,\ \ a_6=kl^2,\ a_7=Pl^2,\ \ a_8=\frac{n\pi v}{l},$$
$$b_1=\frac{a_3}{a_1}, b_2=\frac{a_4}{a_2},\ b_3=\frac{a_5}{a_1},$$
$$b_4=\frac{a_6}{a_1},\ and\ \ b_5=\frac{a_7}{a_1}. \qquad \textbf{(12)}$$

Equations (10) and (11) represent the finite Fourier transformed governing equations of the two Rayleigh beams viscoelastically connected together and subjected to a load moving with a constant velocity.

Laplace Transformed Solutions

To solve Equations (10) and (11), we apply the method of the Laplace integral transformation for the time coordinate between 0 and ∞. The operation of Laplace transform is here indicated by the notation:

$$L[f(t)]=\int_0^\infty f(t)e^{-st}dt \qquad \textbf{(13)}$$

where, L=Laplace transform operator
s=Laplace transform variable.

In particular, we use:

$$L[\overline{w}_j(n,t)]=\overline{\overline{w}}_j(n,s)=\int_0^\infty \overline{w}_j(n,t)e^{-st}dt \qquad \textbf{(14)}$$

Using the transformation (14) on Equation (10), we have:

$$\int_0^\infty \ddot{\overline{w}}_1(n,t)e^{-st}dt+b_1\int_0^\infty \dot{\overline{w}}_1(n,t)e^{-st}dt$$
$$+b_2\int_0^\infty \overline{w}_1(n,t)e^{-st}dt+b_3\int_0^\infty \dot{\overline{w}}_2(n,t)e^{-st}dt \qquad \textbf{(15)}$$
$$+b_4\int_0^\infty \overline{w}_2(n,t)e^{-st}dt=b_5\int_0^\infty \sin a_8te^{-st}dt$$

On evaluating each term of Equation (15) by method of integration by parts and using the set of initial conditions (3), we obtain:

$$C_1\overline{\overline{w}}_1(n,s)+C_2\overline{\overline{w}}_2(n,s)=\frac{b_6}{s^2+a_8^2} \qquad \textbf{(16)}$$

with

$$b_6=a_8b_5,\ C_1=s^2+b_1s+b_2,$$
$$C_2=b_3s+b_4 \qquad \textbf{(17)}$$

Similarly, equation (11) reduces to:

$$\overline{\overline{w}}_1(n,s)=\frac{b_6C_1}{(C_1^2-C_2^2)(s^2+a_8^2)} \qquad \textbf{(18)}$$

Substituting Equation (18) into (16), we obtain:

$$\bar{\bar{w}}_2(n,s) \;=\; \frac{-b_6C_2}{\left(C_1^2-C_2^2\right)\left(s^2+a_8^2\right)} \qquad \textbf{(19)}$$

Equations (18) and (19) may be expressed as:

$$\bar{\bar{w}}_1(n,s)=\frac{b_6\left(s^2+b_1s+b_2\right)}{\left(s^2+d_1s+d_2\right)\left(s^2+d_3s+d_4\right)\left(s^2+a_8^2\right)} \qquad \textbf{(20)}$$

and

$$\bar{\bar{w}}_2(n,s)=\frac{-b_6\left(b_3s+b_4\right)}{\left(s^2+d_1s+d_2\right)\left(s^2+d_3s+d_4\right)\left(s^2+a_8^2\right)} \qquad \textbf{(21)}$$

respectively, where,

$$\begin{aligned} d_1&=b_1-b_3, \quad d_2=b_2-b_4,\\ d_3&=b_1+b_3, \quad d_4=b_2+b_4. \end{aligned} \qquad \textbf{(22)}$$

The Inverse Integral Transformation

We proceed in this section to obtain the inverse transformation of the solutions obtained in the previous section. To this effect, equations (20) and (21) are resolved into partial fractions and inverted as follows:

$$\bar{\bar{w}}_1(n,s)=\frac{b_6\left(s^2+b_1s+b_2\right)}{\left(s^2+a_8^2\right)\left(s^2+d_1s+d_4\right)\left(s^2+d_3s+d_4\right)}$$

$$\equiv\frac{As+B}{s^2+a_8^2}+\frac{Cs+D}{s^2+d_1s+d_2}+\frac{Es+F}{s^2+d_3s+d_4} \qquad \textbf{(23)}$$

where,

$$A=\frac{q_1r_2-q_2r_1}{p_2q_1-p_1q_2} \qquad \textbf{(24)}$$

$$B=\frac{p_2r_1-p_1r_2}{p_2q_1-p_1q_2} \qquad \textbf{(25)}$$

$$C=\frac{1}{m_1}\left(n_1-k_1A-l_1B\right) \qquad \textbf{(26)}$$

$$D=\frac{1}{f_2}\left(c_3-g_1A-f_1B-h_1C\right) \qquad \textbf{(27)}$$

$$E=-\left(A+C\right) \qquad \textbf{(28)}$$

$$F=-\left(f_1A+B+f_2C+D\right) \qquad \textbf{(29)}$$

such that,

$b_7=b_1b_6$, $b_8=b_2b_6$, $d_5=d_1+d_3$,
$d_6=d_1d_3+d_2+d_4$, $d_7=d_1d_4+d_2d_3$,
$d_8=d_2d_4$, $e_1=a_8^2d_1$, $e_2=a_8^2d_2$, $e_3=a_8^2d_3$,
$e_4=a_8^2d_4$, $e_5=a_8^2+d_2$, $e_6=a_8^2+d_4$,
$f_1=d_5-d_1$, $f_2=d_3-d_1$, $f_3=d_6-e_5$,
$f_4=e_6-e_5$, $f_5=d_7-e_1$, $f_6=e_3-e_1$,
$f_7=d_8-e_2$ and $f_8=e_4-e_2$,
$g_1=f_3-d_1f_1$, $g_2=f_5-e_5f_1$,
$g_3=f_7-e_1f_1$, $g_4=-e_2f_1$,
$k_1=f_2g_2-f_4g_1, k_2=f_2g_3-f_6g_1$,
$k_3=f_2g_4-f_8g_1$, $e_1=f_2f_3-f_1f_4$,
$e_2=f_2f_5-f_1f_6$, $l_3=f_2f_7-f_1f_8$
$m_1=f_2h_2-f_4h_1, m_2=f_2h_3-f_6h_1$,
$m_3=f_2h_4-f_8h_1$, $n_1=b_6f_2$,
$n_2=b_7f_2, n_3=b_8f_2$

and

$$\bar{\bar{w}}_2(n,s)=-\frac{b_6\left(b_3s+b_4\right)}{\left(s^2+a_8^2\right)\left(s^2+d_1s+d_2\right)\left(s^2+d_3s+d_4\right)}$$

$$\equiv\frac{Gs+H}{s^2+a_8^2}+\frac{I_xs+J}{s^2+d_1s+d_2}+\frac{Ms+N_x}{s^2+d_3s+d_4} \qquad \textbf{(30)}$$

where,

$$G=\frac{q_1s_2-q_2s_1}{p_2q_1-P_1q_2} \qquad \textbf{(31)}$$

$$H=\frac{p_2s_1-p_1s_2}{p_2q_1-p_1q_2} \qquad \textbf{(32)}$$

$$I_x = -\frac{1}{m_1}\left(k_1 G + l_1 H\right) \quad \textbf{(33)}$$

$$J = -\frac{1}{f_2}\left[g_1 G + f_1 H + h_1 I_x\right] \quad \textbf{(34)}$$

$$M = -\left(G + I_x\right) \quad \textbf{(35)}$$

$$N_x = -\left(f_1 G + H + f_2 I_x + J\right). \quad \textbf{(36)}$$

$$s_1 = m_1 n_4,\ s_2 = m_1 n_5, \quad \textbf{(37)}$$

$$n_4 = -b_3 b_6 f_2,\ n_5 = -b_4 b_6 f_2, \quad \textbf{(38)}$$

Thus, taking inverse Laplace transform of equations (23) and (30), we have:

$$\begin{aligned}\overline{w}_1(n,t) &= A\cos a_8 t + \frac{B}{a_8}\sin a_8 t \\ &+ Ce^{-\frac{1}{2}d_1 t}\left(\cos t_1 t - \frac{d_1}{2t_1}\sin t_1 t\right) + \frac{D}{t_1}e^{-\frac{1}{2}d_1 t} \\ &+ Ee^{-\frac{1}{2}d_3 t}\left(\cos t_2 t - \frac{d_3}{2t_2}\sin t_2 t\right) \\ &+ \frac{F}{t_2}e^{-\frac{1}{2}d_3 t}\sin t_2 t\end{aligned} \quad \textbf{(39)}$$

and

$$\begin{aligned}\overline{w}_2(n,t) &= G\cos a_8 t + \frac{H}{a_8}\sin a_8 t \\ &+ I_x e^{-\frac{1}{2}d_1 t}\left(\cos t_1 t - \frac{d_1}{2t_1}\sin t_1 t\right) \\ &+ \frac{J}{t_1}e^{-\frac{1}{2}d_1}\sin t_1 t \\ &+ e^{-\frac{1}{2}d_3 t}\left(\cos t_2 t - \frac{d_3}{2t_2}\sin t_2 t\right) \\ &+ \frac{N_x}{t_2}e^{-\frac{1}{2}d_3 t}\sin t_2 t\end{aligned} \quad \textbf{(40)}$$

where, $t_1^2 = d_2 - \left(\frac{1}{2}d_1\right)^2$ **(41)**

and

$$t_2^2 = d_4 - \left(\frac{1}{2}d_3\right)^2 \quad \textbf{(42)}$$

Finally, Fourier sine inverses of

$\overline{w}_1(n,t)\ and\ \overline{w}_2(n,t)$

are obtained as:

$$w_1(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} \overline{w}_1(n,t)\sin\frac{n\pi x}{l} \quad \textbf{(43)}$$

and $w_2(x,t) = \frac{2}{l}\sum_{n=1}^{\infty} \overline{w}_2(n,t)\sin\frac{n\pi x}{l}$ **(44)**

respectively. Conclusively, Equation (43) represents the transverse displacement of the upper beam relative to the lower one while the corresponding transverse displacement of the lower beam is given by Equation (44).

NUMERICAL ANALYSIS AND DISCUSSION OF RESULTS

As earlier remarked, the numerical results that follow are for two identical Rayleigh beams that are elastically connected together by a flexible core and subjected to a concentrated moving load. In order to illustrate the theory described in this paper numerically, the following values of the physical constants and parameters are considered. $\pi = \frac{22}{7}$, $EI = 16000Nm^2$, $P = 20N$, $\mu = 0.075$, $\varepsilon_0 = 0.15$, $k = 0.2$, $l = 6.0m$, $\omega_b = 0$, $n = 1$, $x = \frac{1}{2}$ and $t = 0.0(0.2)1$.

The cases when the velocities (v) of the moving load are $3.3ms^{-1}$, $6.3ms^{-1}$, $9.3ms^{-1}$, and $12.3ms^{-1}$ for the following values of r^2 (rotatory inertia effect): 4.0, 8.0 and 10.0 are considered.

The responses of the upper beam relative to the lower beam and that of the lower beam for various values of velocity, rotatory inertia, stiffness coefficient and damping coefficient are shown in the following figures (Figures 1 – 8).

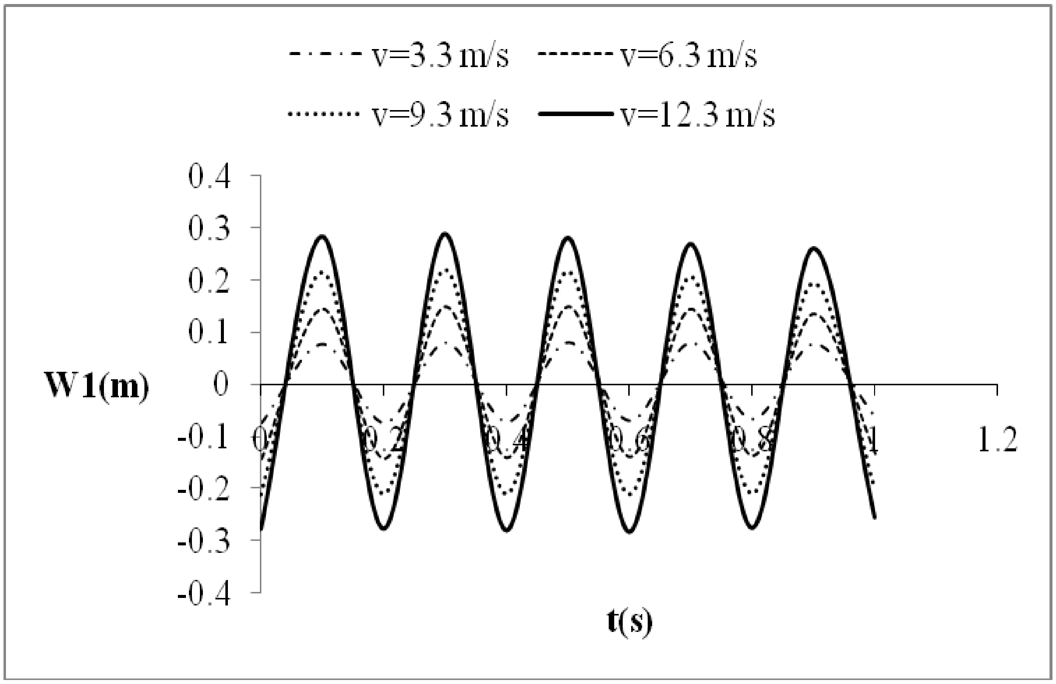

Figure 1: The Response of the Upper Beam Relative to the Lower Beam for Various Values of the Velocity of the Moving Load at a Fixed Value of r^2 (rotatory inertia), i.e., r^2=4.

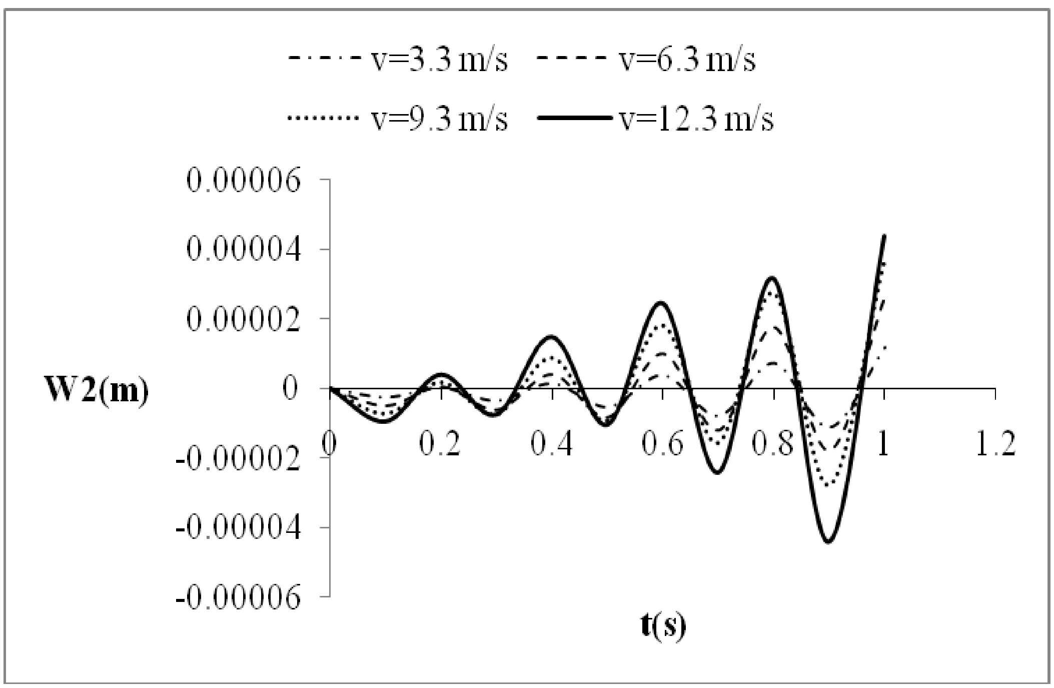

Figure 2: The Response of the Lower Beam for Various Values of the Velocity of the Moving Load at a Fixed Value of r^2 (rotatory inertia), i.e., r^2=4.

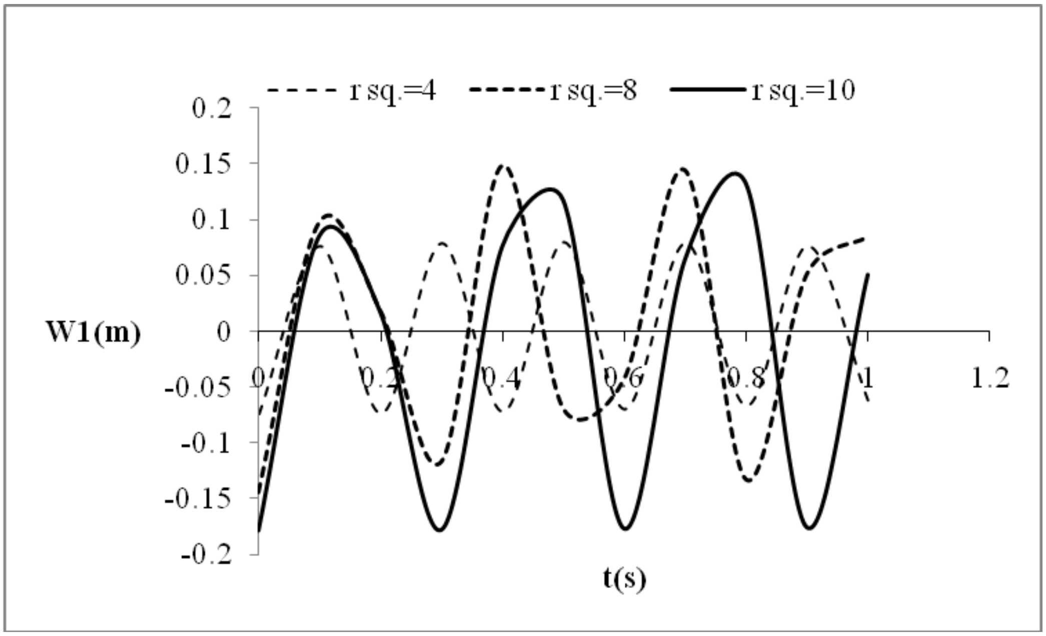

Figure 3: The Response of the Upper Beam Relative to the Lower Beam for Various Values of r^2 (r sq.) when the Load Moves at v = 3.3 m/s

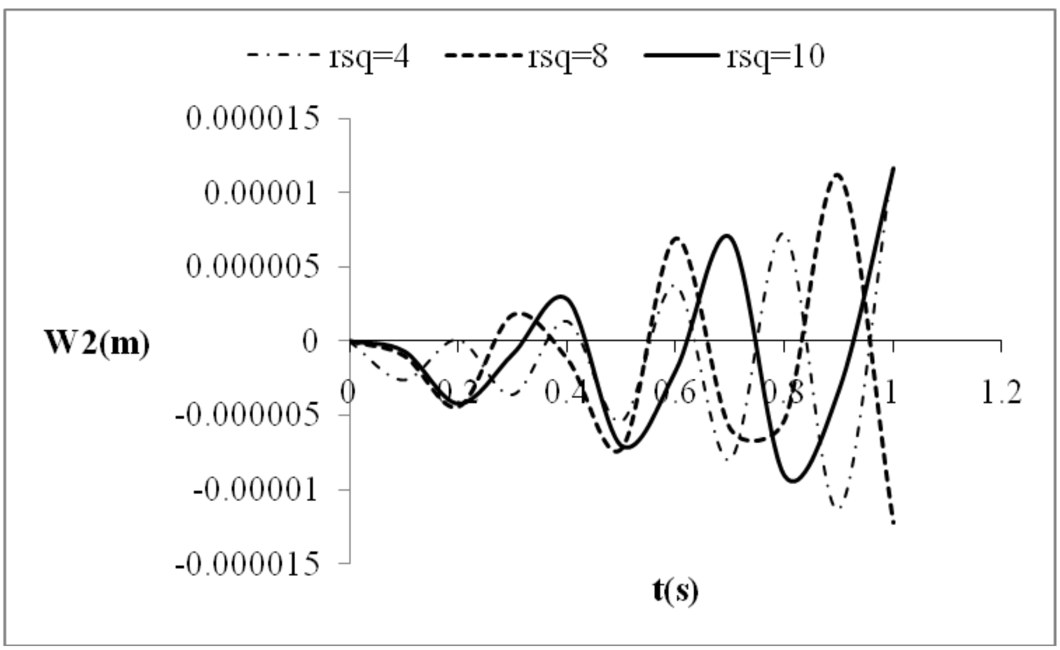

Figure 4: The Response of the Lower Beam for Various Values of r^2 (r sq.) when the Load Moves at v = 3.3 m/s

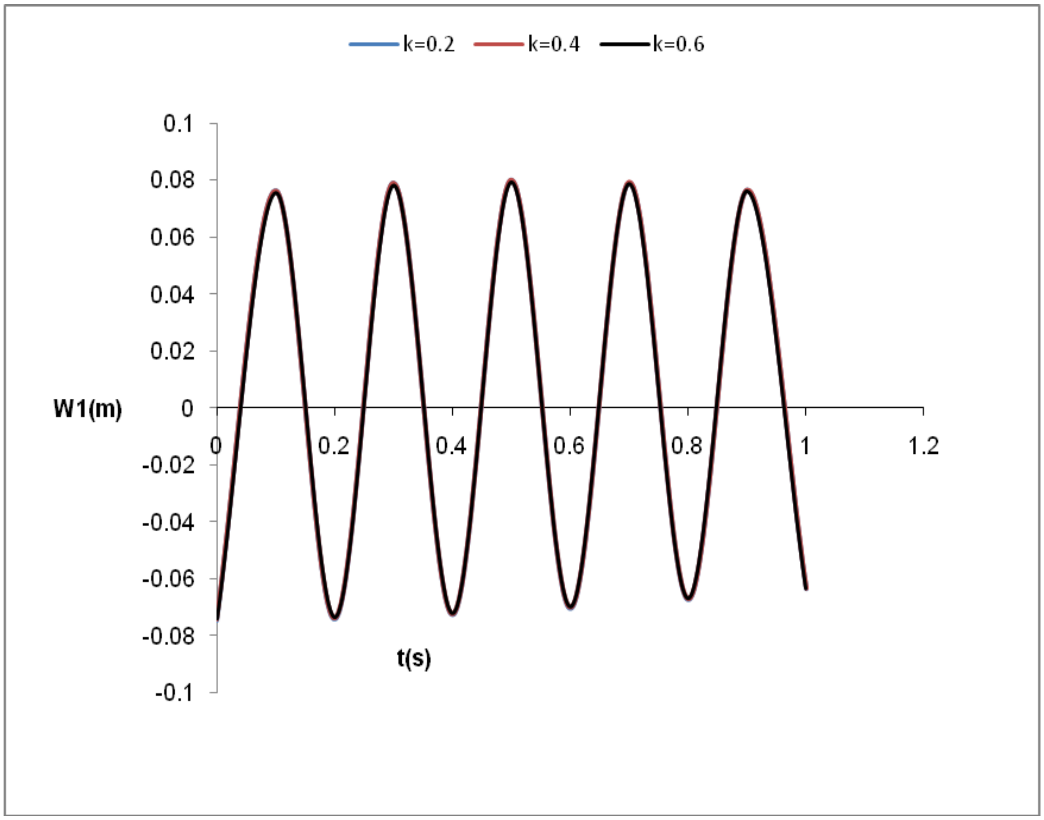

Figure 5: The Response of the Upper Beam Relative to the Lower Beam for Various Values of k (stiffness coefficient).

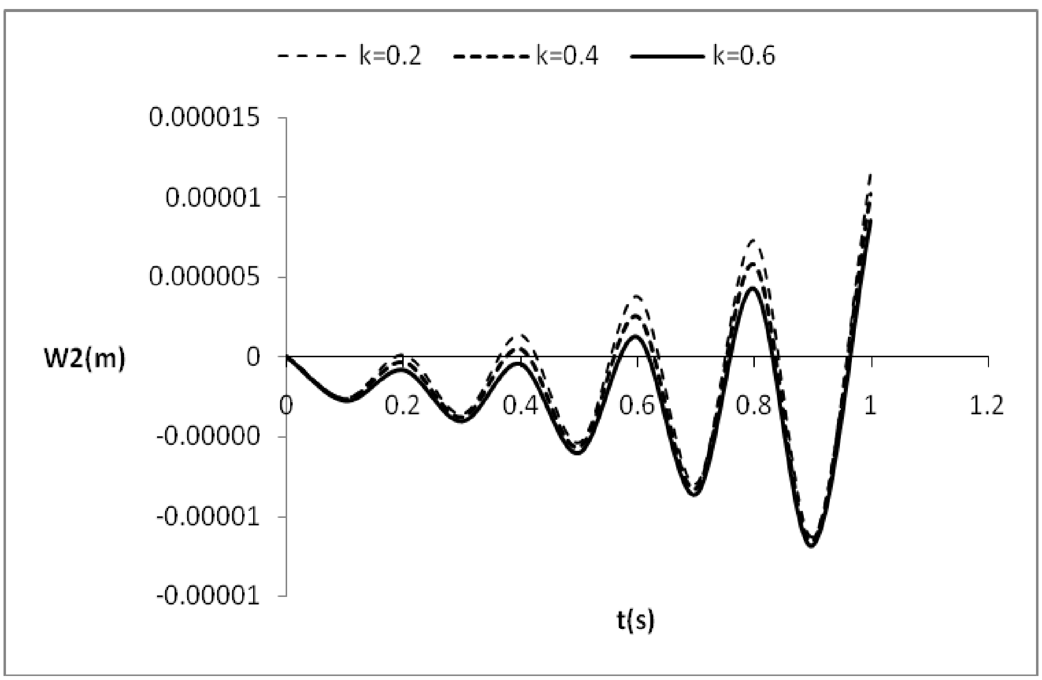

Figure 6: The Response of the Lower Beam for Various Values of k (stiffness coefficient)

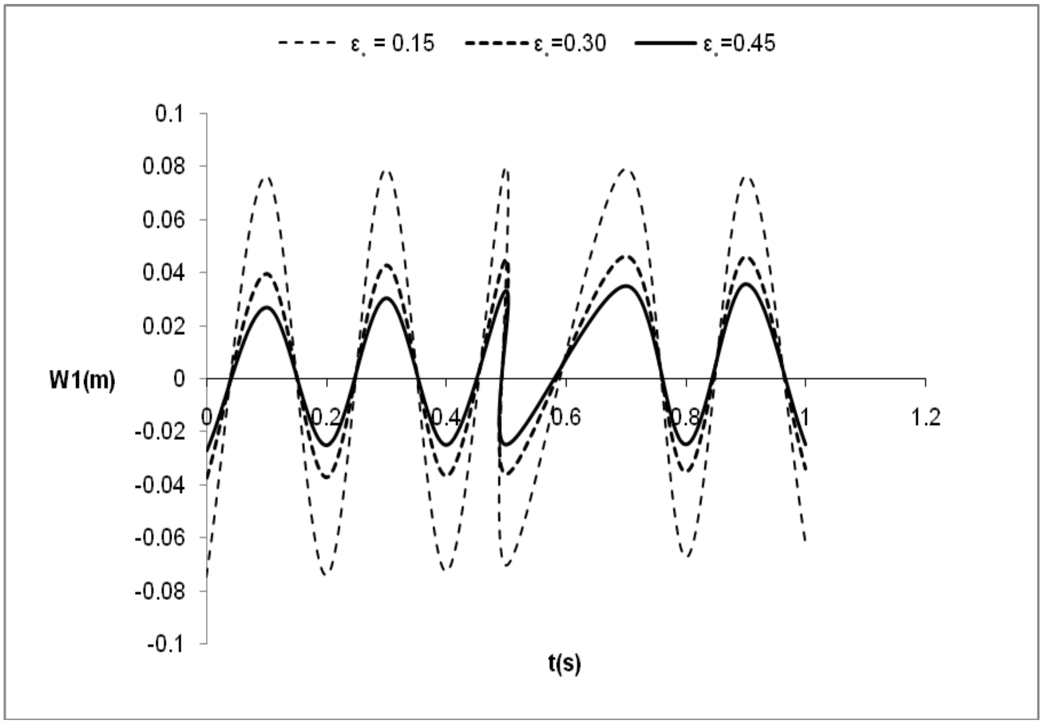

Figure 7: The Response of the Upper Beam Relative to the Lower Beam for Various Values of ε_o (damping coefficient).

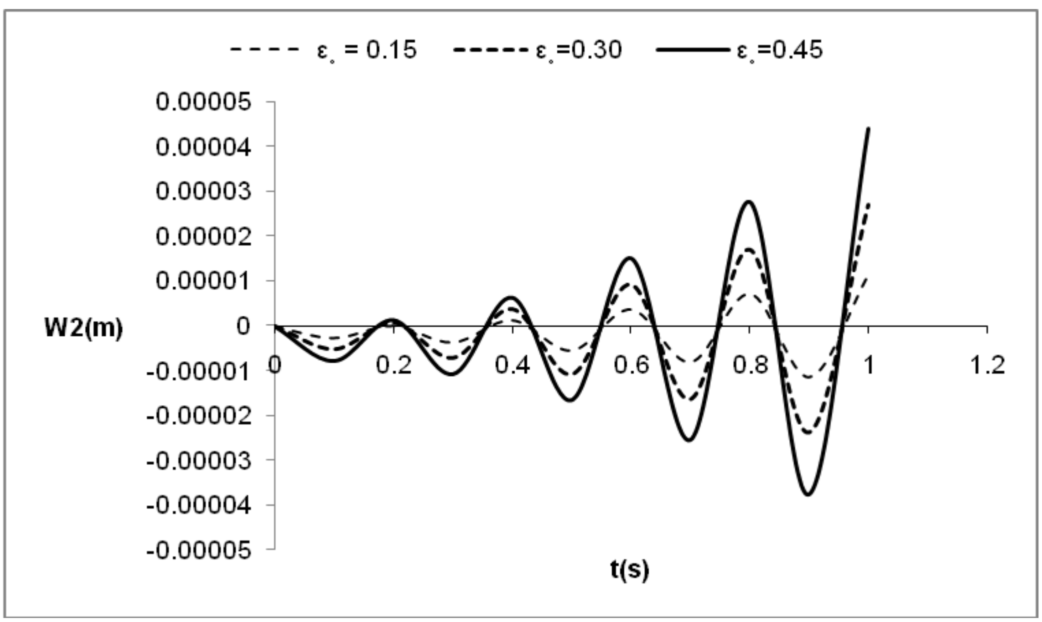

Figure 8: The Response of the Lower Beam for Various Values of ε_o (damping coefficient).

The response of the upper beam relative to the lower beam for various values of the velocity of the moving load at a fixed value of r^2 (rotatory inertia) is shown in Figure 1 while the corresponding one for the lower beam is shown in Figure 2. It is observed that response amplitude of both upper and lower beams increase with increase in the velocity of the moving load at a fixed value of rotatory inertia ($r^2=4$).

The result of the investigation of the effect of the rotatory inertia on the dynamic response of the system is shown in Figures 3 and 4. Figure 3 indicates that the maximum response amplitude of the upper beam increases with increasing values of rotatory inertia. Contrarily, the maximum response amplitude of the lower beam decreases with increasing values of rotatory inertia (Figure 4).

Figure 5 shows the deflections for the upper beam for a velocity of $v = 3.3$ m/s and different values of stiffness coefficient k. From the Figure, it is observed that increasing the values of the stiffness coefficient has irrelevant influence on the response amplitude of the upper beam. On the other hand, as revealed in Figure 6, it is observed that increasing the values of the stiffness coefficient k lead to an increase in the maximum response amplitude of the lower beam.

In the last part of the analysis carried out, we study the dynamic behavior of the beams for various values of damping coefficient as shown in Figures 7 and 8 for the upper and lower beams respectively. Figure 7 shows that increasing the value of the damping coefficient ε_0 decreases the maximum response amplitude of the upper beam. The reverse is the case with the lower beam as depicted by Figure 8.

CONCLUSION

The dynamic response of two identical Rayleigh beams elastically connected together by a flexible core with an attached dash-pot and traversed by a concentrated moving load has been studied in this paper. The governing differential equations of the problem have been solved by using the finite Fourier Integral and Laplace transformation techniques and the prescribed initial and boundary conditions. It is assumed that the beams are of uniform cross-section and of constant mass. Several plots of the response amplitudes of the beams are given. The effects of the velocity of the moving load, rotatory inertia of the beam, stiffness coefficient and damping coefficient of the joining layer on the deflections of the beams are also highlighted. It is found that the amplitude of the deflection of both the upper and lower beams increases with increasing values of velocity of the moving load. Furthermore, it is found that increasing the values of the rotatory inertia leads to an increase in the maximum response amplitude of the upper beam but decreases that of the lower beam. However, the stiffness coefficient has negligible effect on the response amplitude of the upper beam whereas increasing the values of the stiffness coefficient k lead to an increase in the maximum response amplitude of the lower beam.

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ABOUT THE AUTHORS

Professor Jacob A. Gbadeyan is a Professor in the Department of Mathematics, University of Ilorin, Ilorin, Nigeria. He holds a Ph.D. degree in Civil Engineering from the University of Waterloo, Ontario. His research interests are in the areas of moving forces on elastic systems, mathematical modeling of population dynamics and singular perturbation analysis of dynamical systems.

Dr. Moses S. Dada is a Senior Lecturer in the Department of Mathematics of the University of Ilorin, Ilorin, Nigeria. He holds a Ph.D. degree in analytical dynamics. His research areas include moving load problems, Newtonian and Non-Newtonian fluid flow.

Olasunmbo O. Agboola is a member of faculty of the Department of Mathematics, Covenant University, Canaan Land, Ota, Nigeria. He holds a Master of Science (M.Sc.) in Mathematics from the University of Ilorin, Ilorin, Nigeria. His research interests are in analytical dynamics

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