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ORIGINAL ARTICLES

Dynamic Behaviour of a double Rayleigh Beam-System due to uniform partially distributed Moving Load

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ABSTRACT

This paper deals with the dynamic behavior of a double-beam system traversed by a uniform partially distributed moving load. The system is composed of two identical parallel homogeneous simply-supported uniform Rayleigh beams of equal lengths which are continuously connected by a viscoelastic core. The forced vibration problem is solved by the application of the finite Fourier and Laplace integral transformations. Using a numerical example, various plots of the deflections of the beams are presented and discussed for different values of the speed, rotatory inertia and fixed length of the load.

Key words: Double-beam system; Rayleigh beam; Rotatory inertia; and Moving Load.

Introduction

The response problem of a single elastic beam traversed by a moving load at a specified speed has attracted the attention of many researchers since 19^{th} century (See References). Such problems have been extensively studied in connection with machining process, guide-way systems and design of railway bridges (Rao, 2000; Shadnam, *et. al*, 2001). Other practical applications include the modern high speed precision machinery processes (Katz, *et. al*, 1987). Various explicit solutions to different aspects of these problems involving the effect of moving load on a single beam can be found in Fryba (1972), which contains extensive pertinent references. However, and in spite of its practical importance in the fields of civil, mechanical and aerospace engineering, there exists few investigations (when compared to a single-system) involving the vibrations of two beam-models.

Few, notable applications of beam theory to the cases of the vibration of elastically connected two-beam systems include the works of the following scholars (Seelig and Hoppmann II, 1964.a & 1964.b; Kessel, 1966; Kessel and Raske, 1967 &1971; Chonan, 1975; Chonan, 1976; Hamada *et al*., 1983; Yamaguchi and Saito, 1984; Kukla and Skalmierski, 1994; Kukla, 1994; Vu *et al.*, 2000; Oniszczuk, 2000 & 2003; Gbolagade *et al*., 2003; Erol and Gurgoze, 2004; Gbadeyan *et al.*, 2005; Abu-Hilal, 2006 and Gbadeyan *et al.*, 2011).

Specifically, Seelig and Hoppmann II's studies appear to be the first on the vibration of elastically connected double beam systems (Seelig and Hoppmann II, 1964.a, 1964.b). They (1964.a) studied the problem of obtaining the normal mode of a double-beam system. The development and solution of the differential equations of motion of an elastically connected double-beam system subjected to an impulsive load were presented in Seelig and Hoppmann II (1964.b). Kessel (1966) derived resonance conditions for an elastically connected double-beam system in which one of the members is subjected to a moving point load that oscillates longitudinally along the beam about a fixed point. The magnitude of the load is also considered to vary sinusoidally. The analysis indicates that, in addition to the usual case of resonance—which occurs when the frequency of load magnitude variation corresponds to a principal frequency of the system for the *r*th mode of vibration—resonances of the system may occur due to the load-movement frequency. The damped response of an elastically connected double-beam system due to a moving-point load that oscillates longitudinally along one of the members about a fixed point was examined analytically by Kessel and Raske (1967). The deflection equations are determined for a damped double-beam system for two cases. The first being the beams that have individual damping, where the second case deals with introduction of damping as a function of the relative velocity of the two beams.

Chonan (1976) studied the dynamical behaviours of two beams connected with a set of independent springs and subjected to an impulsive load taking the effect of the mass of springs into account. The solution is formulated by the method of the Laplace transformations with respect to both time and space variables, which allows one to analyze all the major performance characteristics in the system subjected to an arbitrarily

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distributed impulsive load. It was found that the amplitudes of deflection and bending moment in the beams decrease as the mass of the springs increases. Vu *et al.* (2000) presented an exact method for solving the forced transverse vibrations of a double-beam system subjected to harmonic excitation. Gbolagade *et al.* (2003) carried out a study on the response of two Euler-Bernoulli beams elastically connected together by a flexible core with an attached dash-pot and traversed by a concentrated force moving with a constant velocity. Oniszczuk (2003) analyzed undamped forced transverse vibrations of an elastically connected double-beam system. The problem was formulated and solved in the case of simply supported beams and the classical modal expansion method was applied. Abu-Hilal (2006) analyzed the dynamic response of a double-beam system traversed by a constant moving load. The system studied consists of two simply supported Euler-Bernoulli beams connected by a viscoelastic layer.

However, in all these previous studies (with the exception of the work in Gbadeyan *et al.* (2005)) the moving load was idealized as a point model, where the interval for the load distribution is neglected and thus the corresponding moving load problems become easier to handle. Nevertheless, Esmailzadeh and Ghorashi (1995) argued that for the moving load problem formulation to be realistic, the interval for the load distribution must be considered. As a matter of fact, no point load exists physically.

The primary motivation for this present paper is derived from this issue which is very important yet unaddressed in most of the previous papers on the subject. In particular, in an earlier paper (Gbadeyan *et. al.*, 2011), the authors carried out an analysis of the dynamics of response of two viscoelastically connected Rayleigh beams under a moving point load for which the load distribution interval is not taken into account. As an extension of that work, this presentation deals with the problem of vibration analysis of the two viscoelastically connected Rayleigh beams traversed by uniform partially distributed moving load. One obvious application for such analysis is the study of vibration of a suspension bridge under a travelling vehicle. Furthermore, the moving load problems involving double beams and a point load can be considered as a special case of the present one, if the load distribution interval is assumed small. The solution technique employed is based on finite Fourier and Laplace integral transformations. Finally, the analysis is illustrated by a numerical example.

Formulation of the Problem:

The Rayleigh beam theory is used here in order to take into account the effect of rotatory inertia, which is usually neglected in Euler-Bernoulli beams.

The structural model of an elastically connected two-beam system, composed of two parallel Rayleigh beams of uniform properties which are joined by a viscoelastic layer, is shown in Fig 1. This layer is simply modelled as a spring-dashpot system, where k and ε_0 are the spring constant and the damping coefficient respectively. The two beams, as shown in Fig. 1, are referred to as upper and lower beam, respectively. The forcing function acting on the upper beam considered here is a uniform partially distributed one as opposed to the concentrated (point) force.

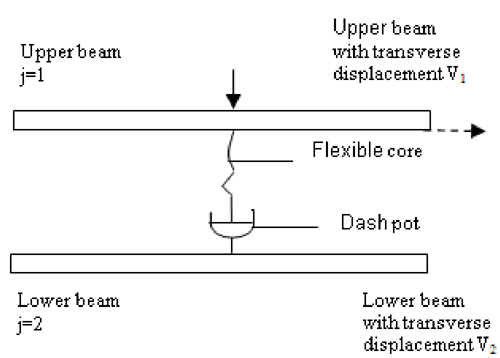

Fig. I: Two uniform Rayleigh Beams connected together by a core.

The transverse vibrations of the double-system shown in Fig. 1 are governed by the following two coupled partial differential equations:

$$EI\frac{\partial^4 V_j(x,t)}{\partial x^4}+\mu\frac{\partial^2 V_j(x,t)}{\partial t^2}+2\mu\omega_b\frac{\partial V_j(x,t)}{\partial t}-r^2\frac{\partial^4 V_j(x,t)}{\partial x^2\partial t^2}$$
$$=\psi_i(x,t)-\left(k+\varepsilon_0\frac{\partial}{\partial t}\right)[V_1(x,t)-V_2(x,t)](-1)^j,\quad j=1,2 \tag{1}$$

where $\Psi_j(x,t)$ is the applied uniform partially distributed moving load defined by

$$\Psi_j(x,t)=\begin{cases}-\dfrac{P}{\varepsilon}\left[H\left(x-\xi+\dfrac{\varepsilon}{2}\right)-H\left(x-\xi-\dfrac{\varepsilon}{2}\right)\right], & j=1\\ 0, & otherwise\end{cases} \tag{2}$$

where $\xi=vt+\dfrac{\varepsilon}{2}$.

The subscript j is attached to the variables associated with the two beams such that with the upper beam, j = 1, and the lower beam, j = 2.
The boundary conditions of the studied simply supported beams are:

$$V_j(0,t)=0=V_j(l,t),\quad j=1,2$$

$$\left.\frac{\partial^2 V_j(x,t)}{\partial x^2}\right|_{x=0}=0=\left.\frac{\partial^2 V_j(x,t)}{\partial x^2}\right|_{x=l},\quad j=1,2 \tag{3}$$

The initial conditions, assumed in a homogeneous form, are

$$V_j(x,0)=0=\frac{\partial V_j(x,0)}{\partial t},\ j=1,2 \tag{4}$$

(since when the beams are at rest they possess neither deflection nor velocity).

Solution of the Forced Vibration Problem:

The forced vibration problem, alluded to, is solved in this section using two integral transform techniques, namely, the Fourier and the Laplace integral transforms as follows:

Analysis I:

Substituting equation (2) into (1) and rewriting the resulting equation, we have

$$EI\frac{\partial^4 V_1(x,t)}{\partial x^4}+\mu\frac{\partial^2 V_1(x,t)}{\partial t^2}+2\mu\omega_b\frac{\partial V_1(x,t)}{\partial t}-r^2\frac{\partial^4 V_1(x,t)}{\partial x^2\partial t^2}$$
$$=-\frac{P}{\varepsilon}\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]+\left(k+\varepsilon_0\frac{\partial}{\partial t}\right)[V_1(x,t)-V_2(x,t)] \tag{5}$$

and

$$EI\frac{\partial^4 V_2(x,t)}{\partial x^4}+\mu\frac{\partial^2 V_2(x,t)}{\partial t^2}+2\mu\omega_b\frac{\partial V_2(x,t)}{\partial t}-r^2\frac{\partial^4 V_2(x,t)}{\partial x^2\partial t^2}=\left(k+\varepsilon_0\frac{\partial}{\partial t}\right)[V_2(x,t)-V_1(x,t)] \tag{6}$$

respectively.

Taking the finite Fourier sine transform of each of the equations (5) and (6) we have

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$$\ddot{\overline{V}}_1(n,t)+b_1\dot{\overline{V}}_1(n,t)+b_2\overline{V}_1(n,t)+b_3\dot{\overline{V}}_2(n,t)+b_4\overline{V}_2(n,t)=b_5\sin\frac{2n\pi\xi}{l} \tag{7}$$

$$\ddot{\overline{V}}_2(n,t)+b_1\dot{\overline{V}}_2(n,t)+b_2\overline{V}_2(n,t)+b_3\dot{\overline{V}}_1(n,t)+b_4\overline{V}_1(n,t)=0 \tag{8}$$

respectively.

Note that to obtain equations (7) and (8), we made use of the following equation:

$$\overline{V_j}(n,t)=\int_0^l V_j(x,t)\sin\frac{n\pi x}{l}dx, n=1,2,3,... \tag{9}$$

whose inverse transform is given as

$$w_j(x,t)=\frac{2}{l}\sum_{n=1}^{\infty}\overline{V}_j(n,t)\sin\frac{n\pi x}{l}dx \tag{10}$$

Also, an overdot denotes differentiation with respect to time t,
and

$$a_1=\mu l^2+r^2(n\pi)^2,\ a_2=\mu l^4+r^2(n\pi l)^2,\ a_3=l^2(2\mu\omega_b-\varepsilon_0),\ a_4=EI(n\pi)^4-kl^4,$$
$$a_5=\varepsilon_0 l^2,\ a_6=kl^2,\ a_7=-\frac{2Pl^3}{n\pi\varepsilon}\sin\frac{2n\pi\varepsilon}{l}, b_1=\frac{a_3}{a_1},\ b_2=\frac{a_4}{a_2},\ b_3=\frac{a_5}{a_1},\ b_4=\frac{a_6}{a_1},\ b_5=\frac{a_7}{a_1} \tag{11}$$

Analysis II:

Next, Laplace integral transform of each of the terms in equations (7) and (8) is taken using the initial conditions (4). To do this end, we apply the following:

$$L\left[\overline{w}_j(n,t)\right]=\overline{\overline{w}}_j(n,s)=\int_0^\infty \overline{w}_j(n,t)e^{-st}dt \tag{12}$$

where, L is the Laplace transform operator and s is the Laplace transform variable, on equations (7) and (8) to obtain

$$c_1\overline{\overline{V}}_1(n,s)+c_2\overline{\overline{V}}_2(n,s)=b_5\left(\frac{b_7s+b_8}{s^2+b_6^2}\right) \tag{13}$$

$$c_1\overline{\overline{V}}_2(n,s)+c_2\overline{\overline{V}}_1(n,s)=0 \tag{14}$$

respectively, (with $c_1=s^2+b_1s+b_2, c_2=b_3s+b_4$) which when solved yields

$$\overline{\overline{V}}_1(n,s)=\frac{b_5(s^2+b_1s+b_2)(b_7s+b_8)}{(s^2+b_6^2)(s^2+d_1s+d_2)(s^2+d_3s+d_4)} \tag{15}$$

And

$$\overline{\overline{V}}_2(n,s)=-\frac{b_5(b_3s+b_4)(b_7s+b_8)}{(s^2+b_6^2)(s^2+d_1s+d_2)(s^2+d_3s+d_4)} \tag{16}$$

such that

$$d_1=b_1-b_3,\ d_2=b_2-b_4,\ d_3=b_1+b_3,\ d_4=b_2+b_4 \tag{17}$$

Analysis III:

The inverse Laplace transformations of the solutions in (15) and (16) are obtained using Maple as follows

$$\overline{V}_1(n,t) = e^{-\frac{1}{2}d_1 t}\left[A\sinh\left(\frac{1}{2}d_5 t\right) + B\cosh\left(\frac{1}{2}d_5 t\right)\right] + e^{-\frac{1}{2}d_3 t}\left[C\sinh\left(\frac{1}{2}d_6 t\right) + D\cosh\left(\frac{1}{2}d_6 t\right)\right] + E\sin b_6 t + F\cos b_6 t \tag{18}$$

and

$$\overline{V}_2(n,t) = e^{-\frac{1}{2}d_1 t}\left[A_1\sinh\left(\frac{1}{2}d_5 t\right) + B_1\cosh\left(\frac{1}{2}d_5 t\right)\right] + e^{-\frac{1}{2}d_3 t}\left[C_1\sinh\left(\frac{1}{2}d_6 t\right) + D_1\cosh\left(\frac{1}{2}d_6 t\right)\right] + E_1\sin b_6 t + F_1\cos b_6 t \tag{19}$$

where,

$$A = \frac{b_5 e_5}{d_5 e_2 e_4},\ B = \frac{b_5 e_8}{e_2 e_4},\ C = \frac{b_5 e_1}{d_6 e_2 e_3},\ D = \frac{b_5 e_9}{e_2 e_3},\ E = \frac{b_5 e_7}{b_6 e_3 e_4},\ F = \frac{b_5 e_6}{e_3 e_4}$$

$$A_1 = \frac{b_5 f_2}{d_5 e_2 e_4},\ B_1 = \frac{b_5 f_5}{e_2 e_4},\ C_1 = \frac{b_5 f_1}{d_6 e_2 e_3},\ D_1 = \frac{b_5 f_6}{e_2 e_3},\ E_1 = \frac{b_5 f_4}{e_2 e_3 e_4},\ F_1 = \frac{b_5 b_6 f_3}{e_3 e_4}$$

$$d_5 = \sqrt{d_1^2 - 4d_2},\ d_6 = \sqrt{d_3^2 - 4d_4},$$

$$\begin{aligned}
e_1 = {} & 2b_8 d_2 d_4^2 + 2b_2 b_8 d_4^2 + b_8 d_3^2 d_4^2 - b_7 d_3 d_4^3 + 2b_7 d_1 d_4^3 + 2b_6^2 b_8 d_4^2 - 2b_1 b_7 d_4^3 + b_2 b_8 d_3^4 - 2b_2 b_8 d_2 d_4 \\
& - b_8 d_1 d_3 d_4^2 - 2b_6^2 b_8 d_2 d_4 - 4b_2 b_8 d_3^2 d_4 + b_2 b_8 d_2 d_3^2 + 3b_2 b_7 d_3 d_4^2 - 2b_2 b_7 d_1 d_4^2 - 2b_2 b_6^2 b_8 d_4 \\
& + 2b_2 b_6^2 b_8 d_2 + 3b_1 b_8 d_3 d_4^2 - 2b_1 b_8 d_1 d_4^2 + 2b_1 b_7 d_2 d_4^2 - 2b_6^2 b_7 d_1 d_4^2 - b_2 b_8 d_1 d_3^3 - b_2 b_7 d_3^3 d_4 \\
& + b_2 b_6^2 b_8 d_3^2 - b_1 b_8 d_3^3 d_4 + b_1 b_7 d_3^2 d_4^2 + 2b_1 b_6^2 b_7 d_4^2 + 3b_2 b_8 d_1 d_3 d_4 - b_2 b_7 d_2 d_3 d_4 - b_1 b_8 d_2 d_3 d_4 \\
& + 3b_6^2 b_7 d_2 d_3 d_4 + b_2 b_7 d_1 d_3^2 d_4 - b_2 b_6^2 b_8 d_1 d_3 - b_2 b_6^2 b_7 d_3 d_4 + 2b_2 b_6^2 b_7 d_1 d_4 + b_1 b_8 d_1 d_3^2 d_4 \\
& - b_1 b_7 d_1 d_3 d_4^2 - b_1 b_6^2 b_8 d_3 d_4 + 2b_1 b_6^2 b_8 d_1 d_4 - 2b_1 b_6^2 b_7 d_2 d_4 - b_2 b_6^2 b_7 d_2 d_3 - b_1 b_6^2 b_8 d_2 d_3 \\
& - 2b_8 d_4^3 - b_7 d_4^2 d_2 d_3 - d_4^2 b_6^2 b_7 d_3 + b_6^2 d_3^2 d_2 b_8 - d_4 b_6^2 d_1 b_1 b_7 d_3 + b_6^2 d_3^2 d_2 b_1 b_7 + d_4 b_6^2 d_1 d_3^2 b_7 \\
& - d_4 b_6^2 d_1 b_8 d_3 - b_6^2 d_3^3 d_2 b_7,
\end{aligned}$$

$$e_2 = d_4 d_1^2 - 2d_2 d_4 - d_4 d_1 d_3 + d_2 d_3^2 - d_1 d_2 d_3 + d_4^2 + d_2^2,$$

$$e_3 = d_4^2 + b_6^2 d_3^2 - 2b_6^2 d_4 + b_6^4,\ e_4 = -2d_2 b_6^2 + b_6^2 d_1^2 + d_2^2 + b_6^4,$$

$$\begin{aligned}
e_5 = {} & -2b_2 b_8 d_2 d_4 - 2b_6^2 b_8 d_2 d_4 + 2b_2 b_6^2 b_8 d_4 - 2b_2 b_6^2 b_8 d_2 - b_2 b_6^2 b_8 d_1 d_3 - b_2 b_6^2 b_7 d_1 d_4 - b_1 b_6^2 b_8 d_1 d_4 \\
& - 2b_1 b_6^2 b_7 d_2 d_4 + 2b_2 b_8 d_2^2 - 2b_1 b_7 d_3^2 + b_8 d_1^2 d_2^2 - b_7 d_1 d_2^3 + b_2 b_8 d_1^4 + 2b_6^2 b_8 d_2^2 + 2b_7 d_2^3 d_3 \\
& + 2b_8 d_2^2 d_4 - 2b_8 d_2^3 + 3b_2 b_8 d_1 d_2 d_3 - b_2 b_7 d_1 d_2 d_4 - b_1 b_8 d_1 d_2 d_4 + b_2 b_7 d_1^2 d_2 d_3 + b_1 b_8 d_1^2 d_3 \\
& - b_1 b_7 d_2^2 d_3 + 3b_6^2 b_7 d_1 d_2 d_4 + 2b_2 b_6^2 b_7 d_2 d_3 - b_2 b_6^2 b_7 d_1 d_2 + 2b_1 b_6^2 b_8 d_2 d_3 - b_1 b_6^2 b_8 d_1 d_2 \\
& + 3b_2 b_7 d_1 d_2^2 + 3b_1 b_8 d_1 d_2^2 - 4b_2 b_8 d_1^2 d_2 - b_2 b_7 d_1^3 d_2 - b_1 b_8 d_1^3 d_2 + b_1 b_7 d_1^2 - b_8 d_1 d_2^2 d_3 \\
& + b_2 b_8 d_1^2 d_4 - 2b_2 b_7 d_2^2 d_3 - 2b_1 b_8 d_2^2 d_3 + 2b_1 b_7 d_2^2 d_4 - b_2 b_8 d_1^3 - 2b_6^2 b_7 d_2^2 d_3 + b_2 b_6^2 b_8 d_1^2 \\
& + 2b_1 b_6^2 b_7 d_2^2 - d_2^2 d_4 d_1 b_7 - b_6^2 d_3 d_2 b_1 b_7 d_1 - b_6^2 b_7 d_2^2 d_1 + d_4 b_6^2 d_1^2 b \ - b_6^2 b_7 d_1^3 d_4 + d_4 b_6^2 d_1^2 b_1 b_7 \\
& + b_6^2 b_7 d_1^2 d_2 d_3 - b_6^2 d_3 d_2 b_8 d_1,
\end{aligned}$$

$$\begin{aligned}
e_6 = {} & b_6^2 d_2 d_3 b_1 b_7 + d_1 d_4 b_6^2 b_1 b_7 + b_6^2 d_1 b_2 b_8 + b_6^2 d_2 d_3 b_8 - d_2 d_3 b_2 b_8 - d_2 b_6^2 b_1 b_8 - d_2 b_6^2 b_2 b_7 + b_6^2 d_3 b_2 b_8 \\
& + d_4 b_6^2 d_1 b_8 - d_4 b_6^2 b_2 b_7 - d_2 d_4 b_6^2 b_7 - d_4 b_6^2 b_1 b_8 - d_1 d_4 b_2 b_8 + d_2 d_4 b_2 b_7 + d_2 d_4 b_1 b_8 - b_6^4 d_3 b_1 b_7 \\
& - b_6^4 d_1 b_1 b_7 + b_6^4 d_1 d_3 b_7 - b_2 b_6^2 b_7 d_1 d_3 - b_1 b_6^2 b_8 d_1 d_3 - b_6^6 b_7 - b_6^4 d_3 b_8 + d_2 b_6^4 b_7 + b_6^4 d_4 b_7 + b_6^4 b_1 b_8 \\
& + b_6^4 b_2 b_7 - b_6^4 d_1 b_8,
\end{aligned}$$

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$$
\begin{aligned}
e_7 =\ & b_2b_8d_2d_4 - b_6^2b_8d_2d_4 - b_2b_6^2b_8d_4 - b_2b_6^2b_8d_2 - b_2b_6^2b_8d_1d_3 + b_2b_6^2b_7d_1d_4 + b_1b_6^2b_8d_1d_4 - b_1b_6^2b_7d_2d_4 \\
& + b_2b_6^2b_7d_2d_3 + +b_1b_6^2b_8d_2d_3 - b_6^4b_7d_2d_3 - b_6^4b_7d_1d_4 + b_6^4b_1b_7d_4 + b_6^4b_1b_7d_2 + b_6^4b_8d_1d_3 - b_2b_6^4b_7d_3 \\
& - b_2b_6^4b_7d_1d_3 - b_6^6b_8 + b_6^4b_2b_8 + d_4b_6^4b_8 - b_1b_6^6b_7 + b_6^6b_7d_1 + b_6^6b_7d_3, \\
e_8 =\ & b_6^2d_3d_2b_1b_7 - d_4b_6^2d_1b_1b_7 - b_8d_2^2d_3 + b_6^2d_1b_2b_8 + b_6^2d_2d_3b_8 + d_4b_6^2b_2b_7 - d_2d_4b_6^2b_7 + d_4b_6^2b_1b_8 \\
& + d_1d_4b_2b_8 - d_2d_4b_2b_7 - d_2d_4b_1b_8 + d_2^2b_2b_7 + d_2^2b_1b_8 + d_1d_2^2b_8 + d_1^3b_2b_8 + b_2b_7d_1d_2d_3 \\
& + b_1b_8d_1d_2d_3 - b_6^2b_7d_1d_2d_3 - 2d_1d_2b_2b_8 - d_1^2d_2b_2b_7 - d_1^2d_2b_1b_8 + d_1d_2^2b_1b_7 - b_2b_8d_1^2d_3 \\
& - b_1b_7d_2^2d_3 + b_6^2b_7d_1^2d_4 + b_7d_2^2d_4 + b_6^2b_7d_2^2 - d_2^3b_7, \\
e_9 =\ & b_6^2d_2d_3b_1b_7 + d_1d_3d_4b_1b_8 + d_1d_4b_6^2b_1b_7 - d_1d_3d_4b_6^2b_7 + d_1d_3d_4b_2b_7 + d_3^2b_2b_8 - b_6^2d_1b_2b_8 + b_6^2d_3^2d_2b_7 \\
& - b_6^2d_2d_3b_1 - d_1d_3^2b_2b_8 + d_2d_3b_2b_8 + d_2b_6^2b_1b_8 + d_2b_6^2b_2b_7 + b_6^2d_3b_1b_2 + d_4^2d_3b_1b_7 - d_6^2d_1b_1b_7 \\
& - 2d_3d_4b_2b_8 + d_4b_6^2d_1b_8 - d_4b_6^2b_2b_7 - d_2d_4b_6^2b_7 - d_4b_6^2b_1b_8 + d_1d_4b_2b_8 - d_4d_3^2b_1b_8 - d_4d_3^2b_2b_7 \\
& - d_2d_4b_2b_7 - d_2d_4b_1b_8 - d_4^3b_7 + d_4^2d_2b_7 + d_4^2b_2b_7 + d_4^2b_1b_8 - d_4^2d_1b_8 + d_4^2b_6^2b_7 + d_4^2d_3b_8, \\
f_1 =\ & -2b_4b_8d_4^2 + 2b_3b_7d_4^3 - b_4b_8d_3^2d_3^4 - 3b_4b_8d_1d_3d_4 + b_4b_7d_2d_3d_4 + b_3b_8d_2d_3d_4 - b_4b_7d_1d_3^2d_4 \\
& + b_4b_6^2b_8d_1d_3 + b_4b_6^2b_7d_3d_4 - 2b_4b_6^2b_7d_1d_4 - b_3b_8d_1d_3^2d_4 + b_3b_7d_1d_3d_4^2 + b_3b_6^2b_8d_3d_4 \\
& - 2b_3b_6^2b_8d_1d_4 + 2b_3b_6^2b_7d_2d_4 + b_4b_6^2b_7d_2d_3 + b_3b_6^2b_8d_2d_3 + 2b_4b_8d_2d_4 + 4b_4b_8d_3^2d_4 \\
& - b_4b_8d_2d_3^2 - 3b_4b_7d_3d_4^2 + 2b_4b_7d_1d_4^2 + 2b_4b_6^2b_8d_4 - 2b_4b_6^2b_8d_2 - 3b_3b_8d_3d_4^2 + 2b_3b_8d_1d_4^2 \\
& - 2b_3b_7d_2d_4^2 + b_4b_8d_1d_3^3 + b_4b_7d_3^3d_4 - b_4b_6^2b_8d_3^2 + b_3b_8d_3^3d_4 - b_3b_7d_3^2d_4^2 - 2b_3b_6^2b_7d_4^2 \\
& + b_6^2b_3b_7d_1d_3d_4 - d_2d_3^2b_6^2b_3b_7, \\
f_2 =\ & b_4b_6^2b_8d_1d_3 + b_4b_6^2b_7d_1d_4 + b_3b_6^2b_8d_1d_4 + 2b_3b_6^2b_7d_2d_4 - 2b_4b_8d_2^2 - 3b_4b_7d_1d_2^2 - 3b_3b_8d_1d_2^2 \\
& + 4b_4b_8d_1^2d_2 + b_4b_7d_1^3d_2 + b_3b_8d_1^3d_2 - b_3b_7d_1^2d_2^2 + b_3b_7d_1d_2^2d_3 - 3b_4b_8d_1d_2d_3 + b_4b_7d_1d_2d_4 \\
& + b_3b_8d_1d_2d_4 - b_4b_7d_1^2d_2d_3 - b_3b_8d_1^2d_2d_3 + 2b_3b_7d_2^3 - b_4b_8d_1^4 - b_4b_8d_1^2d_4 + 2b_4b_7d_2^2d_3 \\
& + 2b_3b_8d_2^2d_3 - 2b_3b_7d_2^2d_4 + b_4b_8d_1^3d_3 - b_4b_6^2b_8d_1^2 - 2b_3b_6^2b_7d_2^2 - 2b_4b_6^2b_7d_2d_3 + b_4b_6^2b_7d_1d_2 \\
& - 2b_3b_6^2b_8d_2d_3 + b_3b_6^2b_8d_1d_2 + 2b_4b_8d_2d_4 - 2b_4b_6^2b_8d_4 + 2b_4b_6^2b_8d_2 + b_6^2b_3b_7d_1d_2d_3 - b_6^2d_1^2b_3b_7d_4, \\
f_3 =\ & b_4b_8d_2d_3 + b_4b_8d_1d_4 - b_4b_7d_2d_4 - b_3b_8d_2d_4 - b_6^2b_4b_8d_3 - b_6^2b_4b_8d_1 + b_6^2b_4b_7d_4 + b_6^2b_4b_7d_2 \\
& + b_6^2b_3b_8d_4 + b_6^2b_3b_8d_2 + b_6^2b_4b_7d_1d_3 - b_6^4b_4b_7 + b_6^2b_3b_8d_1d_3 - b_6^2b_3b_7d_2d_3 - b_6^2b_3b_7d_1d_4 \\
& - b_6^4b_3b_8 + b_6^4b_3b_7d_3 + b_6^4b_3b_7d_1, \\
f_4 =\ & -b_4b_8d_2d_4 + b_6^2b_4b_8d_4 + b_6^2b_4b_8d_2 - b_6^2b_4b_8 + b_6^2b_4b_8d_1d_3 - b_6^2b_4b_7d_2d_3 - b_6^2b_4b_7d_1d_4 + b_6^4b_4b_7d_3 \\
& + b_6^4b_4b_7d_1 - b_6^2b_3b_8d_2d_3 - b_6^2b_3b_8d_1d_4 + b_6^2b_3b_7d_2d_4 + b_6^4b_3b_8d_3 + b_6^4b_3b_8d_1 - b_6^4b_3b_7d_4 \\
& - b_6^4b_3b_7d_2 + b_6^4b_3b_7 - b_6^4b_3b_7d_1d_3, \\
f_5 =\ & -b_4b_8d_2d_3 - b_4b_8d_1d_4 + 2b_4b_8d_1d_2 + b_4b_7d_2d_4 - d_2^2b_4b_7 + b_3b_8d_2d_4 - d_2^2b_3b_8 + d_1^2b_4b_8d_3 - d_1^3b_4b_8 \\
& - b_4b_7d_1d_2d_3 + d_1^2b_4b_7d_2 + b_6^2b_4b_8d_3 - b_6^2b_4b_8d_1 - b_6^2b_4b_7d_4d_4 + b_6^2b_4b_7d_2 - b_3b_8d_1d_2d_3 \\
& + d_1^2b_3b_8d_2 + d_2^2b_3b_7d_3 - d_2^2b_3b_7d_1 - b_2^6b_3b_8d_4 + b_2^6b_3b_8d_2 - b_2^6b_3b_7d_2d_3 + b_2^6b_3b_7d_1d_4, \\
f_6 =\ & -d_2^4b_3b_7d_3 - d_3^3b_4b_8 - d_2^4b_3b_8 - d_2^4b_4b_7 + d_3^2b_4b_7d_4 + d_3^2b_4b_8d_1 + d_3^2b_3b_8d_4 + d_4^2b_3b_7d_1 + b_6^2b_3b_8d_4 \\
& + b_3b_8d_2d_4 + b_6^2b_4b_7d_4 + b_6^2b_4b_8d_1 + b_4b_7d_2d_4 + b_6^2b_3b_7d_2d_3 - b_3b_8d_1d_3d_4 - b_4b_7d_1d_3d_4 \\
& - b_6^2b_3b_7d_1d_4 - b_6^2b_3b_8d_2 - b_6^2b_4b_7d_2 - b_4b_8d_1d_4 - b_6^2b_4b_8d_3 + 2b_4b_8d_3d_4 - b_4b_8d_2d_3
\end{aligned}
$$

Finally, using the inverse Fourier transformation (7) one readily obtains

$$V_1(x,t) = \frac{2}{l}\sum_{n=1}^{\infty}\overline{V}_1(n,t)\sin\frac{n\pi x}{l} \tag{19}$$

and

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$$V_2(x,t)=\frac{2}{l}\sum_{n=1}^{\infty}\overline{V}_2(n,t)\sin\frac{n\pi x}{l} \tag{20}$$

respectively. In conclusion, equation (19) is the desired transverse deflection of the upper beam relative to the lower one while equation (20) represents the corresponding transverse deflection of the lower beam.

Numerical Example:

Making use of the theoretical analysis described in the previous sections and in order to assess the effects of speed of the moving load, rotatory inertia and fixed length of the load on the deflection of the beams, a numerical example is considered here for illustrative purposes. The numerical values representing the geometric and material properties of the two-beam system are chosen as follows: $\pi = 22/7$, $EI = 16000Nm^2$, $P = 20N$, $\mu = 0.075$, $\varepsilon_0 = 0.15$, $k = 0.2$, $l = 6.0m$, $\omega_b = 0$ (*undamped case*), $n = 1$, $x = l/2$ and $\varepsilon = 0.1m$. The cases when the speed (v) of the moving load assumes the values $6.3ms^{-1}$, $9.3ms^{-1}$ and $12.3ms^{-1}$ for the following values of r^2 (rotatory inertia effect): 0.031, 4.0 and 8.0 are considered.

The transverse deflections of the upper beam relative to the lower beam and that of the lower beam due to the uniform partially distributed moving load for various values of speed, rotatory inertia, fixed length of the load are shown in Figs. 2 - 7.

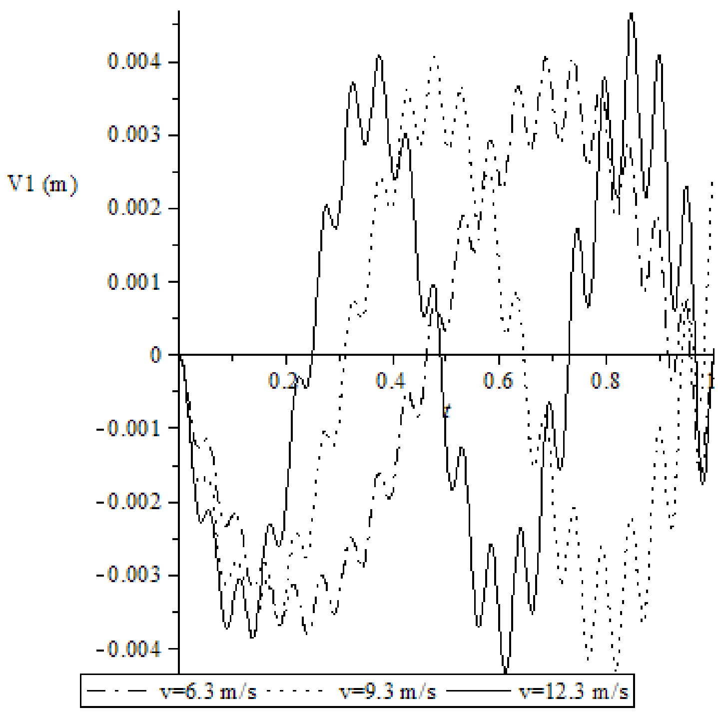

Fig. 2: The deflection of the upper beam relative to the lower beam with varying speed (v) of the moving load at fixed values of r^2 (rotatory inertia), i. e. $r^2 = 4$ and fixed length of the load, $\varepsilon = 0.1$.

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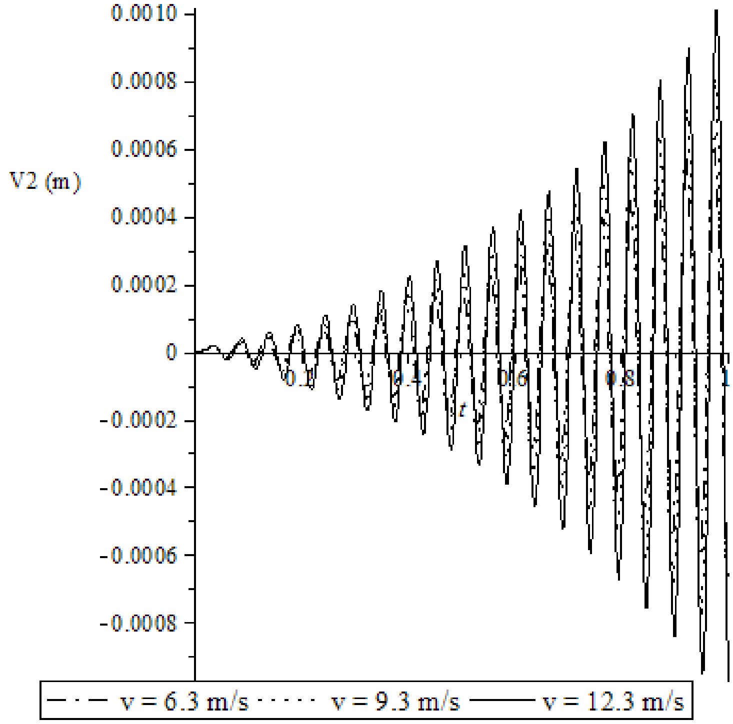

Fig. 3: The response of the lower beam with varying speed of the moving load at a fixed value of r^2 (rotatory inertia), i. e. r^2=4 and fixed length of the load, $\varepsilon = 0.1$.

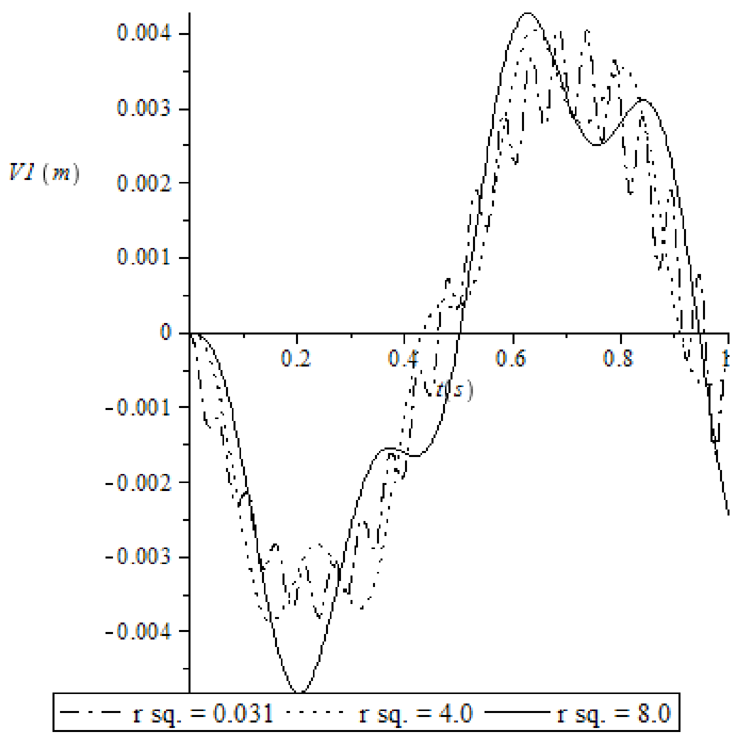

Fig. 4: The response of the upper beam relative to the lower beam with varying rotator inertia when the load moves at v = 6.3 m/s.

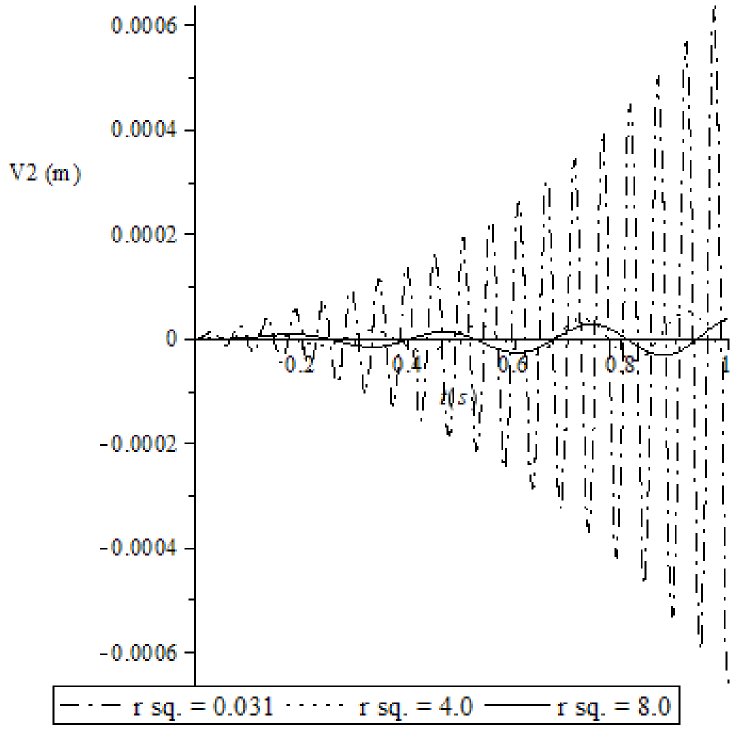

Fig. 5: The response of the lower beam with varying rotatory inertia when the load moves at v = 6.3 m/s

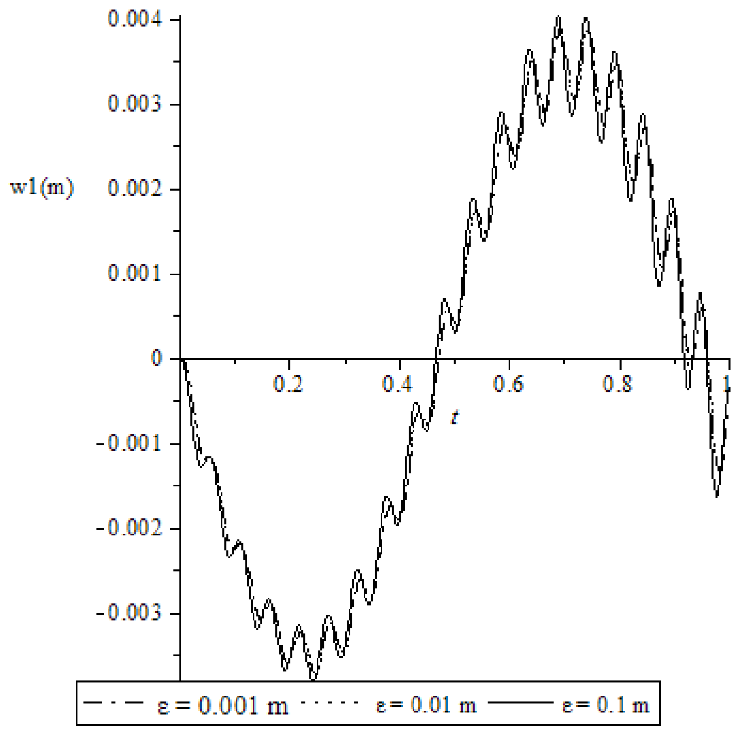

Fig. 6: The response of the upper beam relative to the lower beam with varying length of the load, ε

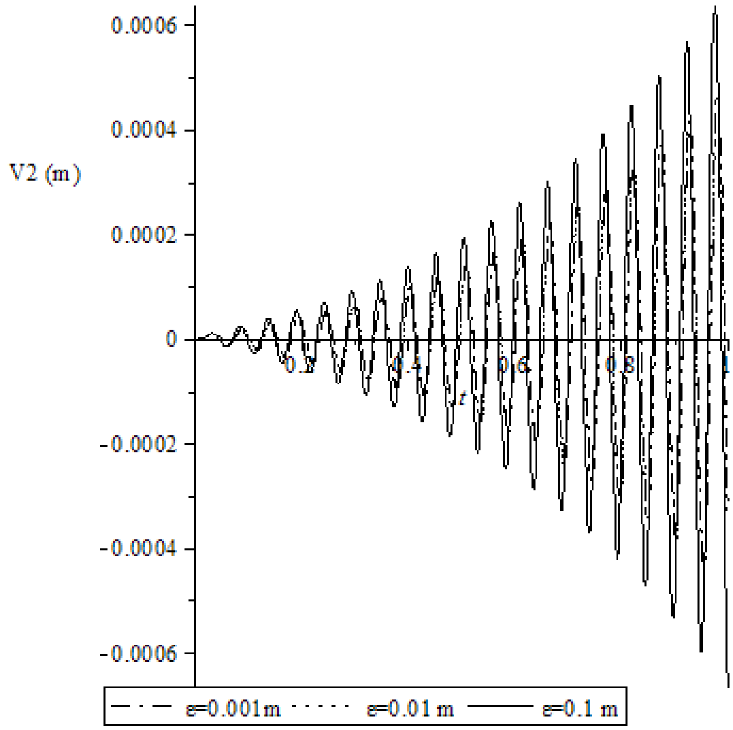

Fig. 7: The response of the lower beam with varying length of the load, ε

Discussion of Results:

Figs. 2 and 3 show the effect of the speed of the moving load on the maximum amplitude of deflection of the beams. As can be observed, the maximum amplitudes of deflections of the two beams (upper and lower beams) increase as the speed of the moving load is increased at fixed values of both the rotary inertia (r^2) and fixed length of the load (ε).

The result of the investigation of the influence of the rotatory inertia on the dynamic response of the system is illustrated in Figs. 4 and 5. Fig. 4 indicates that the maximum amplitude of deflection of the upper beam increases as the value of the rotatory inertia increases. Contrarily, high rotatory inertia causes the maximum amplitude of deflection of the lower beam to increase. (See Fig. 5)

Figs. 6 and 7 presents the effect of the fixed length of the load (ε) on the structural model under investigation. It is cleared that the amplitudes of deflections of both upper and lower beams increase due to the increase of the fixed length of the load.

Conclusion:

This paper is devoted to the dynamic analysis of forced transverse vibrations of a viscoelastically connected simply supported double beam-system using the Rayleigh beam theory. It is assumed that the two beams are identical, parallel and homogeneous. The solutions of the problem formulated for the beams subjected to a uniform partially distributed moving load are found by applying the Fourier and Laplace integral transform techniques. To verify the analysis performed, a numerical example is presented and discussed. It is found that the maximum amplitudes of deflection of the beams increase as the speed of the moving load increases. It is also found that the maximum amplitude of deflections of the upper beam increases while that of the lower beam decreases due to the increase of the rotatory inertia. Finally, it is evident that the amplitudes of deflection of both beams increase with increase in the fixed length of the load.

Nomenclature:

EI : Flexural rigidity of the beam.

E : Young's modulus of elasticity.

I : Moment of inertia of the cross-sectional area of the beam.

μ : Constant mass per unit length of the beam.
k : Spring constant of the viscoelastic layer.
ε_0 : Damping coefficient of the viscoelastic layer.
$V_j(x,t)$: Transverse defelction of the jth beam at point x and time t.
x: Length (axial) co-ordinate with the initial value at the left-hand end of the beam.
t: Time coordinate with the origin at the instant of the load arriving on the beam.
ω_b: Circular frequency of the damping of the beam.
r : Radius of gyration of beam cross-section (m).
$\Psi_i(x,t)$: Uniform partially distributed moving load.
$H(.)$: Heaviside function.
v: Speed of the load motion.
ε: Fixed length of the load.
P: Magnitude of the moving load.
l: Length (span) of the beam.

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