

Trace checking of Metric Temporal Logic with Aggregating Modalities using MapReduce

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Abstract

Modern complex software systems produce a large amount of execution data, often stored in logs. These logs can be analyzed using trace checking techniques to check whether the system complies with its requirements specifications. Often these specifications express quantitative properties of the system, which include timing constraints as well as higher-level constraints on the occurrences of significant events, expressed using aggregate operators.

In this paper we present an algorithm that exploits the MapReduce programming model to check specifications expressed in a metric temporal logic with aggregating modalities, over large execution traces. The algorithm exploits the structure of the formula to parallelize the evaluation, with a significant gain in time. We report on the assesment of the implementation—based on the Hadoop framework—of the proposed algorithm and comment on its scalability.

1 Introduction

Modern software systems, such as service-based applications (SBAs), are built according to a modular and decentralized architecture, and executed in a distributed environment. Their development and their operation depend on many stakeholders, including the providers of various third-party services and the integrators that realize composite applications by orchestrating third-party services. Service integrators are responsible to the end-users for guaranteeing an adequate level of quality of service, both in terms of functional and non-functional requirements. This new type of software has triggered several research efforts that focus on the specification and verification of SBAs.

In previous work [7], some of the authors presented the results of a field study on property specification patterns [11] used in the context of SBAs, both in industrial and in research settings. The study identified a set of property specification patterns specific to service provisioning. Most of these patterns are characterized by the presence of aggregate operations on sequences of events occurring in a given time window, such as “the average distance between pairs of events (e.g., average response time)”, “the number of events in a given time window”, “the average (or maximum) number of events in a certain time interval over a certain time

window”. This study led to the definition of SOLOIST [8] (*SpecificatiOn Language fOr service compoSitions inTeractions*), a metric temporal logic with new temporal modalities that support aggregate operations on events occurring in a given time window. The new temporal modalities capture, in a concise way, the new property specification patterns presented in [7].

SOLOIST has been used in the context of *offline trace checking* of service execution traces. Trace checking (also called *trace validation* [14] or *history checking* [12]) is a procedure for evaluating a formal specification over a log of recorded events produced by a system, i.e., over a temporal evolution of the system. Traces can be produced at run time by a proper monitoring/logging infrastructure, and made available at the end of the service execution to perform offline trace checking. We have proposed procedures [5, 6] for offline checking of service execution traces against requirements specifications written in SOLOIST using bounded satisfiability checking techniques [15]. Each of the procedures has been tailored to specific types of traces, depending on the degree of sparseness of the trace (i.e., the ratio between the number of time instants where significant events occur and those in which they do not). The procedure described in [5] is optimized for sparse traces, while the one presented in [6] is more efficient for dense traces.

Despite these optimizations, our experimental evaluation revealed, in both procedures, an intrinsic limitation in their scalability. This limitation is determined by the size of the trace, which can quickly lead to memory saturation. This is a very common problem, because execution traces can easily get very large, depending on the running time captured by the log, the systems the log refers to (e.g., several virtual machines running on a cloud-based infrastructure), and the types of events recorded. For example, granularity can range from high-level events (e.g., sending or receiving messages) to low-level events (e.g., invoking a method on an object). Most log analyzers that process data streams [9] or perform data mining [16] only partially solve the problem of checking an event trace against requirements specifications, because of the limited expressiveness of the specification language they support. Indeed, the analysis of a trace may require checking for complex properties, which can refer to specific sequence of events, conditioned by the occurrence of other event sequence(s), possibly with additional constraints on the distance among events, on the number of occurrences of events, and on various aggregate values (e.g., average response time). SOLOIST addresses these limitations as we discussed above.

The recent advent of cloud computing has made it possible to process large amount of data on networked commodity hardware, using a distributed model of computation. One of the most prominent programming models for distributed, parallel computing is *MapReduce* [10]. The MapReduce model allows developers to process large amount of data by breaking up the analysis into independent tasks, and performing them in parallel on the various nodes of a distributed network infrastructure, while exploiting, at the same time, the locality of the data to reduce unnecessary transmission over the network. However, porting a traditionally-sequential algorithm (like trace checking) into a parallel version that takes advantage of a distributed computation model like MapReduce is a non-trivial task.

The main contribution of this paper is an algorithm that exploits the MapReduce programming model to check large execution traces against requirements specifications written in SOLOIST. The algorithm exploits the structure of a SOLOIST formula to parallelize its evaluation, with significant gain in time. We have implemented the algorithm in Java using the Apache Hadoop framework [2]. We have evaluated the approach in terms of its scalability and with respect to the state of art for trace checking of LTL properties using MapReduce [3].

The rest of the paper is structured as follows. First we provide some background information, introducing SOLOIST in Sect. 2 and then the MapReduce programming model in Sect. 3. Section 4 presents the main contribution of the paper, describing the algorithm for trace checking of SOLOIST properties using the MapReduce programming model. Section 5

$(w, i) \models p$	iff	$p \in \sigma_i$
$(w, i) \models \neg\phi$	iff	$(w, i) \not\models \phi$
$(w, i) \models \phi \wedge \psi$	iff	$(w, i) \models \phi \wedge (w, i) \models \psi$
$(w, i) \models \phi S_I \psi$	iff	for some $j < i$, $\tau_i - \tau_j \in I$, $(w, j) \models \psi$ and for all $k, j < k < i$, $(w, k) \models \phi$
$(w, i) \models \phi U_I \psi$	iff	for some $j > i$, $\tau_j - \tau_i \in I$, $(w, j) \models \psi$ and for all $k, i < k < j$, $(w, k) \models \phi$
$(w, i) \models \mathfrak{C}_{\bowtie n}^K(\phi)$	iff	$c(\tau_i - K, \tau_i, \phi) \bowtie n$ and $\tau_i \geq K$
$(w, i) \models \mathfrak{L}_{\bowtie n}^{K,h}(\phi)$	iff	$\frac{c(\tau_i - \lfloor \frac{K}{h} \rfloor h, \tau_i, \phi)}{\lfloor \frac{K}{h} \rfloor} \bowtie n$ and $\tau_i \geq K$
$(w, i) \models \mathfrak{M}_{\bowtie n}^{K,h}(\phi)$	iff	$\max \left\{ \bigcup_{m=0}^{\lfloor \frac{K}{h} \rfloor} \{c(lb(m), rb(m), \phi)\} \right\} \bowtie n$ and $\tau_i \geq K$
$(w, i) \models \mathfrak{D}_{\bowtie n}^K(\phi, \psi)$	iff	$\frac{\sum_{(s,t) \in d(\phi, \psi, \tau_i, K)} (\tau_i - \tau_s)}{ d(\phi, \psi, \tau_i, K) } \bowtie n$ and $\tau_i \geq K$

where $c(\tau_a, \tau_b, \phi) = |\{s \mid \tau_a < \tau_s \leq \tau_b \text{ and } (w, s) \models \phi\}|$, $lb(m) = \max\{\tau_i - K, \tau_i - (m+1)h\}$, $rb(m) = \tau_i - mh$, and $d(\phi, \psi, \tau_i, K) = \{(s, t) \mid \tau_i - K < \tau_s \leq \tau_t \text{ and } (w, s) \models \phi, t = \min\{u \mid \tau_s < \tau_u \leq \tau_i, (w, u) \models \psi\}\}$

Figure 1: Formal semantics of SOLOIST

discusses related work. Section 6 presents the evaluation of the approach, both in terms of scalability and in terms of a comparison with the state of the art for MapReduce-based trace checking of temporal properties. Section 7 provides some concluding remarks.

2 SOLOIST

In this section we provide a brief overview of SOLOIST; for the rationale behind the language and a detailed explanation of its semantics see [8].

The syntax of SOLOIST is defined by the following grammar: $\phi ::= p \mid \neg\phi \mid \phi \wedge \phi \mid \phi U_I \phi \mid \phi S_I \phi \mid \mathfrak{C}_{\bowtie n}^K(\phi) \mid \mathfrak{L}_{\bowtie n}^{K,h}(\phi) \mid \mathfrak{M}_{\bowtie n}^{K,h}(\phi) \mid \mathfrak{D}_{\bowtie n}^K(\phi, \phi)$, where $p \in \Pi$, with Π being a finite set of atoms. In practice, we use atoms to represent different events of the trace. I is a nonempty interval over \mathbb{N} ; $\bowtie \in \{<, \leq, \geq, >, =\}$; n, K, h range over \mathbb{N} . Moreover, for the \mathfrak{D} modality, we require that the subformulae pair (ϕ, ψ) evaluate to true in alternation.

The U_I and S_I modalities are, respectively, the metric “Until” and “Since” operators. Additional temporal modalities can be derived using the usual conventions; for example “Next” is defined as $X_I \phi \equiv \perp U_I \phi$; “Eventually in the Future” as $F_I \phi \equiv \top U_I \phi$ and “Always” as $G_I \phi \equiv \neg(F_I \neg\phi)$, where \top means “true” and \perp means “false”. Their past counterparts can be defined using “Since” modality in a similar way. The remaining modalities are called *aggregate* modalities and are used to express the property specification patterns characterized in [7]. The $\mathfrak{C}_{\bowtie n}^K(\phi)$ modality states a bound (represented by $\bowtie n$) on the number of occurrences of an event ϕ in the previous K time instants; it is also called the “counting” modality. The $\mathfrak{L}_{\bowtie n}^{K,h}(\phi)$ (respectively, $\mathfrak{M}_{\bowtie n}^{K,h}(\phi)$) modality expresses a bound on the average (respectively, maximum) number of occurrences of an event ϕ , aggregated over the set of right-aligned adjacent non-overlapping subintervals within a time window K ; it can express properties like “the average/maximum number of events per hour in the last ten hours”. A subtle difference in the semantics of the \mathfrak{L} and \mathfrak{M} modalities is that \mathfrak{M} considers events in the (possibly empty) tail interval, i.e., the leftmost observation subinterval whose length is less than h , while the \mathfrak{L} modality ignores them. The $\mathfrak{D}_{\bowtie n}^K(\phi, \psi)$ modality expresses a bound on the average time elapsed between occurrences of pairs of specific adjacent events ϕ and ψ in the previous K time instants; it can be used to express properties like the average response time of a service.

The formal semantics of SOLOIST is defined on timed ω -words [1] over $2^\Pi \times \mathbb{N}$. A timed sequence $\tau = \tau_0 \tau_1 \dots$ is an infinite sequence of values $\tau_i \in \mathbb{N}$ with $\tau_i > 0$ satisfying $\tau_i < \tau_{i+1}$, for all $i \geq 0$, i.e., the sequence increases strictly monotonically. A timed ω -word over alphabet 2^Π is a pair (σ, τ) where $\sigma = \sigma_0 \sigma_1 \dots$ is an infinite word over 2^Π and τ is a

timed sequence. A timed language over 2^Π is a set of timed words over the same alphabet. Notice that there is a distinction between the integer position i in the timed ω -word and the corresponding timestamp τ_i . Figure 1 defines the satisfiability relation $(w, i) \models \phi$ for every timed ω -word w , every position $i \geq 0$ and for every SOLOIST formula ϕ . For the sake of simplicity, hereafter we express the \mathfrak{U} modality in terms of the \mathfrak{C} one, based on this definition:

$$\mathfrak{U}_{\triangleright n}^{K,h}(\phi) \equiv \mathfrak{C}_{\triangleright n; \lfloor \frac{K}{h} \rfloor}^{\lfloor \frac{K}{h} \rfloor, h}(\phi), \text{ which can be derived from the semantics in Fig. 1.}$$

We remark that the version of SOLOIST presented here is a restriction of the original one introduced in [8]: to simplify the presentation in the next sections, we dropped first-order quantification on finite domains and limited the argument of the \mathfrak{D} modality to only one pair of events; as detailed in [8], these assumptions do not affect the expressiveness of the language.

SOLOIST can be used to express some of the most common specifications found in service-level agreements (SLAs) of SBAs. For example the property: “The average response time of operation A is always less than 5 seconds within any 900 second time window, before operation B is invoked” can be expressed as: $G(B_{start} \rightarrow \mathfrak{D}_{<5}^{900}(A_{start}, A_{end}))$, where A and B correspond to generic service invocations and each operation has a *start* and an *end* event, denoted with the corresponding subscripts.

We now introduce some basic concepts that will be used in the presentation of our distributed trace checking algorithm in Sect. 4. Let ϕ and ψ be SOLOIST formulae. We denote with $\text{sub}(\phi)$ the set of all subformulae of ϕ ; notice that for *atomic* formulae $a \in \Pi$, $\text{sub}(a) = \emptyset$. The set of *atomic* subformulae (or *atoms*) of formula ϕ is defined as $\text{sub}_a(\phi) = \{a \mid a \in \text{sub}(\phi), \text{sub}(a) = \emptyset\}$. The set $\text{sub}_d(\phi) = \{\alpha \mid \alpha \in \text{sub}(\phi), \forall \beta \in \text{sub}(\phi), \alpha \notin \text{sub}(\beta)\}$ represents the set of all *direct subformulae* of ϕ ; ϕ is called the *superformula* of all formulae in $\text{sub}_d(\phi)$. The notation $\text{sup}_\psi(\phi)$ denotes the set of all subformulae of ψ that have formula ϕ as *direct subformula*, i.e., $\text{sup}_\psi(\phi) = \{\alpha \in \text{sub}(\psi), \phi \in \text{sub}_d(\alpha)\}$. The subformulae in $\text{sub}(\psi)$ of a formula ψ form a lattice with respect to the partial ordering induced by the inclusion in sets $\text{sup}_\psi(\cdot)$ and $\text{sub}_d(\cdot)$, with ψ and \emptyset being the *top* and *bottom* elements of the lattice, respectively. We also introduce the notion of the *height* of a SOLOIST formula, which is defined recursively as:

$$h(\phi) = \begin{cases} \max\{h(\psi) \mid \psi \in \text{sub}_d(\phi)\} + 1 & \text{if } \text{sub}_d(\phi) \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

We exemplify these concepts using formula $\gamma \equiv \mathfrak{C}_{\triangleright 3}^{40}(a \wedge b) \cup_{(30,100)} \neg c$.

Hence $\text{sub}(\gamma) = \{a, b, c, a \wedge b, \neg c, \mathfrak{C}_{\triangleright 3}^{40}(a \wedge b)\}$ is the set of all subformulae of γ ; $\text{sub}_a(\gamma) = \{a, b, c\}$ is the set of *atoms* in γ ; $\text{sub}_d(\gamma) = \{\mathfrak{C}_{\triangleright 3}^{40}(a \wedge b), \neg c\}$ is the set of direct subformulae of γ ; $\text{sup}_\gamma(a) = \text{sup}_\gamma(b) = \{a \wedge b\}$ shows that the sets of superformulae of a and b in γ coincide; and the height of γ is 3, since $h(a) = h(b) = h(c) = 0$, $h(\neg c) = h(a \wedge b) = 1$, $h(\mathfrak{C}_{\triangleright 3}^{40}(a \wedge b)) = 2$ and therefore $h(\gamma) = \max\{h(\mathfrak{C}_{\triangleright 3}^{40}(a \wedge b)), h(\neg c)\} + 1 = 3$.

3 The MapReduce programming model

MapReduce [10] is a programming model for processing and analyzing large data sets using a parallel, distributed infrastructure (generically called “cluster”). At the basis of the MapReduce abstraction there are two functions, *map* and *reduce*, that are inspired by (but conceptually different from) the homonymous functions that are typically found in functional programming languages. The *map* and *reduce* functions are defined by the user; their signatures are $\text{map}(k1, v1) \rightarrow \text{list}(k2, v2)$ and $\text{reduce}(k2, \text{list}(v2)) \rightarrow \text{list}(v2)$. The idea of MapReduce is to apply a *map* function to each logical entity in the input (represented by a key/value pair) in order to compute a set of intermediate key/value pairs, and then applying a

reduce function to all the values that have the same key in order to combine the derived data appropriately.

Let us illustrate this model with an example that counts the number of occurrences of each word in a large collection of documents; the pseudocode is:

```

map(String key, String value)                                reduce(String key, Iterator values):
//key: document name                                        //key: a word
//value: document contents                                //values: a list of counts
for each word w in value:                                  int result = 0
    EmitIntermediate(w, "1")                               for each v in values:
                                                            result += ParseInt(v)
                                                            Emit(AsString(result))

```

The *map* function emits list of pairs, each composed of a word and its associated count of occurrences (which is just 1). All emitted pairs are partitioned into groups and sorted according to their key for the reduction phase; in the example, pairs are grouped and sorted according to the word they contain. The *reduce* function sums all the counts (using an iterator to go through the list of counts) emitted for each particular word (i.e., each unique key).

Besides the actual programming model, MapReduce brings in a framework that provides, in a transparent way to developers, parallelization, fault tolerance, locality optimization, and load balancing. The MapReduce framework is responsible for partitioning the input data, scheduling and executing the *Map* and *Reduce* tasks (also called *mappers* and *reducers*, respectively) on the machines available in the cluster, and for managing the communication and the data transfer among them (usually leveraging a distributed file system).

More in detail, the execution of a MapReduce operation (called *job*) proceeds as follows. First, the framework divides the input into splits of a certain size using an *InputReader*, generating key/value (k, v) pairs. It then assigns each input split to Map tasks, which are processed in parallel by the nodes in the cluster. A Map task reads the corresponding input split and passes the set of key/value pairs to the *map* function, which generates a set of *intermediate* key/value pairs (k', v') . Notice that each run of the *map* function is stateless, i.e., the transformation of a single key/value pair does not depend on any other key/value pair. The next phase is called *shuffle and sort*: it takes the intermediate data generated by each Map task, sorts them based on the intermediate data generated from other nodes, divides these data into regions to be processed by Reduce tasks, and distributes these data on the nodes where the Reduce tasks will be executed. The division of intermediate data into regions is done by a *partitioning function*, which depends on the (user-specified) number of Reduce tasks and the key of the intermediate data. Each Reduce task executes the *reduce* function, which takes an intermediate key k' and a set of values associated with that key to produce the output data. This output is appended to a final output file for this reduce partition. The output of the MapReduce job will then be available in several files, one for each Reduce task used.

4 Trace checking with MapReduce

Our algorithm for trace checking of SOLOIST properties takes as input a non-empty execution trace T and the SOLOIST formula Φ to be checked. The trace T is finite and can be seen as a time-stamped sequence of H elements, i.e., $T = (p_1, p_2, \dots, p_H)$. Each of these elements is a triple $p_i = (i, \tau_i, (a_1, \dots, a_{P_i}))$, where i is the position within the trace, τ_i the integer timestamp, and (a_1, \dots, a_{P_i}) is a list of atoms such that $a_{j_i} \in \Pi$, for all $j_i \in \{1, \dots, P_i\}$, $P_i \geq 1$ and for all $i \in \{1, 2, \dots, H\}$.

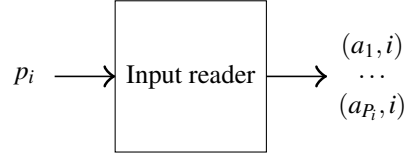
The algorithm processes the trace iteratively, through subsequent MapReduce passes. The number of MapReduce iterations is equal to height of the SOLOIST formula Φ to be checked. The l -th iteration (with $1 < l \leq h(\Phi)$) of the algorithm receives a set of tuples from the $(l-1)$ -th iteration; these input tuples represent all the positions where the subformulae of Φ having

```

function INPUT READER $_{\Phi,k,l}(T_k)$ 
  for all  $(i, \tau_i, A) \in T_k$  do
     $TS(i) \leftarrow \tau_i$ 
    for all  $a \in A$  do
      if  $a \in \text{sub}_a(\Phi)$  then
         $\text{output}(a, i)$ 
      end if
    end for
  end for
end function

```

(a) Input reader algorithm



(b) Data flow of the Input reader

Figure 2: Input reader

height $l - 1$ hold. The l -th iteration then determines all the positions where the subformulae of Φ with height l hold.

Each iteration consists of three phases: 1) reading and splitting the input; 2) (*map*) associating each formula with its superformula; 3) (*reduce*) determining the positions where the superformulae obtained in the previous step hold, given the positions where their subformulae hold. We detail each phase in the rest of this section.

4.1 Input reader

We assume that before the first iteration of the algorithm the input trace is available in the distributed file system of the cluster; this is a realistic assumption since in a distributed setting it is possible to collect logs, as long as there is a total order among the timestamps. The input reader at the first iteration reads the trace directly, while in all subsequent iterations input readers read the output of the reducers of the previous iteration.

The input reader component of the MapReduce framework is able to process the input trace exploiting some parallelism. Indeed, the MapReduce framework exploits the location information of the different fragments of the trace to parallelize the execution of the input reader. For example, a trace split into n fragments can be processed in parallel using $\min(n, k)$ machines, given a cluster with k machines.

Figure 2b shows how the input reader transforms the trace at the first iteration: for every atomic proposition ϕ that holds at position i in the original trace, it outputs a tuple of the form (ϕ, i) . The transformation does not happen in the subsequent iterations, since (as will be shown in Sect. 4.3) the output of the reduce phase has the same form (ϕ, i) . The algorithm in Fig. 2a shows how input reader handles the k -th fragment T_k of the input trace T . For each time point i and for each atom p that holds in position i it creates a tuple (p, i) . Moreover, for each time point i , it updates a globally-shared associative list of timestamps TS . This list is used to associate a timestamp with each time point; its contents are saved in the distributed file system, for use during the reduce phase.

4.2 Mapper

Each tuple generated by an input reader is passed to a mapper at the local node. Mappers “lift” the formula in the tuple by associating it with all its superformulae in the input formula Φ . For example, given the formula $\Phi \equiv (a \wedge b) \vee \neg a$, the tuple $(a, 5)$ is associated with formulae $a \wedge b$ and $\neg a$. The reduce phase will then exploit the information about the direct subformulae to determine all the positions in which a superformula holds.

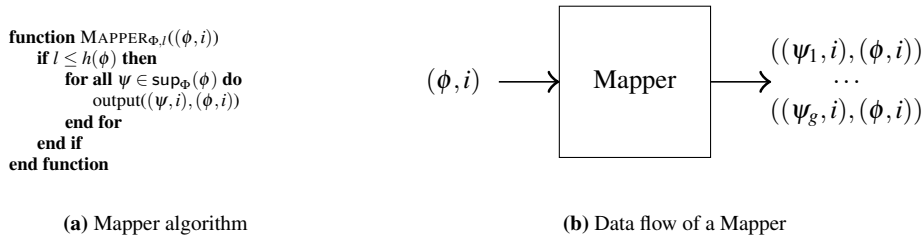


Figure 3: Mapper

As shown in Fig. 3, the output of a mapper are tuples of the form $((\psi, i), (\phi, i))$ where ϕ is a direct subformulae of ψ and i is the position where ϕ holds. For each received tuple of the form (ϕ, i) , the algorithm shown in Fig. 3a loops through all the superformulae ψ of ϕ and emits (using the function *output*) a tuple $((\psi, i), (\phi, i))$.

Notice that the key of the intermediate tuples emitted by the mapper has two parts: this type of key is called a *composite key* and it is used to perform *secondary sorting* of the intermediate tuples. Secondary sorting performs the sorting using multiple criteria, allowing developers to sort not only by the key, but also “by value”. In our case, we perform secondary sorting based on the position where the subformula holds, in order to decrease the memory used by the reducer. To enable secondary sorting, we need to override the procedure that compares keys, to take into account also the second element of the composite keys when their first elements are equal. We have also modified the key grouping procedure to consider only the first part of the composite key, so that each reducer gets all the tuples related to exactly one superformula (as encoded in the first part of the key), sorted in ascending order with respect to the position where subformulae hold (as encoded in the second part of the key).

4.3 Reducer

In the reduce phase, at each iteration l , reducers calculate all positions where subformulae with height l hold. The total number of reducers running in parallel at the l -th iteration is the minimum between the number of subformulae with height l in the input formula Φ and the number of machines in the cluster multiplied by the number of reducers available on each node. Each reducer calls an appropriate reduce function depending on the type of formula used as key in the input tuple. The initial data shared by all reducers is the input formula Φ , the index of the current MapReduce iteration l and the associative map of timestamps TS .

In the rest of this section we present the algorithms of the reduce function defined for SOLOIST connectives and modalities. For space reasons we limit the description to the algorithms for negation (\neg) and conjunction (\wedge), and for the modalities U_l , $\mathcal{C}_{\triangleright n}^K$, $\mathcal{M}_{\triangleright n}^{K,h}$, and $\mathcal{D}_{\triangleright n}^K$. The other temporal modalities can be expressed in a way similar to the *Until* modality U_l . In the various algorithms we use several auxiliary functions whose pseudocode is available in the appendix.

Negation. When the key refers to a negated superformula, the reducer emits a tuple at every position where the subformula does not hold, i.e., at every position that does not occur in the input tuples received from the mappers. The algorithm in Fig. 3e shows how output tuples are emitted. If no tuples are received then the reducer emits tuples at each position. Otherwise, it keeps track of the position i of the current tuple and the position p of the previous tuple and emits tuples at positions $[p + 1, i - 1]$.

Conjunction. We extend the binary \wedge operator defined in Sect. 2 to any positive arity; this extension does not change the language but improves the conciseness of the formulae. With

```

function REDUCER $_{\mathcal{D}_{>n}^K, \Phi, I, TS}(\mathcal{D}_{>n}^K(\phi, \psi), tuples[])$ 
  if  $h(\mathcal{D}_{>n}^K(\phi, \psi)) = l + 1$  then
     $p \leftarrow 0, pairs \leftarrow 0, dist \leftarrow 0$ 
    for all  $(\xi, i) \in tuples$  do
      for  $j \leftarrow p + 1 \dots i - 1$  do
         $updateDistInterval(j)$ 
         $emitDist(j)$ 
      end for
      if  $\xi = \psi$  then
         $pairs \leftarrow pairs + 1$ 
         $dist \leftarrow dist + (TS(i) - TS(subFmas.last))$ 
      end if
       $subFmas.addLast(i)$ 
       $updateDistInterval(i)$ 
       $emitDist(i)$ 
       $p \leftarrow i$ 
    end for
  else
    for all  $(\phi, i) \in tuples$  do
       $output(\phi, i)$ 
    end for
  end if
end function

```

(a) \mathcal{D} modality

```

function REDUCER $_{\wedge, \Phi, I, TS}(\psi, tuples[])$ 
   $p \leftarrow 0, c \leftarrow 1$ 
  while  $(\phi, i) \in tuples$  do
    if  $h(\psi) = l + 1$  then
      if  $i = p$  then
         $c \leftarrow c + 1$ 
      else
        if  $c = |sub_d(\psi)|$  then
           $output(\psi, i)$ 
        end if
         $c \leftarrow 1$ 
      end if
    else
       $output(\phi, i)$ 
    end if
     $p \leftarrow i$ 
  end while
end function

```

(b) Conjunction

```

function REDUCER $_{U_1, \Phi, I, TS}(\phi_1 U_{(a,b)} \phi_2, tuples[])$ 
  if  $h(\phi_1 U_{(a,b)} \phi_2) = l + 1$  then
     $p \leftarrow 0$ 
    for all  $(\xi, i) \in tuples$  do
       $updateLTLBehavior(i)$ 
       $updateMTLBehavior(i)$ 
      if  $\xi = \phi_2$  then
         $emitUntil(i)$ 
      end if
       $p \leftarrow i$ 
    end for
  else
    for all  $(\phi, i) \in tuples$  do
       $output(\phi, i)$ 
    end for
  end if
end function

```

(c) \mathcal{U} modality

```

function REDUCER $_{\mathcal{C}_{>n}^K, \Phi, I, TS}(\mathcal{C}_{>n}^K(\phi), tuples[])$ 
   $p \leftarrow 0, c \leftarrow 0$ 
  for all  $(\phi, i) \in tuples$  do
     $c \leftarrow c + 1$ 
    for  $j \leftarrow p + 1 \dots i - 1$  do
       $updateCountInterval(j)$ 
      if  $c \geq n$  then
         $output(\mathcal{C}_{>n}^K(\phi), j)$ 
      end if
    end for
     $updateCountInterval(i)$ 
    if  $c \geq n$  then
       $output(\mathcal{C}_{>n}^K(\phi), i)$ 
    end if
     $p \leftarrow i$ 
  end for
end function

```

(d) \mathcal{C} modality

```

function REDUCER $_{\neg, \Phi, I, TS}(\neg\phi, tuples[])$ 
   $p \leftarrow 0$ 
  for all  $((\phi, i)) \in tuples$  do
    for  $j \leftarrow p + 1 \dots i - 1$  do
       $output(\neg\phi, j)$ 
    end for
     $p \leftarrow i$ 
  end for
  for  $i \leftarrow p + 1 \dots TS.size()$  do
     $output(\neg\phi, i)$ 
  end for
end function

```

(e) Negation

```

function REDUCER $_{\mathfrak{M}_{>n}^{K,h}, \Phi, I, TS}(\mathfrak{M}_{>n}^{K,h}(\phi), tuples[])$ 
   $p \leftarrow 0$ 
  for all  $(\xi, i) \in tuples$  do
    for  $j \leftarrow p + 1 \dots i - 1$  do
       $updateMaxInterval(j)$ 
       $emitMax(j)$ 
    end for
     $updateMaxInterval(i)$ 
     $emitMax(i)$ 
     $p \leftarrow i$ 
  end for
end function

```

(f) \mathfrak{M} modality

Figure 4: Reduce algorithms

this extension, conjunction $a \wedge b \wedge c$ is represented as a single conjunction with 3 subformulae and has height equal to 1. Tuples (ϕ, i) received from the mapper may refer to any subformula ϕ of a conjunction.

In the algorithm in Fig. 3b we process all the tuples sequentially. First, we check if the height of each subformula is consistent with respect to the iteration in which they are processed. In fact, mappers can emit some tuples before the “right” iteration in which they should be processed, since subformulae of a conjunction may have different height. If the heights are not consistent, the reducer re-emits the tuples that appeared early. Since the incoming tuples are sorted by their position, it is enough to use a counter to record how many tuples there are in each position i . When the value of the counter becomes equal to the arity of the conjunction, it means that all the subformulae hold at i and the reducer can emit the tuple for the conjunction at position i . Otherwise, we reset the counter and continue.

U_I modality. The reduce function for the *Until* modality is shown in Fig. 3c. When we process tuples with this function, we have to check both the temporal behavior and the metric constraints (in the form of an (a, b) interval) as defined by the semantics of the modality.

Given a formula $\phi_1 U_{(a,b)} \phi_2$, we check whether it can be evaluated in the current iteration, since reducer may receive some tuples early. If this happens, reducer re-emits the tuple, as described above.

The algorithm processes each tuple (ϕ, i) sequentially. It keeps track of all the positions in the $(0, b)$ time window in the past with respect to the current tuple. For each tuple it calls two auxiliary functions, `updateLTBehavior` and `updateMTBehavior`. The first function checks whether ϕ_1 holds in all the positions tracked in the $(0, b)$ time window; if this not the case we stop tracing these positions. This guarantee that we only keep track of the position that exhibit the correct temporal semantics of the *Until* formula. Afterwards, function `updateMTBehavior` checks the timing constraints and removes positions that are outside of the $(0, b)$ time window. Lastly, if ϕ_2 holds in the position of the current tuple, we call function `emitUntil`, which emits an *Until* tuple for each position that we track, which is not in the $(0, a)$ time window in the past.

\mathcal{C} modality. The reduce function for the \mathcal{C} modality is outlined in the algorithm in Fig. 3d. To correctly determine if \mathcal{C} modality holds, we need to keep track of all the positions in the past time window $(0, K)$. While we sequentially process the tuples, we use variable p to save the position which appeared in the previous tuple. This allows us to consider positions between each consecutive tuple in the inner “for” loop. We call function `updateCountInterval`, which checks if the tracked positions, together with the current one, occur within the time window $(0, K)$; positions that do not fall within the time interval are discarded. Variable c is used to count in how many tracked positions subformula ϕ holds. At the end, we compare the value of c with n according to the \bowtie comparison operator; if this comparison is satisfied we emit a \mathcal{C} tuple.

\mathfrak{M} modality. The algorithm in Fig. 3f shows when the tuples for the \mathfrak{M} modality are emitted. Similarly to the \mathcal{C} modality, we need to keep track of the all positions in the $(0, K)$ time window in the past. Also, the two nested “for” loops make sure that we consider all time positions. For each position we call in sequence function `updateMaxInterval` and function `emitMax`. Function `updateMaxInterval` is similar to `updateCountInterval`, i.e., it checks whether the tracked positions, together with the current one, occur within the time window $(0, K)$. Function `emitMax` computes, in the tracked positions, the maximum number of occurrences of the subformula in all subintervals of length h . It compares the computed value to the bound n using the \bowtie comparison operator; if this comparison is satisfied it emits the \mathfrak{M} modality tuple.

\mathfrak{D} modality. The reduce function for the \mathfrak{D} modality is shown in Fig. 3a. Similarly to the case of the U_I modality, if the heights of the subformulae are not consistent with the index of

the current iteration, the reducer re-emits the corresponding tuples. After that, the incoming tuples are processed in a sequential way and two nested “for” loops guarantee that we consider all time points. We need to keep track of all the positions in the $(0, K)$ time window in the past in which either ϕ or ψ occurred. Differently from the previous aggregate modalities, we have to consider only the occurrences of ϕ for which there exists a matching occurrence ψ ; for each of these pairs we have to compute the distance. This processing of tuples (and the corresponding atoms and time points that they include) is done by the auxiliary function `updateDistInterval`. Variables `pairs` and `dist` keep track of the number of complete pairs in the current time window and their cumulative distance (computed accessing the globally-shared map `TS` of timestamps). Finally, by means of the function `emitDist`, if there is any pair in the time window, we compare the average distance computed as $\frac{dist}{pairs}$ with the bound n using the \bowtie comparison operator. If the comparison is satisfied, we emit a \mathcal{D} modality tuple.

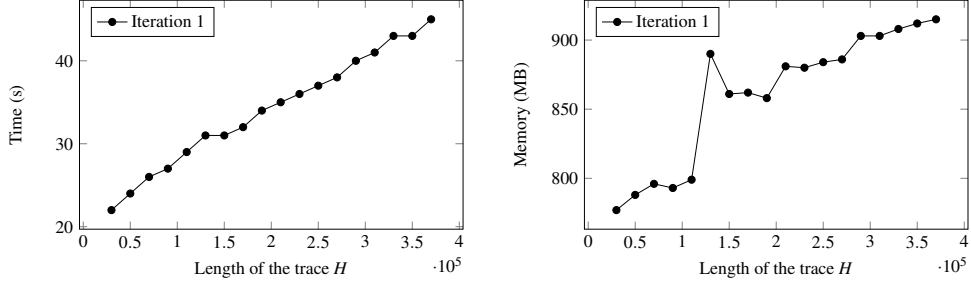
5 Related work

To the best of our knowledge, the approach proposed in [3] is the only one that uses MapReduce to perform offline trace checking of temporal properties. The algorithm is conceptually similar to ours as it performs iterations of MapReduce jobs depending on the height of the formula. However, the properties of interest are expressed using LTL. This is only a subset of the properties that can be expressed by SOLOIST. Their implementation of the conjunction and disjunction operators is limited to only two subformulae which increases the height of the formula and results in having more iterations. Intermediate tuples exchanged between mappers and reducers are not sorted by the secondary key, therefore reducers have to keep track of all the positions where the subformulae hold, while our approach tracks only the data that lies in the relevant interval of a metric temporal formula.

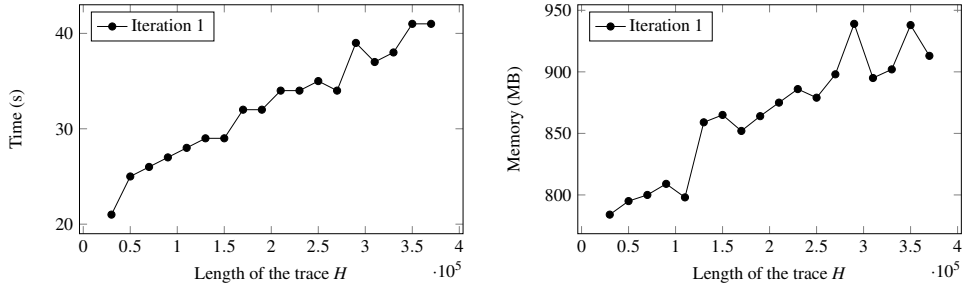
Distributed computing infrastructures and/or programming models have also been used for other verification problems. Reference [13] proposes a distributed algorithm for performing *model checking* of LTL *safety properties* on a network of interconnected workstations. By restricting the verification to safety properties, authors can easily parallelize a bread-first search algorithm. Reference [4] proposes a parallel version of the well-known fixed-point algorithm for CTL model checking. Given a set of states where a certain formula holds and a transition relation of a Kripke structure, the algorithm computes the set of states where the superformula of a given formula holds through a series of MapReduce iterations, parallelized over the different predecessors of the states in the set. The set is computed when a fixed-point of a predicate transformer is reached as defined by the semantics of each specific CTL modality.

6 Evaluation

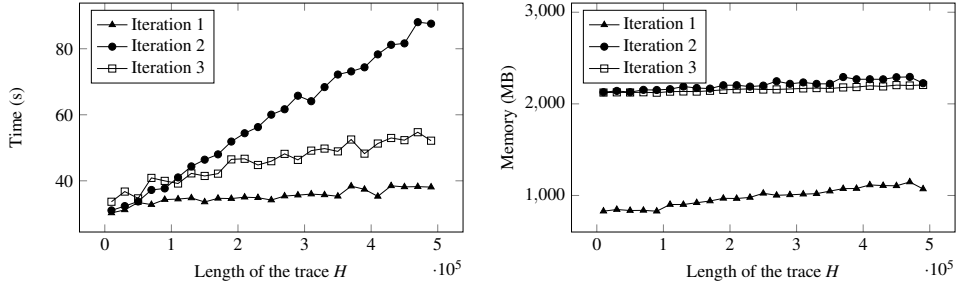
We have implemented the proposed trace checking algorithm in Java using the Hadoop MapReduce framework [2] (version 1.2.1). We executed it on a Windows Azure cloud-based infrastructure where we allocated 10 small virtual machines with 1 CPU core and 1.75 GB of memory. We followed the standard Hadoop guidelines when configuring the cluster: the number of map tasks was set to the number of nodes in the cluster multiplied by 10, and the number of reducers was set to the number of nodes multiplied by 0.9; we used 100 mappers and 9 reducers. We have also enabled JVM reuse for any number of jobs, to minimize the time spent by framework in initializing Java virtual machines. In the rest of this section, we first show how the approach scales with respect to the trace length and how the height of the formula affects



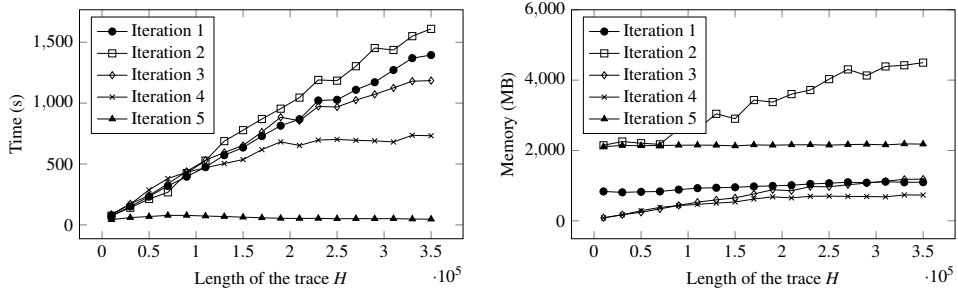
(a) Formula: $e_{<10}^{50000}(a_0)$



(b) Formula: $\mathcal{D}_{<10}^{50000}(a_1, a_2)$



(c) Formula: $(a_0 \wedge (a_1 \wedge a_2))U_{(50,200)}((a_1 \wedge a_2) \vee a_1)$



(d) Formula: $\exists j \in \{0 \dots 9\} \forall i \in \{0 \dots 8\} : G_{(50,500)}(a_{i,j} \rightarrow X_{(50,500)}(a_{i+1,j}))$

Figure 5: Scalability of the algorithm

Table 1: Average processing time per tuple for the four properties.

	Property 1		Property 2		Property 3		Property 4	
	SOLOIST	LTL	SOLOIST	LTL	SOLOIST	LTL	SOLOIST	LTL
Number of tuples	16,121	55,009	24,000	119,871	215,958	599,425	1,747,360	4,987,124
Time per event (μ s)	1.172	19	1.894	21	3.707	14	7.200	30

the running time and memory. Afterwards, we compare our algorithm to the one presented in [3], designed for LTL.

Scalability.

To evaluate scalability of the approach, we considered 4 formulae, with different height: $\mathfrak{C}_{<10}^{50000}(a_0)$, $\mathfrak{D}_{<10}^{50000}(a_1, a_2)$, $(a_0 \wedge (a_1 \wedge a_2))U_{(50,200)}((a_1 \wedge a_2) \vee a_1)$ and $\exists j \in \{0 \dots 9\} \forall i \in \{0 \dots 8\} : G_{(50,500)}(a_{i,j} \rightarrow X_{(50,500)}(a_{i+1,j}))$. Here the \forall and \exists quantifiers are used as a shorthand notation to predicate on finite domains: for example, $\forall i \in \{1, 2, 3\} : a_i$ is equivalent to $a_1 \wedge a_2 \wedge a_3$. We generated random traces with a number of time instants varying from 10000 to 350000. For each time instant, we randomly generated with a uniform distribution up to 100 distinct events (i.e., *atomic* propositions). Hence, we evaluated our algorithm for a maximum number of events up to 35 millions. The time span between the first and the last timestamp was 578.7 days on average, with a granularity of one second.

Figure 5 shows the total time and the memory used by the MapReduce job run to check the four formulae on the generated traces. Formulae $\mathfrak{C}_{<10}^{50000}(a_0)$ and $\mathfrak{D}_{<10}^{50000}(a_1, a_2)$ needed one iteration to be evaluated (shown in Fig. 5a and Fig. 5b). In both cases, the time taken to check the formula increases linearly with respect to the trace length; this happens because reducers need to process more tuples. As for the linear increase in memory usage, for modalities \mathfrak{C} and \mathfrak{D} reducers have to keep track of all the tuples in the window of length K time units and the more time points there are the more *dense* the time window becomes, with a consequent increase in memory usage. As for the checking of the other two formulae (shown in Fig. 5c and Fig. 5d), more iterations were needed because of the height of the formulae. Also in this case, the time taken by each iteration tends to increase as the length of the trace increases; the memory usage is constant since the formulae considered here do not contain aggregate modalities. Notice the increase of time and memory from Fig. 5c to Fig. 5d: this is due to the expansion of the quantifiers in formula $\exists j \in \{0 \dots 9\} \forall i \in \{0 \dots 8\} : G_{(50,500)}(a_{i,j} \rightarrow X_{(50,500)}(a_{i+1,j}))$.

Comparison with the LTL approach [3].

We compare our approach to the one presented in [3], which focuses on trace checking of LTL properties using MapReduce; for this comparison we considered the LTL layer included in SOLOIST by means of the *Until* modality. Although the focus of our work was on implementing the semantics of SOLOIST aggregate modalities, we also introduces some improvements in the LTL layer of SOLOIST. First, we exploited composite keys and secondary sorting as provided by the MapReduce framework to reduce the memory used by reducers. We also extended the binary \wedge and \vee operators to support any positive arity.

We compared the two approaches by checking the following formulae: 1) $G_{(50,500)}(\neg a_0)$; 2) $G_{(50,500)}(a_0 \rightarrow X_{(50,500)}(a_1))$; 3) $\forall i \in \{0 \dots 8\} : G_{(50,500)}(a_i \rightarrow X_{(50,500)}(a_{i+1}))$; and 4) $\exists j \in \{0 \dots 9\} \forall i \in \{0 \dots 8\} : G_{(50,500)}(a_{i,j} \rightarrow X_{(50,500)}(a_{i+1,j}))$. The height of these formulae are 2, 3, 4 and 5, respectively. This admittedly gives our approach a significant advantage since in [3] the restriction for the \wedge and \vee operators to have an arity fixed to 2 results in a larger height

for formulae 3 and 4. We randomly generated traces of variable length, ranging from 1000 to 100000 time instants, with up to 100 events per time instant. With this configuration, a trace can contain potentially up to 10 million events. We chose to have up to 100 events per time instant to match the configuration proposed in [3], where there are 10 parameters per formula that can take 10 possible values. We generated 500 traces. The time needed by our algorithm to check each of the four formulae, averaged over the different traces, was 52.83, 85.38, 167.1 and 324.53 seconds, respectively. We do not report the time taken by the approach proposed in [3] since the article does not report any statistics from the run of an actual implementation, but only metrics determined by a simulation. Table 1 shows the average number of tuples generated by the algorithm for each formulae. The number of tuples is calculated as the sum of all input tuples for mappers at each iterations in a single trace checking run. The table also shows the average time needed to process a single event in the trace. This time is computed as the total processing time divided by the number of time instants in the trace, averaged over the different trace checking runs. The SOLOIST column refers to the data obtained by running our algorithm, while the LTL column refers to data reported in [3], obtained with a simulation. Our algorithm performs better both in terms of the number of generated tuples and in terms of processing time.

7 Conclusion and Future Work

In this paper we present an algorithm based on the MapReduce programming model that checks large execution traces against specifications written in SOLOIST. The experimental results in terms of scalability and comparison with the state of the art are encouraging and show that the algorithm can be effectively applied in realistic settings.

A limitation of the algorithm is that reducers (that implement the semantics of temporal and aggregate operators) need to keep track of the positions relevant to the time window specified in the formula. In the future, we will investigate how this information may be split into smaller and more manageable parts that may be processed separately, while preserving the original semantics of the operators.

Acknowledgments.

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A Auxiliary functions

In this section we present the pseudocode of the auxiliary functions used in the reduce steps of *until*, *count*, *maximum* and *distance* operators presented in Sect. 4.3.

As explained in Sect. 4.3, the reduce function for the *until* modality of the form $\phi_1 U_{(a,b)} \phi_2$ keeps track of all the positions in the past $(0, b)$ time window with respect to the timestamp of the current tuple. For clarity, the positions are partitioned into arrays `int0A` and `intAB` that store the positions in the past $(0, a]$ and (a, b) time windows, respectively. Function `clearIntervals`, shown in Fig. 5e, removes all positions from both arrays. Function `updateLTLBehavior`, shown in Fig. 5b, appends the current position to array `int0A` and checks whether the LTL condition of the *until* operator holds for the stored positions. If the condition is violated,

the arrays are cleared. More specifically, the arrays are cleared if there exist some positions, between the current position i and the previous position p , which were not received by the reducer. The arrays are also cleared if ϕ_1 does not hold in all the consecutive positions currently stored by the reducer. Function `updateMTLBehavior`, shown in Fig. 5a, is used to check the timing conditions of the *until* operator. Since `updateLTLBehavior` inserts the newly received position into the arrays, `updateMTLBehavior` updates the arrays with respect to the timestamp related to the new position. If some positions from `int0A` are not in the $(0, a]$ interval anymore, function `updateMTLBehavior` transfers them to `intAB`. Next, the function removes all tuples from `intAB` that are not in the (a, b) interval. Function `emitUntil`, shown in Fig. 5f, emits *until* tuples for all positions stored in `intAB`.

The reduce function for the $\mathcal{C}_{\succ n}^K(\phi)$ modality keeps track of all the positions in the past time window $(0, K)$. The positions are stored in the array `intK`. Function `updateCountInterval`, shown in Fig. 5c, adds the position from the current tuple to `intK` and then checks if all stored positions are in the past $(0, K)$ time window. In practice, the function compares the difference between the timestamps of the first and the last position in `intK`. As long as this difference is greater than K , the function removes the positions from the beginning of the array. It checks if the subformula ϕ holds at every removed position and, if it is the case, it decrements variable c .

The reduce function for modality $\mathcal{M}_{\succ n}^{K,h}(\phi)$ updates its own corresponding array of positions `intK` using function `updateMaxInterval`. It also computes, using `emitMax`, the maximum number of occurrences of subformula ϕ in subintervals of length h over a window K . Function `updateMaxInterval`, shown in Fig. 5g, checks whether the stored positions occur within the time window $(0, K)$ in the same way as its *count* counterpart. Function `emitMax` calculates, for each position z from `intK`, the subinterval wc it belongs to. This is done by calculating the difference between the timestamp at the last position in `intK` and the one at the position z and then dividing it by the length of the subinterval h . Variable c counts the number of occurrences of subformula ϕ in the current subinterval wc . We increment c for every position where subformula ϕ holds and wc does not change with respect to the previous position. When wc changes, we update variable max and reset variable c to 0 or 1, depending on whether subformula ϕ holds in the current position. When the function terminates, variable max holds the maximum number of ϕ occurrences in all subintervals. Finally, variable max is compared to the bound n and a tuple is emitted in case the condition is satisfied.

The reduce function for the $\mathcal{D}_{\succ n}^K(\phi, \psi)$ modality uses the `updateDistInterval` and `emitDist` functions, in a similar way as the previous modality. Function `updateDistInterval`, shown in Fig. 5d, updates the array of past positions `intK`. If subformula ϕ holds at a position that is removed from the array, we decrement the pairs variable that holds the current number of (ϕ, ψ) pairs in `intK`. We also update the cumulative distance `dist` between complete pairs `intK`. Finally, if there is at least one pair in the current time window, function `emitDist` (shown in Fig. 5i) compares the average distance computed as $\frac{dist}{pairs}$ to the bound n . If the condition is satisfied, it emits a *distance* modality tuple.

Global variables: $a, b, \text{int0A}, \text{intAB}$
function UPDATEMTLBEHAVIOR(i)
 $\tau \leftarrow TS(i)$
 $\tau_c \leftarrow TS(\text{int0A.first}())$
while $\tau - a \geq \tau_c$ **do**
 $p \leftarrow \text{int0A.removeFirst}()$
 $\text{intAB.addLast}(p)$
 $\tau_c \leftarrow TS(\text{int0A.first}())$
end while
 $\tau_c \leftarrow TS(\text{intAB.first}())$
while $\tau - b \geq \tau_c$ **do**
 $p \leftarrow \text{intAB.removeFirst}()$
 $\tau_c \leftarrow TS(\text{intAB.first}())$
end while
end function

(a) Update MTL behavior function

Global variables: intK, ϕ, c, K
function UPDATECOUNTINTERVAL(i)
 $\text{intK.addLast}(i)$
while $TS(\text{intK.last}) - TS(\text{intK.first}) > K$ **do**
 $z \leftarrow \text{intK.removeFirst}()$
if $(\phi, z) \in \text{tuples}$ **then**
 $c \leftarrow c - 1$
end if
end while
end function

(c) Update count interval function

Global variables: $\text{int0A}, \text{intAB}$
function CLEARINTERVALS()
 $\text{intAB.clear}()$
 $\text{int0A.clear}()$
end function

(e) Clear intervals function

Global variables: $\text{intK}, \phi, K, n, h$
function EMITMAX(i)
 $\tau_r \leftarrow TS(\text{intK.last})$
 $wc \leftarrow 0, c \leftarrow 0, \text{max} \leftarrow 0$
for all $z \in \text{intK}$ **do**
if $wc = \lfloor \frac{(\tau_r - TS(z))}{h} \rfloor$ **then**
if $(\phi, z) \in \text{tuples}$ **then**
 $c \leftarrow c + 1$
end if
else
 $\text{max} \leftarrow \max(c, \text{max})$
 $wc \leftarrow (\tau_r - TS(z))/h$
if $(\phi, z) \in \text{tuples}$ **then**
 $c \leftarrow 1$
else
 $c \leftarrow 0$
end if
end if
end for
if $\text{max} \geq n$ **then**
 $\text{output}(\mathcal{M}_{>n}^{K,h}(\phi), i)$
end if
end function

(h) Emit maximum tuples function

Global variables: $p, \xi, \phi_1, \text{int0A}, \text{intAB}$
function UPDATERLBEHAVIOR(i)
if $i - p > 1$ **then**
 $\text{clearIntervals}()$
end if
if $\xi = \phi_1$ **then**
if $\max(\text{int0A.last}(), \text{intAB.last}()) \neq i - 1$ **then**
 $\text{clearIntervals}()$
end if
 $\text{int0A.addLast}(i)$
end if
end function

(b) Update LTL behavior function

Global variables: $\text{intK}, \phi, \text{dist}, \text{pairs}, K$
function UPDATEDISTINTERVAL(i)
 $\text{intK.addLast}(i)$
while $TS(\text{intK.last}) - TS(\text{intK.first}) > K$ **do**
 $z \leftarrow \text{intK.removeFirst}()$
if $z = \text{subFmas.first}()$ **then**
 $\text{subFmas.removeFirst}()$
 $\text{pairs} \leftarrow \text{pairs} - 1$
 $\text{dist} \leftarrow \text{dist} - (TS(\text{subFmas.first}) - TS(z))$
end if
end while
end function

(d) Update distance interval function

Global variables: intAB
function EMITUNTIL(i)
for all $z \in \text{intAB}$ **do**
 $\text{output}(\phi_1 \cup_{(a,b)} \phi_2, z)$
 $\text{intAB.remove}(z)$
end for
end function

(f) Emit until tuples function

Global variables: intK, K
function UPDATEMAXINTERVAL(i)
 $\text{intK.addLast}(i)$
while $TS(\text{intK.last}) - TS(\text{intK.first}) > K$ **do**
 $\text{intK.removeFirst}()$
end while
end function

(g) Update maximum interval function

Global variables: $\text{dist}, \text{pairs}, n$
function EMITDIST(i)
if $\text{pairs} > 0$ **then**
if $\frac{\text{dist}}{\text{pairs}} \geq n$ **then**
 $\text{output}(\mathcal{D}_{>n}^K(\phi, \psi), i)$
end if
end if
end function

(i) Emit distance tuples function

Figure 6: Auxiliary functions