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A Decentralized Formation Control Method Including Self-Organization Around A Target ⋆

M. A. El Kamel [*,**] **L. Beji** [*] **A. Abichou** [**]

[*] *Informatics, Integrative Biology and Complex Systems, university of Evry, France*
(e-mail: Anouar.Elkamel,Lotfi.Beji@iup.univ-evry.fr)
[**] *Mathematical Engineering Laboratory Polytechnic School of Tunisia*
(e-mail: azgal.abichou@ept.rnu.tn)

Abstract: This paper presents a new strategy for formation control of multiple mobile robots to capture a target including self organization. A decentralized formation control is proposed to make the system more reliable and fault-tolerant. Acting on a hostile environnement, each robot of the formation has to avoid an obstacle. For that, we propose a new technique that modifies the trajectory behavior while preserving the formation convergence. LaSalle's theorem is applied to construct the proposed smooth continuous feedback control, to surround the target. The validity of convergence and obstacle avoidance is supported by computer simulation.

Keywords: Formation control, Decentralized control, Obstacle avoidance, Target surrounding.

1. INTRODUCTION

Noticing that the study of moving agent's formation and its control has gained attention in several applications inter alia in the medical field (behavior of a drug in the body), the study of movement of the cells, the road traffics, the migration of a group of animals (bird, fish) in addition to military goals. Hence, this research area have gained increasing attention both in robotics and control communities.

In this paper we focus on study in finding a command for multi mobile robots/agents moving in formation in hostile $2D$ space with one obstacles. The first problems that we meet are:

How each vehicle of the formation can avoid an obstacle in the plane? and at the same time how does it converge to its predefined target?

In a previous work El Kamel et al. (2009) we developed a control law allowing to one vehicle to avoid an obstacle while reaching a desired position. This work is an extension to that previous one to the case of a group of vehicles. Other solutions are given for this type of problems, one can cite the works Kowalczyk et al. (2005) who used the artificial potential function which cause force between the vehicle or vehicle and obstacle, Chen et al. (2007) was considered the navigation function approach with an analytical switch among different cases due to the limited sensing zone of the UAVs. Dimos et al. consider in Dimos et al. (2008), the problem of convergence of robot's formations to a desired configuration, driven by the negative gradient of a potential field. In Ikeda et al. (2003) and Melikyan et al. (2003),an optimal control minimizing the distance between the aircrafts is proposed. The problem which arises in all these works is that the implementation of the control law uses either an algorithm test or turn to an optimal control problem which are both costly and very painful for execution.

Hence we are interested in constructing a regular command without changing the form during time, and which ensures the collision avoidance and convergence to the neighborhood of a fixed target. For the navigation of the formation one considered a decentralized control for the free displacement of each agent and increase reliability of the system.

The paper is organized as follows. After a description of the model in *section*2, one proposes new control approach, in a theorem proposed in *section*3. The control law which decomposed to an attractive part and a behavior regulator function . These lasts are proposed in *section*4 by our main theorem. They guarantee to a group of agent to converge to attractive set while avoiding one obstacle, without switching control over time. The trajectory behavior, will be explained by a corollary in the same section. Simulations are included in *section*5 to illustrate the main theorem.

2. PROBLEM DESCRIPTION

Consider n vehicles moving in the plane where the kinematic behavior of the i^{th} vehicle is described by

$$\begin{aligned} \dot{r}_{x_i} &= v_i \cos\theta i \\ \dot{r}_{y_i} &= v_i \sin\theta i \\ \dot{\theta} i &= w_i \end{aligned} \quad (1)$$

r_{x_i} and r_{y_i} denote the cartesian coordinates (positions), θi is the orientation and v_i , w_i is the linear and angular velocities, respectively.

⋆ This work is integrated within the framework of CIRTA project.

Hence, our main issue is close to that given in Ren et al. (2008) where a feedback linearizing procedure is adequate under the following inputs

$$\begin{pmatrix} v_i \\ w_i \end{pmatrix} = \begin{pmatrix} \cos\theta i & \sin\theta i \\ -\dfrac{1}{d_i}\sin\theta i & \dfrac{1}{d_i}\cos\theta i \end{pmatrix} \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix} \tag{2}$$

which bring the behavior of the i^{th} vehicle to

$$x_i = r_{x_i} + d_i \cos\theta i \quad ; \quad y_i = r_{y_i} + d_i \sin\theta i \tag{3}$$

d_i is a nonzero constant and (x_i, y_i) belongs to the vehicle central axis. As a preliminary result, we obtain

$$\begin{pmatrix} \dot{x}_i \\ \dot{y}_i \end{pmatrix} = \begin{pmatrix} u_{x_i} \\ u_{y_i} \end{pmatrix} \tag{4}$$

u_{x_i} and u_{y_i} are the new inputs that should be defined with respect to the formation stabilizing problem and the regulation control including obstacles avoidance for one or more targets capturing.

More generally, one substitutes the behavior of the i^{th} vehicle by this writing

$$\dot{q}_i = u_i,\ i \in \mathcal{N} = [1, ..., n] \tag{5}$$

with $q_i = (x_i, y_i) \in \mathbb{R}^2$ denotes the position and $u_i = (u_{x_i}, u_{y_i}) \in \mathbb{R}^2$ denotes the velocity which also are the control inputs of the *agent i*. In formation regulation/tracking control, intercommunication between vehicles is necessary to success the mission, hence, the *agent* notion is introduced substituting those of the vehicles.

3. STABILIZING/REGULATING CONTROL METHOD

Let us recall the general cases of behavior which describes the following dynamic control problem.

$$\dot{q} = u \tag{6}$$

the state $q \in \mathbb{R}^{2n}$ and the control law $u \in \mathbb{R}^{2n}$.
As we can see it is straightforward to stabilize this dynamic as it behaves like an integrator. Our main result is to extend the stabilizing control problem such that it incorporates a regulating function. This is necessary for navigation in presence of perturbations. Consequently, we add another term, to the initial stabilizing controller, that ensures regulation and preserves the stability results.
Before stating our main result let us recall the following definition

Definition 1. (Invariant set). A set Ω is an invariant set for a dynamic system $\dot{x} = f(x)$ if every trajectory $x(t)$ which starts from a point in Ω remains in Ω for all time.

Theorem 1. Let $q \in \Omega \subset \mathbb{R}^{2n}$. Consider the differential equation system (6). Assume that for $u = u_a$,

$$\Omega = \{q \in \mathbb{R}^{2n} / 0 \leq V(q) \leq p)\}$$

is an invariant set with respect to $\dot{q} = u_a$, where $V : \mathbb{R}^{2n} \to \mathbb{R}_+$ is such that $\frac{\partial V}{\partial q} u_a \leq 0$

Under the following control law:

$$u = u_a - \begin{pmatrix} \nu_1(q) & 0 & \dots \\ 0 & \ddots & \\ \vdots & 0 & \nu_n(q) \end{pmatrix} \otimes I_2 \begin{pmatrix} \begin{pmatrix} (\dfrac{\partial V}{\partial q})_{x_1} \\ (\dfrac{\partial V}{\partial q})_{y_1} \end{pmatrix}^{\perp} \\ \vdots \\ \begin{pmatrix} (\dfrac{\partial V}{\partial q})_{x_n} \\ (\dfrac{\partial V}{\partial q})_{y_n} \end{pmatrix}^{\perp} \end{pmatrix} \tag{7}$$

the states of (6) and equation $\dot{q} = u_a$ starting in Ω approaches the same set.
$\otimes$ is the Kronecker product, I_2 is the identity matrix $\in \mathcal{M}_{2\times 2}(\mathbb{R})$ and ν_i is a function from $\mathbb{R}^{2n}$ to $\mathbb{R}$ and

$$\frac{\partial V}{\partial q} = [(\frac{\partial V}{\partial q})_{x_1}, (\frac{\partial V}{\partial q})_{y_1}, ..., (\frac{\partial V}{\partial q})_{x_n}, (\frac{\partial V}{\partial q})_{y_n}] \tag{8}$$

□

Proof One have $\Omega = \{q \in \mathbb{R}^{2n} / 0 \leq V(q) \leq p)\}$ is invariant for $\dot{q} = u_a$ and

$$\dot{V} = \frac{\partial V}{\partial q} u_a \leq 0$$

then According to *LaSalle's theorem* the solutions of the equation cited above converge to the greatest invariant set of

$$E = \{q \in \mathbb{R}^{2n} / \dot{V} = 0\}$$

we use the same function V for the system $(6, 7)$, Its differential with respect to t becomes:

$$\begin{aligned} \dot{V} &= \frac{\partial V}{\partial q}\dot{q} \\ &= \frac{\partial V}{\partial q} u_a - \frac{\partial V}{\partial q} \mathfrak{M} \otimes I_2 \begin{pmatrix} F_1^{\perp} \\ \vdots \\ F_n^{\perp} \end{pmatrix} \end{aligned} \tag{9}$$

with $F_i = \begin{pmatrix} (\dfrac{\partial V}{\partial q})_{x_i} \\ (\dfrac{\partial V}{\partial q})_{y_i} \end{pmatrix}$

$\mathfrak{M}$ is the diagonal matrix, their elements are the components of $\nu = [\nu_1(q), \nu_2(q), ...\nu_n(q)]$. The bloc matrix corresponding to $\mathfrak{M} \otimes I_2$ is as following:

$$\mathfrak{M} \otimes I_2 = \begin{pmatrix} \mathfrak{A}_1 & 0 & \dots \\ 0 & \ddots & \vdots \\ \vdots & 0 & \mathfrak{A}_n \end{pmatrix} \quad where \quad \mathfrak{A}_i = \begin{pmatrix} \nu_i & 0 \\ 0 & \nu_i \end{pmatrix}$$

the quantity

$$\frac{\partial V}{\partial q} \mathfrak{M} \otimes I_2 \begin{pmatrix} F_1^{\perp} \\ \vdots \\ F_n^{\perp} \end{pmatrix} = \sum_{i=1}^{n} F_i^t \mathfrak{A}_i F_i^{\perp} = \sum_{i=1}^{n} \nu_i F_i^t F_i^{\perp} = 0$$

then

$$\dot{V} = \frac{\partial V}{\partial q} u_a \leq 0$$

hence

$$\Omega = \{q \in \mathbb{R}^{2n} / 0 \leq V(q) \leq V(q_0)\}$$

is invariant for the solution of (6,7) which implies that the state of system (6,7) converges to the greatest invariant set of E

■

Remark 1. This theorem enables us to break up the control law on sum of two parts. The first is called attractive and the second represents new inputs for our system. Changing $\nu = [\nu_1(q), \nu_2(q), ...\nu_n(q)]$ will modify the trajectory behavior while conserving the convergence.

4. STABILIZING/REGULATION OF A GROUP OF AGENTS

In this section we are interested to n robots or agents navigating towards a static target $C = (C_x, C_y)$ in a planar environment, which are considered as the origin of the terrestrial frame , in presence of the obstacle $O = (O_x, O_y)$. The kinematic of the i^{th} mobile robot/agent is considered by (6).
For the formation, we have this written

$$\dot{q} = u \tag{10}$$

with $q = [q_1, q_2, ..., q_n] \in \mathbb{R}^{2n}$ and $u = [u_1, u_2, ..., u_n] \in \mathbb{R}^{2n}$.

Based on *theorem 1*, our aim is to find the control input that stabilizes the formation through u_a and to avoid a localized obstacle through the regulating control using ν

4.1 Decentralized control law for target surrounding

In this part one is interested in constructing the attractive part $u_a = (u_{a1} u_{a2}, ..u_{an})$ to surround a fixed target by n agents.
Proposition 1. Consider n agents with kinematics given by system (10) and defined in Ω, then under the control inputs

$$u_i = u_{ai} \tag{11}$$

where

$$u_{ai} = -(\parallel q_i - C \parallel^2 - l^2)(q_i - C) \tag{12}$$

For all initial conditions within Ω, where $\Omega \subset \mathbb{R}^{2n}$:

$$\Omega = \{q \in \mathbb{R}^{2n} / l \leq \parallel q_i - C \parallel \leq K\}$$

the solutions of (12) approaches M where

$$M = \{q \in \Omega / \parallel q_i - C \parallel = l\}$$

and $K \geq \parallel q_{i0} - C \parallel$.

■

According to the LaSalle's theorem, the proof of theorem 1 is divided into three steps that we state in the following lemmas. In lemma 1, we show that Ω is an invariant set for the kinematic equations (10,12). The lemma 2 traces the way that a decreasing function can be found and proved. Finally, lemma 3 determines the largest invariant set M.

Lemma 1. let
$\Omega = \{q \in \mathbb{R}^{2n} / l \leq \parallel q_i - C \parallel \leq K\}$
Every solution of (10,12) which starts in Ω remains for all future time in Ω i.e Ω is an invariant set for the kinematic equation (10,12). Moreover, the set Ω is compact.

■

Proof. Assume that $q_0 \in \Omega$ and consider this function

$$S(q_i) = (\parallel q_i - C \parallel^2 - l^2) \tag{13}$$

The derivative of $S(q_i)$ along the trajectories of system (10,12) is given by

$$\begin{aligned} \dot{S}(q_i) &= 2 < \dot{q}_i, q_i - C > = 2 < u_i, q_i - C > \\ &= -2(\parallel q_i - C \parallel^2 - l^2) \parallel q_i - C \parallel^2 \end{aligned} \tag{14}$$

then

$$\dot{S}(q_i) = -2S(q_i)(S(q_i) + l^2) \tag{15}$$

consequently

$$\frac{S(q_i)}{S(q_i) + l^2} = \frac{S(q_{i0})}{S(q_{i0}) + l^2} exp(-2l^2(t - t_0)) \tag{16}$$

As $S(q_{i0}) = \parallel q_{i0} - C \parallel^2 - l^2 \geq 0$ due to the fact that $q_{i0} \in \Omega$ therefore

$$S(q_i) = \parallel q_i - C \parallel^2 - l^2 \geq 0 \quad \Leftrightarrow \quad \parallel q_i - C \parallel \geq l. \tag{17}$$

In the other hand, let $F(q_i)$ as defined as following:

$$F(q_i) = \parallel q_i - C \parallel^2 \tag{18}$$

The differential of F with respect to t is:

$$\begin{aligned} \dot{F}(q_i) &= 2 < \dot{q}_i, q_i - C > = 2 < u_i, q_i - C > \\ &= -2(\parallel q_i - C \parallel^2 - l^2) \parallel q_i - C \parallel^2 \leq 0 \end{aligned} \tag{19}$$

then F is decreasing with respect to time t,
hence

$$\parallel q_i - C \parallel \leq \parallel q_{i0} - C \parallel \leq k \tag{20}$$

therefore, if $q_0 \in \Omega$ then $q \in \Omega$, which implies that Ω is an invariant set for the kinematic equation (10,12).

□

Lemma 2. Assume that q is the vector of components the solutions of $(10, 12)$ and consider the following function candidate

$$V(q) = \sum_{i=1}^{n} (\parallel q_i - C \parallel^2 - l^2) \tag{21}$$

The differential of V with respect to t is negative on Ω.

Proof. The derivative of V along the trajectories of system (10) With the control laws given by theorem ((12) is given by

$$\begin{aligned} \dot{V}(q) &= \sum_{i=1}^{n} < \dot{q}_i, q_i - C > \\ &= -\sum_{i=1}^{n} (\parallel q_i - C \parallel^2 - l^2) \parallel q_i - C \parallel^2 \end{aligned} \tag{22}$$

we have $q_0 \in \Omega$, according to lemma1 Ω is an invariant set with respect to the equation $(10, 12)$, then $q \in \Omega$ hence $(\parallel q_i - C \parallel^2 - l^2) \geq 0$
therefor

$$\dot{V}(q) \leq 0 \tag{23}$$

□

Lemma 3. Assume that q is the vector of components the solutions of $(10, 12)$

$$M = \{q \in \Omega / \parallel q_i - C \parallel = l\}$$

is the greatest invariant set in $E = \{q \in \Omega / \dot{V} = 0\}$

Proof. One have

$$\dot{V}(q) = -\sum_{i=1}^{n} (\parallel q_i - C \parallel^2 - l^2) \parallel q_i - C \parallel^2 = 0$$

then

$$E = \{\parallel q_i - C \parallel = l\} = M \tag{24}$$

because $q \in \Omega$
let $q_0 \in M$ and $S(q_i) = \parallel q_i - C \parallel^2 - l^2$, then according to the proof of lemma1 then

$$\frac{S(q_i)}{S(q_i) + l^2} = \frac{S(q_{i0})}{S(q_{i0}) + l^2} exp(-2l^2(t - t_0)) \tag{25}$$

as $q_0 \in M$ then $S(q_{i0}) = 0$ implies $S(q_i) = \parallel q_i - C \parallel^2 - l^2 = 0$
therefore,

$$q \in M$$

Consequently M is the greatest invariant set in E.

□

Now we are able to prove the results in proposition 1.

Proof. *(Proposition 1)* According to results given by lemma 1, lemma 2, lemma 3 and LaSalle's theorem, Ω is invariant with respect to (10,12) and $\dot{V} \leq 0$ in Ω then every solution starting in Ω approaches M as $t \to \infty$, with M is defined in lemma 3.

□

Consequently one proved that $u_a = [u_{a_1}, u_{a_2}, ..., u_{a_n}]$, where u_{a_i} was defined on proposition 1, do approached the solutions of (10,12)) to the set M which the greatest set of $E = \{q \in \Omega / \dot{V} = 0\}$.

4.2 Obstacle avoidance

In the next section one defines the set of points to be avoided by each agent.

Obstacle localization In this work, we suppose that any obstacle in the plane can be circumscribed by a circle centered in a certain O and of radius r. The obstacle can move inside but the circle is supposed to be fixe. When a vehicle approaches an obstacle, this means that its position q_i approaches O_{q_i}; the intersection point between the circle and the line joining O to q_i (see figure (1)).

$$O_{qi} = O + r \frac{q_i - O}{\|q_i - O\|} \tag{26}$$

where $\|.\|$ denotes the Euclidian norm,

O_{q_i} moves while q_i is moving and depends on the vehicle coordinates. Then the vehicle should avoid all the points on the circle circumscribing the obstacle over time.

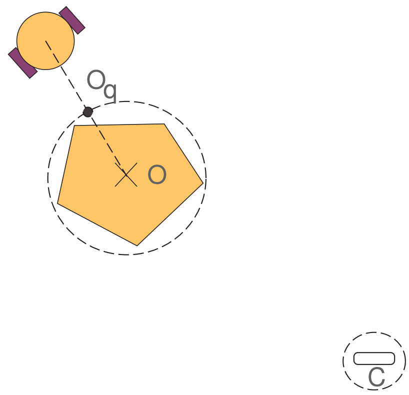

Fig. 1. Vehicle in front of an obstacle

Repulsive command construction The following theorem ensures that the position of each agent position converge to a point of the circle, taken as an attractive set, and avoid the obstacle.

Theorem 2. Consider N unicycles with kinematic given by system (10), defined in Ω. Let $q_{i0} = (x_{i0}, y_{i0})$ denotes the initial position of the vehicle at $t = t_0$, and let $L(x)$ the equation of the line joining the center of the target $C = (C_x, C_y)$ to O. O is the center of the circle circumscribing the obstacle. Then Under the control inputs

$$u_i = u_{ai} + \nu_i (q_i - C)^{\perp} \tag{27}$$

where

$$u_{ai} = -(\parallel q_i - C \parallel^2 - l^2)(q_i - C) \tag{28}$$

and

$$\nu_i = -\frac{sign([y_{i0} - L(x_{i0})][C_x - O_x])}{\|q_i - O_{q_i}\|} (\parallel q_i - C \parallel^2 - l^2) \tag{29}$$

$$sign : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto \begin{cases} 1 & if \;\; x \geq 0 \\ -1 & if \;\; x < 0 \end{cases}$$

for all initial conditions within Ω, where $\Omega \subset \mathbb{R}^{2n}$:

$$\Omega = \{q \in \mathbb{R}^{2n} / l \leq \parallel q_i - C \parallel \leq K\}$$

the solutions of (10,27,28,29) approaches M where

$$M = \{q \in \Omega / \parallel q_i - C \parallel = l\}$$

and $K \geq \parallel q_{i0} - C \parallel$. And the i^{th} agent avoid the dynamic point O_{q_i}.

■

Proof. One have

$$\begin{aligned} \Omega &= \{q \in \mathbb{R}^{2n} / l \leq \parallel q_i \parallel \leq K\} \\ &= \{q \in \mathbb{R}^{2n} / 0 \leq \parallel q_i \parallel^2 - l^2 \leq K^2 - l^2\} \\ &= \{q \in \mathbb{R}^{2n} / 0 \leq V(q) \leq p\} \end{aligned} \tag{30}$$

According to lemma1 and lemma2 Ω is invariant and $\dot{V} \leq 0$ with respect (10,12) and $\frac{\partial V}{\partial q} = q - C$ then according to the theorem 1 the solutions of equations (10,27,28,29) and (10,12) approaches the same set, in another hand according to proposition 1 the solution of (10,12) converge to M then the equation (10,27,28,29) converge to M, which is the circle surrounding the target C
Now we will proof that the trajectory founded after inject-

ing the control law (27,28,29)avoid the circle obstacle:
We have the norme of the vector velocity of i^{th} vehicle is:

$$\|\dot{q}_i\|^2 = (1+\nu_i^2)\|q_i\|^2(\|q_i\|^2 - l^2)^2 \quad (31)$$

In another hand we have:

$$\begin{aligned} \frac{\langle \dot{q}_i / q_i^{\perp} \rangle}{\|q_i\|^2(\|q_i\|^2 - l^2)} &= \nu_i \\ then\ \|\dot{q}_i\| \frac{\cos(\beta)}{\|q_i\|(\|q_i\|^2 - l^2)} &= \nu_i \end{aligned}$$

with β is the angle between $\dot{q}_i$ and $q_i^{\perp}$. Then if we multiply the system (31) by $\frac{\cos^2(\beta)}{\|q_i\|^2(\|q_i\|^2 - l^2)^2}$, we obtain:

$$\begin{aligned} \nu_i^2 &= (1+\nu_i^2)\cos^2(\beta) \\ then\ \cos^2(\beta) &= \frac{\nu_i^2}{(1+\nu_i^2)} \end{aligned}$$

So if there exist a time t_e such that $\lim_{t \to t_e} \|q_i - O_{qi}\| = 0$ then $\lim_{t \to t_e} \cos^2(\beta) = 1$ then $\lim_{t \to t_e} \beta = k\pi$, $\forall\ k \in \mathbb{Z}$, that imply that there exist a constant B such that $\lim_{t \to t_e} \|\dot{q}_i - Bq_i^{\perp}\| = 0$ hence

$$0 = \lim_{t \to t_e} \|(\nu_i - B)q_i^{\perp} - q_i\| \geq \lim_{t \to t_e} (|\nu_i - B| - 1)\|q_i\| = +\infty$$

because $\|q_i\|$ is bounded. Which is an absurdity, hence it not exist a time t such that the trajectory of every vehicle collide with the obstacle.

■

The next corollary ensures that the navigation trajectory have a fluid like behavior with the respect to the obstacle. That is to say, if a vehicle starts in the right side of the line L it will avoid the obstacle while remaining in the right side, if not it avoids it on the left side.

Corollary 1. Let us define two sets K and H

$$K = \{q \in \Omega / y_i \geq L(x_i)\}$$
$$H = \{q \in \Omega / y_i \leq L(x_i)\}$$

where $L(x)$ defined above. These sets are invariants with respect to system (10,27,28,29)

□

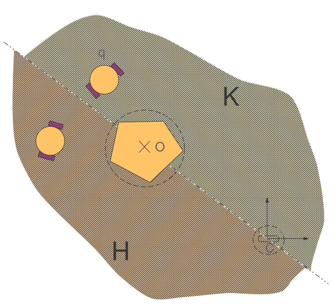

Fig. 2. the invariant sets K and H

Proof. We consider the following differential equation:

$$\dot{q}_i = -(\| q_i \|^2 - l^2)q_i - \nu_i q_i^{\perp} \quad (32)$$

In polar coordinates (ρ_i, σ_i), we get the following transformation

$$\begin{aligned} \dot{\rho}_i &= -(\rho_i^2 - l^2)\rho_i \\ \dot{\sigma}_i &= \nu_i \end{aligned} \quad (33)$$

Where the state σ_i, which is the angle between $\overrightarrow{Cq}$ and the horizontal. The increasing/decreasing of σ_i according to the equation above depends on the sign of the quantity $[y_{i0} - L(x_{i0})][C_x - O_x]$. Further, one introduces α the angle between L and the horizontal. 4 cases arise

<u>-1st**case**</u> if $C_x \geq O_x$, we obtain two cases
<u>**1/** If $y_{i0} \geq L(x_{i0})$</u>

$$\dot{\sigma_i} = -\frac{\rho_i^2 - l^2}{|\sqrt{(\rho_i\cos\sigma_i - O_x)^2 + (\rho_i\sin\sigma_i - O_y)^2} - r|} \leq 0,$$

then $\sigma_i \leq \sigma_{i0}$. However, as $y_{i0} \geq L(x_{i0})$, what implies that $\sigma_i \leq \sigma_{i0} \leq \alpha$, then at every time t, the state $q(t)$ belongs to the half plan (superior) defined by L which proves $y_i \geq L(x_i)$.

<u>**2/** If $y_{i0} \leq L(x_{i0})$</u> then $\dot{\sigma_i} \geq 0$, consequently $\sigma_i \geq \sigma_{i0}$, further $y_{i0} \leq L(x_{i0})$ what implies that $\sigma_i \geq \sigma_{i0} \geq \alpha$ meaning that q belongs to the half plan (inferior) defined by L leading to $y \leq L(x)$.

<u>-2nd**case**</u> if $C_x \leq O_x$
Similarly, two cases arise.
<u>**3/** If $y_{i0} \geq L(x_{i0})$</u>, $\dot{\sigma_i} \leq 0$ and using the same proof given by 1/ of 1st**case**, we find that $y_i \geq L(x_i)$.
<u>**4/** If $y_{i0} \leq L(x_{i0})$</u> following the same analysis of **2/**-1st**case**, then $y_i \leq L(x_i)$.

These 4 studied cases, affirm that if $y_{i0} \geq L(x_{i0})$ then $y_i \geq L(x_i)$ will be conserved over time. Otherwise, $y_i \leq L(x_i)$ for all t. Finally, we can conclude that K and H are invariant sets with respect to the differential system (10,27,28,29).

■

5. SIMULATION RESULTS

In this section one presents the simulation results of navigation of a formation containing six agents. One note, according to the figure 4, that the command, given in the theorem 2, ensures that the distance between each vehicle and the obstacle is greater than $20cm$. In the figure 5 the distance between each vehicle and the target converge to $10cm$, which implies that all the vehicles converge to the circle centered on the target as we can see in figure 3. In figure 3 we can note the behavioral change of the trajectory in front of the obstacle. In fact, one observes the fluid behavior with respect to the line L which join target and obstacle while avoiding it. Finally all the agents surround the target.

6. CONCLUSION

One propose in this paper a new strategy for obstacle avoidance, and convergence to the target. With this strategy we can choose any control law for the convergence and construct the regulator function ν for the obstacle avoidance. The function ν considered in this work ensures a fluid behavior in front obstacle. In another hand we give

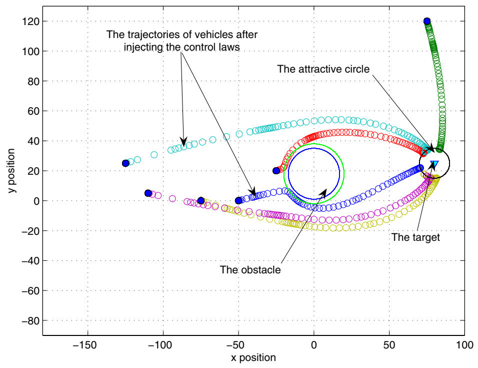

Fig. 3. Simulation of formation navigation

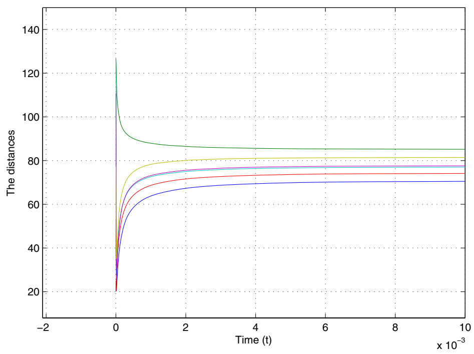

Fig. 4. The distances between each vehicle and the obstacle

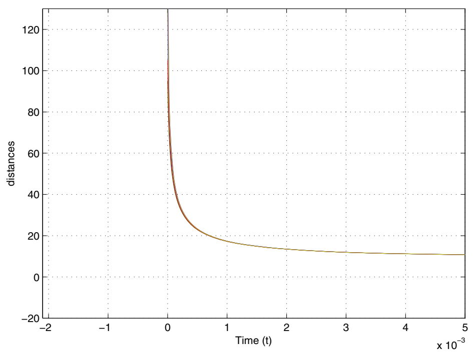

Fig. 5. The distances between each vehicle and the target

in this paper a decentralized control law that ensure the convergence of the formation to a attractive set which is a circle surrounding the fixed target.

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