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# Bond Graph Representation of Standard Interconnection Model

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**Keywords:** Bond graph, parametric uncertainties, Standard Interconnection Model, 1-Port element, multiport element.

## Abstract

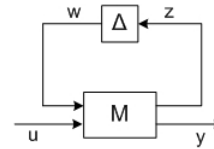
The study of the robustness of a system's parametric uncertainties is based on state representations which separate the nominal part of the system from the uncertain part. The most used form is the standard interconnection model. Recent works have been formulated so as to find this representation graphically by the bond graph approach. A new procedure is proposed in this paper to determine an uncertain model adapted to the study of robustness and for robust control. The advantage of this procedure is in simplifying the resulting graphical model.

## 1. INTRODUCTION

Robustness reflects the ability of a controlled process to absorb a certain number of model imprecisions as parametric uncertainties or external disturbances that have not been taken into account. In this paper, we are interested only in the robustness study of linear time-invariant systems which present parametric uncertainties (or structured uncertainties). Parametric uncertainties are due to the mis-identification or variations of physical parameters of the system. In fact, some parameters of a system can change significantly with time or with operating conditions. There are several types of parametric uncertainty [1]. In this work, only uncertainties of type  $\alpha$  are considered. These uncertainties are variations about nominal values  $\alpha_n$  which are different from zero ( $\alpha_n \neq 0$ ). So, the order of the model is unchanged. The causes of these uncertainties are related to manufacturing (tolerances on the dimensions of mechanical components or on the values of electronic components, etc.).

The most used form of state equations to describe linear time-invariant systems in the presence of parametric uncertainties is the standard interconnection model or internal feedback loop [2–4]. This form is used to study robustness and to apply a robust control to this type of system. In fact, the Standard Interconnection Model (SIM) is a particular case of the Linear Fractional Transformation (LFT). The LFT is a mathematical representation used in the modelling of a number of problems and in the control of dynamic systems. The standard interconnection model enables to separate the nominal values

of the parameters in a matrix  $M$ , called the interconnection matrix, from all the relative uncertainties in a diagonal matrix  $\Delta$  called the uncertainty matrix (Figure 1). To do this, it is necessary to add fictitious inputs and outputs to represent the uncertainties as an internal feedback loop to the system.



**Figure 1.** Standard interconnection model.

The standard interconnection model is described by the following state equations, with  $\Delta = \text{diag}(\delta_i I_{n_i})$ :

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_n \mathbf{x} + \mathbf{B}_1 \mathbf{w} + \mathbf{B}_2 \mathbf{u} \\ \mathbf{z} = \mathbf{C}_1 \mathbf{x} + \mathbf{D}_{11} \mathbf{w} + \mathbf{D}_{12} \mathbf{u} \\ \mathbf{y} = \mathbf{C}_2 \mathbf{x} + \mathbf{D}_{21} \mathbf{w} + \mathbf{D}_{22} \mathbf{u} \\ \mathbf{w} = \Delta \mathbf{z} \end{cases} \quad (1)$$

$\mathbf{x} \in \mathbb{R}^n$  denotes the state vector,  $\mathbf{u} \in \mathbb{R}^m$  (resp.  $\mathbf{y} \in \mathbb{R}^p$ ) is the input (resp. output) vector and  $\mathbf{w} \in \mathbb{R}^l$  (resp.  $\mathbf{z} \in \mathbb{R}^l$ ) is the auxiliary input (resp. output) vector.  $\delta_i$  represents the  $i^{\text{th}}$  relative uncertainty ( $|\delta_i| \leq 1$ ) and  $n_i$  is the dimension of this uncertainty : the number of auxiliary input/output pairs associated with this uncertainty. Before formulating the standard interconnection model, some conditions must be verified on the initial model (before considering uncertainties):

- The model must be proper: the verification of this property is equivalent to showing that the degree of the denominator of the transfer matrix is greater or equal to the degree of the corresponding numerator.
- The model must be stabilizable: if the model is controllable then it is stabilizable.
- The model must be detectable: if the model is observable then it is detectable.

In the bond graph (BG) context, two different bond graph approaches exist to study the robustness of linear models with parametric uncertainties: the uncertain bond graph approach and the incremental bond graph one. In fact, Borutzky and Granda [5, 6] introduced the incremental bond graph for the

bond graph determination of the first order unnormalised frequency domain sensitivities in a symbolic form. Also, it was shown in [7, 8] that the SIM can be obtained from the initial and the associated incremental bond graph. Furthermore, the uncertain bond graph was developed in [9–11]. The aim was to develop a bond graph-based method to express the standard interconnection model. Nevertheless, some assumptions must be checked before adopting this approach:

- Linear system with parametric (structured) uncertainties of type  $\alpha$  (model order unchanged);
- Only uncertainties in the R, I, C, TF, GY elements are considered;
- Independent parameters.

In this work, the same assumptions will be made. The aim of this paper is to develop an alternative BG procedure for the determination of the standard interconnection model on a BG model. The organization of the paper is as follows. In the first section, the existing procedure (uncertain bond graph) for finding the standard interconnection model with a bond graph approach is presented<sup>1</sup>. In the second section, a new procedure is proposed. As an example, a DC motor with parametric uncertainties will be treated. The conclusion is given in the last section.

## 2. THE EXISTING PROCEDURE

A BG procedure was developed in [9–11] to find directly the standard interconnection model from the BG model. Before formulating this model, the property, controllability and observability have to be verified on the initial model. With bond graphs, these properties can be checked by the analysis of the causal paths between the storage elements in integral causality and the control sources, and, between the storage elements in integral causality and the detectors [12, 13]<sup>2</sup>. So, if the controllability and the observability of the system are verified, then it can be concluded that the system is stabilizable and detectable.

The SIM of each BG element depends on the assigned causality. Thus, for the same element, the model changes when the causality is changed. The principle of modelling each BG element is the same: make the relative uncertainties appear in the constitutive(s) relationship(s) of the element and translate this relationship into the BG model. To do this, it is necessary to add modulated sources ( $MS_e$  and  $MS_f$ ) and detectors ( $D_e$  and  $D_f$ ); the added sources correspond to the auxiliary input signals. The SIM described by equation 1 can be

<sup>1</sup>The incremental bond graph will not be discussed in this paper because the first aim of this approach was the determination of the sensitivity functions in the frequency domain.

<sup>2</sup>The bond graph definitions represent sufficient conditions

deduced from the uncertain model after replacing all uncertain elements with their corresponding BG-SIM. The matrices of the standard interconnection model are computed in the same way as the matrices of the nominal system. The matrix  $\Delta$  is a diagonal matrix which contains all the relative uncertainties expressed by the signal part of the BG model. The rank of this matrix is equal to the number of added source/detector pairs in the model.

### 2.1. 1-Port R elements

Let us consider an uncertain R-element in resistance causality where the parameter is the sum of the nominal value  $R_n$  and the uncertain value  $\Delta R$ . The constitutive relationship of this element can be written as:

$$e_R = (R_n + \Delta R) f_R = R_n(1 + \delta_R) f_R \quad (2)$$

with  $\delta_R = \frac{\Delta R}{R_n}$ . The associated BG-SIM is represented in Figure 2. For 1-Port R element in conductance causality, see [10].

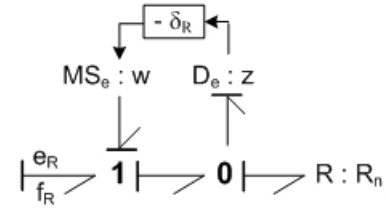


Figure 2. BG-SIM of R-element in resistance causality.

### 2.2. 1-Port I elements

For an uncertain I-element in integral causality where the parameter is the sum of the nominal value  $I_n$  and the uncertain value  $\Delta I$ , the constitutive relationship can be written<sup>3</sup>:

$$f_I = \frac{1}{I_n + \Delta I} \int e_I dt = \frac{1}{I_n} (1 + \delta_{1/I}) \int e_I dt \quad (3)$$

with  $\delta_{1/I} = -\frac{\Delta I}{I_n + \Delta I}$ . The associated BG-SIM is represented in Figure 3. For 1-Port I element in derivative causality and C element see [10].

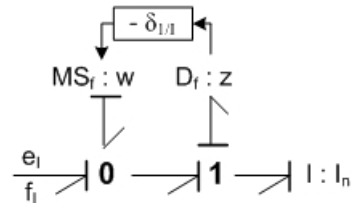


Figure 3. BG-SIM of I-element in integral causality.

<sup>3</sup> $\int e_I dt$  is a notation which represents the generalized momentum  $p(t)$  defined by:  $\int_0^t e_I(\tau) d\tau + p(0)$ .

### 2.3. 2-Port TF elements

Let us consider an uncertain TF-element in  $m$  causality described by the following constitutive relationships:

$$\begin{cases} e_1 = (m_n + \Delta m)e_2 = m_n(1 + \delta_{TF})e_2 \\ f_2 = (m_n + \Delta m)f_1 = m_n(1 + \delta_{TF})f_1 \end{cases} \quad (4)$$

The associated BG-SIM is illustrated by Figure 4 with  $\delta_{TF} = \frac{\Delta m}{m_n}$ . For TF element in  $1/m$  causality and for GY element see [10].

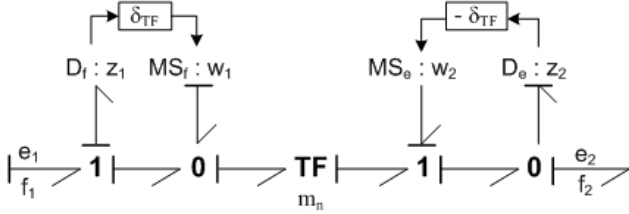


Figure 4. BG-SIM of TF-element in  $m$  causality.

It can be deduced from these bond graph standard interconnection models that the matrix  $\Delta$  contains a sign (-) which appears in the signal parts of the models and consequently this affects the matrices  $B_1$ ,  $C_1$ ,  $D_{11}$ ,  $D_{12}$  and  $D_{21}$ .

## 3. THE NEW PROCEDURE

In this section, an alternative BG procedure is proposed to simplify the graphical model.

### 3.1. 1-Port elements

The BG standard interconnection modelling of 1-Port elements is treated element by element:

- For an uncertain R-element in resistance causality, the relative uncertainty is equal to  $\delta_R = \frac{w}{z} = \frac{\Delta R}{R_n}$  (equation 2) and the associated BG-SIM is illustrated by Figure 5(a). For an uncertain R-element in conductance causality, the constitutive relationship can be written as:

$$f_R = \frac{1}{R_n + \Delta R} e_R = \frac{1}{R_n} (1 + \delta_{1/R}) e_R \quad (5)$$

with  $\delta_{1/R} = -\frac{\Delta R}{R_n + \Delta R}$ . The associated BG-SIM is represented in Figure 5(b).

- For an I-element in integral causality, the relative uncertainty is equal to  $\delta_{1/I} = \frac{w}{z} = -\frac{\Delta I}{I_n + \Delta I}$  (equation 3) and the associated BG-SIM is illustrated by Figure 6(a). For an uncertain I-element in derivative causality, the constitutive relationship can be written as:

$$e_I = (I_n + \Delta I) \frac{df_I}{dt} = I_n (1 + \delta_I) \frac{df_I}{dt} \quad (6)$$

with  $\delta_I = \frac{\Delta I}{I_n}$ . The associated BG-SIM is represented in Figure 6(b).

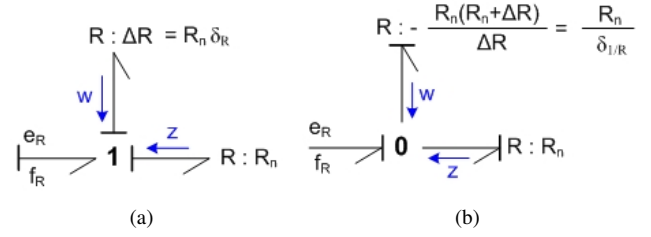


Figure 5. BG-SIM of R-element in (a) resistance and (b) conductance causality.

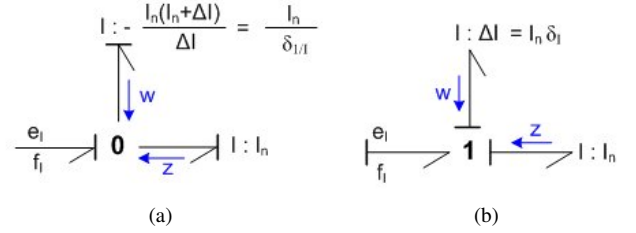


Figure 6. BG-SIM of I-element in (a) integral and (b) derivative causality.

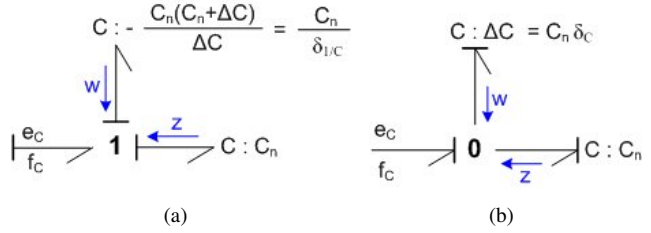


Figure 7. BG-SIM of C-element in (a) integral and (b) derivative causality.

- For an uncertain C-element in integral causality where the parameter is the sum of the nominal value  $C_n$  and the uncertain value  $\Delta C$ , the constitutive relationship is defined by<sup>4</sup>:

$$e_C = \frac{1}{C_n + \Delta C} \int f_C dt = \frac{1}{C_n} (1 + \delta_{1/C}) \int f_C dt \quad (7)$$

with  $\delta_{1/C} = -\frac{\Delta C}{C_n + \Delta C}$ . The associated BG-SIM is represented in Figure 7(a). The constitutive relationship of the uncertain C-element in derivative causality can be written as:

$$f_C = (C_n + \Delta C) \frac{de_C}{dt} = C_n (1 + \delta_C) \frac{de_C}{dt} \quad (8)$$

with  $\delta_C = \frac{\Delta C}{C_n}$ . The associated BG-SIM is represented in Figure 7(b).

<sup>4</sup>  $\int f_C dt$  is a notation which represents the generalized displacement  $q(t)$  defined by:  $\int_0^t f_C(\tau) d\tau + q(0)$ .

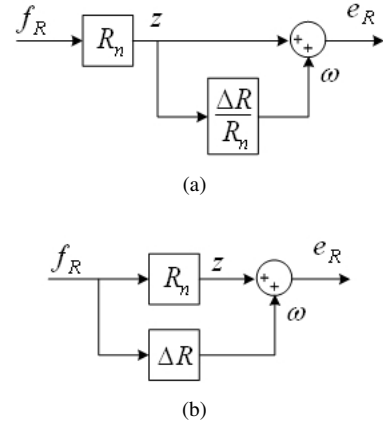
**Interpretation.** Compared with the existing procedure, the first step is unchanged: make the relative uncertainties appear in the constitutive relationships. The auxiliary input/output are internal variables, so it is sufficient to display these variables on the BG model. The auxiliary input/output appear when the BG-element is dissociated into a nominal element and an uncertain element (of the same nature as the initial element). The auxiliary output corresponds to a power variable associated to the nominal element: it is the outgoing variable of the nominal element. The auxiliary input corresponds to an outgoing power variable of the uncertain element. The BG-SIM of every BG element must verify the constitutive relationship of the initial element. So, the BG-SIM depends on the causality assigned to the initial element. The two elements (nominal and uncertain) have the same causality which corresponds to the causality of the initial element.

The relative uncertainty does not appear explicitly in the BG model but it is equal to the fraction  $\frac{w}{z}$  and this because only one junction is considered in the BG standard interconnection model. Let us take as an example the R-element in resistance causality. This element can be represented by two block diagrams (Figure 8(a) and Figure 8(b)). The first representation makes the relative uncertainty appear explicitly. The equivalent BG model necessarily contains two junctions and the relative uncertainty is represented by a signal part because it is equal to a relation between two power variables of the same nature (effort variables in this case). So, the equivalent BG model corresponds to the BG-SIM found by the existing procedure. In the second block diagram, the relative uncertainty does not appear explicitly. The equivalent BG model contains only one junction but the relation between  $w$  and  $z$  cannot be found directly in the BG model. So, the equivalent BG model corresponds to the BG-SIM found by the new procedure. Thus, the simplification of the model structure engenders the loss of a direct reading of the relative uncertainty from the BG model. But, because the relative uncertainty is already known, this result is still interesting.

**Note.** This new procedure allows to duplicate the energy storage elements (I and C) in integral causality and consequently the model order can be increased. In a BG model, the model order is computed from definition 1.

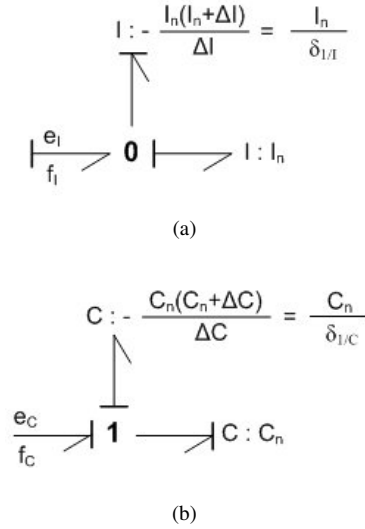
**Definition 1.** [13] On the BG model, the rank of the state matrix  $A$  is given by the difference between the number of energy storage elements  $I$  and  $C$  in integral causality on the BG model in integral causality and the number of energy storage elements  $I$  and  $C$  in integral causality on the BG model in derivative causality.

Nevertheless, if the derivative causality is assigned to the BG-SIM of the energy storage elements in integral causality (Figure 6(a) and 7(a)) in order to compute the model order,



**Figure 8.** Block diagram representations of R-element in resistance causality.

then we point out that this order is unchanged (Figure 9) because only one energy storage element is involved in derivative causality. So, it can be concluded that the nominal element and the uncertain element are dynamically dependent.



**Figure 9.** Derivative causality assignment to the dynamic element (a) I and (b) C initially in integral causality.

### 3.2. 2-Port TF and GY elements

As for 1-Port elements, the BG standard interconnection modelling of TF and GY elements is treated one at a time according to the assigned causality:

- For the uncertain TF-element in m causality, the relative uncertainty is equal to  $\delta_{TF} = \frac{\omega_1}{z_1} = \frac{\omega_2}{z_2} = \frac{\Delta m}{m_n}$  (deduced from the system of equations 4) and the associated BG-SIM is illustrated by Figure 10(a). For an uncertain TF-

element in  $1/m$  causality, the constitutive relationships are defined by:

$$\begin{cases} e_2 = \frac{1}{m_n + \Delta m} e_1 = \frac{1}{m_n} (1 + \delta_{1/TF}) e_1 \\ f_1 = \frac{1}{m_n + \Delta m} f_2 = \frac{1}{m_n} (1 + \delta_{1/TF}) f_2 \end{cases} \quad (9)$$

The associated BG-SIM is illustrated by Figure 10(b) with  $\delta_{1/TF} = -\frac{\Delta m}{m_n + \Delta m}$ .

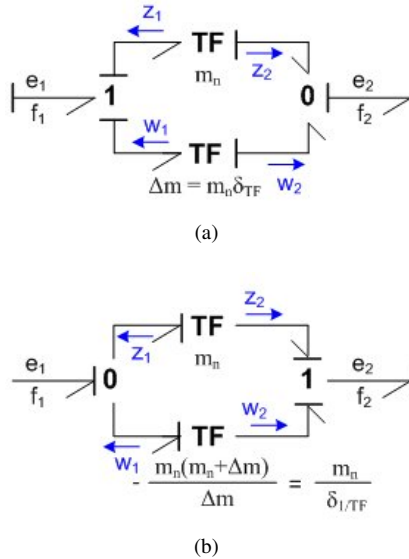
- For an uncertain GY-element in  $r$  causality, the constitutive relationships are defined by:

$$\begin{cases} e_1 = (r_n + \Delta r) f_2 = r_n (1 + \delta_{GY}) f_2 \\ e_2 = (r_n + \Delta r) f_1 = r_n (1 + \delta_{GY}) f_1 \end{cases} \quad (10)$$

The associated BG-SIM is illustrated by Figure 11(a) with  $\delta_{GY} = \frac{\Delta r}{r_n}$ . For an uncertain GY-element in  $1/r$  causality, the constitutive relationships are written as follows:

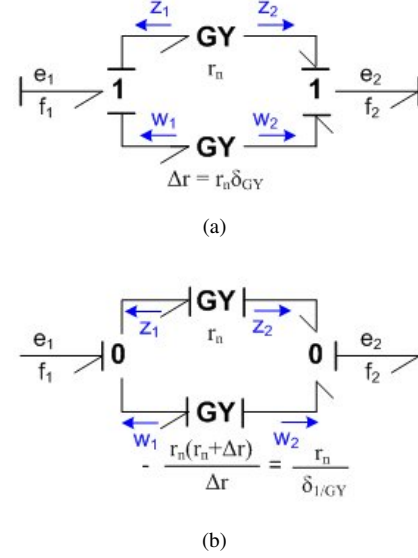
$$\begin{cases} f_1 = \frac{1}{r_n + \Delta r} e_2 = r_n (1 + \delta_{1/GY}) e_2 \\ f_2 = \frac{1}{r_n + \Delta r} e_1 = r_n (1 + \delta_{1/GY}) e_1 \end{cases} \quad (11)$$

The associated BG-SIM is illustrated by Figure 11(b) with  $\delta_{1/GY} = -\frac{\Delta r}{r_n + \Delta r}$ .



**Figure 10.** BG-SIM of TF-element in (a)  $m$  and (b)  $1/m$  causality.

**Interpretation.** First, as for the 1-Port elements, the TF and GY elements are dissociated into nominal and uncertain elements. But, for these BG-elements, despite the fact that they have a unique uncertain parameter, a unique auxiliary



**Figure 11.** BG-SIM of GY-element in (a)  $r$  and (b)  $1/r$  causality.

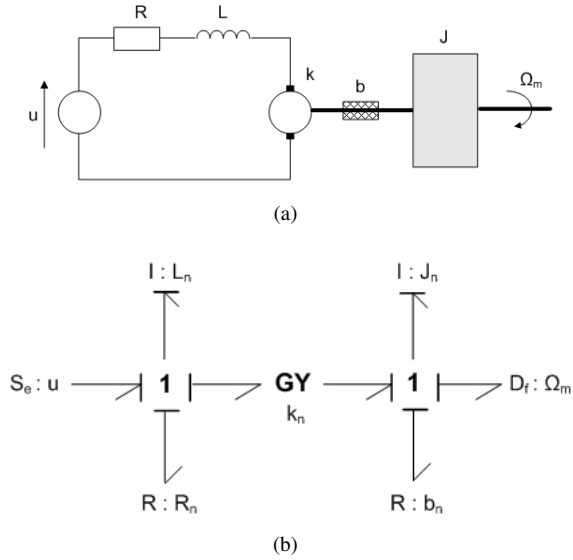
input/output pair is not sufficient. In fact, if a unique auxiliary input/output pair is considered, the uncertainty value will appear in the interconnection matrix  $M$  and consequently the principle of standard interconnection modelling will not be respected. So, the BG-SIM of TF and GY elements necessarily contains two pairs of auxiliary input/output. The outgoing power variables of the nominal element corresponds to auxiliary outputs and the outgoing power variables of the uncertain element corresponds to auxiliary inputs. The nominal and uncertain elements have the same causality which corresponds to the causality of the initial element. The relative uncertainty is not deduced by exploiting causality but it is equal to the fraction  $\frac{w_1}{z_1} = \frac{w_2}{z_2}$ . The type of added junctions (0 or 1) depends on the assigned causality and is such that the BG-SIM verifies the constitutive relationships.

**Note.** The standard interconnection model described by state equations 1 can be deduced directly from the uncertain model after substitution of all the uncertain elements by the associated BG standard interconnection models. The rank of the matrix  $\Delta$  is equal to the number of pairs of auxiliary inputs/outputs associated with the relative uncertainties.

#### 4. EXAMPLE: DC MOTOR

Let us consider a classical DC Motor illustrated in Figure 12(a). It consists of the electrical circuit composed of a voltage source  $u$ , resistance  $R$  and windings  $L$ . Electromechanical coupling is characterized by the torque constant  $k$ . On the mechanical side, only the rotor inertia  $J$  and the viscous friction  $b$  on the rotor are considered. The voltage source

$u$  represents the control input of the system. The motor velocity  $\Omega_m$  represents the output of the system. The state vector  $x = [p_1 \ p_2]^T$  represents the energy storage variables of the windings  $L$  and of the rotor inertia  $J$ . All the parameters ( $R$ ,  $L$ ,  $k$ ,  $J$  and  $b$ ) are uncertain parameters:  $R=R_n+\Delta R$ ;  $L=L_n+\Delta L$ ;  $k=k_n+\Delta k$ ;  $J=J_n+\Delta J$  and  $b=b_n+\Delta b$ . The associated nominal BG model is represented in Figure 12(b). The property, controllability and observability of this BG model are verified (for more details, see [12–14]). The theorems cited in these references represent sufficient conditions for these properties.



**Figure 12.** (a) DC Motor and (b) associated nominal BG model.

The existing procedure and the new procedure will be applied to find the standard interconnection model.

#### 4.1. Uncertain BG model according to the existing procedure

By applying the existing procedure, the obtained BG model is represented in Figure 13. The auxiliary input/output vectors are defined by:  $\omega = [\omega_R \ \omega_b \ \omega_L \ \omega_J \ \omega_1 \ \omega_2]^T$  and  $z = [z_R \ z_b \ z_L \ z_J \ z_1 \ z_2]^T$ . The associated standard interconnection model is described by the state equations 1 with:

$$A_n = \begin{bmatrix} -\frac{R_n}{L_n} & -\frac{k_n}{J_n} \\ \frac{k_n}{L_n} & -\frac{b_n}{J_n} \end{bmatrix}; B_1 = \begin{bmatrix} 1 & 0 & R_n & k_n & 0 & k_n \\ 0 & 1 & -k_n & b_n & k_n & 0 \end{bmatrix};$$

$$B_{2n} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; C_{2n} = \begin{bmatrix} 0 & \frac{1}{J_n} \end{bmatrix}; D_{12} = D_{22n} = 0;$$

$$C_1 = \begin{bmatrix} \frac{R_n}{L_n} & 0 \\ 0 & \frac{b_n}{J_n} \\ \frac{1}{L_n} & 0 \\ 0 & \frac{1}{J_n} \\ \frac{1}{L_n} & 0 \\ 0 & \frac{1}{J_n} \end{bmatrix}; D_{11} = \begin{bmatrix} 0 & 0 & -R_n & 0 & 0 & 0 \\ 0 & 0 & 0 & -b_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$$; D_{21} = [0 \ 0 \ 0 \ -1 \ 0 \ 0].$$

$$\Delta = \text{diag}(-\delta_R, -\delta_b, -\delta_{1/L}, -\delta_{1/J}, \delta_{GY}, -\delta_{GY}); \text{ with } : \delta_R = \frac{\Delta R}{R_n}; \delta_b = \frac{\Delta b}{b_n}; \delta_{1/L} = -\frac{\Delta L}{L_n + \Delta L}; \delta_{1/J} = -\frac{\Delta J}{J_n + \Delta J}; \delta_{GY} = \frac{\Delta k}{k_n}.$$

#### 4.2. Uncertain BG model according to the new procedure

By applying the new procedure, the BG model obtained is represented in Figure 14. The same auxiliary inputs/outputs vectors are considered. The associated standard interconnection model is described by the state equations 1 with:

$$B_1 = \begin{bmatrix} -1 & 0 & -R_n & -k_n & -1 & 0 \\ 0 & -1 & k_n & -b_n & 0 & 1 \end{bmatrix};$$

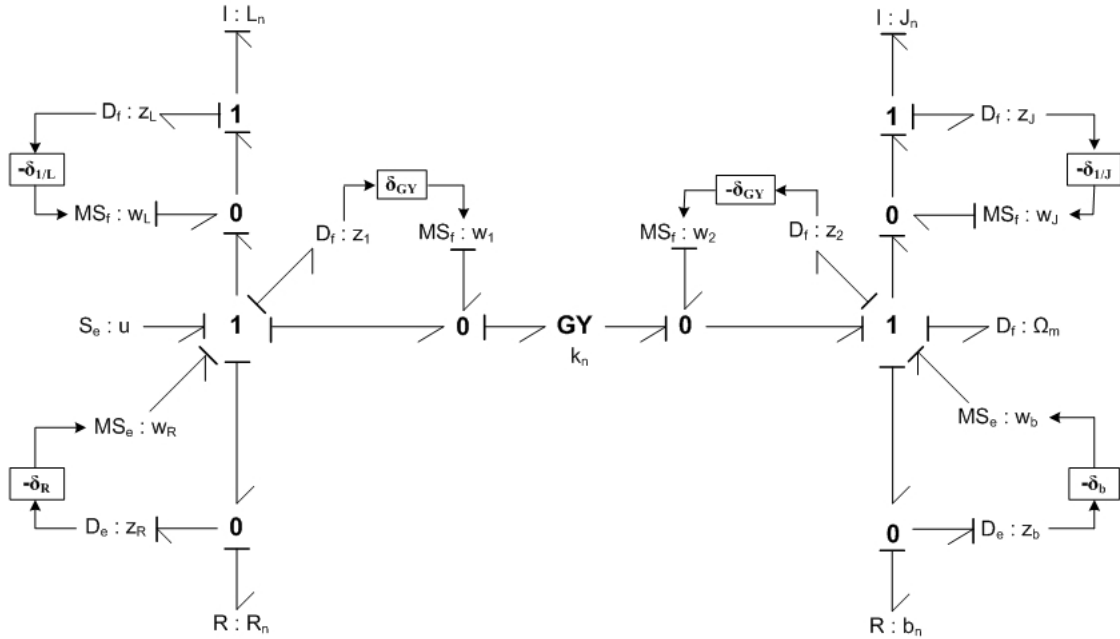
$$D_{11} = \begin{bmatrix} 0 & 0 & R_n & 0 & 0 & 0 \\ 0 & 0 & 0 & b_n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_n & 0 & 0 \\ 0 & 0 & k_n & 0 & 0 & 0 \end{bmatrix}; C_1 = \begin{bmatrix} \frac{R_n}{L_n} & 0 \\ 0 & \frac{b_n}{J_n} \\ \frac{1}{L_n} & 0 \\ 0 & \frac{1}{J_n} \\ 0 & \frac{k_n}{J_n} \\ \frac{k_n}{L_n} & 0 \end{bmatrix};$$

$$D_{21} = [0 \ 0 \ 0 \ 1 \ 0 \ 0].$$

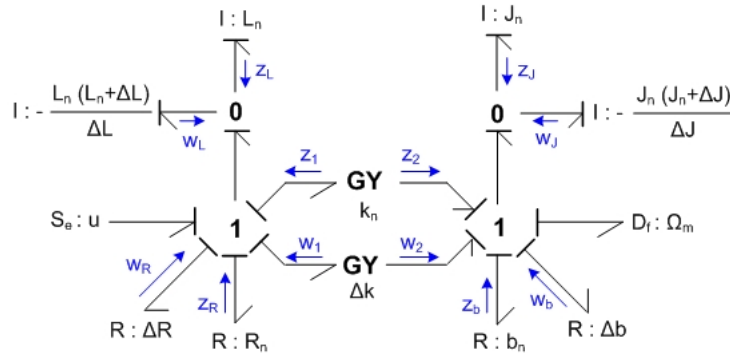
$$\Delta = \text{diag}(\delta_R, \delta_b, \delta_L, \delta_J, \delta_1, \delta_2); \text{ with } : \delta_R = \frac{\Delta R}{R_n}; \delta_b = \frac{\Delta b}{b_n}; \delta_L = -\frac{\Delta L}{L_n + \Delta L}; \delta_J = -\frac{\Delta J}{J_n + \Delta J}; \delta_1 = \delta_2 = \frac{\Delta k}{k_n}. \text{ The matrices } A_n, B_{2n}, C_{2n}, D_{12}, D_{22n} \text{ remain unchanged.}$$

#### 4.3. Comparison

If we compare the results found by the two procedures, we can observe that there are some differences in the matrices  $B_1$ ,  $C_1$ ,  $D_{11}$  and  $D_{21}$ . To explain this difference, let us establish the block diagrams associated to the BG models found by the existing and the proposed procedures. The BG model found by existing procedure is equivalent to the block diagram illustrated in Figure 15(a) with  $\Delta = \text{diag}\left(-\frac{\Delta R}{R_n}, -\frac{\Delta b}{b_n}, \frac{\Delta L}{L_n + \Delta L}, \frac{\Delta J}{J_n + \Delta J}, \frac{\Delta k}{k_n}, -\frac{\Delta k}{k_n}\right)$  and the BG model found by new procedure is equivalent to the block diagram illustrated in Figure 15(b) with  $\Delta = \text{diag}\left(\frac{\Delta R}{R_n}, \frac{\Delta b}{b_n}, -\frac{\Delta L}{L_n + \Delta L}, -\frac{\Delta J}{J_n + \Delta J}, \frac{\Delta k}{k_n}, \frac{\Delta k}{k_n}\right)$ . The differences between the two block diagram representations correspond to the sign (-) which is associated to the auxiliary inputs of the elements  $R$ ,  $L$ ,  $J$  and  $b$  and to the block diagrams of the electromechanical coupling  $k$ . Indeed, in the Figure 15(a) if we zoom on the part representing the electromechanical coupling (characterized by  $k$ ) then it can be remarked that the sum is before the block of parameter  $k_n$  contrarily to the second block diagram (Figure 15(b)) where the block of parameter



**Figure 13.** The BG model of the DC Motor found by the existing procedure.



**Figure 14.** The BG model of the DC Motor found by the new procedure.

$k_n$  is behind the sum. These different arrangements explain the differences obtained in the state-space models.

## 5. CONCLUSION

In this paper, a new procedure was proposed to determine the standard interconnection model describing a linear time-invariant system with parametric uncertainties. For the determination of the standard interconnection model, in comparison with the existing procedure, the new procedure simplifies the uncertain BG model and makes the auxiliary inputs/outputs appear as internal power variables. But, in the new procedure, the relative uncertainty does not appear explicitly in the BG model.

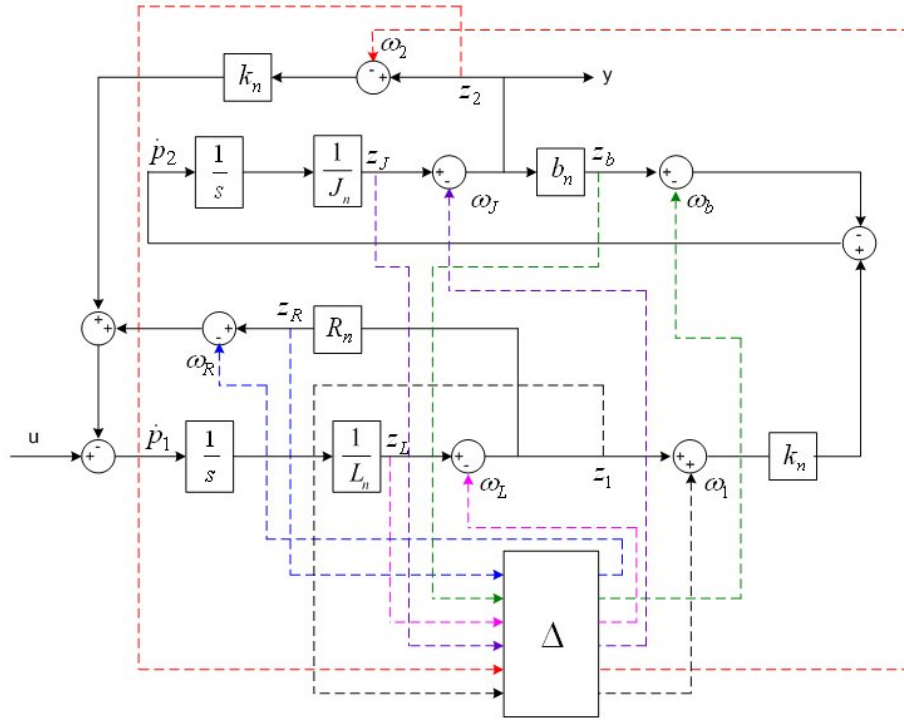
A perspective concerns the TF and GY elements. In fact, they are particular cases of multiport elements because they have

a unique uncertain parameter but, in the general case, a multiport element has a characteristic matrix which contains different parameters. The first step in finding the BG-SIM of a multiport element is the decomposition of the characteristic matrix into several matrices such that on every column and row of each matrix there appears one and only one parameter. Afterwards, every matrix will be dissociated into nominal and uncertain matrices and finally the auxiliary inputs/outputs will be identified. The BG representation of the standard interconnection model of multiport elements will be the topic of a coming paper.

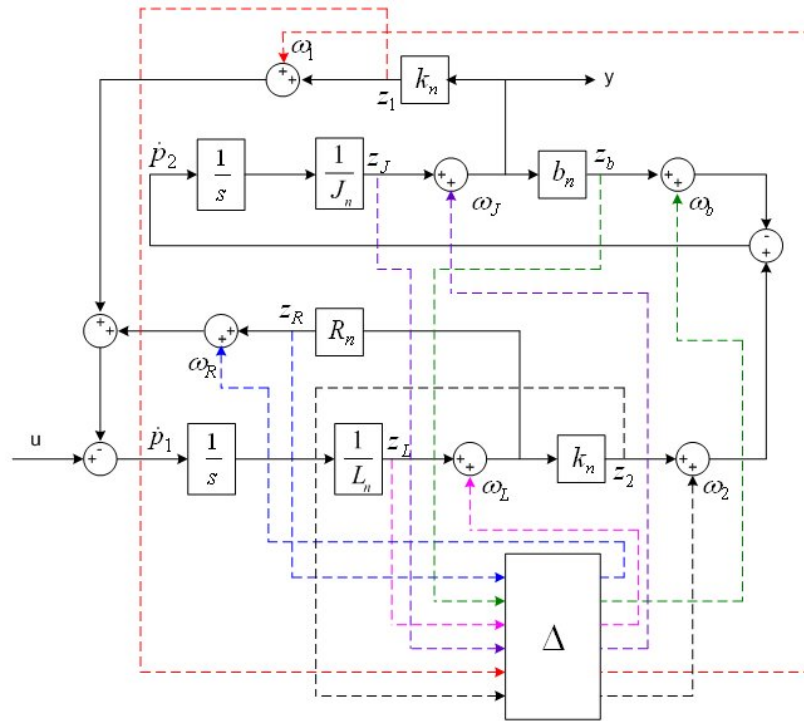
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(a)



(b)

**Figure 15.** Block diagram representations of the DC motor with parametric uncertainties.

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