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Abstract

This paper extends a generic method to design a port-Hamiltonian formulation modeling all geometric interconnection structures of a physical switching system with varying constraints. A non-minimal kernel representation of this family of structures (named Dirac structures) is presented. It is derived from the parameterized incidence matrices which are a mathematical representation of the primal and dual dynamic network graphs associated with the system. This representation has the advantage of making it possible to model complex physical switching systems with varying constraints and to fall within the framework of passivity-based control.

Keywords: modeling, port-Hamiltonian systems, network graph, family of geometric interconnection structures, incidence matrix, energy exchanges.

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1. Introduction

Network graphs have been used to model physical switching systems in various domains, such as energy, information, formation flying or people transportation. Topological changes such as edges or vertices addition or removal, happen in the graph in case of disturbances, for example when equipment fails or railway lines are unavailable. In a physical switching system with varying constraints (PSS), the switches are seen as ideal elements whose function is to change the interconnection of the functional elements, according to certain discrete parameters. Then, the topology of the PSS may change instantaneously depending on the discrete parameters (Van der Schaft & Schumacher, 2000).

We shall consider in this paper a class of systems where the topology is associated with a physical modeling approach based on the use of energy, using the port-Hamiltonian framework (Maschke, Van der Schaft & Breedveld, 1992). Passivity-based control methods can thus be developed within this port-Hamiltonian framework.

This energy-based approach used here, is related to other works on linear switched Hamiltonian systems (Gerritsen, Van der Schaft & Heemels, 2002), hybrid Hamiltonian systems for electrical circuits (Jeltsema, Scherpen & Klaassens, 2001), mechanical systems (Haddad, Nersesov & Chellaboina, 2003) or various power converters (Escobar, Van der Schaft & Ortega, 1999). It is also related to hybrid models based on bond graphs, which are another graphical representation where the switches are modeled by effort or flow sources (Buisson, 1993; Cormerais, Buisson, Leirens & Richard, 2002). The design of passivity-based control can also be developed within the Euler-Lagrange framework, as in (Scherpen, Jeltsema and Klassens, 2003).

In this paper, we extend the results in (Magos, Valentin & Maschke, 2004-2) to give a graph theoretic construction of the port-Hamiltonian formulation for physical switching systems with varying constraints.

We use the example of power converters, but they may be seen as equivalent physical systems from a different field. Indeed, it is important to point out that some mechanical systems or hydraulic systems have an equivalent network representation. They may be represented as circuits or more generally by bond graphs (Paynter, 1961; Karnopp, Margolis & Rosenberg, 1990).

The paper is organized as follows. Section 2 provides some background on network graphs, dual network graphs, the minimal and non-minimal formulations of generalized Kirchhoff's laws and the associated Dirac structures. The port-Hamiltonian formulation is supposed to be known and we refer to (Lozano, Brogliato, Egeland & Maschke, 2000) and (Van der Schaft, 2000). Section 3 introduces the

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dynamic network graphs for systems with variable topology. It presents the main result of this paper, which is a hybrid incidence matrix that is used for the formulation of a unique model, valid for all the configurations of a physical system with switches. This matrix is parameterized by the discrete state of the switches. Section 4 gives the constructive procedure for deriving an algebraic representation of the family of geometric interconnection structures (Dirac structures) associated with the dynamic network graph. The non-minimal implicit port-Hamiltonian formulation is directly deduced from this algebraic representation. This result is applied to the electric power converter of Cuk in section 5.

2. Dirac structure for network models

We shall recall some graph theoretical definitions in relation to the network modeling of physical systems (Paynter, 1961; Recski, 1989; Narayanan, 1997). The network model consists of a set of n_e dipoles interconnected by a so-called network graph. This **network graph** is an oriented graph, $G = (V(G), E(G))$ where $V(G)$ is a nonempty finite set of n_v *vertices* ($v_x \in V(G)$) and $E(G)$ is a nonempty set of n_e ordered pairs of elements of $V(G)$ called *edges* ($e_{Gi} \in E(G)$ / $e_{Gi} = (v_x , v_y)$, v_x being the start vertex and v_y being the end vertex). If $v_x = v_y$, the edge is a **self-loop**. A network graph is said to be **cyclically connected** if, and only if, there is a circuit subgraph (connected subgraph with each vertex v_x having degree two i.e. in contact with two graph edges) containing any pair of vertices. This last concept is independent of any orientation.

Every edge of the network graph is associated with a pair (f, e) of conjugated variables, called *power variables* because their product has the unit of power. The set of power variables is represented by two vectors: $\underline{f}$ the cocycle variables vector and $\underline{e}$ the cycle variables vector. In the sequel, we shall also call f and e, *flow and effort variables,* in line with bond-graph terminology. For electrical circuits, the cycle variables are voltages and the cocycle variables, currents. The network graph may be partitioned into a maximal tree and its cotree (this partition is not unique). The network graph describes the interconnection constraints between the power variables due to Kirchhoff's laws which may be formulated in a generalized form (Recski, 1989).

The term "generalized" indicates that the cycle and cocycle variables are not necessarily voltages and currents but may also be forces and velocities or pressure and mass flow (Maschke, Van der Schaft & Breedveld, 1992). In the sequel we assume the sign convention is that of the receptor/passive elements, i.e. the edges orientation, corresponds to the sign convention of the cocycle variables and the opposite sign convention of the cycle variables.

Various mathematical representations of network graphs exist (Recski, 1989). In the sequel, we shall use the following matrix representations: the fundamental cutset and loop matrices associated with any maximal tree (defined in equation (1)), and the incidence matrix (defined in Definition 3). Generally the fundamental loop and cutset matrices are most naturally used to model network graphs because they lead to a minimal system of equations.

2.1. Minimal representation of generalized Kirchhoff's interconnection structure

From the choice of a maximal tree in the network graph, one may write the following relations between the cycle and cocycle variables (Recski, 1989):

$$\begin{bmatrix} I & Q_c \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \underline{f_t} \\ \underline{f_c} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -Q_c^T & I \end{bmatrix}\begin{bmatrix} \underline{e_t} \\ \underline{e_c} \end{bmatrix} = 0 \qquad (1)$$

where I is the Identity matrix, $[I \quad Q_c]$ is the fundamental cutset matrix and $[-Q_c^T \quad I]$ is the fundamental loop matrix with coefficients in {-1, 0, 1}, matching the selected maximal tree. The vector $\underline{f_t}$ (respectively $\underline{f_c}$) is the subvector of cocycle variables related to the tree (respectively cotree) elements. The vector $\underline{e_t}$ (respectively $\underline{e_c}$) is the subvector of cycle variables related to the tree (respectively cotree) elements.

A simple permutation of the variables and relations leads to:

$$\begin{bmatrix} \underline{f_t} \\ \underline{e_c} \end{bmatrix} = \begin{bmatrix} 0 & -Q_c \\ Q_c^T & 0 \end{bmatrix}\begin{bmatrix} \underline{e_t} \\ \underline{f_c} \end{bmatrix} \qquad (2)$$

Generalized Kirchhoff's laws have been related to a geometric structure, called Dirac structure on the space of power variables, (Maschke, Van der Schaft & Breedveld, 1995; Maschke & Van der Schaft, 1998; Bloch & Crouch, 1999). A Dirac structure on a vector space may be defined in terms of various linear maps, related to six various representations: the kernel, image, input-output, constrained effort, constrained flow and canonical representations (Dalsmo & Van de Schaft, 1998; Golo, 2002). In this paper, we shall use the kernel representation which directly leads to an implicit port-Hamiltonian representation.

Definition 1. *Minimal kernel representation of a Dirac structure.*
Every Dirac structure $D \subset V \times V^*$ is uniquely defined in a basis $B = (b_1, ..., b_n)$ by the couple of real valued $(n \times n)$ matrices (F,E), called structure matrices, satisfying the conditions:

$$EF^T + FE^T = 0 \text{ and } rank[E \vdots F] = n .$$

$D = \{(f, e) \in V \times V^* / F\underline{f} + E\underline{e} = 0\}$, where $\underline{f}$ is the coordinate vector of f in the basis B of V and $\underline{e}$ is the coordinate vector of e in the dual basis B^* of V^*. D is defined by exactly n equations.

It should be noted that the equation (1) is a minimal kernel representation of a Dirac structure of dimension n_e and the equation (2) is a minimal input-output representation of the same Dirac structure (Golo, 2002). In the sequel we shall use duality to express the generalized Kirchhoff's cycle and cocycle laws: the generalized Kirchhoff's cocycle laws will be expressed using the primal network graph and the generalized Kirchhoff's cycle laws using the dual network graph.

2.2. Dual network graph

We shall furthermore make the following assumption:

Assumption 1: the network graphs are planar so that it is possible to obtain a dual graph G^* for a graph G.

Let us recall Euler's formula.

Euler's formula. *A planar representation of a graph $G = (V(G), E(G))$ divides the plane into $n_e - n_v + 2$ regions named faces. If G is a finite planar graph, one of the faces is not bounded. It is called the external face.*

Then, a dual graph G^* is deduced from the set of faces associated with the graph G as defined below.

Definition 2. *Let $G = (V(G), E(G))$ be a planar graph. A dual graph $G^* = (V(G^*), E(G^*))$ of G is a graph where a vertex set $V(G^*)$ replaces the faces defined by G. Two vertices of $V(G^*)$ are connected by one edge of $E(G^*)$ (are adjacent) if their corresponding faces in G have a boundary edge in common.*

Let define the incidence matrix which gives a simple representation of the interconnection between edges and vertices of an oriented network graph with no self-loop. It is less frequently used than fundamental matrices because it leads to a non-minimal representation, but it is extremely well-adapted to systems with variable topology as explained in Section 3.

2.3. Non-minimal representation of Kirchhoff's interconnection structure

Definition 3. *The incidence matrix of the network graph G, is the $(n_v \times n_e)$ matrix $M_I(G)$ with:*

$$M_I(G)_{i,j} = \begin{cases} -1 \text{ if } e_{Gj} = (v_k, v_i) \text{ and } v_i \neq v_k \\ 1 \text{ if } e_{Gj} = (v_i, v_k) \text{ and } v_i \neq v_k \\ 0 \text{ otherwise} \end{cases} \quad (3)$$

For $i \in \{1,\dots, n_v\}$, $j \in \{1,\dots, n_e\}$ and $k \in \{1,\dots, n_v\}$.

Each row of the incidence matrix gives the edges connected to the corresponding vertex and each column gives the two vertices connected to the corresponding edge. The incidence matrix is a mathematical representation of a network graph if, and only if, the network graph does not include any self-loops. A self-loop produces a null column in the incidence matrix.

Definition 4. *Non-minimal kernel representation of a Dirac structure.*
Every Dirac structure $D \subset V \times V^*$ may also be defined in a basis $B = (b_1, \dots, b_n)$ by a non-minimal kernel representation which is characterized by the couple of real valued $(n' \times n)$ matrices (F,E), called structure matrices, satisfying the conditions:

$$EF^T + FE^T = 0 \text{ and } rank[E \vdots F] = n.$$

$D = \{(f, e) \in V \times V^* \,/\, F\underline{f} + E\underline{e} = 0\}$, where $\underline{f}$ is the coordinate vector of f in the basis B of V and $\underline{e}$ is the coordinate vector of e in the dual basis B^* of V^*. D is defined by n' equations, with n' > n.

The incidence matrix representation of a cyclically connected network graph with no self-loop, G, leads to a non-minimal kernel representation of the generalized Kirchhoff's laws, using incidence matrices of both the network graph G and a dual graph G^*.

$$\begin{bmatrix} M_I(G) \\ 0 \end{bmatrix} \underline{f} + \begin{bmatrix} 0 \\ M_I(G^*) \end{bmatrix} \underline{e} = 0 \quad (4)$$

This representation is non-minimal in the sense that it is a set of $(n_e + 2)$ equations whereas a minimal representation would have 2 less. For a fixed topology circuit, this representation can immediately be made minimal by removing one row of $M_I(G)$ and one row in $M_I(G^*)$. These rows correspond to the choice of a reference vertex for the primal graph and a reference vertex in the dual graph (often, the latter corresponds to the external cycle).

But, for a variable topology circuit, the choice of rows to remove changes according to the differing topology of the circuit, it is therefore impossible to reduce the non-minimal formulation to a unique minimal representation for all the configurations which is defined in section 3.

The equation (4) is a particular case of a non-minimal representation of a Dirac structure corresponding to a terminal formulation of generalized Kirchhoff's laws. It has been called a relaxed kernel representation in (Van der Schaft, Cervera & Baños, 2004) for the interconnection of port-Hamiltonian systems.

3. Dynamic network graph and its matrix representation

In this section we shall present the graphical formulation of a switching interconnection in terms of a dynamic network graph and the transformations between incidence matrices corresponding to the set of its configurations. More detailed motivations to use the incidence matrix as a mathematical representation of the network graphs and to define a dynamic network graph are presented in (Valentin, Magos & Maschke, 2006-b).

Definition 5: A dynamic network graph *$G_w = (V(G_w), E(G_w), E_w(G_w))$ consists of an oriented graph where:*
** $V(G_w)$ is a nonempty finite set of n_v vertices, $V(G_w)=\{ v_x , x \in \{1,..,n_v\}\}$,*
** $E(G_w)$ is a nonempty finite set of n_{ef} ordered pairs of elements of $V(G_w)$, called* ***functional edges****, $E(G_w) = \{e_{Gi} / e_{Gi} = (v_x , v_y), i \in \{1,.., n_{ef} \}, v_x$ being the start vertex, v_y being the end vertex and $(x,y) \in \{1,..,n_v\}^2\}$. The port of a functional element is associated with every of the n_{ef} oriented functional edges of this graph.*
** $E_w(G_w)$ is a nonempty finite set of n_s ordered pairs of elements of $V(G_w)$ called* ***virtual edges,*** *$E_w(G_w) = \{e_{Gwj} / e_{Gwj} = (v_x , v_y), j \in \{1,.., n_s \}, v_x$ being the start vertex, v_y being the end vertex and $(x,y) \in \{1,..,n_v\}^2\}$. The port of a switching element is associated with every of the n_s oriented virtual edges of this graph.*

To motivate the proposed approach, let us consider the simple example of the Cuk converter (Escobar, Van der Schaft & Ortega, 1999) which is represented in figure 1 and controls the power provided to the load by the voltage source through the control of the switches Sw_i.

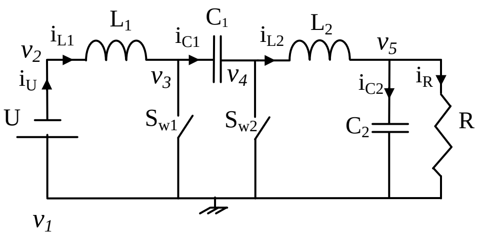

Fig. 1. the Cuk converter

A dynamic network graph Ga_w of the Cuk converter is given in figure 2 (functional edges are represented as thick lines and virtual edges as thin lines.):

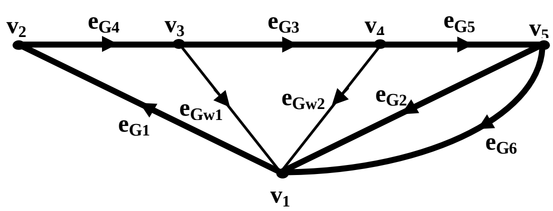

Fig. 2. dynamic network graph Ga_w of the Cuk converter

Denote $M_I(G_w)_{i\bullet}$ the i-row of the incidence matrix $M_I(G_w)$: $M_I(G_w)_{i\bullet} = \{M_I(G_w)_{ij}, j \in \{1,\ldots, n_{ef}\}\}$ and $M_I(G_w)_{\bullet j}$ the j-column of $M_I(G_w)$: $M_I(G_w)_{\bullet j} = \{M_I(G_w)_{ij}, i \in \{1,\ldots, n_v\}\}$.

If the first configuration G_1 (with all the switches open) is compared to one of the others, G', where a switch connected between vertices v_i and v_j is closed, then, $M_I(G')_{i\bullet} = 0$ and $M_I(G')$ is obtained from $M_I(G_1)$ after a linear transformation. We suggest calling this transformation disconnection-reconnection of the nodes v_i and v_j. Thus, the incidence matrix $M_I(G')$ of the new network graph G' can be obtained from $M_I(G_1)$ after a linear transformation.

Definition 6. *Let Σ_w be a physical switching system with varying constraints whose dynamic network graph is $G_w = (V(G_w), E(G_w), E_w(G_w))$. Its* ***reference graph****, G_r, is defined as being the subgraph $(V(G_w), E(G_w))$ with all vertices and functional edges of G_w. The $(n_v \times n_{ef})$ incidence matrix of the reference graph, G_r, is denoted by $M_I(G_r)$.*

Let us define a discrete parameter $w_k \in \{0, 1\}$, for each switch, S_{wk}, so that: w_k=1 if the switch is closed and w_k=0 if the switch is open. Thus, the discrete state (configuration) of the model is given by: $W = [w_1, w_2, ..., w_{ns}]^T$

In order to present, in Theorem 1, an expression for the family of incidence matrices associated with all the configurations of a physical system with variable topology, let us give two preliminary definitions:

Definition 7. *Let Σ_w be a physical switching system with varying constraints with n_s switches, whose dynamic network graph is $G_w = (V(G_w), E(G_w), E_w(G_w))$. Its* ***virtual graph*** *is defined as being the subgraph $(V(G_w), E_w(G_w))$ with all n_v vertices and all n_s virtual edges of G_w. n_s* ***disconnection – reconnection matrices****, $M_{DR1}(G_w)(w_1)$ to $M_{DRns}(G_w)(w_{ns})$, are associated with the n_s oriented virtual edges, $\{e_{Gwk} = (v_i, v_j), (i, j)\in\{1,..,n_v\}^2$ and $k\in\{1,..,n_s\}\}$, of this virtual graph. They are defined by:*

$$M_{DRk}(G_w)(w_k)_{m,n} = \begin{cases} w_k & \text{if} \quad m = j,\ n = i \text{ and } i \neq j \\ -w_k & \text{if} \quad m = n = i \text{ and } i \neq j \\ 0 & \text{Otherwise} \end{cases} \quad (5)$$

For $(m, n) \in \{1,..,n_v\}^2$.

Remark: $M_{DRk}(G_w)(w_k)$ models the transformation of the reference graph G_r when closing the switch S_{wk}.

Thus, a dynamic network graph, $G_w = (V(G_w), E(G_w), E_w(G_w))$ is mathematically represented both by the incidence matrix of the reference configuration, $M_I(G_r)$, and by the n_s disconnection - reconnection matrices $M_{DR1}(G_w)(w_1)$ to $M_{DRns}(G_w)(w_{ns})$. $M_I(G_r)$ represents the static functional part of the dynamic graph whilst the matrices $M_{DRk}(G_w)(w_k)$ represent the variable part.

Using these definitions, one may now give an expression of the incidence matrix parameterized with the state of the switches, for physical switching systems which fulfill the following assumption 2.

Assumption 2*. The physical switching system can be represented by a dynamic planar circuit which is cyclically connected with no self-loop.*

This assumption guarantees that the incidence matrices give an accurate mathematical representation of the primal and dual network graphs of this circuit.

Theorem 1. *The (n_v x n_{ef}) parameterized incidence matrix, $M_I(G_w)(W)$, of the 2^{ns} configurations of a dynamic oriented network graph, G_w with n_s virtual edges is given by:*

$$M_I(G_w)(W) = M_T(G_w)(W)\, M_I(G_r) \quad (6)$$

$$= \prod_{k=1}^{n_s}\left(M_{T[k]}(G_w)(W)\right) M_I(G_r) \quad (7)$$

where the (n_v x n_v) matrix $M_{T[k]}(G_w)(W)$ is defined as following by recurrent series:

$$\text{For } n_s,\ M_{T[n_s]} = I_{n_v} + M_{DRn_s}(G_w)(w_{n_s})$$

$$\text{For } k \in \{1,.., n_s\text{-}1\},$$

$$M_{T[k]} = I_{n_v} + \left[\prod_{i=k+1}^{n_s}\left(M_{T[i]}\right)\right] M_{DRk}(G_w)(w_k) \left[\prod_{j=k+1}^{n_s}\left(M_{T[j]}\right)\right]^T$$

Remark: to simplify the notations $M_{T[k]}(G_w)(W)$ and $M_{DRk}(G_w)(w_k)$ are denoted $M_{T[k]}$ and M_{DRk}.

$M_{T[k]}$ represents the transformations in the geometric interconnections between the elements of the system, produced by the switch Sw_k, taking into account the states of the switches Sw_{ns} to Sw_{k+1}.

The proof of Theorem 1 is given in the appendix and uses properties of the disconnection-reconnection transformation matrices.

4. Implicit port-Hamiltonian representation

In this section, we shall give the port-Hamiltonian representation of a circuit with varying topology obtained by adding to the dynamic graph G_w, the definition of a set of elements connected to its "functional edges". These elements may be inductors, capacitors, effort or flow sources or resistors. Considering source elements does however add constraints to the definition of the configurations, for instance, two effort sources may not be in parallel. This leads to consider a subset of the configurations, called admissible configurations and defined as follows.

Definition 8*: a non-admissible configuration corresponds to:*

i) An effort source in short-circuit or several independent effort sources connected in a cycle with no other functional elements (effort-sources-only cycle).

ii) A flow source connected in an open-circuit or several independent flow sources connected in a cocycle with no other functional elements (flow-sources-only cocycle or cutset).

The set of admissible configurations of the physical switching system Σ_w is denoted $A(\Sigma_w)$. $A(\Sigma_w) \subset \{0, 1\}^{ns}$.

It has been shown in (Valentin, Magos & Maschke, 2006-a) how the set of admissible configurations can be defined by the analysis of the parameterized incidence matrices associated with a PSS.

In the sequel, we shall firstly use the parameterized incidence matrices defined in section 3 to define a set of Dirac structures and then define the port-Hamiltonian system.

4.1. Parameterized Dirac structure

We shall define this Dirac structure by its kernel representation according to the Definition 4. It can be noted that in (Magos, Valentin & Maschke, 2004-1) the Dirac structure was defined by its constrained flow representation.

Definition 9. *Consider a physical switching system with a dynamic network graph G_w, its parameterized incidence matrix $M_I(G_w)(W)$, the parameterized incidence matrix $M_I(G_w^*)(W)$ of a dual graph and $A(\Sigma_w)$ its set of admissible configurations. The generalized Kirchhoff's laws define a parameterized Dirac structure $D(G_w)(W)$ on the vectors of cycle variables $\underline{e} \in \Re^{nef}$ and cocycle variables $\underline{f} \in \Re^{nef}$ which admit the following kernel representation:*

$$\begin{bmatrix} M_I(G_w)(W) \\ 0 \end{bmatrix} \underline{f} + \begin{bmatrix} 0 \\ M_I(G_w^*)(W) \end{bmatrix} \underline{e} = 0\ , W \in A(\Sigma_w)\}.$$

$D(G_w)(W)$ represents a family of geometric interconnection structures. It depends explicitly on the state W of the switches. This structure is non-minimal in the sense that it is a set of (n_{ef}+n_s+2) equations while a minimal representation would have (n_s+2) less. It has a variable rank depending on the state of the switches but has a constant dimension $((n_{ef}+n_s+2)\times n_{ef})$.

This non-minimal representation of the parameterized Dirac structure models the configurations of the PSS in a uniform way. In this context, the roles of the incidence matrices of the network graph and of a dual graph, less frequently used, are significant.

4.2. Implicit hybrid port-Hamiltonian formulation

The port-Hamiltonian system is defined on the space of the energy variables *p* and *q* (for electrical circuits: the inductances' flux, ϕ and capacitors' charges, *q*). The Hamiltonian function is the total electromagnetic energy *H(p, q)* of the circuit.

Assume moreover that the network has some ports (edges) to which sources are connected and denote the corresponding power variables by (i_S, u_S) and others to which resistors are connected and denote the corresponding power variables by (i_R, u_R). This leads to an algebro-differential system defined as a non-minimal implicit parameterized port-Hamiltonian system by:

$$\begin{bmatrix} M_I(G_w)(W) \\ 0 \end{bmatrix} \underline{f} + \begin{bmatrix} 0 \\ M({}_I G_w{}^*)(W) \end{bmatrix} \underline{e} = 0 \qquad (8)$$

with: $(\underline{f}, \underline{e}) \in D(G_w)(W)$, $W \in A(\Sigma_w)$,

$$\underline{f} = \begin{bmatrix} i_S & \dot{q} & \dfrac{\partial H}{\partial p} & i_R \end{bmatrix}^T \text{ and}$$

$$\underline{e} = \begin{bmatrix} u_S & \dfrac{\partial H}{\partial q} & \dot{p} & u_R \end{bmatrix}^T$$

5. Application on the Cuk converter

We shall consider the example of the Cuk converter represented on the figure 1. This example was also used in (Escobar, Van der Schaft & Ortega, 1999) for which a port-Hamiltonian system was formulated and in (Scherpen, Jeltsema and Klassens, 2003) in the Lagrangian formulation where only the unconstraint configurations are considered. This excludes the cases when $w_1=w_2=0$ or $w_1=w_2=1$. The formulation proposed here deals with the full set of configurations.

♦ The incidence matrix $M_I(Ga_w)(W)$ of the Cuk converter represented by the dynamic network graph given in figure 2 is:

$$M_I(Ga_w)(W) = \begin{bmatrix} 1 & -1 & w_1 - w_2 & -w_1 & w_2 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1-w_1 & -1+w_1 & 0 & 0 \\ 0 & 0 & -1+w_2 & 0 & 1-w_2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

with $(w_1, w_2) \in \{0, 1\}^2$.

♦ The dual incidence matrix $M_I(Ga_w{}^*)(W)$ is obtained from the dual graph $Ga_w{}^*$ represented in figure 3. The dotted lines indicate the dual functional edges, $e_{Gi}*$ and the thin dotted lines the dual virtual edges, $e_{Gwi}*$. As $n_e - n_v + 2 = 5$, a dual network graph $Ga_w{}^*$ has 5 dual vertices $v_i{}^*$.

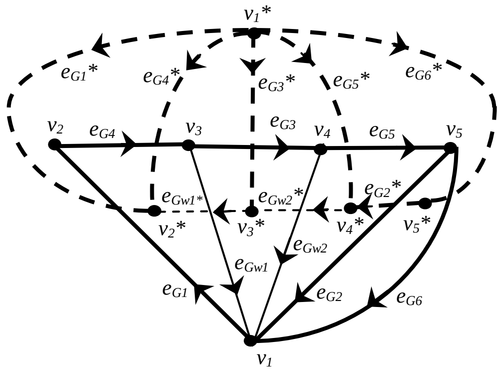

Fig. 3. Network graph Ga_w, dual network graph $Ga_w{}^*$

The incidence matrix of the reference configuration of this dual dynamic network graph of the Cuk converter, $Ga_r{}^*$, is:

$$M_I(Ga_r^*) = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Hence, the parameterized incidence matrix, $M_I(Ga_w{}^*)(W^*)$, associated with this dual network graph $Ga_w{}^*$, is obtained from Theorem 1:

$$M_I(Ga_w{}^*)(W^*) = M_T(Ga_w{}^*)(W^*)M_I(Ga_r^*)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ -1 & -w_1^*w_2^* & -w_1^* & -1 & -w_1^*w_2^* & 0 \\ 0 & -(1-w_1^*)w_2^* & -1+w_1^* & 0 & -(1-w_1^*)w_2^* & 0 \\ 0 & -1+w_2^* & 0 & 0 & -1+w_2^* & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

with $(w_1{}^*, w_2{}^*) \in \{0, 1\}^2$.

♦ The Kirchhoff's cocycle laws for the Cuk converter are $M_I(Ga_w)(W)\underline{f} = 0$ with:

$$\underline{f} = \begin{bmatrix} f_U & f_{C_1} & f_{C_2} & f_{L_1} & f_{L_2} & f_R \end{bmatrix}^T$$

Notice that $w_i{}^*=1-w_i$, then: $M_I(Ga_w{}^*)(W) =$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ -1 & -(1-w_1)(1-w_2) & -1+w_1 & -1 & -(1-w_1)(1-w_2) & 0 \\ 0 & -w_1(1-w_2) & -w_1 & 0 & -w_1(1-w_2) & 0 \\ 0 & -w_2 & 0 & 0 & -w_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

with $(w_1, w_2) \in \{0, 1\}^2$.

Then, the Kirchhoff's cycle laws for the Cuk power converter are: $M_I(Ga_w{}^*)(W)\underline{e} = 0$ with:

$$\underline{e} = \begin{bmatrix} e_U & e_{C_1} & e_{C_2} & e_{L_1} & e_{L_2} & e_R \end{bmatrix}^T$$

♦ Therefore, as all configurations are admissible, the algebro-differential system of the Cuk converter is defined as an implicit non-minimal parameterized port-Hamiltonian system by:

$$\begin{bmatrix} M_I(Ga_w)(W) \\ 0 \end{bmatrix} \underline{f} + \begin{bmatrix} 0 \\ M_I(Ga_w^{*})(W) \end{bmatrix} \underline{e} = 0$$

with: $W \in \{0, 1\}^2$,

$$\underline{f} = \begin{bmatrix} i_U & \dot{q}_1 & \dot{q}_2 & \frac{\partial H}{\partial \phi_1} & \frac{\partial H}{\partial \phi_2} & i_R \end{bmatrix}^T$$

and: $\underline{e} = \begin{bmatrix} U & \frac{\partial H}{\partial q_1} & \frac{\partial H}{\partial q_2} & \dot{\phi}_1 & \dot{\phi}_2 & u_R \end{bmatrix}^T$

These incidence matrices $M_I(Ga_w)(W)$ and $M_I(Ga_w^{*})(W)$ can be analyzed to draw conclusions concerning the constrained configurations of the system. Two constrained configurations are created when the capacitor C_1 is short-circuited ($w_1=w_2=1$) or when the inductances L_1 and L_2 are connected in series ($w_1=w_2=0$). Indeed, it is of prime importance to remove non-admissible configurations from the control design procedure and to be aware of constrained configurations which may lead to state discontinuities in the trajectory of the system.

Note that this implicit port-Hamiltonian system entirely encompasses the model presented in (Escobar, Van der Schaft & Ortega, 1999) and includes two additional constraints.

7. Conclusions and perspectives

In this paper, we have proposed a port-Hamiltonian formulation of physical systems with switching interconnection. The switching topology is defined by a dynamic network graph to which energy conserving, energy dissipating, sources and switching elements are connected. In a first step, we have defined a parameterized incidence matrix corresponding to the set of configurations of the switches. In a second step, we have used this matrix to define a parameterized port-Hamiltonian formulation of the admissible configurations of the PSS. It is based on a parameterized non-minimal kernel representation of the Dirac structure associated with generalized Kirchhoff's laws. The formal design of a hybrid automaton model of the autonomous physical switching system is presented in (Valentin, Magos & Maschke, 2006-a).

An attractive feature is that the discrete state of the switching part is explicit as well as the interconnections between elements storing, providing or dissipating energy in the system. The class of models presented in this paper encompass a great variety of non-linear switched systems. The analysis of such systems is quite complex to be handled in general and often, one needs to restrict the class to obtain results, for example when resolving the existence of solutions (Gerritsen, Van der Schaft & Heemels, 2002).

This parameterized port-Hamiltonian formulation has the advantages of being well-structured (a primal network graph gives the cocycle relations and a dual network graph gives the cycle relations from duality considerations) and completely formalized (from the system to the final representation) without requiring *a-priori* knowledge of a specific physical field. The advantages of the Euler-Lagrange framework (Scherpen, Jeltsema and Klassens, 2003) are that the graph of the circuit is not necessarily planar. The formulation proposed here has the advantage of dealing with constraints with varying rank and to falls into the framework of passivity-based control.

This work may be continued by extending the solution concepts and trajectory calculation developed in (Gerritsen, Van der Schaft & Heemels, 2002) to dissipative physical switching system with effort sources. Another perspective of this paper is to extend control design methods based on continuous Hamiltonian systems such as Interconnection Damping Assignment Passivity Based Control (Ortega, van der Schaft, Maschke & Escobar, 2002) and continuous control design method for parameterized port-controlled Hamiltonian systems with autonomous switching as impacts (Haddad, Nersesov & Chellaboina, 2003) to dissipative physical switching system with sources and controlled switches. Optimal control methods such as (Manon, Valentin-Roubinet & Gilles, 2002; Sussmann, 1999; Zaytoon, 2001) can also be applied.

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Appendix. The proof of Theorem 1.

The proof is based on recursion.

If the number of virtual edges in G_w^1 is equal to 1, then $M_I(G_w^1)(w_1) = [I_{n_v} + M_{DR1}(G_w^1)(w_1)]M_I(G_r)$ which is an immediate conclusion of the definitions of an incidence matrix and of the disconnection-reconnection transformation (definitions 3 and 7).
Now, suppose that the Theorem 1 is true for a dynamic graph $G_w^{n_s-1}$ *with* n_s-1 virtual edges.

Then: $M_I(G_w^{n_s-1})(W) = \prod_{k=1}^{n_s-1} \left(M_{T[k]}(G_w^{n_s-1})(W) \right) M_I(G_r)$.

Hence, in the sequel, we prove that, for a dynamic network graph $G_w^{n_s}$ = $(V(G_w^{n_s}), E(G_w^{n_s}), E_w(G_w^{n_s}))$, *with* n_s virtual edges:

$$M_I(G_w^{n_s})(W) = \prod_{k=1}^{n_s}\left(M_{T[k]}(G_w^{n_s})(W)\right)M_I(G_r)$$

which can be written:

$$M_I(G_w^{n_s}) = \prod_{k=1}^{n_s-1}\left(M_{T[k]}(G_w^{n_s})\right)\underbrace{\left[I_{n_v} + M_{DRn_s}\right]M_I(G_r)}_{} \quad (10)$$

Following definitions 3 and 7, $[I_{n_v} + M_{DRn_s}(G_w^{n_s})(1)]M_I(G_r) = M_I(G_1'')$ is the incidence matrix of the subgraph $(V(G_w), E(G_w))$ with all vertices and functional edges of G_w and with the virtual edge e_{Gwns} closed.

As well, $[I_{n_v} + M_{DRn_s}(G_w^{n_s})(0)]M_I(G_r) = M_I(G_0'')$ is the incidence matrix of the subgraph $(V(G_w^{n_s}), E(G_w^{n_s}))$ with all vertices and functional edges of $G_w^{n_s}$ and with the virtual edge e_{Gwns} open.

Consequently, equation (10) calculates the parameterized incidence matrix of a dynamic graph $G_w^{n_s-1}$ *with* n_s*–1* virtual edges, e_{Gw1} to e_{Gwns-1}, and with the virtual edge e_{Gwns} closed or open through the transformation of the incidence matrix of the reference configuration, $\left[I_{n_v} + M_{DRn_s}\right]M_I(G_r)$ which covers $M_I(G_1'')$ and $M_I(G_0'')$.

This transformation is completed by the product $\prod_{k=1}^{n_{s-1}}\left(M_{T[k]}(G_w^{n_s})(W)\right)$. Now remains the proof that $M_{T[k]}(G_w^{n_s})(W)$ is also obtained by recurrence, by the following expression:

$$M_{T[k]}(G_w^{n_s})(W) = I_{n_v} + \left[\prod_{i=k+1}^{n_s}\left(M_{T[i]}\right)\right]M_{DRk}\left[\prod_{j=k+1}^{n_s}\left(M_{T[j]}\right)\right]^T$$

In the sequel, we consider two cases:

* for $k = n_s-1$,

$$M_{T[ns-1]}(G_w^{n_s}) = I_{n_v} + M_{T[ns]}.M_{DRns-1}.M_{T[ns]}^T$$

* for $k \in \{1,.., n_s-2\}$, $M_{T[k]} =$

$$I_{n_v} + \left[\prod_{i=k+1}^{n_s-1}\left(M_{T[i]}\right)\right]\underbrace{M_{T[ns]}.M_{DRk}.M_{T[ns]}^T}_{}\left[\prod_{j=k+1}^{n_s-1}\left(M_{T[j]}\right)\right]^T$$

For $k \in \{1, ..., n_s-1\}$, let us denote: $M^{\#}_{DRk} = M_{T[ns]}M_{DRk}M_{T[ns]}^T$, which appears in both expressions.

We next prove that, $M^{\#}_{DRk}$ for $k \in \{1,.., n_s-1\}$ are n_s-1 disconnection-reconnection matrices defining the dynamic network graph $G_w^{n_s-1}$ = $(V(G_w^{n_s-1}), E(G_w^{n_s-1}), E_w(G_w^{n_s-1}))$ with n_s*–1* virtual edges e_{Gw1} to e_{Gwns-1} and with the virtual edge e_{Gwns} closed or open.

$$M_{DRk} = \begin{array}{c} p_k \\ \begin{bmatrix} 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \ddots & \vdots & & & & \vdots \\ \vdots & & -w_k & & & & \vdots \\ \vdots & & \vdots & \ddots & & & \vdots \\ \vdots & & \vdots & & \ddots & & \vdots \\ \vdots & & w_k & & & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ p_k \\ \\ \\ q_k \\ \\ \end{array}$$

and

$$M_{T[ns]} = \begin{array}{c} p_{ns} \\ \begin{bmatrix} 1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \ddots & & & \vdots & & \vdots \\ \vdots & & \ddots & & w_{ns} & & \vdots \\ \vdots & & & \ddots & \vdots & & \vdots \\ \vdots & & & & 1-w_{ns} & & \vdots \\ \vdots & & & & \vdots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ q_{ns} \\ \\ p_{ns} \\ \\ \\ \end{array}$$

Five cases must be analyzed to calculate $M^{\#}_{DRk}$:

1/ if ($q_{ns} \neq q_k$) and ($q_{ns} \neq p_k$) and ($p_{ns} \neq q_k$) and ($p_{ns} \neq p_k$), $M^{\#}_{DRk} = M_{DRk}$ whatever $k \in \{1,.., n_s-1\}$.

Indeed, it means that there is no connection between e_{Gwns}, and e_{Gwk} in the dynamic graph $G_w^{n_s-1}$.

2/ if ($q_{ns} = q_k$) and ($q_{ns} \neq p_k$) and ($p_{ns} \neq q_k$) and ($p_{ns} \neq p_k$), then, whatever $k \in \{1,.., n_s-1\}$, $M^{\#}_{DRk} = M_{DRk}$.

Indeed, it means that when e_{Gwns} and e_{Gwk} have the same end vertex, the state of e_{Gwns} does not appear in the disconnection-reconnection matrix $M^{\#}_{DRk}$.

3/ if ($q_{ns} \neq q_k$) and ($q_{ns} = p_k$) and ($p_{ns} \neq q_k$) and ($p_{ns} \neq p_k$), then, whatever $k \in \{1,.., n_s-1\}$, $M^{\#}_{DRk} = M_{DRk}$.

Indeed, it means that when the end vertex of edge e_{Gwns} is the same as the start vertex of edge e_{Gwk}, the influence of the state of e_{Gwns} to the disconnection-reconnection transformation due to e_{Gwk} is taken into account in the product $\prod_{k=1}^{n_s-1}\left(M_{T[k]}\right)$ and not in the term $M^{\#}_{DRk}$. In that case, the orientation of the virtual edges' sequence respects the matrices' product.

4/ if ($q_{ns} \neq q_k$) and ($q_{ns} \neq p_k$) and ($p_{ns} = q_k$) and ($p_{ns} \neq p_k$), then, whatever $k \in \{1,.., n_s\text{-}1\}$, $M^{\#}_{DRk} =$

$$
\begin{array}{cc}
\begin{array}{ccccccc} & & p_k & & & & \end{array} & \\
\begin{bmatrix}
0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & \ddots & \vdots & & & & \vdots \\
\vdots & & -w_k & & & & \vdots \\
\vdots & & w_{ns}w_k & \ddots & & & \vdots \\
\vdots & & \vdots & & \ddots & & \vdots \\
\vdots & & (1-w_{ns})w_k & & & \ddots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0
\end{bmatrix}
&
\begin{array}{c} \\ \\ p_k \\ q_{ns} \\ \\ q_k = p_{ns} \\ \\ \end{array}
\end{array}
$$

If $w_{ns}=0$, then $M^{\#}_{DRk} = M_{DRk}$. Otherwise, edges connected to v_{pk} are disconnected and reconnected to v_{qns}. Indeed, it means that when the end vertex of the edge e_{Gwk} is the same as the start vertex of the edge e_{Gwns}, the reconnection vertex depends on the state of e_{Gwns} because the orientation of the virtual edges' sequence does not respect the matrices' product.

5/ if ($q_{ns} \neq q_k$) and ($q_{ns} \neq p_k$) and ($p_{ns} \neq q_k$) and ($p_{ns} = p_k$), then, whatever $k \in \{1,.., n_s\text{-}1\}$, $M^{\#}_{DRk} =$

$$
\begin{array}{cc}
\begin{array}{ccccccc} & & p_k = p_{ns} & \cdots & q_{ns} & & \end{array} & \\
\begin{bmatrix}
0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & \ddots & \vdots & \vdots & & & \vdots \\
\vdots & & (w_{ns}-1)w_k & \vdots & & & \vdots \\
\vdots & & \vdots & -w_{ns}w_k & & & \vdots \\
\vdots & & \vdots & \vdots & \ddots & & \vdots \\
\vdots & & (1-w_{ns})w_k & w_{ns}w_k & & \ddots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0
\end{bmatrix}
&
\begin{array}{c} \\ \\ p_k = p_{ns} \\ q_{ns} \\ \\ q_k \\ \\ \end{array}
\end{array}
$$

If $w_{ns}=0$, then $M^{\#}_{DRk} = M_{DRk}$. Otherwise, edges initially connected to v_{pk} have been disconnected and reconnected to v_{qns} by the closing of e_{Gwns}. Then, edges connected to v_{qns} are disconnected and reconnected to v_{qk}. It means that, when e_{Gwns} and e_{Gwk} have the same start vertex, the disconnection vertex depends on the state of e_{Gwns}.

Then, we proved by recursion that the $(n_v \times n_v)$ matrices $M^{\#}_{DRk}$ for $k \in \{1,.., n_s\text{-}1\}$ are $n_s\text{-}1$ disconnection-reconnection matrices representing the dynamic graph G_w with n_s-1 virtual edges e_{Gw1} to e_{Gwns-1} and with the virtual edge e_{Gwns} closed or open. Thus, Theorem 1 is proved.

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