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Delay-dependent exponential stability criteria for stochastic systems with polytopic-type uncertainties

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Abstract: This paper considers the problem of delay-dependent exponential stability in mean square for stochastic systems with polytopic-type uncertainties and time-varying delay. Applying the descriptor model transformation and introducing free weighting matrices, a new type of Lyapunov-Krasovskii functional is constructed based on linear matrix inequalities (LMIs), and some new delay-dependent criteria are obtained. These criteria include the delay-independent/rate-dependent and delay-dependent/rate-independent exponential stability criteria. These new criteria are less conservative than existing ones. Numerical examples demonstrate that these new criteria are effective and are an improvement over existing ones.

Keywords: Stochastic system; Exponential stability in mean square; Time-varying state delay; Delay-dependent criteria; Linear matrix inequality (LMI)

1 Introduction

Recently, delay-dependent stability criteria for stochastic delay systems have attracted extensive attention (see [1~8]). In view of the robustness of stochastic stability, the linear and semilinear systems were studied in [1, 2], respectively. [3] investigated the stability of linear and semilinear stochastic differential equation by means of the exponential stability. Verriest [4] presented stability of linear stochastic differential equation via Riccati equations. Based on the LMI approach, [5~8] gave the delay-dependent robust stability criteria of uncertain stochastic systems, respectively. However, the criteria in [5] involved the parameterized model transformation. To determine the stability of system, [6] and [8] used some inequality constraint. [7] used a descriptor integral inequality constraint, and the criteria in [7, 8] with matrix constraint $P \leqslant \alpha I$ ($\alpha > 0$ is a scalar, P is the product of Lyapunov matrix). These results show considerable conservativeness.

This paper presents some new delay-dependent exponential stability criteria for stochastic system with polytopic-type uncertainties and time-varying delay. First, applying descriptor model transformation [9], we set descriptor stochastic system and construct a new type of Lyapunov-Krasovskii functional. Second, based on the idea of [10], some free weighting matrices are introduced to exclude constraint conditions in [6~8]. Finally, using LMI algorithm, we obtain delay-dependent and delay-independent exponential stability criteria for stochastic system with polytopic-type uncertainties and time-varying delay. These criteria include delay-dependent/rate-independent and delay-independent/rate-dependent exponential stability criteria. In contrast with the existing stability criteria, these new criteria are less conservative. Numerical simulation examples show that these results are effective and an improvement over existing ones.

For convenience, we adopt the following notations: $\mathrm{tr}(A)(A^{\mathrm{T}})$ denotes trace(transpose) of the matrix A; $A \geqslant 0$ ($A > 0$) denotes positive semidefinite (positive definite) matrix A; $L^2_{F_0}([-\tau, 0]; \mathbb{R}^n)$ is the family of $\mathbb{R}^n-$valued stochastic processes $\eta(s)$, $-\tau \leqslant s \leqslant 0$ such that $\eta(s)$ is F_0- measurable for every second and $\int_{-\tau}^{0} E\,\|\eta(s)\|^2\,\mathrm{d}s < \infty$; and $E\{\cdot\}$ denotes mathematical expectation operator with respect to the given probability measure P.

2 Preliminaries

Consider the robust stability of system (1) with polytopic-type uncertainties, that is, assume that system (1) has the following form:

$$\begin{aligned}\mathrm{d}x(t) &= [Ax(t) + A_d x(t-h(t))]\mathrm{d}t \\ &\quad + [Cx(t) + C_d x(t-h(t))]\mathrm{d}\beta(t), \qquad (1)\\ x(t) &= \varphi(t),\ \ \forall t \in [-\tau, 0],\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, the system matrices A, A_d, C, and C_d are assumed to be uncertain but belong to a known convex compact set of polytopic type, namely

$$(A, A_d, C, C_d) \in \Omega, \tag{2}$$

where Ω is a given convex bounded polyhedral domain described by q vertices as follows:

$$\begin{aligned}\Omega := \{(A, A_d, C, C_d) &= \sum_{k=1}^{q} \xi_k (A_k, A_{kd}, C_k, C_{kd}) \\ \xi_k \geqslant 0; \sum_{k=1}^{q} \xi_k = 1\}.\end{aligned} \tag{3}$$

The time delay $h(t)$ is a time-varying continuous function that satisfies

$$0 \leqslant h(t) \leqslant \tau \tag{4}$$

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and

$$\dot{h}(t) \leqslant \mu \leqslant 1, \tag{5}$$

where τ and μ are constants, $\varphi(t)$ is a continuous vector-valued initial function, and $\varphi := \{\varphi(s) : -\tau \leqslant s \leqslant 0\} \in L^2_{F_0}([-\tau, 0], \mathbb{R}^n)$. It is well known that (1) has a unique solution, denoted by $x(t,\varphi)$, which is square integrable. So (1) admits a trivial solution $x(t,0) \equiv 0$. A_k, A_{kd}, C_k, C_{kd} are known constant matrices with appropriate dimensions. The variables $\beta(t)$ are an m-dimensional Brownian motion defined on a complete probability space (Ω, F, P) with a natural filtration $\{F_t\}_{t\geqslant 0}$ (i.e., $F_t = \sigma\{\varpi(s) : 0 \leqslant s \leqslant t\}$).

Definition 1 System (1) is said to be exponentially stable in mean square if there exists a positive constant α_0 such that

$$\lim_{t\to\infty} \sup \frac{1}{t} \log E\,\|x(t)\|^2 \leqslant \alpha_0.$$

2.1 Delay-dependent robust exponential stability

To discuss the stability of system (1), first, we introduce the descriptor system approach, set

$$q(t) = Ax(t) + A_d x(t-h(t)), \tag{6}$$

$$g(t) = Cx(t) + C_d x(t-h(t)). \tag{7}$$

Then system (1) becomes the following descriptor stochastic system

$$\mathrm{d}x(t) = q(t)\mathrm{d}t + g(t)\mathrm{d}\beta(t). \tag{8}$$

Moreover, equations in (6) and (7) ensure the following zero equations

$$\begin{aligned} &2\left[x^{\mathrm T}(t)N_1 + x^{\mathrm T}(t-h(t))N_2 + q^{\mathrm T}(t)N_3 + g^{\mathrm T}(t)N_4\right] \\ &\times\left[Ax(t) + A_d x(t-h(t)) - q(t)\right] \equiv 0, \end{aligned} \tag{9}$$

$$\begin{aligned} &2\left[x^{\mathrm T}(t)T_1 + x^{\mathrm T}(t-h(t))T_2 + q^{\mathrm T}(t)T_3 + g^{\mathrm T}(t)T_4\right] \\ &\times\left[Cx(t) + C_d x(t-h(t)) - g(t)\right] \equiv 0, \end{aligned} \tag{10}$$

where N_r and T_r $(r = 1,2,3,4)$ are appropriately dimensioned matrices. On the other hand, the Newton-Leibniz formula provides

$$x(t) - x(t-h(t)) = \int_{t-h(t)}^{t} \dot{x}(s)\mathrm{d}s = \int_{t-h(t)}^{t} q(s)\mathrm{d}s + \varsigma, \tag{11}$$

where $\varsigma^{\mathrm T} = [\int_{t-h(t)}^{t} g(s)\mathrm{d}\beta(s)]^{\mathrm T}$ by (11), get

$$\int_{t-h(t)}^{t} q(s)\mathrm{d}s = x(t) - x(t-h(t)) - \varsigma, \tag{12}$$

then, we obtained the following Theorem.

Theorem 1 Consider system (1) with polytopic-type uncertainties (3) and a time-varying delay satisfying (4) and (5). Given scalars $\tau > 0$ and $\mu < 1$, system (1) is robust exponentially stable in mean square, if there exist symmetric positive definite matrices $P_k > 0, Q_k \geqslant 0, Z_k > 0$, and $R^{(k)}_{ij}$ $(i,j = 1,2,3)$ and appropriately dimensioned matrices N_r and T_r $(r = 1,2,3,4)$ such that $R^{(k)}_{ij} = R^{(k)}_{ji}$ and the following LMIs hold for $k = 1,\cdots,q$:

$$\varPhi^{(k)} = \begin{bmatrix} \varPhi^{(k)}_{11} & \varPhi^{(k)}_{12} & \varPhi^{(k)}_{13} & \varPhi^{(k)}_{14} & R^{(k)}_{13} \\ * & \varPhi^{(k)}_{22} & \varPhi^{(k)}_{23} & \varPhi^{(k)}_{24} & R^{(k)}_{23} \\ * & * & \varPhi^{(k)}_{33} & \varPhi^{(k)}_{34} & 0 \\ * & * & * & \varPhi^{(k)}_{44} & 0 \\ * & * & * & * & -Z_k \end{bmatrix} < 0 \tag{13}$$

and

$$R^{(k)} = \begin{bmatrix} R^{(k)}_{11} & R^{(k)}_{12} & R^{(k)}_{13} \\ * & R^{(k)}_{22} & R^{(k)}_{23} \\ * & * & R^{(k)}_{33} \end{bmatrix} > 0,$$

where an asterisk $*$ denotes a block induced easily by symmetry and

$$\begin{aligned}
\varPhi^{(k)}_{11} &= N_1A_k + A_k^{\mathrm T}N_1^{\mathrm T} + T_1C_k + C_k^{\mathrm T}T_1^{\mathrm T} + Q_k + \tau R^{(k)}_{11} \\
&\quad + R^{(k)}_{13} + R^{(k)T}_{13}, \\
\varPhi^{(k)}_{12} &= N_1A_{dk} + A_k^{\mathrm T}N_2^{\mathrm T} + T_1C_{dk} + C_k^{\mathrm T}T_2^{\mathrm T} + \tau R^{(k)}_{12} \\
&\quad - R^{(k)}_{13} + R^{(k)T}_{23}, \\
\varPhi^{(k)}_{13} &= P_k - N_1 + A_k^{\mathrm T}N_3^{\mathrm T} + C_k^{\mathrm T}T_3^{\mathrm T}, \\
\varPhi^{(k)}_{14} &= A_k^{\mathrm T}N_4^{\mathrm T} - T_1 + C_k^{\mathrm T}T_4^{\mathrm T}, \\
\varPhi^{(k)}_{22} &= N_2A_{dk} + A_{dk}^{\mathrm T}N_2^{\mathrm T} + T_2C_{dk} - (1-\mu)Q_k + C_{dk}^{\mathrm T}T_2^{\mathrm T} \\
&\quad + \tau R^{(k)}_{22} - R^{(k)}_{23} - R^{(k)T}_{23}, \\
\varPhi^{(k)}_{23} &= -N_2 + A_{dk}^{\mathrm T}N_3^{\mathrm T} + C_{dk}^{\mathrm T}T_3^{\mathrm T}, \\
\varPhi^{(k)}_{24} &= A_{dk}^{\mathrm T}N_4^{\mathrm T} - T_2 + C_{dk}^{\mathrm T}T_4^{\mathrm T}, \\
\varPhi^{(k)}_{33} &= -N_3 - N_3^{\mathrm T} + \tau R^{(k)}_{33}, \\
\varPhi^{(k)}_{34} &= -N_4^{\mathrm T} - T_3,\ \ \varPhi^{(k)}_{44} = P_k - T_4^{\mathrm T} - T_4 + \tau Z_k.
\end{aligned}$$

Proof Choose a Lyapunov-Krasovskii functional for system (1) to be

$$V(t) = \sum_{i=1}^{5} V_i(t),$$

in which

$$\begin{aligned}
V_1(t) &= \sum_{k=1}^{q} x(t)^{\mathrm T}P_k x(t), \\
V_2(t) &= \sum_{k=1}^{q} \int_{t-h(t)}^{t} x^{\mathrm T}(s)Q_k x(s)\mathrm{d}s, \\
V_3(t) &= \sum_{k=1}^{q} \int_{-\tau}^{0}\int_{t+\theta}^{t} q^{\mathrm T}(s)R^{(k)}_{33}q(s)\mathrm{d}s\mathrm{d}\theta, \\
V_4(t) &= \sum_{k=1}^{q} \int_{-\tau}^{0}\int_{t+\theta}^{t} \mathrm{tr}[g^{\mathrm T}(s)Z_k g(s)]\mathrm{d}s\mathrm{d}\theta, \\
V_5(t) &= \sum_{k=1}^{q} \int_{0}^{t}\int_{\alpha-h(\alpha)}^{\alpha} \delta^{\mathrm T}R_k\delta\mathrm{d}s\mathrm{d}\alpha,
\end{aligned}$$

where $\delta^{\mathrm T} = [x^{\mathrm T}(\alpha), x^{\mathrm T}(\alpha-h(\alpha)), q^{\mathrm T}(s)]$,

$$R^{(k)} = \begin{bmatrix} R^{(k)}_{11} & R^{(k)}_{12} & R^{(k)}_{13} \\ * & R^{(k)}_{22} & R^{(k)}_{23} \\ * & * & R^{(k)}_{33} \end{bmatrix}.$$

$P_k, Q_k, Z_k, R^{(k)}_{ij}$ $(i,j = 1,2,3)$ are positive definite matrices with appropriate dimensions. Let L be the weak infinitesimal operator of (8), then, by Itô differential formula,

$$L_{v=0}V_1 = \sum_{k=1}^{q}\left\{2x^{\mathrm T}(t)P_k q(t) + \mathrm{tr}\left[g^{\mathrm T}(t)P_k g(t)\right]\right\}, \tag{14}$$

$$\begin{aligned} L_{v=0}V_2 \leqslant \sum_{k=1}^{q}&\{x^{\mathrm T}(t)Q_k x(t) \\ &-(1-\mu)x^{\mathrm T}(t-h(t))Q_k x(t-h(t))\}, \end{aligned} \tag{15}$$

$$L_{v=0}V_3 \leqslant \sum_{k=1}^{q}\{\tau q^{\mathrm T}(t)R^{(k)}_{33}q(t) - \chi\}, \tag{16}$$

$$L_{v=0}V_4 \leqslant \sum_{k=1}^{q}\{\tau \mathrm{tr}\,[g^{\mathrm{T}}(t)Z_k g(t)] - \int_{t-h(t)}^{t} \mathrm{tr}\,[g^{\mathrm{T}}(s)Z_k g(s)]\,\mathrm{d}s\}, \tag{17}$$

$$L_{v=0}V_5 = \sum_{k=1}^{q}\{h(t)\xi^{\mathrm{T}}\begin{bmatrix} R_{11}^{(k)} & R_{12}^{(k)} \\ R_{12}^{(k)T} & R_{22}^{(k)} \end{bmatrix}\xi + 2\int_{t-h(t)}^{t}\xi^{\mathrm{T}}\begin{bmatrix} R_{13}^{(k)} \\ R_{23}^{(k)} \end{bmatrix} q(s)\mathrm{d}s + \chi\}, \tag{18}$$

where $\chi = \int_{t-h(t)}^{t} q^{\mathrm{T}}(s)R_{33}^{(k)}q(s)\mathrm{d}s$.

Substitute (12) into (18), get

$$\begin{aligned} &L_{v=0}V_5 \\ &= \sum_{k=1}^{q}\Bigg\{h(t)\xi^{\mathrm{T}}\begin{bmatrix} R_{11}^{(k)} & R_{12}^{(k)} \\ R_{12}^{(k)T} & R_{22}^{(k)} \end{bmatrix}\xi \\ &\quad +2\xi^{\mathrm{T}}\begin{bmatrix} R_{13}^{(k)} & -R_{13}^{(k)} \\ R_{23}^{(k)} & -R_{23}^{(k)} \end{bmatrix}\xi + \chi - 2\xi^{\mathrm{T}}\begin{bmatrix} R_{13}^{(k)} \\ R_{23}^{(k)} \end{bmatrix}\varsigma\Bigg\}. \end{aligned} \tag{19}$$

By Lemma [11], for any matrix $Z_k > 0$,

$$-2\xi^{\mathrm{T}}\begin{bmatrix} R_{13}^{(k)} \\ R_{23}^{(k)} \end{bmatrix}\varsigma \leqslant \xi^{\mathrm{T}}\begin{bmatrix} R_{13}^{(k)} \\ R_{23}^{(k)} \end{bmatrix} Z_k^{-1}\begin{bmatrix} R_{13}^{(k)} \\ R_{23}^{(k)} \end{bmatrix}^{\mathrm{T}}\xi + \varsigma^{\mathrm{T}}Z_k\varsigma.$$

Obviously,

$$\begin{aligned} &L_{v=0}V_5 \\ &\leqslant \sum_{k=1}^{q}\Bigg\{\tau\xi^{\mathrm{T}}\begin{bmatrix} R_{11}^{(k)} & R_{12}^{(k)} \\ R_{12}^{\mathrm{T}(k)} & R_{22}^{(k)} \end{bmatrix}\xi + 2\xi^{\mathrm{T}}\begin{bmatrix} R_{13}^{(k)} & -R_{13}^{(k)} \\ R_{23}^{(k)} & -R_{23}^{(k)} \end{bmatrix}\xi \\ &\quad +\chi + \varsigma^{\mathrm{T}}Z_k\varsigma + \xi^{\mathrm{T}}\begin{bmatrix} R_{13}^{(k)} \\ R_{23}^{(k)} \end{bmatrix} Z_k^{-1}\begin{bmatrix} R_{13}^{(k)} \\ R_{23}^{(k)} \end{bmatrix}^{\mathrm{T}}\xi\Bigg\}, \end{aligned} \tag{20}$$

where $\xi^{\mathrm{T}} = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-h(t))]$. Combining $L_{v=0}V_i$ $(i = 1,2,3,4,5)$ and adding the terms on the left of (9)-(10) to $L_{v=0}V$, we can express $L_{v=0}V$ as

$$L_{v=0}V \leqslant \sum_{k=1}^{q}\{\eta^{\mathrm{T}}(t)\xi_k\Phi^{(k)}\eta(t) - \int_{t-h(t)}^{t}\mathrm{tr}\,[g^{\mathrm{T}}(s)Z_k g(s)]\,\mathrm{d}s + \varsigma^{\mathrm{T}}Z_k\varsigma\},$$

where $\eta^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-h(t)), q^{\mathrm{T}}(t), g^{\mathrm{T}}(t)]$, $\sigma^{\mathrm{T}} = [(R_{13}^{(k)})^{\mathrm{T}}, (R_{23}^{(k)})^{\mathrm{T}}, 0, 0]$,

$$\Phi^{(k)} = \begin{bmatrix} \Phi_{11}^{(k)} & \Phi_{12}^{(k)} & \Phi_{13}^{(k)} & \Phi_{14}^{(k)} \\ * & \Phi_{22}^{(k)} & \Phi_{23}^{(k)} & \Phi_{24}^{(k)} \\ * & * & \Phi_{33}^{(k)} & \Phi_{34}^{(k)} \\ * & * & * & \Phi_{44}^{(k)} \end{bmatrix} + \sigma Z_k^{-1}\sigma^{\mathrm{T}}.$$

Since $E\left(\varsigma^{\mathrm{T}}Z_k\varsigma\right) = E\int_{t-h(t)}^{t}\mathrm{tr}\,[g^{\mathrm{T}}(s)Z_k g(s)]\,\mathrm{d}s$, it follows that

$$EL_{v=0}V(t) \leqslant \sum_{k=1}^{q} E\eta^{\mathrm{T}}(t)\xi_k\Phi^{(k)}\eta(t). \tag{21}$$

By Schur's complement, $\Phi^{(k)} < 0$ is equivalent to LMI (13). From the proof of Theorem 1 [12], there exist a scalar α such that

$$\lim_{t\to\infty}\sup\frac{1}{t}\log E\|x(t)\|^2 \leqslant -\alpha,$$

which implies that system (1) is exponentially stable in mean square. The proof of Theorem 1 is completed.

Remark 1 By constructing an output feedback controller, we can obtain the stability criteria for system with output feedback in the same methods. So, it is without loss of generality for the discussion of system (1).

Note that a delay-dependent and rate-independent exponential stability criterion for system (1) with polytopic-type uncertainties (3) and a delay satisfying (4) and (5) can be derived from Theorem 1 by choosing $Q_k = 0$ as follows.

Corollary 1 Given scalar $\tau > 0$, system (1) with polytopic-type uncertainties (3) and a time-varying delay satisfying (4) is exponentially stable in mean square if there exist symmetric positive-definite $P_k, Z_k, R_{ij}^{(k)}$ $(k = 1,\cdots,q)$ and appropriately dimensioned matrices N_r and T_r $(r = 1,2,3)$ such that $R_{ij}^{(k)} = R_{ji}^{(k)}$ $(i,j = 1,2,3)$ and the following LMI holds for $k = 1,2,\cdots,q$:

$$\hat{\Phi}^{(k)} = \begin{bmatrix} \hat{\Phi}_{11}^{(k)} & \Phi_{12}^{(k)} & \Phi_{13}^{(k)} & \Phi_{14}^{(k)} & R_{13}^{(k)} \\ * & \hat{\Phi}_{22}^{(k)} & \Phi_{23}^{(k)} & \Phi_{24}^{(k)} & R_{23}^{(k)} \\ * & * & \Phi_{33}^{(k)} & \Phi_{34}^{(k)} & 0 \\ * & * & * & \Phi_{44}^{(k)} & 0 \\ * & * & * & * & -Z_k \end{bmatrix} < 0 \tag{22}$$

and

$$R^{(k)} = \begin{bmatrix} R_{11}^{(k)} & R_{12}^{(k)} & R_{13}^{(k)} \\ * & R_{22}^{(k)} & R_{23}^{(k)} \\ * & * & R_{33}^{(k)} \end{bmatrix},$$

where

$$\begin{aligned} \hat{\Phi}_{11}^{(k)} &= N_1A_k + A_k^{\mathrm{T}}N_1^{\mathrm{T}} + T_1C_k + C_k^{\mathrm{T}}T_1^{\mathrm{T}} \\ &\quad +\tau R_{11}^{(k)} + R_{13}^{(k)} + R_{13}^{(k)T}, \\ \hat{\Phi}_{22}^{(k)} &= N_2A_{dk} + A_{dk}^{\mathrm{T}}N_2^{\mathrm{T}} + T_2C_{dk} + C_{dk}^{\mathrm{T}}T_2^{\mathrm{T}} \\ &\quad +\tau R_{22}^{(k)} - R_{23}^{(k)} - R_{23}^{(k)T}, \end{aligned}$$

and $\Phi_{ij}^{(k)}$ $(i = 1,2,3;\ j = 2,3,4)$ are defined in (13).

2.2 Delay-independent rate-dependent robust exponential stability

If we set the matrices Z_k and $R_{ij}^{(k)}$ $(i = j = 1,2,3)$ to zero, then we can obtain a delay-independent and rate-dependent exponential stability criterion for system (1) with polytopic-type uncertainties (3) and a time-varying delay satisfying (4) and (5). In this case, Theorem 1 becomes the following corollary.

Corollary 2 Given scalar $\mu < 1$ and system (1) with polytopic-type uncertainties (3) and a time-varying delay satisfying (5) is robust exponentially stable in mean square, if there exist symmetric positive-definite matrices P_k and Q_k and appropriately dimensioned matrices N_r and T_r $(r = 1,2,3,4)$ such that the following LMI holds for $k = 1,2,\cdots,q$:

$$\tilde{\Phi}^{(k)} = \begin{bmatrix} \tilde{\Phi}_{11}^{(k)} & \tilde{\Phi}_{12}^{(k)} & \tilde{\Phi}_{13}^{(k)} & \tilde{\Phi}_{14}^{(k)} \\ * & \tilde{\Phi}_{22}^{(k)} & \tilde{\Phi}_{23}^{(k)} & \tilde{\Phi}_{24}^{(k)} \\ * & * & \tilde{\Phi}_{33}^{(k)} & \tilde{\Phi}_{34}^{(k)} \\ * & * & * & \tilde{\Phi}_{44}^{(k)} \end{bmatrix} < 0, \tag{23}$$

where

$$\tilde{\Phi}_{11}^{(k)} = N_1A_k + A_k^{\mathrm{T}}N_1^{\mathrm{T}} + T_1C_k + C_k^{\mathrm{T}}T_1^{\mathrm{T}} + Q_k,$$
$$\tilde{\Phi}_{12}^{(k)} = N_1A_{kd} + A_k^{\mathrm{T}}N_2^{\mathrm{T}} + T_1C_{kd} + C_k^{\mathrm{T}}T_2^{\mathrm{T}},$$
$$\tilde{\Phi}_{13}^{(k)} = P_k - N_1 + A_k^{\mathrm{T}}N_3^{\mathrm{T}} + C_k^{\mathrm{T}}T_3^{\mathrm{T}},$$
$$\tilde{\Phi}_{14}^{(k)} = A_k^{\mathrm{T}}N_4^{\mathrm{T}} - T_1 + C_k^{\mathrm{T}}T_4^{\mathrm{T}},$$
$$\tilde{\Phi}_{22}^{(k)} = N_2A_{kd} + A_{kd}^{\mathrm{T}}N_2^{\mathrm{T}} + T_2C_{kd} - (1-\mu)Q_k + C_{kd}^{\mathrm{T}}T_2^{\mathrm{T}},$$
$$\tilde{\Phi}_{23}^{(k)} = -N_2 + A_{kd}^{\mathrm{T}}N_3^{\mathrm{T}} + C_{kd}^{\mathrm{T}}T_3^{\mathrm{T}},$$
$$\tilde{\Phi}_{24}^{(k)} = A_{kd}^{\mathrm{T}}N_4^{\mathrm{T}} - T_2 + C_{kd}^{\mathrm{T}}T_4^{\mathrm{T}},\quad \tilde{\Phi}_{33}^{(k)} = -N_3 - N_3^{\mathrm{T}},$$
$$\tilde{\Phi}_{34}^{(k)} = -N_4^{\mathrm{T}} - T_3,\quad \tilde{\Phi}_{44}^{(k)} = 2P_k - T_4^{\mathrm{T}} - T_4.$$

In addition, if there is no stochastic uncertainty in system (1), that is, $\beta(t)$ is assumed to be zero, system (1) degenerate into system [10]. We obtain the following corollary.

Corollary 3 Given scalars $\tau > 0$, and $\mu < 1$, system (1) with $\beta(t) = 0$ and with time-varying delay satisfying (4) and (5) is asymptotically stable, if there exist symmetric positive-definite matrix $P_k > 0, Q_k \geqslant 0$, and $R_{ij}^{(k)}$ $(i, j = 1, 2, 3)$ and appropriately dimensioned matrices N_r $(r = 1, 2, 3, 4)$ such that $R_{ij}^{(k)} = R_{ji}^{(k)}$ and the following LMIs hold for $k = 1, 2, \cdots, q$:

$$\Sigma^{(k)} = \begin{bmatrix} \Sigma_{11}^{(k)} & \Sigma_{12}^{(k)} & \Sigma_{13}^{(k)} \\ * & \Sigma_{22}^{(k)} & \Sigma_{23}^{(k)} \\ * & * & \Sigma_{33}^{(k)} \end{bmatrix} < 0 \tag{24}$$

and

$$R^{(k)} = \begin{bmatrix} R_{11}^{(k)} & R_{12}^{(k)} & R_{13}^{(k)} \\ * & R_{22}^{(k)} & R_{23}^{(k)} \\ * & * & R_{33}^{(k)} \end{bmatrix} > 0,$$

where

$$\Sigma_{11}^{(k)} = N_1A_k + A_k^{\mathrm{T}}N_1^{\mathrm{T}} + Q_k + \tau R_{11}^{(k)} + R_{13}^{(k)} + R_{13}^{(k)T},$$
$$\Sigma_{12}^{(k)} = N_1A_{dk} + A_k^{\mathrm{T}}N_2^{\mathrm{T}} + \tau R_{12}^{(k)} - R_{13}^{(k)} + R_{23}^{(k)T},$$
$$\Sigma_{13}^{(k)} = P_k - N_1 + A_k^{\mathrm{T}}N_3^{\mathrm{T}},$$
$$\Sigma_{22}^{(k)} = N_2A_{dk} + A_{dk}^{\mathrm{T}}N_2^{\mathrm{T}} - (1-\mu)Q_k + \tau R_{22}^{(k)} - R_{23}^{(k)} - R_{23}^{(k)T},$$
$$\Sigma_{23}^{(k)} = -N_2 + A_{dk}^{\mathrm{T}}N_3^{\mathrm{T}},\quad \Sigma_{33}^{(k)} = -N_3 - N_3^{\mathrm{T}} + \tau R_{33}^{(k)}.$$

Remark 2 Corollary 3 is equivalent to Theorem 2 [10]. Moreover, because the new Lyapunov-Krasovskii functional is different from that in [10], the results obtained from Corollary 3 are less conservative than existing ones [10].

Besides, if we assume $A_{dk} = C_{dk} = 0$, system (1) turns into a stochastic system without time delay, the following corollary can be acquired.

Corollary 4 Given scalars $\tau > 0$ and $\mu < 1$, system (1) with time-varying delay satisfying (4) and (5) and with $A_{dk} = C_{dk} = 0$ exponentially stable in mean square, if there exist symmetric positive-definite matrix $P_k > 0$, and appropriately dimensioned matrices N_r and T_r $(r = 1, 3, 4)$ such that the following LMI holds for $k = 1, 2, \cdots, q$:

$$\hat{\Sigma}^{(k)} = \begin{bmatrix} \hat{\Sigma}_{11}^{(k)} & \hat{\Sigma}_{13}^{(k)} & \hat{\Sigma}_{14}^{(k)} \\ * & \hat{\Sigma}_{33}^{(k)} & \hat{\Sigma}_{34}^{(k)} \\ * & * & \hat{\Sigma}_{44}^{(k)} \end{bmatrix} < 0, \tag{25}$$

where

$$\hat{\Sigma}_{11}^{(k)} = N_1A_k + A_k^{\mathrm{T}}N_1^{\mathrm{T}} + T_1C_k + C_k^{\mathrm{T}}T_1^{\mathrm{T}},$$
$$\hat{\Sigma}_{13}^{(k)} = P_k - N_1 + A_k^{\mathrm{T}}N_3^{\mathrm{T}} + C_k^{\mathrm{T}}T_3^{\mathrm{T}},$$
$$\hat{\Sigma}_{14}^{(k)} = A_k^{\mathrm{T}}N_4^{\mathrm{T}} - T_1 + C_k^{\mathrm{T}}T_4^{\mathrm{T}},\quad \hat{\Sigma}_{33}^{(k)} = -N_3 - N_3^{\mathrm{T}},$$
$$\hat{\Sigma}_{34}^{(k)} = -N_4^{\mathrm{T}} - T_3,\quad \hat{\Sigma}_{44}^{(k)} = P_k - T_4^{\mathrm{T}} - T_4.$$

Remark 3 When $\mu = 0$, the delay is time invariant. From Theorem 1 and Corollary 2, we can easily obtain the delay-dependent and delay-independent robust exponential stability criteria for continuous-time linear stochastic system with polytopic-type uncertainties and with time-invariant state delay, respectively.

3 Numerical simulation

In this section, for the purpose of illustrating the usefulness and flexibility of the methods in this paper, we present some simulation examples.

Example 1 Consider the following time-varying delay system Σ_1 with polytopic-type uncertainties ([10] Example 2), where

$$A = \begin{bmatrix} 0 & -0.12 + 12\rho \\ 1 & -0.465 - \rho \end{bmatrix},\ A_d = \begin{bmatrix} -0.1 & -0.35 \\ 0 & 0.3 \end{bmatrix}$$

and $\|\rho\| \leqslant 0.035$ [12]. Let $\rho_m = 0.035$ [10] and set

$$A_1 = \begin{bmatrix} 0 & -0.12 + 12\rho_m \\ 1 & -0.465 - \rho_m \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & -0.12 - 12\rho_m \\ 1 & -0.465 + \rho_m \end{bmatrix},$$
$$A_{1d} = A_{2d} = A_d = \begin{bmatrix} -0.1 & -0.35 \\ 0 & 0.3 \end{bmatrix}.$$

When $\mu = 0$, the upper bound on the time delay obtained in [10] is 0.863. However, by Corollary 3, the system Σ_1 is robustly stable for delay $\tau = 0.8758$, which is better than the values in [10]. Table 1 shows a comparison of the upper bounds for $\mu \neq 0$ obtained by Fridman and Shaked's method [13], He's method [10], and our methods (Corollary 3). It is clear that the upper bounds obtained by Corollary 3 are larger than those given in [13] and [10].

Table 1 Calculation results for Example 1.

μ	0	0.1	0.9	any μ
Fridman [13]	0.782	0.736	0.454	0.454
He [10]	0.863	0.786	0.454	0.454
Corollary 3	0.8758	0.8041	0.603	0.4548

Example 2 Consider the robust stability of the uncertain stochastic delay system Σ_2 with the following parameters:

$$A = \begin{bmatrix} -1 & 0.3\rho \\ 1 & -1 + 0.3\sigma \end{bmatrix},\ A_d = \begin{bmatrix} -1 & 0.3\rho \\ -0.6 & -1.5 + 0.3\sigma \end{bmatrix},$$
$$C = \begin{bmatrix} 0.1 & -0.1 + 0.3\rho \\ 0 & 0.1 + 0.3\sigma \end{bmatrix},\ C_d = \begin{bmatrix} -0.2 & 0.3\rho \\ 0 & 0.1 + 0.3\sigma \end{bmatrix},$$

and $|\rho| \leqslant 1, |\sigma| \leqslant 1$.

The parameter uncertainty can be represented by a four-vertex polytope and the upper bound of the time delay τ, which guarantees that the given system is exponentially stable in mean square, as given in Table 2. Set the initial conditions as $x(0) = [0.04, 0]^{\mathrm{T}}$; Figs.1 and 2 show the state response of polytopic model with four-vertex systems (each system has two states), as $\mu = 0.5$, $\tau_{\max} = 0.5598$, and μ is any value, $\tau_{\max} = 0.3893$, respectively.

Table 2 The upper bound of τ for Example 2, as a function of the bound μ.

μ	0	0.1	0.5	0.9	any μ
$\tau_{\max}$	0.7498	0.7093	0.5598	0.4223	0.3893

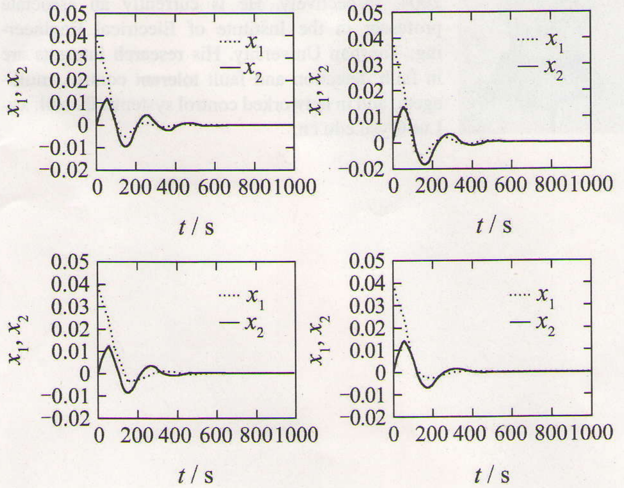

Fig. 1 The state response of system with 4 vertices (as $\mu = 0.5$, $\tau_{\max} = 0.5598$).

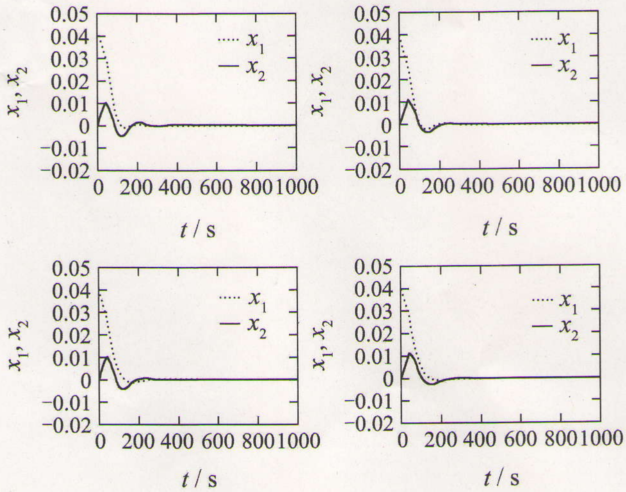

Fig. 2 The state response of system with 4 vertices (as μ is any value, $\tau_{\max} = 0.3893$).

4 Conclusions

This paper presents some new stability criteria for stochastic time-varying delay systems with polytopic-type uncertainties. Based on the equivalent descriptor stochastic system and some free weighing matrices, a new type of Lyapunov-Krasovskii functional is constructed, and new techniques are developed to make the criteria less conservative. Finally, numerical examples demonstrate that the criteria presented here perform much better than the existing stable one.

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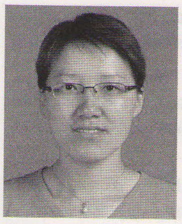

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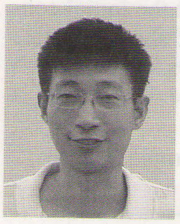

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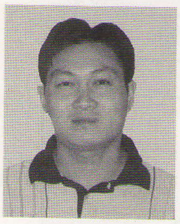

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