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A Regular Perturbation Analysis Of The Non-Linear Contaminant Transport Equation With An Initial And Instantaneous Point Source.

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Abstract: In this research work, we provide a regular perturbation analysis of a non – linear equation arising in contaminant transport. The equation is characterised by advection, diffusion and absorption. Assuming the adsorption term is modelled by a Freundlich isotherm it can be non-linear in concentration and non-differentiable as the concentration approaches zero. We consider the approximation of this equation using a regular perturbation and thereby solving the resulting linear equations analytically.

Key words: perturbation, contaminant, advection, diffusion, adsorption.

INTRODUCTION

The flow and spread of fluids through porous media such as soils, bead packing, ceramics and concrete plays an important role in a variety of environmental and technological processes. Examples include the spreading and clean up of underground hazardous waste, oil recovery separation processes such as chromatography and catalysis, and degradation of building materials.

Also in many fluid pollution accidents, early detection is important for taking swift and appropriate measures. To predict the fate of such pollutants during their transport has become an ever increasing job for hydro geologist and applied scientist. The problem is to define the flow lines of ground water in the aquifers, the travel times of water along the flow lines and to predict the chemical reaction which alters the concentrations during transport.

Several studies has been carried out in the study of hydro dynamics dispersion with various boundary conditions, these include the work of Batu (1989) which provides a generalized analytical solution for the two dimensional case, Aiyesimi (2004) studied the mathematical analysis of environmental pollution of the Freudlich non-linear contaminant transport formulation. Gideon and Aiyesimi (2005) studied the influence of retardation factor on the non-linear contaminant flow. Makinde and Chinyoka (2010) transient analysis of pollutant dispersion in a cylindrical pipe with a nonlinear waste discharge concentration. Most of the work done in the past either neglects the nonlinear term or considers it a constant.

In this paper, we provide a regular perturbation analysis for constant initial concentration for the non – linear contaminant transport equations by expanding the nonlinearity as a series of perturbation thereby obtaining more accurate results.

Formulation of the Problem:

We consider the flow of an incompressible fluid through a homogenous, saturated porous medium where the fluid is contaminated by a solute with concentration $c(x,t)$. Assume that flow is steady and that transport is described by advection, molecular diffusion and mechanical dispersion. Also, flow is one dimensional and in the direction of the positive $x-axis.$ under the assumptions, mass conservation of the contaminant gives the equation

$$c_t + \frac{p_b}{n} S_t + uc_x - Dc_{xx} = 0 \qquad 0 < x < 1, \qquad t > 0 \qquad (1.0)$$

Where $S(x,t)$ is the mass of the contaminant absorbed on the solid matrix per unit mass of the solid. $P_b > 0$ is the bulk density of the porous medium, $n > 0$ is the porosity, it is the effective fluid velocity and $D > 0$ accounts for the molecular diffusion and mechanical dispersion.

The chemical reactions describing adsorption may be fast, that is at equilibrium or slow that is not at equilibrium Dawson (1989).

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Problem Solution:

Consider the non-linear flow equation (1.0)

$$c_t + Uc_x + \frac{P_b}{n} S_t - Dc_{xx} = 0 \qquad (1.0)$$

Which can be written as

$$\frac{\partial c}{\partial t} + \frac{\partial \phi(c)}{\partial t} + U \frac{\partial c}{\partial t} - D \frac{\partial^2 c}{\partial x^2} = 0 \qquad (1.1)$$

Which can be written as

$$\frac{\partial c}{\partial t} + \varepsilon \frac{\partial c}{\partial t} + U \frac{\partial c}{\partial x} - D \frac{\partial^2 c}{\partial x^2} = 0 \qquad (1.2)$$

Expanding (1.2) it terms of ε *we have*

$$c(x,t) = C^0(x,t) + \varepsilon C^1(x,t) \qquad (1.3)$$

We have the following order ε^0 *and* ε^1 equations

$$C_t^0 + UC_x^0 - DC_{xx}^0 = 0 \qquad (1.4)$$

And

$$C_t^0 + C_t^1 + UC_x^1 - DC_{xx}^1 = 0$$

$$\Rightarrow C_t^1 + UC_x^1 - DC_{xx}^1 = -C_t^0 \qquad (1.5)$$

Solution of the Order ε^0 Equation:

We shall proceed to provide an analytical solution to the order ε^0 equation

$$\frac{\partial C^0}{\partial t} + U \frac{\partial C^0}{\partial x} - D \frac{\partial^2 C}{\partial x^2} = 0$$

$$\begin{aligned} &subject\ \ to \\ &C(0,t) = 1 \\ &C(L,t) = 0 \\ &C(x,0) = 1 \end{aligned} \qquad (1.6)$$

Applying the method of separation of variables we have

$$C^0(x,t) = \sum_{n=1}^{\infty} \frac{-2}{n\pi}\left[(-1)^n - 1\right]\ell^{\frac{u}{2D}x - \lambda^2 t} \sin\frac{n\pi}{L} + \frac{1}{\ell^{\frac{u}{D}}} - 1\left[\ell^{\frac{u}{D}L} - \ell^{\frac{u}{D}x}\right] \qquad (1.7)$$

Solution of the Order ε^1 Equation:

By putting $C^0(x,t)$ in (1.5) we have

$$\frac{\partial c^{(1)}}{\partial t} + U \frac{\partial^2 c^{(1)}}{\partial x} - D \frac{\partial^2 c^1}{\partial x^2} = \lambda^2 \sum_{n=1}^{\infty} \left\{ \frac{-2}{n\pi}\left[(-1)^n - 1\right]\ell^{\frac{ux}{2D} - \lambda^2 t} \sin\frac{n\pi x}{L} \right\} \qquad (1.8)$$

Solving (1.8) analytically we have subject to the homogenous conditions

$$C(0,t) = 0 \qquad and \qquad C(x,0) = 1$$

$$C(L,t) = 0$$

We have

$$C^1(x,t) = \sum_{n=1}^{\infty}\left[\frac{4\lambda^2 \ell^{-\lambda^2 t}}{\pi\left(k-\lambda^2\right)} + \frac{\pi\left(k-\lambda^2\right) - \lambda^2 \ell^{-kt}}{\pi\left(k-\lambda^2\right)}\right]\ell^{\frac{u}{2D}x}\sin\frac{n\pi x}{L}$$

Where $k = \dfrac{3u^2}{4D} - \dfrac{Dn^2\pi^2}{L}$

$$\lambda^2 = \frac{D}{4}\left[\frac{n^2\pi^2}{L^2} - \frac{u^2}{D^2}\right].$$

Therefore the approximate solution becomes

$$C(x,t) = C^0(x,t) + \varepsilon C^1(x,t).$$

RESULTS AND DISCUSSION

The graph of concentration against distance is shown in *fig.1* for various values of dispersion coefficient. The values for the parameters are $t = 0.3,\ u = 0.05,\ L = 10,\ \pi = 3.142$ and the perturbation parameter $\varepsilon = 0.001.$ It is observed that (1) as the dispersion coefficient D increases from 0.2 to 1 the concentration profile also increases. (2) the concentration decreases steadily to zero until a steady state is achieved along the length of the channel.

In *fig.2* we have a plot of concentration against distance for various values of the perturbation term; it is also observed that the concentration profile increases as the perturbation term also increases. And fig.3 shows the influence of the advection term u, it is observed that as the value of u increases from 0.03 to 0.11 the concentration decreases.

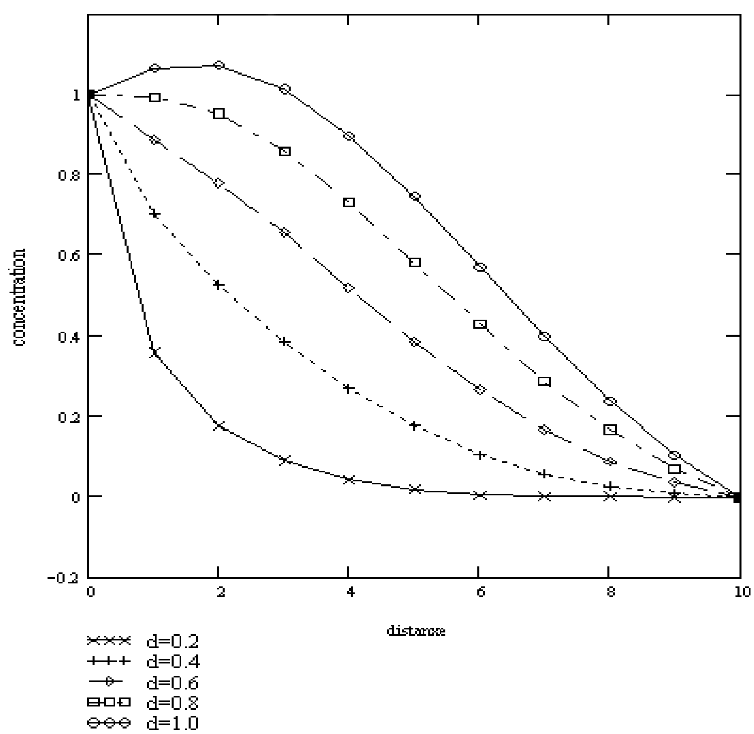

Fig. 1: Concentration Versus Distance

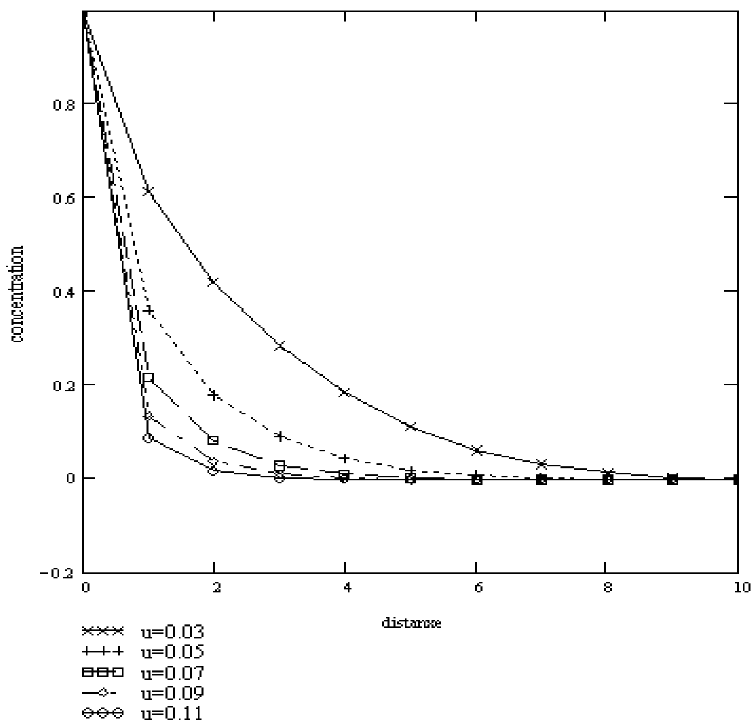

Fig. 2: Concentration Versus Distance

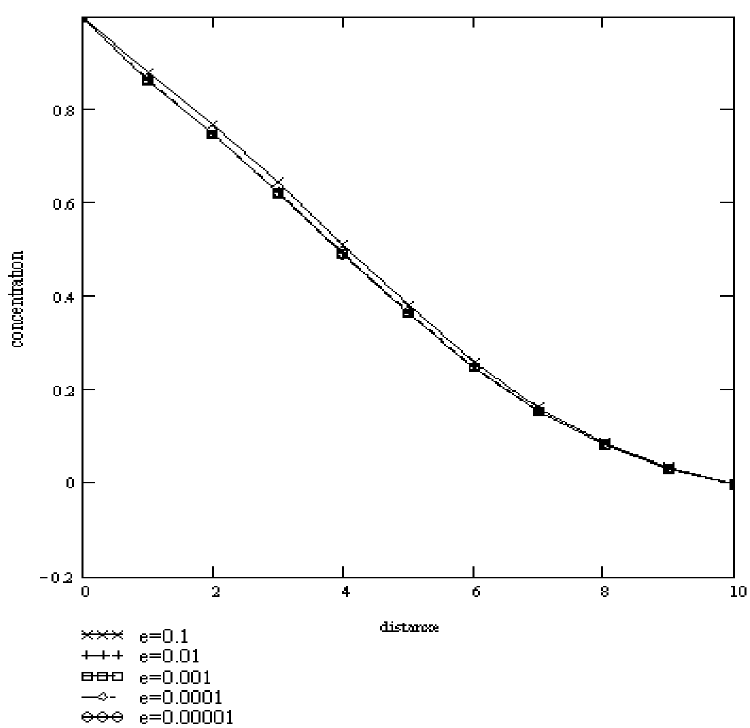

Fig. 3: Concentration Versus Distance

Conclusions:

The problem of nonlinear contaminant transport with an initial and instantaneous point source has been solved using the method of regular perturbation and it is observed that the concentration increases with the dispersion coefficient and decreases with advection term.

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