Alternative mass sensing techniques based on nonlinear phenomena in M/NEMS resonators

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Summary. This paper investigates alternative mass sensing techniques that take advantage of multi-stability and bifurcations observed in the nonlinear regime. A finite-degree-of-freedom reduced-order model of an electrostatically-actuated microbeam with small added mass is considered. Strategies are proposed for robust real-time detection, quantification and localization of the added mass.

Introduction

Mass sensing is one important application of M/NEMS resonators. The principle of a resonant sensor consists in electrostatically actuating a microbeam on its fundamental mode and in measuring the shift in frequency induced by an external perturbation. For this purpose, the resonator can be driven in the linear or in the nonlinear regime. In the linear regime, the model is simpler, however vibrations are limited to small amplitudes which may not exceed thermo-mechanical noise. One way to improve the sensitivity of the resonator is to improve the signal-to-noise ratio and the quality factor. Another way is to downscale the beam's dimension to the nanoscale [1]. A third way consists in working in the nonlinear regime and in using nonlinear phenomena [2] such as parametric resonance [3, 4], pull-in [5] or multi-stability and bifurcations [6, 7]. An alternative detection technique that takes advantage of multi-stability and bifurcations observed in hysteretic cycles is investigated here.

Nonlinear micro/nanobeam-based electromechanical resonator with added mass

The model for the nonlinear electrostatically-actuated clamped-clamped microbeam with added mass of Figure 1 is based on the model developed by Kacem *et al.* for inertial resonant sensors [8].The nondimensional equation of motion is given by

$$\frac{\partial^4 w}{\partial x^4}\mathrm{d}x + \frac{\partial^2 w}{\partial t^2}\mathrm{d}x + \delta_{x_0}(x)m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t}\mathrm{d}x - \left(N + \alpha_1\int_0^1\left[\frac{\partial w}{\partial x}\right]^2\mathrm{d}x\right)\frac{\partial^2 w}{\partial x^2}\mathrm{d}x = \alpha_2\frac{\left[V_{dc} + V_{ac}\cos(\Omega t)\right]^2}{(1-w)^2}\mathrm{d}x \quad (1)$$

where $w(x,t)$ is the nondimensional bending displacement of the beam, m is the ratio between the added mass and the microbeam mass, i.e. $m = m_{added}/m_{beam}$ and $\delta_{x_0}(x)$ is the Dirac function used to locate the added mass. The left-hand side of Equation (1) represents the inertial, damping, elastic and geometrically nonlinear mechanical forces, whereas the right-hand side stands for the nonlinear electrostatic forces with V_{dc}, V_{ac}, Ω being the DC polarization voltage, the amplitude of the AC voltage, and the excitation frequency respectively.

A reduced-order finite-degree-of-freedom model is generated by modal decomposition with the undamped linear mode shapes of the straight microbeam as basis functions for the Galerkin procedure. Response curves of the resonator (see Figure 2) are then computed by the Harmonic Balance combined with the Asymptotic Numerical Method for continuation analysis by means of the ManLab software [9].
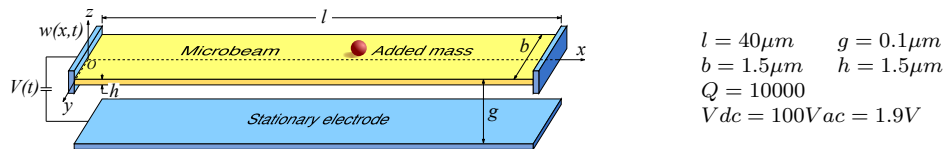

$l = 40\mu m$	$g = 0.1\mu m$
$b = 1.5\mu m$	$h = 1.5\mu m$
$Q = 10000$	
$Vdc = 100$	$Vac = 1.9V$

Figure 1: Geometry and data of the resonant microbeam with added mass.

Mass detection techniques

The softening response curve of the nonlinear resonator with and without added mass is shown in Figure 2. Classical mass sensing relies on the shift in frequency of the response curve in presence of an added mass. However, this shift becomes all the smaller and thus more difficult to detect as the amount of added mass decreases. An alternative bifurcation-based detection technique that exploits nonlinear dynamical phenomena such as the bi-stability of a piezoelectrically actuated microcantilever was recently proposed by Kumar *et al.* [6]. This technique is illustrated in Figure 2. When operating at a constant excitation frequency Ω_{op} slightly lower than the frequency Ω_{lim} of the fold bifurcation point A_{lim}, the frequency response curve without added mass (red curve) is multi-valued with two stable solutions A_1 and A_2. When a mass m is added, the response curve is shifted to the left (blue curve) and becomes single-valued at frequency Ω_{op}. As a consequence, a jump from point A_1 to point B is observed, resulting in a sudden increase in amplitude.

This detection technique was numerically investigated in [7] and it was shown that after the removal of the mass the solution can either return to point A_1 or jump up to point A_2 depending on the amount of added mass. For small mass
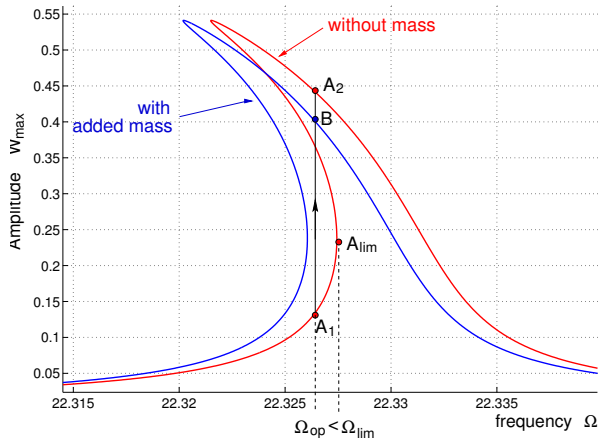
Figure 2: Shift in frequency of the due to added mass $m = 5.10^{-5}$ and jump phenomenon from A_1 to B.
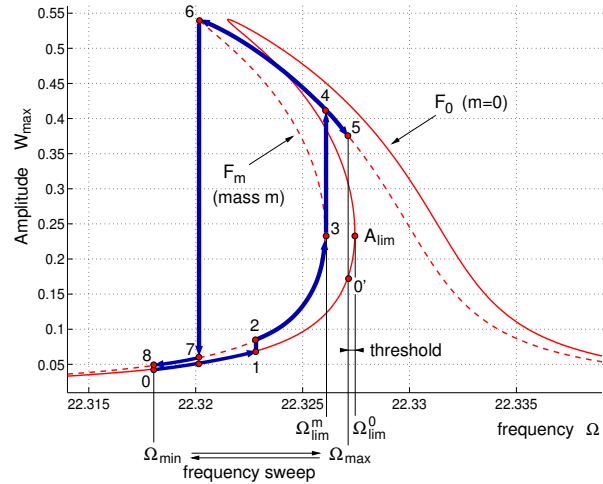
Figure 3: Hysteretic cycle obtained by frequency sweep for $m = 5.10^{-5}$.

ratios the jump always occurs towards the upper point A_2 and consequently only a small jump in amplitude from point A_2 to point B happens when another mass arrives on the beam, which is not satisfying.

In order to ensure the robustness of this detection technique, the system must always return to its initial stable position A_1, so that a large jump in amplitude takes place each time a mass hits the resonator. A simple solution for enforcing the reinitialization, based on a sequential decrease and increase of the operating frequency, was proposed in [7] but it suffers from many shortcomings. For instance, the instant when this reinitialization has to be operated is unknown.

This problem can be overcome by using a slow time-varying frequency sweep such as $\Omega(t) = \Omega_{op} + \Delta\Omega \cos(\epsilon\pi t + \phi)$ where $\epsilon << \Omega_{op}$ is the sweep velocity and $\Omega_{max} = \Omega_{op} + \Delta\Omega$ and $\Omega_{min} = \Omega_{op} - \Delta\Omega$ are the frequency sweep boundaries. When t changes, successive sweep-up and sweep-down in frequency are performed. Without added mass, the response goes back-and-forth between 0 and $0'$. There is no hysteretic cycle nor associated amplitude jump. When an added mass arrives on the resonator (at point 1) the response goes through the hysteretic cycle shown in Figure 3 according to the path defined by the following points: 1-2/3-4-5-6-7-8/3-4-5-6-7-8/..., and there are amplitude jumps from 3 to 4 and from 6 to 7 which can be used for detection and quantification. When the added mass leaves the resonator, the response goes back between 0 and $0'$ and the jumps disappear.

This new mass sensing technique enables automatic reinitialization as well as real-time detection. Moreover, it allows for a fine tuning of the minimal and maximal mass that can be detected, $\Omega_{lim}^0 - \Omega_{max}$ being the threshold of mass detection for instance. Experimental validation is under progress to obtain a proof of concept and demonstrate the interest of the proposed strategy for ultrasensitive mass sensing.

Conclusions

An alternative mass sensing technique based on the nonlinear phenomena in M/NEMS resonators has been numerically investigated. This detection takes advantage of the bistability and bifurcations of the hysteretic nonlinear responses and permits real-time mass detection and quantification.

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