Working Paper Series, N. 19, December 2009

The Enhanced Denton's Benchmarking Procedure for Extrapolation

Matrix Formulation and Practical Issues

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This version (2009-12-31)

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1 Introduction

Benchmarking techniques are typically used in National Accounts (NA) to derive quarterly estimates of an annual aggregate. The benchmarking problem can be split into two parts: (i) *distribution* and (ii) *extrapolation*. In the former case the procedure is used to generate quarterly data which are both consistent with annual data (i.e. the sum of the quarters is equal to the annual level) and close to the movements of a quarterly preliminary series. Extrapolation refers to the calculation of quarterly forecasts of the target variable basically based on the movements of the quarterly preliminary series.

To avoid steps between contiguous years, benchmarking techniques based on some movement preservation principle are normally employed. A widely used solution of this type is given by Cholette (1984), based on the original proposal of Denton (1971). The procedure considers an objective function according to which the proportional period-to-period changes (or Proportional First Differences, PFD) of the benchmarked series be as close as possible to those of the preliminary figures. Hereafter, this procedure will be referred to as the Denton PFD benchmarking.

The Denton PFD technique is aimed at preserving at the best the movements of the preliminary series. When it comes to extrapolation, this procedure might not be fully satisfactory. The problem is clearly outlined in the International Monetary Fund (IMF) manual on Quarterly National Accounts (QNA) (Bloem *et al.*, 2001; see also Quenneville *et al.*, 2004 for applications of benchmarking techniques to extrapolation). To avoid possible biases and improve the estimates' quality of the most recent periods, the IMF proposes an 'enhanced version' of the Denton PFD soluton. Such enhancement corresponds to a forecasting mechanism of the annual *Benchmark-to-Indicator* (BI) ratio for the unknown year, derived by the user in accordance with statistical evidences (i.e. ARIMA forecasts) or economical considerations (i.e. correlation with the business cycle).

The criterion minimized by the enhanced Denton PFD procedure is suggested by Bloem *et al.* (2001). However, a matrix representation of the problem and the resulting analytical solution is lacking and not yet available in the literature. A shortcut version is offered in place of it, which turns out to give "similar results for less volatile series" (Bloem *et al.*, 2001, p. 93). In this paper we provide a matrix formulation of the enhanced Denton PFD solution, derive the analytical solution, and compare it to both the shortcut version and the classical Denton's solution. Our main interest is to evaluate whether the simplification of the shortcut version might distort the informative content of the preliminary series in extrapolation[1].

The paper is structured as follows. Section 2 presents the Denton PFD benchmarking solution. Section 3 provides the matrix formulation of the enhanced solution, whereas section 4 illustrates the approximate solution suggested in Bloem *et al.* (2001). A comparison of the different benchmarking techniques for extrapolation is presented in section 5 using a simple numerical example. Conclusions are drawn in the final section.

2 The Denton PFD benchmarking technique

Denote with $\mathbf{y}$ the $(Ns \times 1)$ vector of unknown "true" values to be estimated, and with $\mathbf{y}_0$ the $(N \times 1)$ vector of known aggregated values. The two vectors are linked by the linear temporal aggregation relationship

$$\mathbf{J}\mathbf{y} = \mathbf{y}_0, \tag{1}$$

where $\mathbf{J}$ is the $(N \times Ns)$ temporal aggregation matrix [2]

$$\mathbf{J} = \mathbf{I}_N \otimes \mathbf{1}_s',$$

[1] This work originates from the activity undertaken in the OECD-NBS of China project "Improving the quality of monthly/quarterly statistics of China" (Marini and Zollino, 2009). The NBS of China is going to produce (discrete) quarterly estimates of National Accounts aggregates through an indirect approach, choosing the shortcut version of the enhanced Denton PFD solution as the main benchmarking technique.

[2] For the sake of clarity, we only consider in this paper the case of temporal aggregation by sum.

s is the aggregation order (for example, $s = 4$ when quarterly figures are yearly aggregated). Let us define $\mathbf{p}$ the $(Ns \times 1)$ vector of preliminary values to be adjusted, for which

$$\mathbf{J}\mathbf{p} \neq \mathbf{y}_0.$$

Moving from the original proposal by Denton (1971), Cholette (1984) worked out a *modified Denton's Proportional First Differences (PFD) benchmarking procedure*, by solving a constrained quadratic minimization problem, where the sum of squared proportional differences between the benchmarked values and the preliminary values is minimized under the constraint that the benchmarked estimates be in line with the temporally aggregated counterparts.

Formally, the procedure minimizes the objective function

$$\sum_{t=2}^{Ns} \left(\frac{y_t}{p_t} - \frac{y_{t-1}}{p_{t-1}} \right)^2 \tag{2}$$

subject to the constraint (1).

Using matrix notation, the PFD criterion can be written as

$$(\mathbf{y} - \mathbf{p})' \mathbf{Q} (\mathbf{y} - \mathbf{p}) \tag{3}$$

with

$$\mathbf{Q} = \hat{\mathbf{p}}^{-1} \mathbf{D}' \mathbf{D} \hat{\mathbf{p}}^{-1},$$

where $\hat{\mathbf{p}} = diag(\mathbf{p})$, and $\mathbf{D}$ is the exact $(Ns - 1 \times Ns)$ first differences matrix

$$\mathbf{D} = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Di Fonzo (2003) provides the following solution of the system (1-2):

$$\begin{bmatrix} \mathbf{y} \\ \lambda \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{p}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{Ns} \end{bmatrix} \begin{bmatrix} \mathbf{D}'\mathbf{D} & \hat{\mathbf{p}}\mathbf{J}' \\ \mathbf{J}\hat{\mathbf{p}} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{y}_0 \end{bmatrix} \tag{4}$$

The benchmarking solution (4) can also be used for extrapolation. Suppose k additional preliminary observations are available for the most recent high-frequency periods (e.g. months or quarters), with $k = 1, \dots, s$. The length[3] of the preliminary vector $\mathbf{p}$ becomes $n = Ns + k$. To obtain the corresponding estimates of $\mathbf{y}$, it is only necessary to enlarge the aggregation matrix $\mathbf{J}$ by a sequence of k columns of zero:

$$\mathbf{J} = \begin{bmatrix} \mathbf{I}_N \otimes \mathbf{1}'_s & \mathbf{0}_{N \times k} \end{bmatrix}$$

[3] Obviously, the extrapolation can be done for periods longer than a year. Here it is convenient to limit our formulation to deal with the extrapolation of a single year.

and solve the system (4). This formulation implies that for the extrapolated periods the Benchmark-to-Indicator (BI)[4] ratios are kept constant and equal to the last available ratio y_{Ns}/p_{Ns} (Bloem *et al.*, 2001, p. 88), leading to an implicit forecast of the annual BI ratio. If the BI ratios can be modeled somehow, a better forecast can be made. For example if the annual growth rate of the preliminary series is unbiased as compared to the annual data, then, on average, the best forecast would be the previous annual BI ratio. On the contrary, if one knows that a bias is present in the annual growth rate of the indicator, the best forecast would be to use the previous year's BI ratio multiplied by the bias correction factor.

3 The enhanced version for extrapolation

The IMF has proposed a modification to the Denton PFD method for extrapolation (Bloem *et al.*, 2001) that permits to introduce an explicit forecast of the BI ratio. Using the same objective function (2) under constraint (1), and assuming $s = 4$, the following constraint is added to the behaviour of the extrapolated BI ratios:

$$\sum_{t=Ns+1}^{n} \frac{y_t}{p_t} w_{t-4} = b_{N+1} \qquad (5)$$

where

$$w_t = \frac{p_t}{p_{0,T}}, \qquad t = 1, \ldots, n, \qquad T = \left[\frac{t-1}{s}\right] + 1, \qquad (6)$$

where $[a]$ is the integer part of the real number a. The weight w_t is the share of each sub-period with respect to the relevant yearly aggregated preliminary series, and

$$b_{N+1} = \frac{y_{0N}}{p_{0N}} q,$$

where q is the forecasted change of the BI ratio for year $N + 1$.

In matrix notation, constraint (5) can be expressed as:

$$\left[\mathbf{R}\hat{\mathbf{p}}^{-1}\hat{\mathbf{p}}_E\hat{\mathbf{p}}_{0E}^{-1}\right]\mathbf{y} = b_{N+1}, \qquad (7)$$

[4] Bloem *et al.* (2001) denote the preliminary data as the indicator. We maintain this taxonomy for practical reasons. In our opinion the concept of "indicator" is better suited in a temporal disaggregation framework, where a statistical/econometric relationship is established between the *related series* (i.e., the indicator) and the variable to be estimated.

with

$$\begin{aligned}
\mathbf{R} &= \begin{bmatrix} 0'_{Ns} & 1'_k \end{bmatrix} \\
\hat{\mathbf{p}}^{-1} &= diag(\mathbf{p}^{-1}) = \begin{bmatrix} p_1^{-1} & 0 & \dots & 0 \\ 0 & p_2^{-1} & \dots & 0 \\ 0 & 0 & \dots & p_n^{-1} \end{bmatrix} \\
\hat{\mathbf{p}}_E &= diag(\mathbf{E}\mathbf{p}) \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & p_{n-s+1} & 0 \\ 0 & 0 & \dots & 0 & p_{n-s} \end{bmatrix} \\
\mathbf{E} &= \begin{bmatrix} \mathbf{0}_{s\times n-s} & \mathbf{0}_{s\times s} \\ \mathbf{I}_{n-s} & \mathbf{0}_{n-s\times s} \end{bmatrix} \\
\hat{\mathbf{p}}_{0E}^{-1} &= diag(\mathbf{E}\mathbf{p}_0^{*-1}) \\
\mathbf{p}_0^* &= \begin{bmatrix} \mathbf{p}_0 \otimes 1_s \\ 1_k. \end{bmatrix}.
\end{aligned}$$

Denoting $\mathbf{k} = \left[\mathbf{R}\hat{\mathbf{p}}^{-1}\hat{\mathbf{p}}_E\hat{\mathbf{p}}_{0E}^{-1}\right]$ the n-dimensional row vector in (7), the constraints for the enhanced version of Denton PFD for extrapolation can be expressed as:

$$\begin{bmatrix} \mathbf{J} \\ \mathbf{k} \end{bmatrix} \mathbf{y} = \begin{bmatrix} \mathbf{y}_0 \\ b_{N+1} \end{bmatrix},$$

that is $\mathbf{J}^*\mathbf{y} = \mathbf{y}_0^*$. Both the benchmarked and extrapolated values are thus given by expression (4), replacing $\mathbf{J}$ with $\mathbf{J}^*$ and $\mathbf{y}_0$ with $\mathbf{y}_0^*$, respectively.

4 An approximation of the enhanced solution

A shortcut version of the enhanced Denton PFD method is presented in the IMF manual on QNA for illustrative purposes (Bloem *et al.*, 2001, 6.35). Let q_t be the BI ratio from the original Denton method

$$q_t = \frac{y_t}{p_t}, \qquad t = 1, \dots, Ns + k.$$

The enhanced method explicitly requires a forecast of the annual BI ratio for year $N + 1$, denoted as b_{N+1} in the previous section. To avoid the step problem, it is necessary that the transition from the fourth quarter of the last year to the first quarter of the current year be as smooth as possible. Denote with η the quantity

$$\eta = \frac{1}{3}\left(q_{Ns} - b_{N+1}\right) \tag{8}$$

which will ensure, as shown later, that the average of the extrapolated quarterly BI ratios be approximately equal to the annual forecasted BI ratio b_{N+1}.

This quantity is used to adjust the extrapolated quarters of the Denton benchmarking solution. The values of the quarters of the last available year are modified firstly, starting from the second quarter:

$$\begin{aligned} q^*_{Ns-2} &= q_{Ns-2} + \frac{1}{4}\eta \\ q^*_{Ns-1} &= q_{Ns-1} + \frac{1}{4}\eta \\ q^*_{Ns} &= q_{Ns} - \frac{1}{2}\eta. \end{aligned}$$

Then, the following recursion is used to calculate the BI ratios for the extrapolated quarters:

$$q^*_{Ns+k} = q^*_{Ns+k-1} - \eta \qquad k = 1, \ldots, 4.$$

To understand the properties of this approximation, it is useful to aggregate the quarterly BI ratios at the annual level. Two years are involved, the last available one (N) and the extrapolated one ($N+1$). The annual average of the modified BI ratios for year N is given by:

$$\frac{1}{4}\sum_{k=0}^{3} q^*_{Ns-k}.$$

Replacing each term of the sum with the original BI ratios we have

$$\frac{1}{4}\left[q_{Ns-3} + q_{Ns-2} + \frac{1}{4}\eta + q_{Ns-1} + \frac{1}{4}\eta + q_{Ns} - \frac{1}{2}\eta\right]$$

that is

$$\frac{1}{4}\sum_{k=0}^{3} q_{Ns-k}.$$

Then, the original annual BI ratio is preserved; this implies that for year N the sum of the quarterly benchmarked series is equal to the annual (observed) value. Now, consider the annual average of the extrapolated BI ratios

$$\frac{1}{4}\sum_{k=0}^{3} q^*_{Ns+k}.$$

It can be transformed into

$$\begin{aligned} \frac{1}{4} \quad [\quad & q_{Ns} - \frac{1}{2}\eta - \eta + \\ & q_{Ns} - \frac{1}{2}\eta - 2\eta + \\ & q_{Ns} - \frac{1}{2}\eta - 3\eta + \\ & q_{Ns} - \frac{1}{2}\eta - 4\eta] \end{aligned}$$

that corresponds to

$$\frac{1}{4}\left[4q_{Ns} - 12\eta\right].$$

Replacing η according to expression (8) we have:

$$\frac{1}{4}\left[4q_{Ns} - 4(q_{Ns} - b_{N+1})\right] = b_{N+1}.$$

The shortcut version guarantees therefore that the (implicit) annual extrapolated BI ratio is exactly the one imposed by the user. Albeit this characteristic is preserved, the shortcut version provides different results in terms of quarter-to-quarter movements with respect to the analytical solution. Such differences are investigated in the following section through a numerical example with artificial data.

5 An example with artificial data

We consider the data in the examples 6.2-6.4 of the IMF manual on QNA. Table 1 shows the indicator series in the first column for the period 1998Q1-2000Q4. The second column contains the annual target series for years 1998 and 1999. The annual BI ratios are shown in the third column. The quarterly BI ratios from the Denton benchmarking solution (PFD variant) are given in the fifth column, followed by the benchmarked series. According to the modified Denton PFD solution (Cholette, 1984), the extrapolated quarters of year 2000 are obtained by multiplying the values of the indicator series by the last available BI ratio (10.355 of quarter 1999Q4).

The enhanced Denton PFD requires an explicit forecast of the annual BI ratio of year 2000. It is assumed that this ratio increases of 2.0% over the previous year. The BI ratio of year 2000 is therefore 10.486 ($10.280\times$ 1.02). According to (8), the constant η is equal to

$$\eta = \frac{1}{3}(10.355 - 10.486) = -0.044.$$

Using the approximation illustrated in section 4, η is used to obtain the enhanced quarterly BI ratios from 1999Q2 to 2000Q4. The corresponding benchmarked values are given in the last column of table 1.

Table 2 compares the quarter-to-quarter growth rates of the benchmarked series of the original Denton PFD and the enhanced version. The last row shows the annual rate from the four extrapolated quarters. The original Denton PFD yields an annual rate of 1.6%, while the shortcut version of the enhanced method gives 2.9%. Compared to the growth of 0.9% in the indicator series, both methods provide an upward extrapolation of the quarterly benchmarked series. In the first case, the correction factor is implicitly given by the use of the BI ratio of 1999Q4. In the latter case, the annual BI ratio is explicitly defined. In fact, the differences between the rates of change of the indicator and the enhanced Denton PFD is exactly 2.0%, the same increase of the BI ratio assumed for 2000.

Table 1: Extrapolation using forecast BI ratios (Example 6.2, Bloem *et al.*, 2001)

Date	Indicator	Annual data	Annual BI ratios	Original BI ratios	Original Modified Denton PFD	Enhanced BI ratios (shortcut)	Enhanced Denton PFD (shortcut)
1998Q1	98.2			9.876	969.8	9.876	969.8
1998Q2	100.8			9.905	998.4	9.905	998.4
1998Q3	102.2			9.964	1018.3	9.964	1018.3
1998Q4	100.8	4000.0	9.950	10.054	1013.4	10.054	1013.4
1999Q1	99.0			10.174	1007.2	10.174	1007.2
1999Q2	101.6			10.264	1042.8	*10.253*	*1041.7*
1999Q3	102.7			10.325	1060.3	*10.314*	*1059.2*
1999Q4	101.5	4161.4	10.280	10.355	1051.0	*10.377*	*1053.2*
2000Q1	100.5			10.355	1040.6	*10.420*	*1047.2*
2000Q2	103.0			10.355	1066.5	*10.464*	*1077.8*
2000Q3	103.5			10.355	1071.7	*10.508*	*1087.5*
2000Q4	101.5		*10.486*	10.355	1051.0	*10.551*	*1071.0*

Table 2: Quarter-to-quarter growth rates and annual rate for year 2000

Date	Indicator	Original Modified Denton PFD	Enhanced Denton PFD (shortcut)
1998Q1			
1998Q2	2.6	3.0	3.0
1998Q3	1.4	2.0	2.0
1998Q4	-1.4	-0.5	-0.5
1999Q1	-1.8	-0.6	-0.6
1999Q2	2.6	3.5	3.4
1999Q3	1.1	1.7	1.7
1999Q4	-1.2	-0.9	-0.6
2000Q1	-1.0	-1.0	-0.6
2000Q2	2.5	2.5	2.9
2000Q3	0.5	0.5	0.9
2000Q4	-1.9	-1.9	-1.5
2000	**0.9**	**1.6**	**2.9**

So far, we have considered the shortcut version of the enhanced method. Table 3 shows the benchmarked series derived from the analytical solution, formalized in section 3 of this work. Differently from the shortcut version, the analytical solution differs from the original Denton PFD in each quarter. The quarterly growth rates change consequently, with a maximum discrepancy of -0.3% in 2000Q4. As expected, the annual rate of change of 2000 of the two versions is the same (2.9%).

It is interesting to compare the extrapolated rates of change with those of the indicator (see table 4). The last two columns of the table show such differences. Concerning the quarters of 2000, the shortcut version provides forecasts that are 0.4% higher than the indicator's rates of change. The analytical solution shows a higher distance in the first quarter (0.5%), while differences are reduced in the other quarters (0.4%, 0.2% and 0.1%).

Table 3: Enhanced Denton PFD: comparison between shortcut and analytical solutions

Date	Enhanced BI ratios shortcut	Enhanced Denton PFD shortcut	Enhanced BI ratios analytical	Enhanced Denton PFD analytical	Quarter to quarter changes Enhanced Denton PFD shortcut	Quarter to quarter changes Enhanced Denton PFD analytical
1998Q1	9.876	969.8	9.883	970.5		
1998Q2	9.905	998.4	9.909	998.9	3.0	2.9
1998Q3	9.964	1018.3	9.963	1018.2	2.0	1.9
1998Q4	10.054	1013.4	10.045	1012.5	-0.5	-0.6
1999Q1	10.174	1007.2	10.153	1005.1	-0.6	-0.7
1999Q2	10.253	1041.7	10.247	*1041.1*	3.4	3.6
1999Q3	10.314	1059.2	10.326	*1060.5*	1.7	1.9
1999Q4	10.377	1053.2	10.391	*1054.7*	-0.6	-0.5
2000Q1	10.420	1047.2	10.441	*1049.3*	-0.6	-0.5
2000Q2	10.464	1077.8	10.479	*1079.3*	2.9	2.9
2000Q3	10.508	1087.5	10.504	*1087.2*	0.9	0.7
2000Q4	10.551	1071.0	10.517	*1067.5*	-1.5	-1.8
2000	**10.486**	**1070.9**	**10.485**	**1070.8**	**2.9**	**2.9**

Table 4: Enhanced Denton PFD: comparison with indicator series

Date	Quarter to quarter changes Indicator	Quarter to quarter changes Enhanced Denton PFD shortcut	Quarter to quarter changes Enhanced Denton PFD analytical	Differences with indicator Enhanced Denton PFD shortcut	Differences with indicator Enhanced Denton PFD analytical
1998Q1					
1998Q2	2.6	3.0	2.9	0.3	0.3
1998Q3	1.4	2.0	1.9	0.6	0.5
1998Q4	-1.4	-0.5	-0.6	0.9	0.8
1999Q1	-1.8	-0.6	-0.7	1.2	1.1
1999Q2	2.6	3.4	3.6	0.8	1.0
1999Q3	1.1	1.7	1.9	0.6	0.8
1999Q4	-1.2	-0.6	-0.5	0.6	0.6
2000Q1	-1.0	-0.6	-0.5	0.4	0.5
2000Q2	2.5	2.9	2.9	0.4	0.4
2000Q3	0.5	0.9	0.7	0.4	0.2
2000Q4	-1.9	-1.5	-1.8	0.4	0.1
2000	**0.9**	**2.9**	**2.9**	**2.0**	**2.0**

To summarize such differences, table 5 shows the Mean Squared Differences (MSD) of the three benchmarking procedures under review with respect to the indicator series. The MSD is calculated on the whole sample and on the extrapolated quarters only (2000Q1-2000Q4). The smallest MSD is achieved by the original Denton PFD (0.5946%). Concerning the enhanced methods, the analytical solution is slightly closer to the movements of the preliminary series (0.6392% *vs.* 0.6523% of the shortcut version). The improvement of the analytical solution is even higher if only the differences of the extrapolated quarters are considered: 0.3312% *vs.* 0.4184%. Notice that in this case the MSD of the original Denton PFD is zero, that

Table 5: Enhanced Denton PFD: MSD of growth rates

	Original Modified Denton PFD	Enhanced Denton PFD shortcut	Enhanced Denton PFD analytical
MSD total	0.5946	0.6523	0.6392
MSD year 2000	0.0000	0.4184	0.3312

is the quarterly growth rates of the benchmarked series is exactly the same as the indicator series.

The comparison done in the previous tables might be somehow affected by the particular BI ratio used. Then, we made a simulation exercise by varying the value of b_{2000} in the range (0.94, 1.06) (from -6% to +6%), with a step of 0.02 (+2%). The same indicator series is used in each scenario. Table 6 displays the following statistics:

- the PFD criteria, as defined into (2);
- the MSD with respect to the growth rates of the indicator series.

Both statistics are calculated for the whole period and the extrapolated quarters only. Finally, the last column shows the annual rates of change of 2000. The original Denton PFD method is shown in the last row of the table for comparison.

From this exercise we see that:

- the analytical solution always outperforms the shortcut version in terms of closeness to the movements of the indicator series. Improvements are higher as the change of the extrapolated BI ratio is farther from 1.0 (no change);
- the annual extrapolated rate of change of the two versions of the enhanced method is the same (except in two cases for rounding errors, when b_{2000} is equal to 0.94 and 1.04);
- the original Denton PFD is the method that preserves "at the best" the movements of the indicator, also in extrapolation.

6 Conclusion

With the original Denton PFD, the implicit annual forecast based on the last quarterly BI ratio can introduce a bias in the benchmarked series. The advantage of the enhanced benchmarking method proposed by the IMF is that the user can explicitly define the annual extrapolated rate of change of the BI ratio. Despite the highly subjective nature of this extrapolation method, it certainly gives more flexibility to national accounts' compilers.

The analytical solution of the enhanced Denton PFD is formalized in this paper and can be easily implemented in any computing software. The shortcut version is thus unnecessary and might be even dangerous. Despite it guarantees the desired

Table 6: Enhanced Denton PFD: comparison of shortcut and analytical solutions with different BI ratios

b_{2000}	Benchmarking method	PFD total	PFD_{2000}	MSA total	MSA_{2000}	Δ_{2000}
0.94	EPFD - Analytical solution	0.222	0.132	1.413	1.823	-5.1
	EPFD - Shortcut version	0.285	0.212	1.618	2.332	-5.2
0.96	EPFD - Analytical solution	0.130	0.065	1.074	1.267	-3.1
	EPFD - Shortcut version	0.161	0.105	1.203	1.615	-3.1
0.98	EPFD - Analytical solution	0.070	0.022	0.785	0.723	-1.1
	EPFD - Shortcut version	0.080	0.035	0.843	0.919	-1.1
1.00	EPFD - Analytical solution	0.042	0.002	0.610	0.191	0.9
	EPFD - Shortcut version	0.042	0.003	0.616	0.241	0.9
1.02	EPFD - Analytical solution	0.046	0.005	0.639	0.331	2.9
	EPFD - Shortcut version	0.048	0.008	0.652	0.418	2.9
1.04	EPFD - Analytical solution	0.083	0.031	0.845	0.842	4.9
	EPFD - Shortcut version	0.098	0.050	0.909	1.061	5.0
1.06	EPFD - Analytical solution	0.152	0.081	1.129	1.344	7.0
	EPFD - Shortcut version	0.191	0.131	1.248	1.688	7.0
	Original Denton PFD	0.040	0.000	0.595	0.000	1.6

forecast of the current year, the simplified recursion might distort the quarter-to-quarter movements of the indicator series. This might hamper the analysis of business cycle, mainly focused on QNA estimates. According to some experiments, we have found that the analytical solution preserves much better the movements of the indicator series (even though worse than the original Denton PFD method). For these reasons, when Denton's PFD benchmarking is used to extrapolate quarterly figure, the analytical solution should be preferred to the shortcut version.

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Acknowledgements

Many thanks are due to Lu Feng (NBS of China), Jiemin Guo and Paul Schreier (OECD), and Francesco Zollino (Bank of Italy). We alone are responsible for the errors of the paper. The views contained here are those of the authors and do not necessarily reflect those of the institutions for which they work. Financing from MIUR is also gratefully acknowledged by T. Di Fonzo.

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