View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Archive Ouverte en Sciences de l'Information et de la Communication

HAL
archives-ouvertes.fr

A robust component mode synthesis method for stochastic damped vibroacoustics

Q.H. Tran, Morvan Ouisse, Noureddine Bouhaddi

▶ To cite this version:

Q.H. Tran, Morvan Ouisse, Noureddine Bouhaddi. A robust component mode synthesis method for stochastic damped vibroacoustics. Mechanical Systems and Signal Processing, Elsevier, 2010, 24 (1), pp.164-181. 10.1016/j.ymssp.2009.06.016 . hal-00622982

HAL Id: hal-00622982
https://hal.archives-ouvertes.fr/hal-00622982

Submitted on 23 Mar 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

This document is the author's final manuscript of

Q.H. Tran, M. Ouisse and N. Bouhaddi: A robust component mode synthesis method for stochastic damped vibroacoustics. Mechanical Systems and Signal Processing, 24:164–181, 2010.

This paper has been published by Elsevier and can be found at
http://dx.doi.org/10.1016/j.ymssp.2009.06.016

A Robust Component Mode Synthesis Method for Stochastic Damped Vibroacoustics

Quang Hung Tran[1], Morvan Ouisse[1], Noureddine Bouhaddi[1]

FEMTO-ST Institute, Applied Mechanics - UMR CNRS 6174 - 24, chemin de l'épitaphe - 25000 Besançon - France

---

Abstract

In order to reduce vibrations or sound levels in industrial vibroacoustic problems, the low-cost and efficient way consists in introducing visco- and poro-elastic materials either on the structure or on cavity walls. Depending on the frequency range of interest, several numerical approaches can be used to estimate the behavior of the coupled problem. In the context of low frequency applications related to acoustic cavities with surrounding vibrating structures, the finite elements method (FEM) is one of the most efficient techniques. Nevertheless, industrial problems lead to large FE models which are time-consuming in updating or optimization processes. A classical way to reduce calculation time is the Component Mode Synthesis method (CMS), whose classical formulation is not always efficient to predict dynamical behavior of structures including visco-elastic and/or poro-elastic patches. Then, to ensure an efficient prediction, the fluid and structural bases used for the model reduction need to be updated as a result of changes in a parametric optimization procedure. For complex models, this leads to prohibitive numerical costs in the optimization phase or for management and propagation of uncertainties in the stochastic vibroacoustic problem. In this paper,

the formulation of an alternative CMS method is proposed and compared to classical ($\mathbf{u}$,p) CMS method: the Ritz basis is completed with static residuals associated to visco-elastic and poro-elastic behaviors. This basis is also enriched by the static response of residual forces due to structural modifications, resulting in a so-called robust basis, also adapted to Monte Carlo simulations for uncertainties propagation using reduced models.
Key words: component mode synthesis, vibroacoustics, uncertainties, viscoelastic damping, poro-elastic damping

---

List of symbols

$\mathbf{u}$	structural displacement
$\mathbf{v}$	velocity on a surface
$\mathbf{n}$	normal vector of fluid domain
p	pressure
V_s	structural domain
V_f	fluid domain
S_u	surface of fluid-structure coupling
S_a	acoustic absorbing surface
∂V_s^f	structural surface on which external force is imposed
$\mathbf{f}_s$	external force imposed on structure
c	speed of sound in fluid
ρ_f	fluid density
Z_a	acoustic impedance
ω	angular frequency
f	frequency

U	structural displacement vector, physical coordinates
$\mathbf{q}^s$	structural displacement vector, general coordinates
$\mathbb{U}$	random vector corresponding to U
P	pressure vector, physical coordinates
$\mathbf{q}^f$	pressure vector, general coordinates
$\mathbb{P}$	random vector corresponding to P
M	mass matrix
$\bar{M}$	reduced mass matrix
$\mathbb{M}$	random matrix corresponding to M
K	stiffness matrix
$\bar{K}$	reduced stiffness matrix
$\mathbb{K}$	random matrix corresponding to K
A_f	absorbing matrix
$\bar{A}_f$	reduced absorbing matrix
$\mathbb{A}$	random matrix corresponding to A_f
F_s	external force vector
C	coupling matrix
Y	physical coordinates vector
θ	random variable
G	shear modulus of viscoelastic material
T_s, T_f	reduction bases of structure and fluid domains
T	temperature
$\Re\{.\}$	real part
$E\{.\}$	first statistical moment

1. Introduction

In transports industry, reduction of vibration and acoustic levels using industrial vibroacoustic numerical models leads to large and costly problems. Solving dissipative systems in presence of uncertain parameters is still a challenge. The techniques which are classically used in the low frequency range are the finite/infinite elements or boundary elements methods [1], their frequency limits being directly related to the size of the elements compared to the wavelength and to the computer limits. When the frequency range of interest is becoming too high for these approaches, some specific methods are available, often based on wave approaches or power/energy flow analyses [2]. In this paper we will mainly focus on a specific problem, which is the vibroacoustic analysis of damped closed systems, exhibiting an acoustic cavity surrounded by a vibrating structure. For this kind of problem, the finite element method is clearly the most appropriate technique to deal with industrial geometries, even if it is limited to the low frequency range, which is the domain of interest in this work.

Vibroacoustic conservative problem

Because of the proximity of the problem topology with structural dynamics, the concept of modal analysis has been naturally extended to vibroacoustics. In the low-frequency range, this is of particular interest in the context of engineering design, since some trends can help the designer to make decisions using a fully conservative model, which is easy to implement numerically. Modeling damping terms is clearly the hardest thing during the whole process, so using conservative models avoid a difficult step, which can be acceptable only at early design stage, in particular in applications where noise and vibrations are among the design criteria. In this context, using vibroacoustic normal modes can be interesting in a engineering point of view.

In a numerical point of view, even this non-dissipative case still induces difficulties, in particular because the finite elements method (FEM) based on the classical displacement-pressure ($\mathbf{u}$,p) formulation leads to a coupled problem which is large and not symmetric [3], and the very efficient eigenvalue solvers dedicated to symmetric problems, which have been developed for years, can not be applied. Of course, more general solvers can be used, but an alternative way is to transform the initial problem in a symmetric one, using symmetrization techniques [4, 5]. These techniques can be either based only on mathematical considerations (by transforming unsymmetric matrices into symmetric ones), or on physical considerations, by choosing, instead or added to pressure p, another variable in the fluid domain. Among the available descriptions, it has been shown [3] that using the displacement potential leads to a well-posed problem in the static case. Some other formulations leading to symmetric system are for example field displacement, which is complicated by its irrotationality constraint [6]; velocity potential, whose topology is not classic [7, 8] (the double sized state-space has to be used for eigenvalue problem); or combination of two variables, pressure and displacement potential for example [9, 3], which doubles the number of DOFs.

Vibroacoustic damped problem

In order to practically reduce sound level, the low-cost and efficient way consists in introducing visco- and poro-elastic materials, most of the time after the initial design of the structure. The case of viscoelastic damped structure coupled with compressible fluid is considered here and the finite elements (FE) model of visco-elastic structures which is used in this paper is available in literature [10, 11]. Resonances dominated by fluid cavity are controlled by poro-elastic materials. The two classical ways of using such materials in FE models is either to consider the acoustic impedance of the material (the material being modeled by a boundary condition on fluid domain) or to consider the modeling of porous media using for example the Biot-Allard theory [12, 13, 14] whose FE models need a discretization of the poro-elastic domain. For both approaches, the frequency dependence of material parameters is undoubtedly a key point for efficient representation of physical phenomenon, even if it induces difficulties for the resolution of the problem. This resolution is also affected by the size and the topology of the FE models. For frequency responses evaluations, direct resolution of these models are time-consuming and dynamic reduction method [15] is most of the time necessary, in particular when one is interested in the optimization of the choice of absorbing materials (material characteristics, positioning, uncertainties management...). Normal modes of coupled system could be used, but the topology of the system and the high number of DOFs induce numerical difficulties for finding eigenmodes of the coupled system. Therefore, decoupling of domains (fluid and structure) is often considered, normal modes of *in vacuo* structure and rigid walls cavity are classically used for modal reduction. Unfortunately, it has been shown [16] that these reduction strategies have bad convergence properties that can be physically explained by the velocity discontinuity at the fluid-structure interface, which have been replaced by rigid walls. Even if an infinite number of modes would be used, the exact solution in terms of velocity could not be achieved.

In literature [3], it has been proposed to use the displacement potential as unknown variable in the fluid domain and its decoupled modal basis was enriched by static response of cavity induced by the deformation of structure. This is an efficient approach but difficult to use when an acoustic absorbing material is introduced and modeled using normal impedance boundary condition. Recently [17], an equivalent method has been proposed, using pressure as unknown variable, leading to the same difficulty. Identically, the use of pseudo-static corrections for both decoupled modal bases has been investigated [18]: this technique uses static corrections of Ritz basis for elastic structures and has a limitation due to singularity of fluid matrix and can be difficult to adapt to the component synthesis approach.

The first point which is addressed in this paper is related to the improvement of CMS techniques for vibroacoustics: the classic decoupled bases are used first, and then the fluid basis is enriched by cavity residuals vectors associated to specific boundary conditions on the coupling interface, in order to improve convergence.

Stochastic vibroacoustic damped problem

When dealing with absorbing material for vibroacoustics, uncertainties are of first importance for engineering applications, since corresponding materials are most of the time based on polymers or composites which have complex mechanical behavior (anisotropy, visco elasticity, frequency and temperature dependance...) and most of the time exhibit uncertain behavior, due to material and manufacturing dispersions or environmental conditions. Basically, two approaches can be used to deal with uncertainties in the context of FEM, the parametric approach (stochastic FEM) and the non-parametric approach.

The non-parametric approach has been proposed some years ago [19]. In this approach, which is adapted to complex industrial cases with many uncertainties, the idea is that the whole set of uncertainties (including material, manufacturing, environmental, models uncertainties) can be represented by a single dispersion parameter (or a reduced set of parameters). Some mathematical tools have been developed to build a set of random matrices that are used in a Monte Carlo simulation to estimate the variability of the response. The method, which was first developed for positive definite matrices, has been recently extended to vibroacoustics [20]. This approach is well adapted to uncertainties propagation, but it does not allow one to estimate the impact of a given physical parameter to the global dispersion, which is of first importance in design phase or optimization processes.

The parametric approach [21] is used in this paper. It requires the parametric description of random variables, and some stochastic bases are used to project the uncertain response of the system. The calculation cost can be very large, since many iterations are required, depending on the strategy chosen. In any cases, model reduction can help to reduce calculation cost, providing that the reduced model can represent the behavior of the full model. The classical model reduction strategies which have been discussed above must be updated as soon as one parameter varies. In this paper, a specific effort is made to define a so-called robust basis, which does not require updating after parametric changes, in order to use it efficiently in inverse problems (e.g. in the case of optimization) or during the direct random analysis problem (uncertainties propagation).

The construction of the bases associated to uncertainties propagation is based on parametric approach. Fluctuation of random variables around their nominal values is considered as modifications according to nominal model and this set of modifications induces a set of residual forces which act on the nominal model. Robust basis is established by enriching the Ritz basis of nominal model with dynamic vectors (corresponding to a deterministic frequency) or static responses of nominal model due to modification forces. This strategy allows one to obtain a final reduced problem with a small size that can be efficiently used in iterative procedures.

2. Formulation of vibroacoustic problem

2.1. Coupled formulation

The internal vibroacoustic problem which is considered in this paper is presented in figure 1. Let V_f be the fluid domain, V_s the structural domain, S_u the fluid-structure coupling interface and S_a the acoustic absorbing surface characterized by acoustic impedance $Z_a(\omega)$. The equations describing the permanent harmonic response at frequency ω of fluid domain in terms of

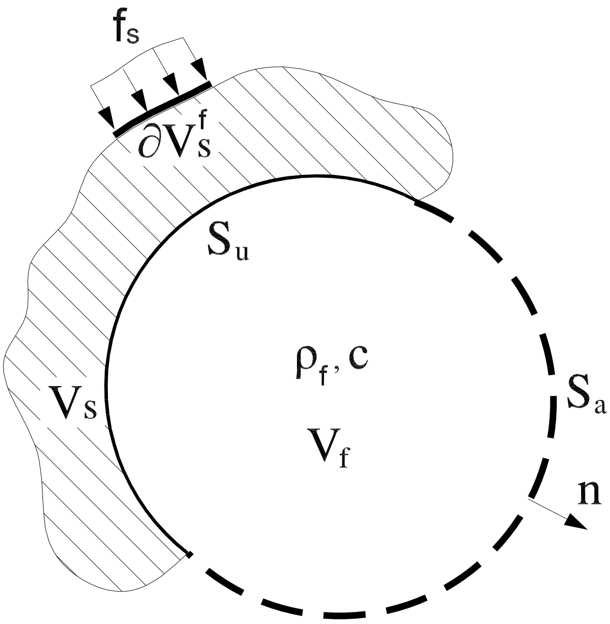

Figure 1: Description of vibroacoustic problem

pressure variable are [3, 9, 22]:

$$\left\{\begin{array}{rcll} \Delta p + \dfrac{\omega^2}{c^2} p & = & 0, & (a) \\ \dfrac{\partial p}{\partial \mathbf{n}} & = & \rho_f \omega^2 u_n \text{ on } S_u \cup S_a, & (b) \\ v_n & = & \dfrac{p}{Z_a(\omega)} \text{ on } S_a. & (c) \end{array}\right. \quad (1)$$

In order to be well-posed in the static case ($\omega = 0$), the following constraint is introduced [17]:

$$\int_{V_f} p dV = -\rho_f c^2 \int_{S_u \cup S_a} u_n dS, \quad (2)$$

which leads to the static solution p^s:

$$p^s = -\frac{\rho_f c^2}{V_f} \int_{S_u \cup S_a} u_n dS. \quad (3)$$

In the dynamic case ($\omega \neq 0$), any solution of (1) always satisfies the constraint (2).

A weak variational formulation of vibroacoustic problem is:

$$
\text{for all admissible } (\delta\mathbf{u}, \delta p), \text{ find } (\mathbf{u}, p) \text{ such that:}
\left\{
\begin{array}{rl}
0 = & k^s(\mathbf{u}, \delta\mathbf{u}) - \omega^2 m^s(\mathbf{u}, \delta\mathbf{u}) - \int_{S_u} p\mathbf{n}\delta\mathbf{u}dS \\
 & - \int_{\partial V_s^f} \mathbf{f}_s \delta\mathbf{u}dS, \\
0 = & \int_{V_f} \nabla p \nabla \delta p dV - \frac{\omega^2}{c^2}\int_{V_f} p\delta p dV \\
 & -\omega^2 \rho_f \int_{S_u} u_n \delta p dS + j\omega \frac{\rho_f}{Z_a(\omega)} \int_{S_a} p\delta p dS, \\
0 = & \int_{V_f} p dV + \rho_f c^2 \int_{S_u \cup S_a} u_n dS.
\end{array}
\right. \tag{4}
$$

k^s and m^s are structural stiffness and mass operators, ∂V_s^f is the structural surface on which the external force is imposed.

The FE discretization of (4) can be written as:

$$K_s U - \omega^2 M_s U - CP = F_s, \tag{5}$$

$$K_f P - \omega^2 M_f P + \frac{j\omega}{Z_a(\omega)} A_f P - \rho_f \omega^2 C^T U = 0, \tag{6}$$

where K_s, M_s are structural stiffness and mass matrices, K_f et M_f are respectively matrices corresponding to the discretization of kinematic energy and compressibility matrix of fluid (named "stiffness" and "mass" matrices of fluid in the following). $K_f \in \mathcal{R}^{N_f \times N_f}$ is symmetric positive semi-definite of rank $N_f - 1$, $M_f \in \mathcal{R}^{N_f \times N_f}$ is symmetric definite positive; C is the coupling matrix; A_f is the absorbing acoustic matrix, symmetric and depending on the geometry of S_a; F_s is the external force vector.

Combining (5) and (6) allows one to obtain the classic unsymmetric system:

$$\left(\begin{bmatrix} K_s & -C \\ 0 & K_f \end{bmatrix} - \omega^2 \begin{bmatrix} M_s & 0 \\ \rho_f C^T & M_f \end{bmatrix} + \frac{j\omega}{Z_a(\omega)} \begin{bmatrix} 0 & 0 \\ 0 & A_f \end{bmatrix}\right) \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix}. \tag{7}$$

This can be written in a compact form:

$$[K - \omega^2 M + \frac{j\omega}{Z_a(\omega)} A]Y = F, \tag{8}$$

where:

$$K = \begin{bmatrix} K_s & -C \\ 0 & K_f \end{bmatrix} \; ; \; M = \begin{bmatrix} M_s & 0 \\ \rho_f C^T & M_f \end{bmatrix} \; ; \; A = \begin{bmatrix} 0 & 0 \\ 0 & A_f \end{bmatrix}$$
$$Y = \begin{Bmatrix} U \\ P \end{Bmatrix} \; ; \; F = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix}. \tag{9}$$

2.2. Acoustic modes

Acoustic modes are solutions of the eigenvalue problem using rigid cavity boundary conditions:

$$\begin{cases} \Delta p + \dfrac{\omega^2}{c^2} p = 0, & (a) \\ \dfrac{\partial p}{\partial \mathbf{n}} = 0 \text{ on rigid walls}, & (b) \\ \displaystyle\int_{V_f} p dV = 0. & (c) \end{cases} \tag{10}$$

The associated weak variational formulation is:

$$\int_{V_f} \nabla p \nabla \delta p dV - \frac{\omega^2}{c^2} \int_{V_f} p \delta p dV = 0. \tag{11}$$

Then, the discretization corresponding to equation (11) is:

$$\left(K_f - \omega^2 M_f\right) P = 0. \tag{12}$$

It is easy to see that ($\omega = 0; p = const$) is a trivial solution of ($10ab$), but it does not satisfy the constraint ($10c$). On the opposite, ($10c$) is automatically verified by all the solutions ($\omega \neq 0$) of ($10ab$).

3. Model reduction of deterministic vibroacoustic problem

3.1. Classical reduction using decoupled basis

One can project now (7) on the decoupled basis T_s and T_f containing *in vacuo* structural modes and rigid wall cavity modes:

$$U = \sum_{\beta=1}^{n_s} U_\beta q_\beta^s = T_s \mathbf{q}^s, \quad P = \sum_{\alpha=1}^{n_f} P_\alpha q_\alpha^f = T_f \mathbf{q}^f, \tag{13}$$

where q_β^s are the modal coordinates. One obtains the reduced system:

$$\left(\begin{bmatrix} \bar{K}_s & -\bar{C} \\ 0 & \bar{K}_f \end{bmatrix} - \omega^2 \begin{bmatrix} \bar{M}_s & 0 \\ \rho_f \bar{C}^T & \bar{M}_f \end{bmatrix} + \frac{j\omega}{Z_a(\omega)} \begin{bmatrix} 0 & 0 \\ 0 & \bar{A}_f \end{bmatrix} \right) \begin{Bmatrix} \mathbf{q}^s \\ \mathbf{q}^f \end{Bmatrix} = \begin{Bmatrix} \bar{F}_s \\ 0 \end{Bmatrix}, \tag{14}$$

where:

$$\begin{aligned} &\bar{K}_s = T_s^T K_s T_s, && \bar{M}_s = T_s^T M_s T_s, \\ &\bar{C} = T_s^T C T_f, && \bar{K}_f = T_f^T K_f T_f, \\ &\bar{M}_f = T_f^T M_f T_f, && \bar{A}_f = T_f^T A_f T_f, \\ &\bar{F}_s = T_s^T F_s. \end{aligned} \tag{15}$$

The fluid basis T_f is defined using rigid wall instead of coupling interface, inducing that the velocity continuity is not satisfied on the coupling interface.

Therefore the response can not converge exactly (in terms of velocity) to the accurate solution even if many modes are introduced [16]. To improve the convergence, T_f is enriched by static response p^s defined by (3). p^s is constant in space at each given ω, so it can be written $p^s = q_0^f p_0$, where p_0 is the static cavity mode [17] corresponding to $\omega = 0$:

$$p = \sum_{\alpha=1}^{n} p_\alpha q_\alpha^f + p^s = \sum_{\alpha=0}^{n} p_\alpha q_\alpha^f, \tag{16}$$

or, in a discretized form:

$$P = \sum_{\alpha=0}^{n} P_\alpha q_\alpha^f. \tag{17}$$

In the case without absorbing area S_a, the system (14) can be transformed to a reduced symmetric system by using decomposition (16). If the cavity modes p_α have been "mass"-normalized, variational equations (4) with p defined by (16), using test function $\delta p = p_\alpha, \alpha = 0, ..., n$ leads to:

$$k^s(\mathbf{u}, \delta\mathbf{u}) - \omega^2 m^s(\mathbf{u}, \delta\mathbf{u}) - q_0^f p_0 \int_{S_u} \mathbf{n}\delta\mathbf{u} dS \\ - \sum_{\alpha=1}^{n} \int_{S_u} q_\alpha^f p_\alpha \mathbf{n}\delta\mathbf{u} dS = \int_{\partial V_s^f} \mathbf{f}_s \delta\mathbf{u} dS, \tag{18}$$

$$q_0^f = -\rho_f p_0 \int_{S_u} u_n dS, \tag{19}$$

$$\left(\omega_\alpha^2 - \omega^2\right) q_\alpha^f - \omega^2 \rho_f \int_{S_u} u_n p_\alpha dS = 0, \ \forall\alpha \in \mathcal{N}^*. \tag{20}$$

The elimination of variable q_0^f leads to:

$$k^s(\mathbf{u}, \delta\mathbf{u}) - \omega^2 m^s(\mathbf{u}, \delta\mathbf{u}) + \rho_f p_0^2 \int_{S_u} u_n dS \int_{S_u} \mathbf{n}\delta\mathbf{u} dS \\ - \sum_{\alpha=1}^{n} \int_{S_u} q_\alpha^f p_\alpha \mathbf{n}\delta\mathbf{u} dS = \int_{S_f} \mathbf{f}_s \delta\mathbf{u} dS, \tag{21}$$

$$\left(\omega_\alpha^2 - \omega^2\right) q_\alpha^f - \omega^2 \rho_f \int_{S_u} u_n p_\alpha dS = 0,\ \forall \alpha \in \mathcal{N}^*, \tag{22}$$

or, after discretization:

$$\begin{aligned}(K_s + K_c)\, U - \omega^2 M_s - \omega^2 \sum_{\alpha=1}^{n} \frac{1}{\omega_\alpha^2} C P_\alpha q_\alpha^f \\ -\omega^2 \sum_{\alpha=1}^{n} \frac{\rho_f}{\omega_\alpha^2} C P_\alpha P_\alpha^T C^T U \quad &= F_s,\end{aligned} \tag{23}$$

$$\left(\frac{1}{\rho_f} - \frac{\omega^2}{\rho_f \omega_\alpha^2}\right) q_\alpha^f - \frac{\omega^2}{\omega_\alpha^2} P_\alpha^T C^T U = 0,\ \forall \alpha \in \mathcal{N}^*, \tag{24}$$

where K_c is obtained by the discretization of $p_0^2 \int_{S_u} u_n dS \int_{S_u} \mathbf{n} \delta \mathbf{u} dS$. Using the fluid basis T_f containing normal modes P_α, the combination of the two equations below leads to the following symmetric system:

$$\left(\begin{bmatrix} K_s + K_c & 0 \\ 0 & diag\left(\frac{1}{\rho_f}\right) \end{bmatrix} - \omega^2 \begin{bmatrix} M_s + M_c & C T_f diag\left(\frac{1}{\omega_\alpha^2}\right) \\ sym & diag\left(\frac{1}{\rho_f \omega_\alpha^2}\right) \end{bmatrix} \right) \begin{Bmatrix} U \\ \mathbf{q}^f \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix}, \tag{25}$$

in which the matrix $M_c = \sum_{\alpha=1}^{n} \frac{\rho_f}{\omega_\alpha^2} C P_\alpha P_\alpha^T C^T$ is symmetric. The reduced symmetric system is expressed in hybrid coordinates (physic: U, modal: $\mathbf{q}^f$).

To complete the reduction on modal basis, U is now projected on the solutions of the following eigenvalue problem [3, 17]:

$$(K_s + K_c)\, U_\beta = \omega_\beta^2 \left(M_s + M_c\right) U_\beta, \tag{26}$$

where U_β is a structural mode of the structure including added mass and stiffness effects of fluid, associated to K_c and M_c. The projection of U is:

$$U = \sum_{\beta=1}^{m} U_\beta q_\beta^s = T_s \mathbf{q}^s. \tag{27}$$

This gives the reduced model expressed with generalized coordinates:

$$\left(\begin{bmatrix} diag(\omega_\beta^2) & 0 \\ 0 & diag\left(\frac{1}{\rho_f}\right) \end{bmatrix} \right. \tag{28}$$

$$\left. -\omega^2 \begin{bmatrix} \mathbf{I} & \bar{C} diag\left(\frac{1}{\omega_\alpha^2}\right) \\ sym & diag\left(\frac{1}{\rho_f \omega_\alpha^2}\right) \end{bmatrix} \right) \left\{ \begin{array}{c} \mathbf{q}^s \\ \mathbf{q}^f \end{array} \right\} = \left\{ \begin{array}{c} T_s^T F_s \\ 0 \end{array} \right\}.$$

It should be emphasized that the above equations have been obtained without acoustic absorbing area S_a. When S_a, characterized by Z_a, is present, the variables in equations leading to (21) and (22) are linearly dependent (because of the A_f terms), therefore that transformation leads to a complex reduced system which is not easy to implement. An alternative way to obtain a reduced model in such a situation is presented in the next section.

3.2. A CMS method using decoupled basis enriched by residual response vectors

In this section, a simple and efficient modal synthesis method is proposed, based on the enrichment of decoupled fluid basis by selected residual vectors which are responses of fluid cavity caused by interface operators (fluid-structure coupling surface and absorbing area).

Equation (6) can be rewritten as:

$$\left(K_f - \omega^2 M_f\right) P = \omega^2 \rho_f C^T U - \frac{j\omega}{Z_a(\omega)} A_f P = F_{fs} + F_{fa}, \tag{29}$$

in which F_{fs} and F_{fa} denote external forces caused by structure and by absorbing surface:

$$\begin{array}{rcll} F_{fs} & = & \omega^2 \rho_f C^T U & \text{(a)}, \\ F_{fa} & = & -\frac{j\omega}{Z_a(\omega)} A_f P & \text{(b)}. \end{array} \tag{30}$$

These forces are linked to the unknowns of the problem. The objective consists in determining their responses to enrich fluid basis defined by (17). Structural basis is still T_s containing the normal modes of *in vacuo* structure. F_{fs} and F_{fa} can be evaluated using modal projection of displacement and pressure in their expressions:

$$\begin{array}{rcll} F_{fs} & \approx & \omega^2 \rho_f C^T T_s \mathbf{q}^s & \text{(a)}, \\ F_{fa} & \approx & -\frac{j\omega}{Z_a(\omega)} A_f T_f \mathbf{q}^f & \text{(b)}. \end{array} \tag{31}$$

In structural dynamics, static responses are classically used to determine residual vectors. In presence of fluid, the singularity of K_f can induce numerical difficulties, however one can modify it by adding an extra term $\alpha_c M_f$ which is proportional to fluid mass matrix. To well represent the behavior of system, α_c should be within the frequency band of interest: $\alpha_c = \omega_c^2;\ \omega_c \in [\omega_{min}\ \omega_{max}]$. Residual vectors are introduced by:

$$\begin{array}{rcl} \Delta T_{fs} & = & \left(K_f - \omega_c^2 M_f\right)^{-1} C^T T_s, \\ \Delta T_{fa} & = & \left(K_f - \omega_c^2 M_f\right)^{-1} A_f T_f. \end{array} \tag{32}$$

Thus enriched fluid basis is now:

$$T_{fe} = [T_f\ \Delta T_{fs}\ \Delta T_{fa}]. \tag{33}$$

A singular values decomposition (SVD) of T_{fe} can be realized to guarantee good conditioning by selecting the largest directions of the space, resulting

in a reduction of vector numbers [23].
Reduced system is the same one as (14) with T_f replaced by T_{fe}. It should be noted that to count static response of higher structural modes (which are dropped out of T_f) T_f contains now also static mode P_0. Efficiency and performance of this reduced model can be compared to the reduced problem (14) which uses the classical decoupled bases.

3.3. Model reduction of structure with viscoelastic damping

Now, consider the case in which the structure includes damped viscoelastic patches. In this case, stiffness K_s can be separated into two parts, one being purely elastic, constant, and the other one being viscoelastic, frequency and temperature dependent:

$$K_s(\omega) = K_{se} + G(\omega, T)K_{sv}. \tag{34}$$

Thus, the FE model of the structure with viscoelastic damping can be written as

$$[K_{se} + G(\omega, T)K_{sv} - \omega^2 M_s]U = F_s, \tag{35}$$

where $G(\omega, T)$ is the shear modulus of viscoelastic material, mainly depending on frequency ω and temperature T, and possibly to other environmental factors. The figure 2 shows a nomogram in reduced frequency of viscoelastic material $3M\ ISD112$ allowing the synthetic representation of frequency and temperature evolution of the shear modulus $G(\omega, T)$. The reduced frequency is defined by: $\omega_r = \alpha_T \omega$ in which α_T is function of temperature T. More details about visco-elastic aspects can be found in references [10, 11].

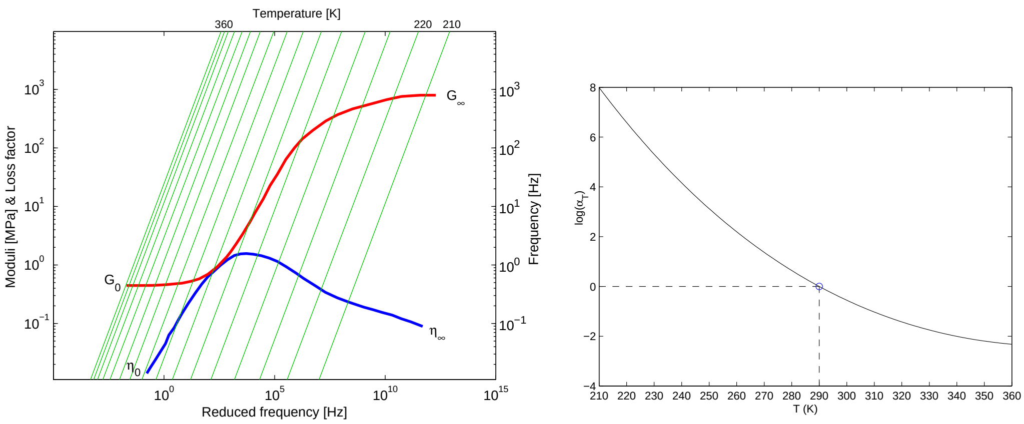

Figure 2: Nomogram in reduced frequency of material $3M\ ISD112^{TM}$ and associated reduced frequency α_T

In the model reduction strategy, it is better to use a basis which is not frequency dependent. To achieve this, one can for example use normal pseudo-modes [24] or multi-model [25]. In this study, one considers the multi-model approach. Let $[\omega_{min}\ ;\ \omega_{max}]$ be the frequency range of interest and $\omega_c \in [\omega_{min}\ ;\ \omega_{max}]$ be a specific value of frequency, then the complex stiffness is $K_s(\omega_c) = K_{se} + G(\omega_c, T)K_{sv}$. The basis T_{s0} containing the modes of associate conservative problem can be calculated easily:

$$[\Re\{K_s(\omega_c)\} - \omega^2 M_s]U = 0, \tag{36}$$

where $\Re\{.\}$ stands for real part. This basis is then enriched by the static response ΔT_{s0} of system (36) to viscoelastic force defined as follow:

$$\Delta T_{s0} = [\Re\{K_s(\omega_c)\}]^{-1} K_{sv} T_{s0}, \tag{37}$$

which defines the enriched basis $T_s = [T_{s0}\ \Delta T_{s0}]$. If the basis T_s is not efficient enough to insure the convergence, in particular if the frequency range is very large, several values of ω_c can be used to build the basis T_{s0}. The reduced model is:

$$\{\bar{K}_{se} + G(\omega, T)\bar{K}_{sv} - \omega^2 \bar{M}_s\}\mathbf{q}^s = \bar{F}_s. \tag{38}$$

Note that with this basis, the reduced matrix $\bar{K}_s$ and $\bar{M}_s$ are not diagonal.

3.4. Extension of Craig-Bampton dynamic substructuring

The classical Craig-Bampton basis of a subdomain k is written for a structural subdomain or a fluid one:

$$T_s^{(k)} = \begin{bmatrix} \mathbf{I} & 0 \\ -(K_{II}^s)^{-1}K_{IF}^s & \Psi_s \end{bmatrix} \text{ for structure,} \tag{39}$$

and

$$T_f^{(k)} = \begin{bmatrix} \mathbf{I} & 0 \\ -(K_{II}^f)^{-1}K_{IF}^f & \Psi_f \end{bmatrix} \quad \text{for fluid.} \tag{40}$$

I index is related to internal DOFs, while F index is associated to fixed DOFs. Ψ_s contains normal modes of structural subdomain fixed on interface, Ψ_f contains normal modes of fluid subdomain with boundary condition $P = 0$ on interface.

If the structure includes viscoelastic damping patches, matrices K_{II}^s and K_{IF}^s will be replaced by $K_{II}^s(\omega = \omega_c)$ and $K_{IF}^s(\omega = \omega_c)$ like proposed in litterature [11]. Associate basis is thus $\Psi_s = [\Psi_{s0}\ \Delta\Psi_{s0}]$ in which Ψ_{s0} contains solutions of the following equation:

$$\left\{\Re\left(K_{II}^s(\omega_c)\right) - \omega^2 M_{II}^s\right\} U = 0, \tag{41}$$

and $\Delta\Psi_{s0}$ is determined by

$$\Delta T_{s0}(\omega_c) = \left[\Re\left(K_{II}^s(\omega_c)\right)\right]^{-1} K_{II}^{sv} T_{s0}(\omega_j). \tag{42}$$

According to section 3.2, for a fluid subdomain which has coupling surfaces $S_u^{(k)}$ with structure and absorbing surface $S_a^{(k)}$, basis Ψ_f is now replaced by enriched basis $\Psi_{fe} = [\Psi_f \ \Delta\Psi_{fs} \ \Delta\Psi_{fa}]$ where $\Delta\Psi_{fs}$ and $\Delta\Psi_{fa}$ are:

$$\begin{aligned} \Delta\Psi_{fs} &= \left(K_{II}^f - \omega_c^2 M_{II}^f\right)^{-1} C_{II}^T \sum T_s, \\ \Delta\Psi_{fa} &= \left(K_{II}^f - \omega_c^2 M_{II}^f\right)^{-1} A_{II}^f \Psi_f, \end{aligned} \tag{43}$$

in which $\sum T_s$ symbolizes the set of all substructural bases coupled with the considered fluid sub-domain.

3.5. Simulations

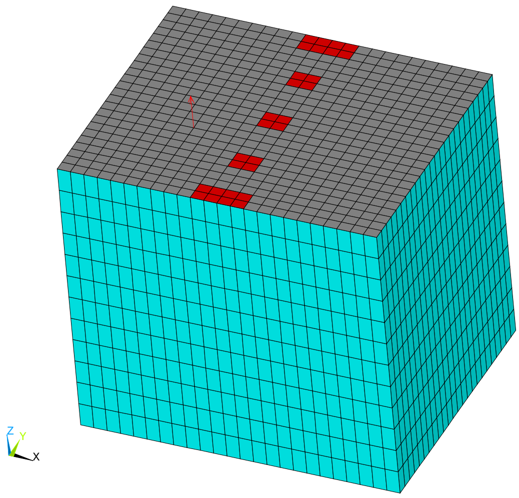

Figure 3: FE model of cavity coupled with plate treated by viscoelastic patches and three rigid walls treated by poroelastic patches

An acoustic cavity (fig. 3) of size $a \times b \times c = 0.654 \times 0.527 \times 0.6\ m^3$ is coupled with a flexible plate which is treated by viscoelastic patches $3M\ ISD112$

(Figure 2) at $T = 25^oC$, the other walls are rigid. On three perpendicular rigid walls, poro-elastic patches are sticked, they are characterized by acoustic impedance $Z_a(\omega)$ given in figure 4. A harmonic point force located in $(x = 0.191m; y = 0.198m)$ is exciting the plate, and the frequency range of interest is $[0;\ 300]Hz$.

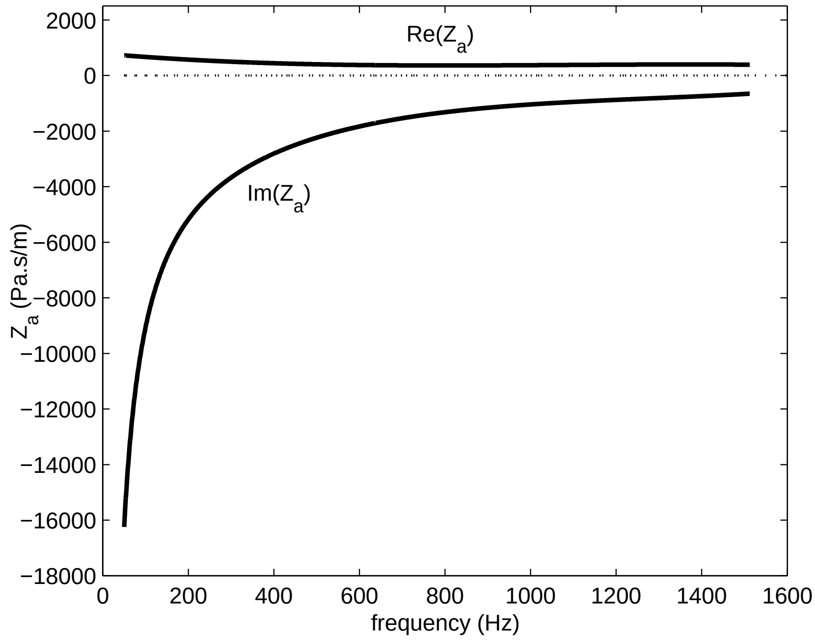

Figure 4: Acoustic impedance of absorbing material $Z_a(\omega)$

The characteristics of the plate are:

- base plate:

$a \times b = 0.654 \times 0.527\ m^2$,

$E_1 = 7 \times 10^{10} Pa;\ \nu_1 = 0.33$,

$h_1 = 3 \times 10^{-3}\ m$;

- viscoelastic layer: $h_2 = 2.54 \times 10^{-5}\ m$;

- constrained layer: $E_3 = 2.5{\times}10^9 Pa$; $\nu_3 = 0.33$; $h_3 = 5{\times}10^{-4}\ m$.

The plate is meshed using 24×24 ANSYS SHELL63 elements (thin plate). For the fluid domain, 24 × 24 × 12 FLUID30 elements are used. This mesh guarantees 6 elements per wavelength in the frequency range of interest (up to 300 Hz).

Two indicators are used to present the results, the acoustic power P_i and the mean of quadratic velocity $\bar{V}_n^2$:

$$P_i = \int_{V_f} \left(\frac{1}{4\rho_f} pp^* + \frac{1}{4\rho_f \omega^2} \nabla p \nabla p^* \right) dV, \tag{44}$$

$$\bar{V}_n^2 = \frac{1}{|S|} \int_S v_n^2 dS, \tag{45}$$

where p^* stands for the conjugate of p.

Two cases are considered. Case 1 corresponds to cavity with viscoelastic plate and case 2 corresponds to cavity with viscoelastic plate and poro-elastic patches (surface S_a).

In order to build the normal modes of multi-model method, one value $\omega_c = 150 \times 2\pi$ (in the middle of band) is used. Basis T_s contains 41 modes: 20 normal modes of T_{s0} (criterion $2 \times f_{max}$), 20 modes of ΔT_{s0} and one vector corresponding to static residual response to force F_s. Basis T_f has 30 modes contained in the frequency band $[0;\ 3 \times f_{max}]$ while ΔT_{fs} contains 36 vectors and ΔT_{fa} contains 29 ones. So enriched basis T_{fe} has 66 modes for case 1 and 95 for case 2 (with or without S_a). The figure 5 shows the limitation of classic decoupled bases even if they have been enriched by p^s: errors are large, even when the number of modes in T_f is increased to 100 and the static mode is included in the basis. In the whole paper, results are presented in dB using a reference level of $10^{-12} S.I.$, and the reference solution is obtained by solving the full FE model with the direct method.

Figures 6 and 7 illustrate the efficiency of the proposed basis in which decoupled fluid basis is enriched using residual vectors with or without absorbing area S_a: the error levels are much lower than with the classical uncoupled bases.

4. Robust dynamic reduction method

4.1. Formulation

In this section, the investigations are only related to the parametric approach of uncertainties. In the following, fonts with double lines will be used for random variables (i.e. $\mathbb{M}$ will be used to describe the random variable associated to M). The FE deterministic model has been defined above (equations 5-7) and it can be written in compact form (8):

$$[K - \omega^2 M + \frac{j\omega}{Z_a} A]Y = F, \tag{46}$$

where $Y^T = [U^T \;\; P^T]$ is the deterministic response vector. In order to validate the method, we consider that uncertain parameters are only those of characteristics of acoustic absorbing material and viscoelastic elements, so there is no uncertainty in matrices K_f, M_f, C and A_f.

Figure 5: Indicators of classical uncoupled basis, cavity without S_a. (a): sound power level, (b): mean of quadratic velocity

Figure 6: Indicators of proposed method, cavity without S_a. (a): sound power level, (b): mean of quadratic velocity

Figure 7: Indicators of proposed method, cavity with S_a. (a): sound power level, (b): mean of quadratic velocity

Let θ be a random variable (vector or scalar), the stochastic FE model function of θ is:

$$\left[\mathbb{K}_s - \omega^2\mathbb{M}_s\right]\mathbb{U} + C\mathbb{P} = F_s, \tag{47}$$

$$\left[K_f - \omega^2 M_f + \frac{j\omega}{\mathbb{Z}_a}A_f\right]\mathbb{P} + \rho_f\omega^2 C^T\mathbb{U} = 0. \tag{48}$$

Or, in compact form:

$$[\mathbb{K} - \omega^2\mathbb{M} + \frac{j\omega}{\mathbb{Z}_a}A]\mathbb{Y} = F. \tag{49}$$

By setting $\mathbb{H} = [\mathbb{K} - \omega^2\mathbb{M} + \frac{j\omega}{\mathbb{Z}_a}A]$ one has:

$$\theta \longmapsto \mathbb{Y} = f(\theta) = \mathbb{H}^{-1}F. \tag{50}$$

The random variable θ can be represented as:

$$\theta = \theta_0 + \Delta\theta, \tag{51}$$

where θ_0 is the nominal value of θ and $\Delta\theta$ is the fluctuation around θ_0. If θ is Gaussian one can map:

$$\theta = \theta_0\left(1 + \delta_\theta\xi\right), \tag{52}$$

where δ_θ represents the dispersion level, and ξ is a central Gaussian variable. Thereby a random matrix, for example $\mathbb{K}_s$, function of θ can be expressed as:

$$\mathbb{K}_s(\theta) = K_s + \Delta\mathbb{K}_s(\theta), \tag{53}$$

in which $K_s = \mathbb{K}_s(\theta_0)$ is its nominal value, $\Delta\mathbb{K}_s(\theta)$ is considered as a random modification around K_s. Consequently, equations (47) and (48), for a set of parameters described by variables θ, can be written as:

$$\begin{aligned}\left[K_s + \Delta\mathbb{K}_s(\theta) - \omega^2(M_s + \Delta\mathbb{M}_s(\theta))\right]\mathbb{U}(\theta) & \\ + C\mathbb{P}(\theta) &= F_s, \quad (54) \\ \left[K_f - \omega^2 M_f + \frac{j\omega}{Z_a + \Delta\mathbb{Z}_a(\theta)}A_f\right]\mathbb{P}(\theta) & \\ + \rho_f\omega^2 C^T\mathbb{U}(\theta) &= 0, \quad (55)\end{aligned}$$

where $\Delta\mathbb{K}_s(\theta) = \Delta\mathbb{K}_{se}(\theta) + \Delta\left[\mathbb{G}(\theta)\mathbb{K}_{sv}(\theta)\right]$ for viscoelastic damping. The term corresponding to acoustic absorbing material can be rewritten as:

$$\frac{1}{Z_a + \Delta\mathbb{Z}_a(\theta)}A_f = \frac{1}{Z_a}A_f + \mathbf{a}(\theta)A_f, \quad (56)$$

in which $\mathbf{a}(\theta)$ is a random scalar - also function of Z_a and $\Delta\mathbb{Z}_a(\theta)$. Equations (54)-(56) show that the random model has random perturbation terms around deterministic model.

Using the robust reduction bases $\mathbb{T}_s$ and $\mathbb{T}_{fe}$ leads to robust reduced model of (49):

$$[\bar{\mathbb{K}} - \omega^2\bar{\mathbb{M}} + \frac{j\omega}{\mathbb{Z}_a}\bar{\mathbb{A}}]\bar{\mathbb{Y}} = \bar{\mathbb{F}}. \quad (57)$$

Problem now resides in the construction of robust bases $\mathbb{T}_s$ and $\mathbb{T}_{fe}$. During Monte Carlo simulation, for any one sample of θ, solving new eigenvalue problems and calculating residual vectors are theoretically required to find them, as indicated by equations (32), (36) and (37), which is time expensive. In order to avoid this kind of reactualization, an alternative approximate method is the robust basis [23, 26], based on the deterministic model whose reduction basis are $T_s = [T_{s0}\ \Delta T_{s0}]$ and $T_{fe} = [T_f\ \Delta T_{fs}\ \Delta T_{fa}]$. These deterministic bases will be enriched by the response vectors of deterministic model to the forces corresponding to random modifications. For the structure, the static responses to these forces are considered [23, 11] (respectively mass, elastic and viscoelastic stiffness):

$$\begin{cases} \Delta\mathbb{T}_{s1}^{M_s} = [\Re(K_s(\omega_c))]^{-1}\Delta\mathbb{M}_s(\theta)T_{s0}, \\ \Delta\mathbb{T}_{s1}^{K_{se}} = [\Re(K_s(\omega_c))]^{-1}\Delta\mathbb{K}_{se}(\theta)T_{s0}, \\ \Delta\mathbb{T}_{s1}^{K_{sv}} = [\Re(K_s(\omega_c))]^{-1}\Delta\mathbb{K}_{sv}(\theta)T_{s0}. \end{cases} \tag{58}$$

The deterministic basis T_s is then enriched by the mean structural modification random vectors $\Delta\mathbb{T}_{s1}$:

$$\Delta\mathbb{T}_{s1} = E\left([\Delta\mathbb{T}_{s1}^{M_s}\ \Delta\mathbb{T}_{s1}^{K_{se}}\ \Delta\mathbb{T}_{s1}^{K_{sv}}]\right), \tag{59}$$

where $E(.)$ is the first statistical moment. The robust basis $\mathbb{T}_s$ is thus:

$$\mathbb{T}_s = [T_s\ \Delta\mathbb{T}_{s1}]. \tag{60}$$

For the fluid domain, the random force due to structural one is represented by the term $\Delta\mathbb{T}_{s1}$: its residual response is approached as follows:

$$\Delta\mathbb{T}_{fs} = \left(K_f - \omega_c^2 M_f\right)^{-1} C^T \Delta\mathbb{T}_{s1}. \tag{61}$$

The random modifications forces associated to absorbing area is $[j\omega\mathbf{a}(\theta)A_f\mathbb{P}(\theta)]$ which is approximated by $[j\omega\mathbf{a}(\theta)A_fT_f\mathbf{q}^f(\theta)]$. Because $\mathbf{a}(\theta)$ is a scalar, the residual response of this force is always estimated by the term ΔT_{fa} of the deterministic basis T_{fe}.

Finally the robust fluid basis can be established:

$$\mathbb{T}_{fe} = [T_{fe}\ \Delta\mathbb{T}_{fs}]. \tag{62}$$

One can then orthogonalize the basis and find the dominant directions of the subspaces $\mathbb{T}_s$ and $\mathbb{T}_{fe}$ by using singular values decompositions [23]. This is of particular interest if a large number of vectors has to be added to the initial basis.

4.2. Simulations

A simplified exhaust line composed by two aluminum tubes, and supported at ends is considered (figure 8). The geometric characteristics of the tube with small diameter are: radius $R_1 = 0.1m$, length $L_1 = 0.4m$. Those of large diameter are: $R_2 = 0.25m$, $L_2 = 1m$. Two ends are closed by rigid walls on which poro-elastic elements are sticked. The tube of large diameter is treated by viscoelastic patches at its extremities on the circumference, like shown in figure 8. Other geometries and material characteristics are given in section 3.5. The frequency band of interest $[100;\ 350]Hz$ is divided into 800 points, and a single value in the middle of the band $\omega_c = \omega_j = 225 \times 2\pi$ is used when required. A point force is exciting the system in the diameter direction.

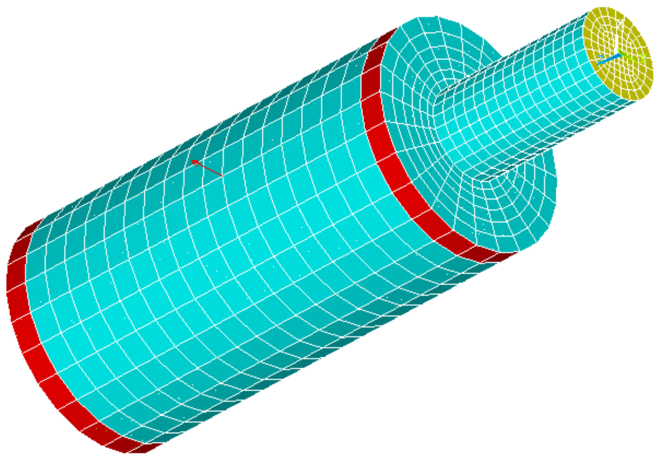

Figure 8: Simplified exhaust line treated by viscoelastic and poro-elastic elements (modeled with ANSYS)

Full FE model has 12899 DOFs (5784 structural DOFs and 7115 fluid DOFs), corresponding to at least 6 elements per wavelength at 350 Hz.

Craig-Bampton method cuts the line in two parts: main part (tube with large diameter and connecting plate) and secondary one (tube with small diameter). Structural nominal basis T_s of main part contains 194 modes (144 static modes which correspond to 144 junction DOFs and 50 normal modes) enriched by 51 modes residual modes associated to viscoelastic forces. The one of secondary part contains 153 modes (144 static modes and 9 normal modes). For the fluid domain, the enriched basis contains 51 acoustic modes and 119 enriched modes in which 106 of ΔT_{fs} and 13 of ΔT_{fa} . Thus, the final deterministic reduced model has 373 DOFs including 203 structural DOFs and 170 fluid DOFs. This corresponds to a reduction ratio of 97%. The acoustic impedance $Z_a(\omega) = Z_R(\omega) + jZ_I(\omega)$ has been obtained by fitting the experimental curves:

$$
\begin{aligned}
Z^R(\omega) &= Z_1^R + Z_2^R f + Z_3^R f^2 + Z_4^R f^3; \quad \omega = 2\pi f, \\
Z^I(\omega) &= \frac{Z_1^I}{f} + Z_2^I + Z_3 f + Z_4^I f^2 + Z_5^I f^3.
\end{aligned}
\tag{63}
$$

The nominal values of coefficients (using international system units with f in hertz) are: $Z_1^R = 788$; $Z_2^R = -1.32$; $Z_3^R = 1.31 \times 10^{-3}$; $Z_4^R = -4,01 \times 10^{-7}$; $Z_1^I = -694 \times 10^3$; $Z_2^I = -2640$; $Z_3^I = 5.49$; $Z_4^I = -4.44 \times 10^{-3}$; $Z_5^I = 1,24 \times 10^{-6}$. All coefficients are considered as random variables of normal distribution with dispersion coefficient of 2% (case 1) and 4% (case 2), respectively corresponding to about 6% and 10% of dispersion on Z^R and Z^I. For the structural part, the chosen random variables are temperature T, thicknesses h_1, h_2 and h_3 of the viscoelastic treatment:

- h_1: Gamma distribution, $h_1 = 3 \times 10^{-3} m$, $\sigma_{\mathrm{h}_1} = 0.01 h_1$ (case 1), $\sigma_{\mathrm{h}_1} = 0.02 h_1$ (case 2);

- h$_2$: Gamma distribution, $h_2 = 2.54 \times 10^{-5} m$, $\sigma_{\mathrm{h}_2} = 0.05 h_2$ (case 1), $\sigma_{\mathrm{h}_2} = 0.1 h_2$ (case 2);
- h$_3$: Gamma distribution, $h_3 = 5 \times 10^{-4} m$, $\sigma_{\mathrm{h}_3} = 0.05 h_3$ (case 1), $\sigma_{\mathrm{h}_3} = 0.1 h_3$ (case 2);
- T: Normal distribution, $T = (273 + 20) K$, $\sigma_{\mathrm{T}} = 0.01 T$ (case 1) and $\sigma_{\mathrm{T}} = 0.02 T$ (case 2).

This paper being oriented on damping devices design, only uncertainties related to poro- and visco-elastic materials are considered. Nevertheless any parameter uncertainty that can be addressed through the element or material parameters of the FE model can be considered. The uncertainties that can not be considered with the proposed approach are those related to mesh changes (i.e. change of cavity size or plate width and length) and modeling uncertainties (i.e. errors in the model itself). As far as the poroelastic layer is concerned, it is clear that the uncertainty distribution on fitting parameters are not easy to use in a practical application, since they are not directly linked to physical properties. An extension of this work could be to consider non-parametric uncertainties for the porous layer, coupled to a parametric description of physical variables for other parameters.

Figures 9-10 show the extreme statistics of sound power level P_i and mean of quadratic normal velocity of structure $\bar{V}_n^2$ with 1000 Monte Carlo simulation (MCS) samples, the solid lines correspond to extreme statistics and mean values, the dashed ones correspond to nominal model. Case 1 is related to low dispersion and case 2 corresponds to medium dispersion. In term of calculation time, the proposed method requires 1280 seconds for the con-

Figure 9: Acoustic indexes spectrums. Uncertainty levels: case 1. 1000 samples. Solid line = mean value; Dashed line = nominal value; Grey area = min-max bounds

Figure 10: Acoustic indexes spectrums. Uncertainty levels: case 2. 1000 samples. Solid line = mean value; Dashed line = nominal value; Grey area = min-max bounds

struction of nominal projection bases, and then each sample consumes 600 secs (processor Pentium IV 3.2GHz, RAM 1Gb). For N=1000 samples, this represents a CPU reduction ratio of $99,5\%$ when using the direct CMS on the full model. In order to validate the statistics given by the reduced model, the full stochastic FE model should be solved without reduction, which obviously requires weeks of calculations, so a complete validation of the statistics given by the reduced model is not easy to perform. An evaluation on a limited frequency band is nevertheless proposed as reference using 1000 samples requiring each 65 direct calculations between 281.2 and 301.5 Hz. The figure 11 exhibits the results of this validation by comparing the statistics of full stochastic approach and proposed robust CMS method. One can clearly observe that, on this limited frequency range, the proposed method can efficiently estimate minimum, maximum and mean values of the indicators: the results are very close to those obtained by the full stochastic method. One can expect the same efficiency on the whole frequency range of interest.

5. Concluding remarks

In this study, a component modes synthesis method for damped vibroacoustic problems which is efficient and easy to implement has been proposed, in order to predict the structural modifications induced by the viscoelastic and poro-elastic materials used as design solution for vibration and noise reduction. The convergence is greatly improved and the efficiency of the proposed Ritz bases is compared to performances of classical uncoupled bases using *in vacuo* structural modes and undamped rigid wall cavity modes, which are commonly used to perform a modal reduction. On the other hand,

Figure 11: Comparison of acoustic indexes spectrums: mean value and min-max bounds. Uncertainty levels: case 1. 1000 samples. Legends: — full stochastic model; * proposed robust CMS.

robust bases have been constructed to solve the problem of uncertainties propagation. Furthermore, it is shown that the proposed robust bases result in significant time reduction compared to direct resolution with the full model.

This robust basis is easily extended to Craig-Bampton CMS method and can greatly reduce the required time on substructured acoustic models. This facilitates the inclusion of these models into large scales automated optimization schemes for robust design.

The poro-elastic material was modeled by an absorbing surface characterized by its acoustic impedance $Z_a(\omega)$ depending on frequency. In order to well understand the coupling phenomena between structure and acoustic domain and also to represent all effects, the next step will be to explicitly model the poro-elastic media and build appropriate reduction bases.

References

[1] N. Atalla, R. Bernhard, Review of numerical solutions for low-frequency structural-acoustic problems, Applied Acoustics 43 (3) (1994) 271–294.

[2] W. Desmet, Mid-frequency vibro-acoustic modelling: challenges and potential solutions, in: ISMA, Vol. 2, Leuven, Belgium, 2002, pp. 835–862.

[3] H. J.-P. Morand, R. Ohayon, Interactions fluide-structure, Masson, 1992.

[4] B. M. Irons, Role of part-inversion in fluid-structure problems with mixed variables, AIAA Journal 8(3) (1970) p.568.

[5] C. A. Felippa, Symmetrization of the contained compressible-fluid vibration eigenproblem, Communications in Applied Numerical Methods 1 (1985) 241–247.

[6] M. A. Hamdi, Y. Ousset, G. Verchery, A displacement method for the analysis of vibrations of coupled fluide-structure systems, International Journal for Numerical Methods in Engineering 13 (1978) 139–150.

[7] G. C. Everstine, A symmetric potential formulation for fluid-structure interactions, Journal of Sound and Vibration 107 (1981) 121–129.

[8] L. G. Olson, K. J. Bathe, Analysis of fluid-structure. a direct symmetric coupled formulation based on the fluid velocity potential, Computers & Structures 21 (1985) 21–32.

[9] C. Lesueur, Rayonnement acoustique des structures, vibroacoustique, interactions fluide-structures, Eyrolles, 1988.

[10] T. P. Khatua, Y. K. Cheung, Bending and vibration of multilayer sandwich beams and plates, International Journal for Numerical Methods in Engineering 6 (1973) 11–24.

[11] A. M. G. de Lima, Modélisation et optimisation robuste de l'amortissement viscoélastique de systèmes mécaniques, Ph.D. thesis, Thèse de l'Université de Franche-Comté et de l'Université Fédérale d'Ulberlandia (2007).

[12] J.-F. Allard, Propagation of sound in porous media; Modelling sound absorbing materials, Elsevier Science Publishers Ltd., London, 1993.

[13] Y. J. Kang, J. S. Bolton, Finite elements modelling of isotropic elastic porous materials coupled with acoustical finite elements, Journal of Acoustical Society of America 98(1) (1995) 635–643.

[14] N. Atalla, R. Panneton, P. Debergue, A mixed displacement pressure formulation for poroelastic materials, Journal of the Acoustical Society of America 104(3) (1998) 1444–1452.

[15] R. R. Craig, M. C. C. Bampton, Substructure methods in vibration, ASME Journal of Vibration and Acoustics 117 (1995) 207–213.

[16] S. Boily, F. Charron, The vibroacoustic reponse of a cylindrical shell structure with viscoelastic and poroelastic materials, Applied Acoustics 58 (1999) 131–152.

[17] R. Ohayon, Linear vibrations of structures coupled with an internal fluid, in: Eccomas School, 3-7 May 2006, Ibiza Spain, 2006.

[18] M. Tournour, N. Atalla, Pseudostatic corrections for the forced vibroacoustic response of a structure-cavity system, Journal of the Acoustical Society of America 170 (2000) 2379–2386.

[19] C. Soize, A nonparametric model of random uncertainties for reduced matrix models in structural dynamics, Probabilistic Engineering Mechanics 15 (2000) 277–294.

[20] C. Fernandez, C. Soize, L. Gagliardini, Fuzzy structure theory modeling of sound-insulation layers in complex vibroacoustic uncertain systems: Theory and experimental validation, The Journal of the Acoustical Society of America 125 (2009) 138.

[21] R. Ghanem, P. Spanos, Stochastic finite elements: a spectral approach, Dover publications, 2003.

[22] R. Ohayon, C. Soize, Structural acoustics and vibration, AP, 1998.

[23] G. Masson, B. Ait-Brik, S. Cogan, N. Bouhaddi, Component mode synthesis (cms) based on an enriched ritz approach for efficient structural optimization, Journal of Sound and Vibration 296 (2006) 845–860.

[24] A. Plouin, E. Balmès, Pseudo-modal representation of large models with vicoelastic behavior, Proceedings of International Modal Analysis Conference (1998) 1440–1446.

[25] E. Balmès, Model reduction for system with frequency dependent damping properties, Proceedings of International Modal Analysis Conference (1997) 223–229.

[26] M. Guedri, N. Bouhaddi, R. Majed, Reduction of the stochastic finite element models using a robust dynamic condensation method, Journal of Sound and Vibration 297 (1-2) (2006) 123–145.