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Market potential dynamics in innovation diffusion: modelling the synergy between two driving forces

Renato Guseo
Department of Statistical Sciences
University of Padua
Italy

Mariangela Guidolin
Department of Statistical Sciences
University of Padua
Italy

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Keywords: Co–evolutionary diffusion process, Dual–effect market, Communication network, Slowdown, Saddle, Chasm, New drugs, Likelihood ratio order.

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Department of Statistical Sciences
Via Cesare Battisti, 241
35121 Padova
Italy

tel: +39 049 8274168
fax: +39 049 8274170
http://www.stat.unipd.it

Corresponding author:
Renato Guseo
tel: +39 049 827 4146
renato.guseo@unipd.it
http://www.stat.unipd.it/~guseo

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1 Introduction

Some recent literature on innovation diffusion has followed the idea that the market for new products needs to be divided into two major segments, usually termed "visionaries and pragmatists" (see Moore (1991)), "early market and main market" (see Goldenberg et al. (2002), Muller and Yogev (2006), Karmeshu and Goswami (2001), Vakratsas and Kolsarici (2008)), "influentials and imitators" (see Van den

Bulte and Joshi (2007)).

In particular Moore, building on the well–known categorization of adopters proposed by Rogers (2003), suggested that the market for innovations is initially just represented by early adopters and that the main market develops in a second stage of diffusion. Early and main markets are different in their attitudes and expectations towards novelties, and this difference may result in a precise separation between the two, implying a different treatment in terms of marketing strategies (see Mahajan and Muller (1998)). Such separation has been theorized as a possible explanation for the slowdown pattern –also known with minor differences as chasm, saddle or dip– that many diffusion processes show when, after a rapid takeoff, a product's sales reach an initial peak followed by a decline –whose length and depth may vary– and eventually by a resumption that may exceed the initial peak. The difference between early market and main market may be so deep and the slowdown so intense as to provoke product failure. Grounding on Moore's intuition, Goldenberg et al. (2002) have suggested that the existence of a saddle may be seen as a dual–market symptom. Their analysis has been based, first, on two exploratory studies on artificial markets realized with Cellular Automata models in order to verify the frequency of the saddle phenomenon in simulated situations, and then on an aggregate model –the details of which are not provided in the paper– to tie the dual–market explanation to saddle phenomena emerging in real situations. From the reported values of the determination index, R^2, which range from 79% to 99.1% with an average of 92.1%, we may argue that the performance of this aggregate model is quite unsatisfactory. Moreover, the parameters' estimates are not presented, which makes it difficult to understand the theoretical conclusions.

In the spirit of the work by Goldenberg et al. (2002), Muller and Yogev (2006) have developed a dual–market diffusion model, in which the dynamics of the early market are expressed in Equation (1)

$$\frac{dI(t)}{dt} = \left(p_i + q_i \frac{I(t)}{N_i}\right)(N_i - I(t)). \tag{1}$$

As one may observe the early market's cumulative adoptions, $I(t)$, are described through a simple Bass model, where parameters p_i and q_i have the usual meaning and N_i is the market potential of the early market. Instead, cumulative adoptions of the main market, $M(t)$, present a more complex structure,

$$\frac{dM(t)}{dt} = \left(p_m + q_m \frac{M(t)}{N_i + N_m} + q_{im} \frac{I(t)}{N_i + N_m}\right)(N_m - M(t)). \tag{2}$$

Equation (2) proposes a bipartition of the word-of-mouth effect, which would be partly due to communication among the main market's individuals, q_m, and partly to cross-market communications between the early and the main markets, q_{im}.

Although the authors do not provide much detail about the statistical implementation of the model, they recognize the difficulty in working with a model of seven parameters and thus the need to reduce it to five by assuming $p_m = 0$ and estimating N_m (the market potential of the main market) "from external sources". Whereas the external estimation of N_m represents a weakness of the model, we observe the

interesting assumption that each segment, namely early and main market, has its specific market potential, and the growth of the main market includes the early market potential; in other words, it implicitly admits the possibility of a market potential that changes over time.

Karmeshu and Goswami (2001) have introduced a different methodology in order to take into account heterogeneity of agents in a standard *normalized* Bass model

$$\frac{dX(t)}{dt} = \alpha(1 - X) + \beta X(1 - X), \qquad X(0) = X_0 \tag{3}$$

by modifying its basic structure via a general assumption about the stochastic nature of α and β parameters. The solution process corresponding to model (3) is the usual one conditionally on α, β and X_0. The authors study particular moments of the previous process describing a general joint mixing distribution $\phi(\alpha, \beta)$ by the means of the so–called "two–point–distribution" (TPD) formalism allowing an approximate representation, through six parameters μ_i, σ_i, ν_i, $(i = \alpha, \beta)$, i.e., local means, standard deviations and skewness. This is an innovative approach which allows a formal definition of the dynamic mean value $\mathcal{M}(t)$ of the cumulative adoption process as a linear combination of four Bass standard cumulative distributions. Studying the variation of $\frac{d\mathcal{M}(t)}{dt}$ with reference to time t for various choices of parameters, it is possible to describe unimodal and bimodal life cycles where the latter is obtained for increasing values of standard deviations σ_α and σ_β. This decomposition allows a flexible description of diffusion evolution, but not a clear and interpretable origin of the components dominating over time.

Following a different path, another work that has recently dealt with the existence of a dual market and the slowdown in diffusion arguably related to it is that of Van den Bulte and Joshi (2007). The authors have developed a two-segment mixture model, to account for the presence of two distinct segments, namely influentials and imitators, whose adoption behaviour is captured by the following hazard functions,

$$h_1(t) = p_1 + q_1 F_1(t), \tag{4}$$

$$h_2(t) = p_2 + q_2[wF_1(t) + (1 - w)F_2(t)]. \tag{5}$$

Consistently with the influentials–imitators hypothesis, Equations (4) and (5) show an asymmetry; in fact, type 1 may influence type 2, but the reverse cannot occur. The overall adoption process is the weighted sum of the adoption of the two segments, under the assumption that these may not have the same importance, i.e.

$$F_m(t) = \vartheta F_1(t) + (1 - \vartheta)F_2(t), \tag{6}$$

where $F_1(t)$ and $F_2(t)$ are probability distribution functions. Similarly, the weighted sum of the corresponding densities yields

$$f_m(t) = \vartheta f_1(t) + (1 - \vartheta)f_2(t). \tag{7}$$

The so–called Asymmetric Influence Model (AIM) by Van den Bulte and Joshi is defined by calculating closed-form solutions of $F_1(t)$ and $F_2(t)$. The solution of $F_1(t)$ is that of the standard Bass model, while $F_2(t)$ presents a much more complex

structure, referable to a Riccati equation. Though we do not report the details of such a solution, the impressive mathematical effort of the overall construction is noteworthy indeed. At the same time we must notice that the closed–form solution depends on the Gaussian hypergeometric function

$$ {}_2F_1(1;b;c;k) = \sum_{n=0}^{\infty} \frac{\Gamma(b+n)\Gamma(c)k^n}{\Gamma(b)\Gamma(c+n)} \tag{8} $$

which is convergent for arbitrary b, c if $|k| < 1$ and for $c > 1 + b$ if $k \pm 1$. As the authors themselves recognize, the estimation of function (6) is very troublesome, since function (8) is a series with infinite terms which may converge slowly. Abandoning the "closed-form" solution of F_2 they have proposed a numerical solution of Equation (9)

$$ \frac{dX(t)}{dt} = M[\vartheta f_1(t) + (1-\vartheta) f_2(t)] + \varepsilon(t). \tag{9} $$

Van den Bulte and Joshi's model proposes a mixture of two sub–populations of adopters as a possible explanation of the chasm (or dip) exhibited by several diffusion processes: specifically such a pattern appears when considering Equation (7), i.e., the weighted sum of two densities.

Another model recently proposed by Guseo and Guidolin (2009) shows that a slowdown in diffusion emerges as a consequence of the sum of "two densities" i.e., the existence of a "dual-effect" in market evolution; however, the approach adopted in this model radically differs from that of Van den Bulte and Joshi (2007), since such a duality does not come from a separation into segments of adopters, but rather the interpretation of diffusion as composed of two distinct, yet co–evolving processes: communication and adoption. In particular, communication dynamics are seen as determinants of the market potential, whose structure is not fixed, but generated through time as a function of the spread of knowledge about an innovation. As we have noticed, the work of Muller and Yogev (2006) seems to admit a variable market potential too, assuming the existence of two sub–populations of adopters with their own potential: in Guseo and Guidolin (2009) there is not an arbitrary division into two major segments and the structure of the market potential does not depend on an assumed heterogeneity of individuals, but rather on the aggregate process of communication that leads to the formation of a social acceptance of an innovation, or vice versa. The "dual-effect" modelling allows the recognition of a more interesting aspect than the pure determination of a slowdown, expressing a dynamic ranking between the two co–evolving processes with different managerial implications. In some situations, probably the majority, the communication component is dominant, while in others we empirically identify an opposite behaviour, where the adoption process has a driving role.

The paper is organized as follows. In Section 2 we present some definitions and basic properties of the Guseo and Guidolin (2009) co–evolutionary model with a particular specification of a dynamic market potential. In Section 3 we consider six different pharmaceutical drug diffusions in Italian geographic areas that exhibit slowdowns and saddle effects well–recognized by previous model. In Section 4 we

propose a natural decomposition of model density which allows a simple interpretation of the drivers in evolution due to main effects exerted by communication and adoption forces. The *Likelihood ratio order* explains the different time position of these forces. In Section 5 a further *weak ordering*, based on simple location indexes, is proposed and compared with the likelihood ratio order or the *usual stochastic order*, confirming a direct managerial usefulness of discovering the driving role between adoption and communication. Final comments and discussion are presented in Section 6.

2 Co–evolution of Market Potential and Diffusion of an Innovation

We have highlighted in Section 1 that some interesting research effort was done in the last decade to represent heterogeneity of agents, by specifying mixture models that take into account different local structures of market responses, due to an assumed latent decomposition of adopters in sub–populations over time (see, for instance, Van den Bulte and Joshi (2007), Karmeshu and Goswami (2001), Muller and Yogev (2006), Goldenberg et al. (2002) among others). A recurring assumption of previous mixtures or mixed models is the existence of different local market potentials.
In Guseo and Guidolin (2009) this discrete, probably unnecessary, taxonomy has been overcome through a special Cellular Automaton description whose aggregate mean–field approximation, in continuous time, is expressed by the following Equation

$$y'(t) = m(t)\left\{-r_s\frac{y(t)}{m(t)} + \left(p_s + q_s\frac{y(t)}{m(t)}\right)\left(1 - \frac{y(t)}{m(t)}\right)\right\} + y(t)\frac{m'(t)}{m(t)}, \qquad (10)$$

where $y'(t)$ represents instantaneous adoptions at time t, $y(t)$ denotes the corresponding cumulative adoptions, p_s and q_s are the usual Bass like parameters depicting innovation (external) and imitation (internal) effects, and r_s accounts for a possible decay effect due to not retained adoptions. In this model particular attention is devoted to a general representation of the market potential via a non–negative flexible function $m(t) \geq 0$. We highlight that it is not a function of a special family.

A characteristic claim in Equation (10), the "self–reinforcing" term, $y(t)\frac{m'(t)}{m(t)}$, emphasizes the instantaneous variations in $y'(t)$ due to a collective or inertial movement of global market potential. An expanding $m(t)$ induces a benefit in instantaneous adoptions and, vice versa, a declining $m(t)$ implies a shrinkage. Under a constant market potential, $m(t) = m$, the self–reinforcing effect is absent.

An extension of Equation (10) is based on the modification over time of uniform dynamics due to *exogenous intervention* effects (source of external heterogeneity) during the diffusion process. A similar approach is developed by Bass et al. (1994) in the Generalized Bass Model (GBM) under an assumed fixed potential m.

This more flexible context is modelled in Guseo and Guidolin (2009) through a multiplicative intervention function, $x(t)$, whose neutral level is $x(t) = 1\,\forall t$, which may incorporate exogenous factors, like marketing mix strategies, different political

regulations or incentive measures,

$$y'(t) = m(t)\left\{-r_s\frac{y(t)}{m(t)} + \left(p_s + q_s\frac{y(t)}{m(t)}\right)\left(1 - \frac{y(t)}{m(t)}\right)\right\}x(t) + y(t)\frac{m'(t)}{m(t)}. \quad (11)$$

Notice that function $x(t)$ only exerts its effect on the first component of Equation (11),which is a function of the future, and not on the self–reinforcing term, which depends on the past.

The original GBM is a particular sub–model in Equation (11), because two special constraints apply: the decay parameter is excluded, $r_s = 0$, and the market potential is constant, $m(t) = m$.

Equation (11) defines a nested co–evolutionary model as a special non–autonomous Riccati equation. Its closed form solution is determined on the basis of Equation (25) (see Appendix A), with an initial condition $C = 0$, for $g(\cdot) = m(\cdot)$ and $f(\cdot) = x(\cdot)$

$$y(t) = m(t)\frac{1 - e^{-D_s\int_0^t x(\tau)d\tau}}{\frac{1}{{}_sr_2} - \frac{1}{{}_sr_1}e^{-D_s\int_0^t x(\tau)d\tau}}, \quad D_s = \sqrt{(q_s - p_s - r_s)^2 + 4q_sp_s} > 0, \quad (12)$$

where ${}_sr_i = (-(q_s - p_s - r_s) \pm D_s)/(-2q_s)$, $i = 1, 2$, with ${}_sr_2 > {}_sr_1$.

The obtained solution highlights the natural multiplicative role of the market potential $m(t)$, a very simple extension of the corresponding GBM based on the substitution of m with $m(t)$. The time dependent market potential $m(t)$ penalizes or magnifies with different emphases the evolution of the natural purchase process. Notice that the $m(t)$ flexible shape may be modelled according to different perspectives.

In Guseo and Guidolin (2009) we can find a special proposal based on a formal description of knowledge dynamics regarding a specific innovation, and interpreted as a growing network constituting a basic precursor to market potential. Communication dynamics are represented by a Network Automata model whose mean–field approximation, in continuous time, is proportional to an autonomous Riccati equation, namely,

$$\nu'(t) = -(q_c + w_c)\nu^2(t) + (q_c - p_c - e_c)\nu(t) + p_c, \qquad q_c > p_c > 0, \quad (13)$$

where p_c denotes the external or innovative component of the communication process while q_c and w_c represent positive and negative word–of–mouth effects and e_c is a decay effect representing the natural loss of information due to ageing. If we exclude e_c and w_c components, we obtain a standard Bass model referring to network growth. This is a technical way to express knowledge dynamics or increasing awareness about an innovation translating a qualitative rationale –*absorptive capacity*– due to Cohen and Levinthal (1990).

Equation (13) may be solved by recognizing in it a special version of Equation (25) (see Appendix A). For initial conditions $\nu(0) = 0$, $f(\cdot) = 1$ and $g(\cdot) = 1$, we obtain

$$\nu(t) = \frac{1 - e^{-D_ct}}{\frac{1}{{}_cr_2} - \frac{1}{{}_cr_1}e^{-D_ct}}, \quad D_c = \sqrt{(q_c - p_c - e_c)^2 + 4(q_c + w_c)p_c} > 0, \quad (14)$$

where ${}_cr_i = (-(q_c - p_c - e_c) \pm D_c)/(-2(q_c + w_c)),\ i = 1, 2$, with ${}_cr_2 > {}_cr_1$. If, for instance, $e_c > 0$ then the limit of $\nu(t)$ for $t \to +\infty$ may be less than 1.

2.1 Market potential definition

Function $U\nu(t)$ defines an aggregate temporal evolution of the knowledge about an innovation within the proposed communication network. This knowledge may be transformed in a dynamic market potential in order to define a potential boundary to the nested adoption process. This potential boundary is a latent structure that we cannot measure directly.

Without loss of generality, we can consider the positive squared root of $U\nu(t)$, i.e., $h(t) = \sqrt{U}\sqrt{\nu(t)}$, that depicts the number of *informed individuals*. Notice that $h(t)$ is proportional to $\sqrt{\nu(t)}$, so that we may assume

$$m(t) = K\sqrt{\nu(t)} \tag{15}$$

as the actual market potential, where K is a free parameter useful for repeated adoptions.

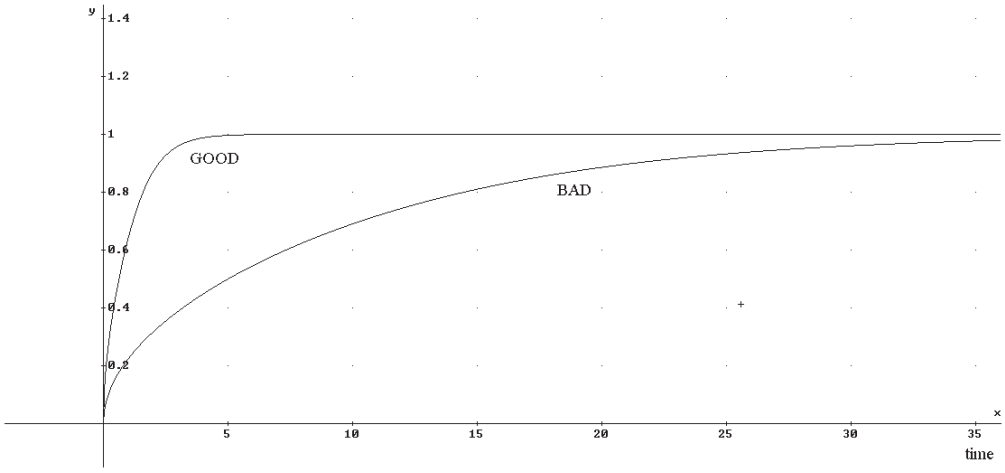

Figure 1: Two normalized ($K = 1$) dynamic market potentials over time (x). Good communication: $p_c = 0.15$, $q_c = 0.90$. Bad communication: $p_c = 0.01$, $q_c = 0.06$.

If communication effects are *persistent*, i.e. with no decay effect, $e_c = 0$, and no negative word–of–mouth, $w_c = 0$, then $D_c = q_c + p_c$ and ${}_cr_1 = -p_c/q_c$, ${}_cr_2 = 1$ so that

$$m(t) = K\sqrt{\frac{1 - e^{-(p_c+q_c)t}}{1 + \frac{q_c}{p_c}e^{-(p_c+q_c)t}}}. \tag{16}$$

In Equation (16), the limiting behaviour of $m(t)$ for $t \to +\infty$ equals the constant carrying capacity K.

In Figure 1 we compare two different situations concerning communication. Higher parameters q_c and p_c allow a steeper–ascent behaviour towards the stationary upper asymptotic limit.

Figure 2 shows three different co–evolutionary processes for the instantaneous case by assuming, for simplicity, exit parameters, e_c, w_c and r_s, set to zero. Cases A and B have two peaks with a good slowdown and a saddle. In case B, the higher value of parameter p_c (0.045) increases earlier adoptions. Case C, characterized by higher adoption parameters, generates a unimodal behaviour without slowdown and with a weaker right tail.

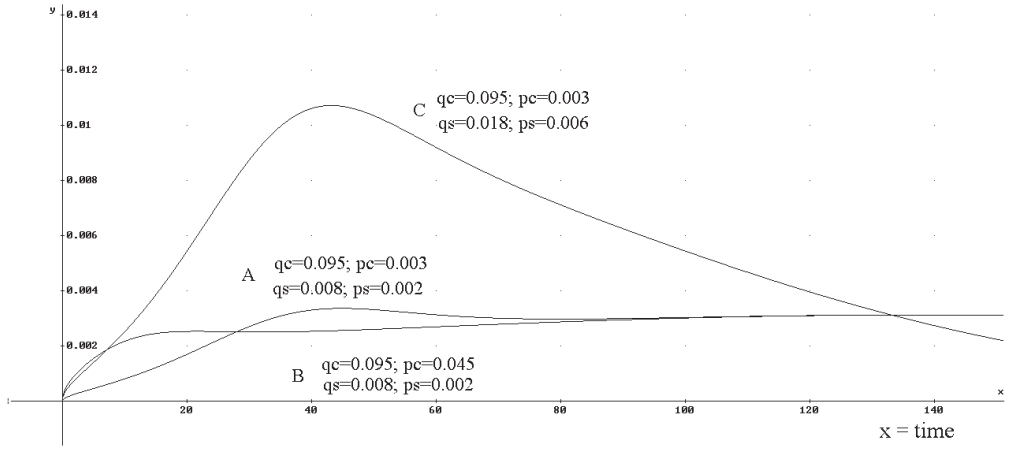

Figure 2: Three different co–evolutionary instantaneous adoption processes. Case B depicts earlier adoptions than case A in the first part of the cycle due to a higher p_c parameter. Both A and B present slowdown and saddle effects. Case C shows a unimodal distribution over time with a short right tail.

The statistical implementation of model (12) may require alternative error structures. In a nonlinear regressive approach we consider a particular model for observations, $w(t) = y(t) + \varepsilon(t)$, with an i.i.d. residual $\varepsilon(t)$. A more realistic approach is based on ARMAX representation with a standard nonlinear estimation as a first step that acts as a "covariate" parallel to the autocorrelated residual components (see, for instance Guseo (2004), Guseo and Dalla Valle (2005) and Guseo et al. (2007)).

As is well–known, joint identifiability of parameters in Equation (13) is not possible because the autonomous Riccati Equation (25), under $f(\cdot) = g(\cdot) = 1$, is characterized by three independent parameters. We have to set one of the four parameters to a specified level, and a reasonable choice may be e_c or w_c exclusion.

3 Pharmaceutical Drugs' Diffusion in Italian Geographic Areas

In this section we present some applications of the Guseo and Guidolin model to the diffusion of six new pharmaceutical drugs, introduced in the Italian market in 2005.

The data, provided by IMS Health, cover the period between August 2005 and July 2007 with a spatial disaggregation by areas ("NordEst", "NordOvest", "Cen-

tro", "Sud"). This kind of information allows interesting comparisons between areas and considerations on their different level of receptiveness.

The aim of these applications is to show the performance of the model, to discuss its main results in terms of statistical identification, fitting and parameter estimates, and to highlight the recurring presence of a slowdown in all of the cases considered. The need to account for a dynamic market potential in the pharmaceutical context has been justified by Guseo and Guidolin by stressing that communication, in the institutional form of detailing, physician meetings, medical journals, advertising and in the form of word-of-mouth, within the medical community and among patients, plays a conclusive role in new drugs' diffusion, facilitating the formation of the market potential. Thus, pharmaceutical products appear to be an ideal candidate for modelling the co-evolution of communication and adoption, where the first usually precedes and pulls the second. However, the general behaviour according to which communication precedes adoption may be contradicted in some cases, where adoption dynamics have a driving role. A recent work on the diffusion of new drugs by Vakratsas and Kolsarici (2008) interestingly remarks that the market for pharmaceuticals is typically created from patients' need for treatment, as diagnosed by physicians, and that this need will be unfulfilled if no prescription is available at the time of the diagnosis. This will result in an accumulation of demand, prior to product launch. Vakratsas and Kolsarici use this observation to motivate their dual-market model for new prescription drugs, with an early and main market, respectively formed by patients with severe and easily diagnosed pathologies and by patients with mild problems or lack of persistent symptoms. Instead, we will employ the severity of pathology and the lack of an adequate treatment as a criterion to evaluate the different nature of prescription drugs and the related diffusion process. In particular, we may expect that a new drug, treating a severe pathology, will be characterized by a diffusion process where the accumulated demand of patients determines an early dominance of adoptions.

For simplicity and space reasons we only examine the following six different configurations: "FOL–NordEst", "FOL–Centro", "LIB–NordEst", "REX–Italy", "KEP–NordEst" and "LYR–Italy" where the first part of the code refers to specific drugs: "FOL", "LIB", "REX", "KEP" and "LYR".

"FOL" was introduced in Italy in August 2005 and prescribed by physicians to prevent fetus malformations such as exencephaly and neoplasms. Based on *Folic Acid*, this drug is prescribed by physicians to expectant mothers during early gestation. The consumption of Folic Acid by expectant mothers and by women who are planning a pregnancy has been the subject of educational campaigns in many countries. The objective of a daily consumption of Folic Acid has required an increasing commitment by both physicians and patients. In addition to the availability of food fortified with it, the recent possibility to take a specific drug that guarantees a quantity of Folic Acid considered adequate for preventing malformations is seen as a fundamental step forward.

Based on the active principle of *Barnidipine*, "LIB" is a new calcium–antagonist introduced in Italy in April 2005 for the treatment of mild–to–moderate hypertension. Several studies have documented that high blood pressure must be controlled

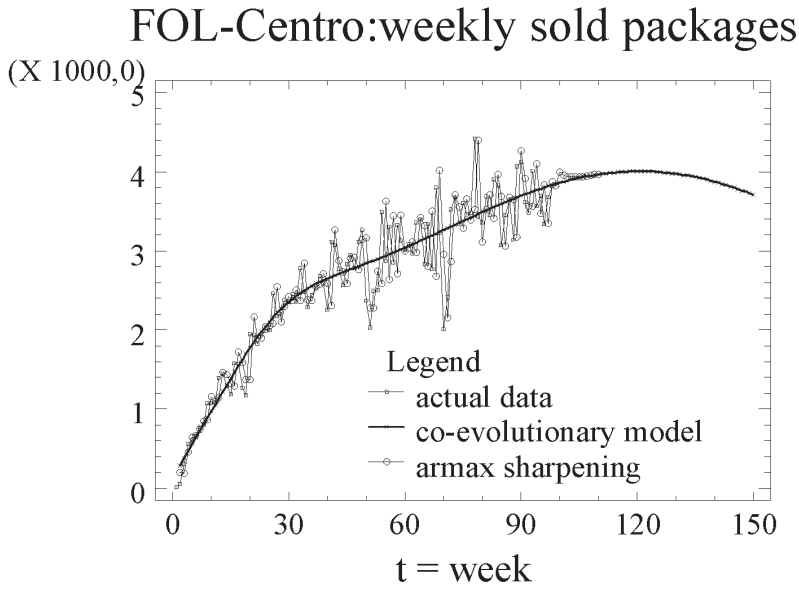

Figure 3: FOL-Centro, Italy: Co–evolutionary non–cumulative model with no exit rule, ARMAX sharpening, and actual weekly "FOL" packages sales data.

both with life style modifications, such as weight reduction, dietary sodium reduction and physical activity, and with an appropriate pharmacological treatment that helps to prevent the risks of heart trouble. The innovativeness of LIB would just rely on the active principle based on a new molecule, whose effect lasts for 24 hours. Indeed, Barnidipine is the 12th calcium–antagonist put into commerce and has been proven to be essentially equivalent to other, less recent calcium–antagonists. The current modified release product is based on barnidipine hydrochloride. Some recent applications refer to atherosclerosis (atheroma) and related atherogenesis, oxidative stress and clotting activity.

"REX" was introduced in Italy in August 2005. Its active principle is *Lovastatin*. The most commonly covered diseases are hypercholesterolemia, familial hypercholesterolemia and hyperlipoproteinemias when a prudent diet is not sufficient or effective. This product also provides a useful option in the management of patients with dislipidemia and in prevention of coronary heart diseases. Lovastatin is the most recent statin introduced in Italy. If taken in dose of 20–40 mg/day, it has been proven effective in lowering cholesterol level, and in reducing the risk of infarction and cardiac arrest.

"KEP" was launched in Italy in April 2005. The new active principle on which the drug is base is *Ketoprofen*, which is commonly employed for treating pain and inflammation. In particular, the topical Ketoprofen patch appears an effective and safe option for the treatment of painful inflammations.

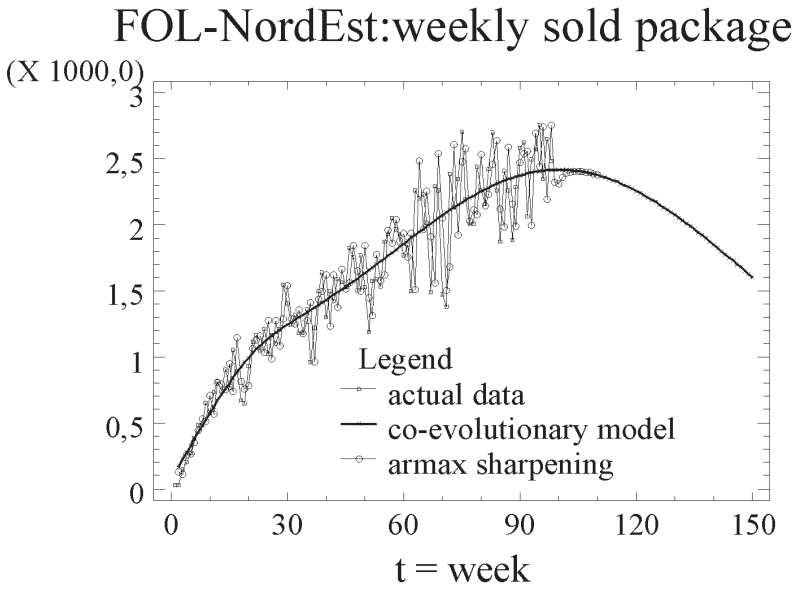

Figure 4: FOL-NordEst, Italy: Co–evolutionary non–cumulative model with no exit rule, ARMAX sharpening, and actual weekly "FOL" packages sales data.

Based on the active principle of *Pregabalin*, "LYR" was initially approved for treating epilepsy (as adjunctive therapy), neuropathic pain and post-herpetic neuralgia pain, that is, to treat pain caused by nerve damage due to diabetes and herpes zoster infection. In particular, Pregabalin is considered a valuable addition to the still–limited options to the treatment of neuropathic pain, proving to be effective in patients who have previously failed to respond to other active principles (e.g., Gabapentin). In 2006 the European Commission approved Pregabalin for the treatment of generalized anxiety disorder. More recently, some studies have shown that Pregabalin is effective in treating chronic pain in disorders such as fibromylagia; in June 2007 the Food and Drug Administration approved "LYR" as the first treatment for fibromyalgia.

We choose to compare all six cases by applying the model in its reduced form, without considering exit rates (parameters $w_c = e_c = r_s = 0$) and exogenous interventions (function $x(t) = 1$),

$$w(t) = K\sqrt{\frac{1-e^{-(p_c+q_c)t}}{1+\frac{q_c}{p_c}e^{-(p_c+q_c)t}}\;\frac{1-e^{-(p_s+q_s)t}}{1+\frac{q_s}{p_s}e^{-(p_s+q_s)t}}} + \varepsilon(t). \tag{17}$$

In Table 1 we summarize the estimation results for the proposed cases under a standard nonlinear least squares approach (Levemberg–Marquardt; see, for instance, Seber and Wild (1989)) in the co–evolutionary model as expressed in Equation (17).

As we may notice from the values of the determination index, R_1^2, the models present very high levels of global fitting and all the involved parameters are significant. Recall that in S–shaped models it is quite common to obtain high levels for

Table 1: Pharmaceutical drugs' diffusion. Parameters' estimates of co–evolutionary models for "FOL-NordEst", "FOL-Centro", "LIB-NordEst", "REX-Italy", "KEP-NordEst", "LYR-Italy" areas of Italy with no exit rule. ( ) marginal linearized asymptotic 95% confidence limits

"FOL-NordEst"						
K	q_c	p_c	q_s	p_s	R_1^2	$D-W$
339352	0.09430	0.01969	0.02487	0.00175	0.999961	0.556
(320070)	(0.07664)	(0.01657)	(0.02385)	(0.00170)	SSE :	
(358633)	(0.11196)	(0.02282)	(0.02590)	(0.00180)	[8.39324E6]	
"FOL-Centro"						
763867	0.08190	0.01192	0.01728	0.00175	0.999967	0.476
(638683)	(0.07342)	(0.01099)	(0.01549)	(0.00154)	SSE :	
(889051)	(0.09038)	(0.01229)	(0.01908)	(0.00197)	[1.99934E7]	
"LIB-NordEst"						
647061	0.08114	0.00385	0.01853	0.00100	0.999939	0.264
(533821)	(0.07483)	(0.00345)	(0.01696)	(0.00088)	SSE :	
(760300)	(0.08746)	(0.00427)	(0.02010)	(0.00112)	[2.88209E7]	
"REX-Italy"						
1748620	0.04429	0.00026	0.08186	0.00938	0.999957	0.328
(1664410)	(0.04288)	(0.00025)	(0.07727)	(0.00889)	SSE :	
(1832830)	(0.04571)	(0.00027)	(0.08646)	(0.00986)	[4.24433E8]	
"KEP-NordEst"						
1652380	0.05573	0.01699	0.00409	0.00116	0.99964	0.047
(–)	(0.03446)	(0.01361)	(-0.00608)	(-0.00385)	SSE :	
(9000760)	(0.07699)	(0.02038)	(0.01426)	(0.00617)	[2.80659E8]	
"LYR-Italy"						
5116020	0.05322	0.00090	0.09451	0.03408	0.999908	0.172
(5066630)	(0.05207)	(0.00087)	(0.07987)	(0.03056)	SSE :	
(5165410)	(0.05438)	(0.00093)	(0.10914)	(0.03759)	[1.77303E10]	

the determination index R^2. For instance, $R^2 = 0.95$ is a low goodness–of–fit value because the competing model is too elementary: the constant one.

As already observed in Guseo and Guidolin (2009), for "FOL" the communication parameters have a higher value in the area of "NordEst", while they are much lower in "Centro", despite the apparently better diffusion process in this area (see market potential K).

We may simply test wether the difference is significant by observing the confidence intervals of the estimated parameters: for example, if we consider parameter estimates for "NordEst" we will see that, except for parameter p_s, the others have a higher value than that of the upper confidence interval of correspondent parameters for "Centro".

In both cases, the presence of a slowdown in data evolution strongly departs from a classical bell–shaped Bass model (BM). This effect is quite perfectly absorbed by the model. See Figures 3 and 4.

The Durbin-Watson statistic in both cases (0.556 and 0.476) suggests the presence of autocorrelated residuals that may be seen by observing the original instantaneous data plotted in Figures 3 and 4.

This problem may be overcome by implementing an appropriate ARMAX procedure. The main results are outlined in Table 2 for the co–evolutionary model *folCcoevCM* of the"Centro" area, and in Table 3 for the co–evolutionary model

folNEcoevCM of the "NordEst" area.

Table 2: FOL-Centro, Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(1,0,1) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	MA(1)	$folCcoevCM$	$mean$	SSE	indexes
0.62285	-0.365964	1.0000	-9.10578	7.78466E6	$R_2^2 = 0.999987$
(6.09150)	(-3.06161)	(776.197)	(-0.05328)	$\{d.f.94\}$	$P^2 = 0.610$
[0.000000]	[0.002871]	[0.000000]	[0.957614]		$F \simeq 71$

The significance of ARMAX sharpening with respect to the co–evolutionary model *folCcoevCM* may be easily determined via a squared multiple partial correlation coefficient, $P^2 = (R_2^2 - R_1^2)/(1 - R_1^2)$ and the corresponding F–ratio, i.e., $F = P^2(N - k)/[(1 - P^2)s]$. In particular, we obtain $P_C^2 = 0.61$ and $F_C \simeq 71$.

Similarly, for the "NordEst" area we obtain $P_{NE}^2 = 0.58$ and $F_{NE} \simeq 62$.

Both ARMAX extensions are significant. For a graphical comparison among actual data, co–evolutionary modelling and ARMAX sharpening for "FOL" in "NordEst" and "Centro", see Figures 3 and 4.

Table 3: FOL-NordEst, Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(1,0,1) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	$MA(1)$	$folNEcoevCM$	$mean$	SSE	indexes
0.51913	-0.47710	1.00075	-58.562	3.555071E6	$R_2^2 = 0.999983$
(4.57612)	(-4.07077)	(794.163)	(-0.603505)	$\{d.f.94\}$	$P^2 = 0.576436$
[0.000014]	[0.000098]	[0.000000]	[0.547626]		$F \simeq 62$

With reference to "LIB–NordEst" we observe a very good behaviour of diffusion with a noticeable NLS fitting, $R_1^2 = 0.999939$. A low level of the Durbin–Watson statistic, $D - W = 0.264$, denotes a high level of autocorrelation of residuals. With an ARMAX procedure we obtain a significant improvement, as one may see by the squared partial correlation, $P^2 = 0.76$ and F-ratio, $F \simeq 114$. Detailed results are reported in Table 4. Figure 5 highlights a weak but visible slowdown effect.

Table 4: LIB–NordEst, Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(1,0,1) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	MA(1)	$libNEcoevCM$	$mean$	SSE	indexes
0.84483	-0.20266	1.0101	1051.21	7.027450E6	$R_2^2 = 0.999985$
(14.75)	(-1.99748)	(375.787)	(3.7364)	$\{d.f.114\}$	$P^2 = 0.756168$
[0.000000]	[0.048155]	[0.000000]	[0.000293]		$F \simeq 114$

"REX-Italy" presents a good behaviour at the national level with an excellent NLS fitting, $R_1^2 = 0.999957$ and its instantaneous evolution emphasizes a pronounced slowdown. See, in particular, Figure 6. A corresponding ARMAX(1,0,1) sharpening allows a significant improvement, confirmed by squared the partial correlation $P^2 = 0.727$ and a high F–ratio, $F \simeq 124$ (see Table 5).

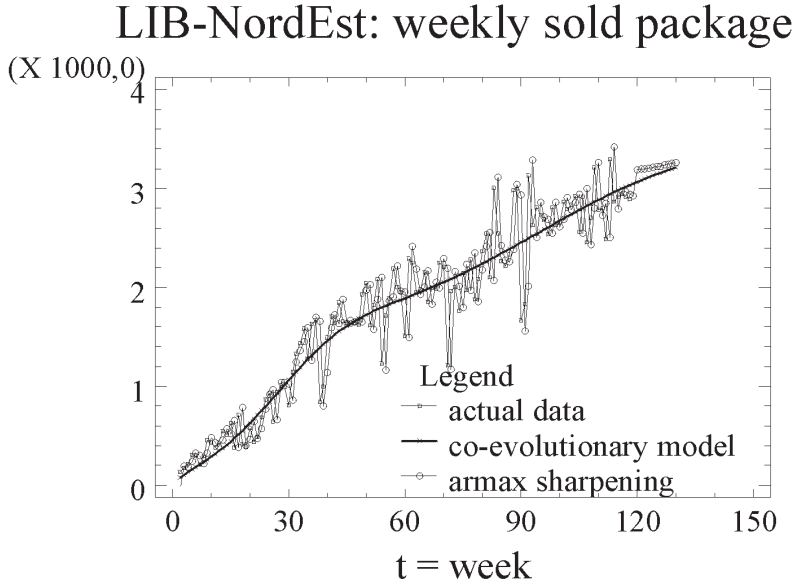

Figure 5: LIB-NordEst, Italy: Co–evolutionary non–cumulative model with no exit rule, ARMAX sharpening, and actual weekly "LIB" packages sales data.

Table 5: "REX-Italy", Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(1,0,1) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	MA(1)	$rexItcoevCM$	$mean$	SSE	indexes
0.731513	-0.396113	1.00061	-228.84	1.1580927E8	$R_2^2 = 0.9999883$
(9.02578)	(-3.72637)	(594.591)	(-0.255711)	$\{d.f.\ 97\}$	$P^2 = 0.7271436$
[0.000000]	[0.000327]	[0.000000]	[0.798715]		$F \simeq 124$

"KEP" denotes an acceptable behaviour in the "NordEst" area with a good NLS fitting, $R^2 = 0.99964$. We can see the origin of such a non-smooth shape by inspecting Figure 7, exactly in the earlier stages where the product presents "stop–and–go" movements. We notice a consistent slowdown generating a proper saddle effect. In this case the ARMAX(2,0,2) sharpening is strongly effective, $R_2^2 = 0.99999178$, with a very high squared partial correlation, $P^2 = 0.9772$, and a very large F–ratio, $F \simeq 933$. See, in particular, the results reported in Table 6.

"LYR-Italy" presents a mature life cycle with some evidence of market contraction. The NLS fitting is excellent, $R_1^2 = 0.99991$, and the subsequent ARMAX(2,0,2) sharpening is quite effective, $P^2 = 0.8952$ and $F \simeq 188$ (see Table 7). The presence of a bimodal behaviour is well-recognized by the co–evolutionary model with an evident saddle effect (see Figure 8).

Previous results highlight different dynamics in diffusion of these new pharmaceutical drugs. In all the examined cases we report the presence of a slowdown effect in the earlier part of diffusion. In some cases its shape is quite pronounced and defines a saddle. The next Section will prove that these effects may be easily explained

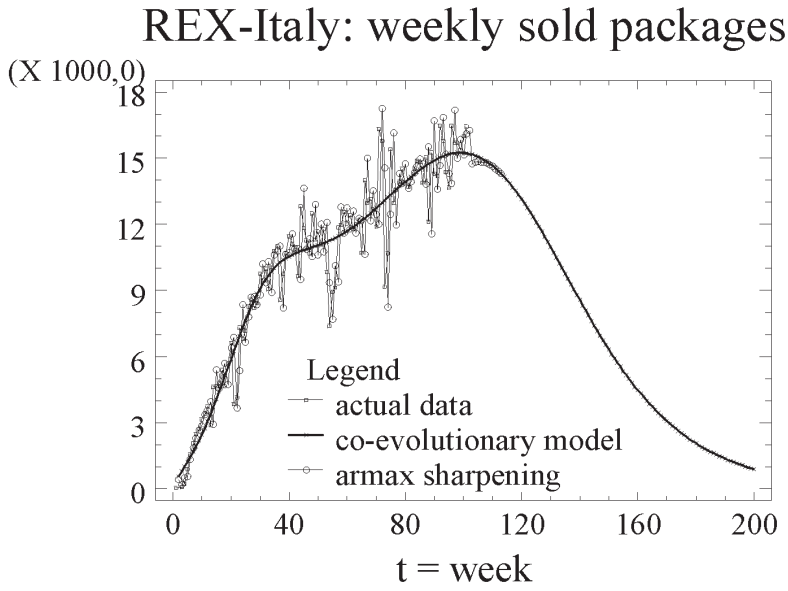

Figure 6: "REX-Italy", Italy: Co–evolutionary non cumulative model with no exit rule, ARMAX sharpening, and actual weekly "REX" packages sales data.

Table 6: "KEP-Italy", Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(2,0,2) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	$AR(2)$	$MA(1)$	$MA(2)$	$kepNEcoevCM$	$mean$	SSE	indexes
1.9705	-0.9908	0.6563	0.3352	0.99685	354.02	6.40772E6	$R_2^2 = 0.9999918$
(101.7)	(-51.04)	(8.624)	(4.321)	(1075.5)	(3.021)	$\{d.f.\ 113\}$	$P^2 = 0.9771690$
[0.0000]	[0.0000]	[0.0000]	[0.0001]	[0.00000]	[0.0031]		$F \simeq 933$

through an interaction between variable market potential and corresponding adoption process. The multiplicative model allows a viable interpretation of the basic components that may be sufficiently separated over time, not requiring the hypothesis of a dual market or dual segment market characterizing agents' heterogeneity.

4 Some statistical aspects of a co–evolutionary model

In this Section we analyze some behavioural aspects of the Guseo and Guidolin co–evolutionary model. For the sake of clarity we will refer to the simplest version, i.e., Equation (17), without considering exit rates (parameter $w_c = e_c = r_s = 0$) and external intervention (function $x(t) = 1$). We may consider a simple reparameteri-

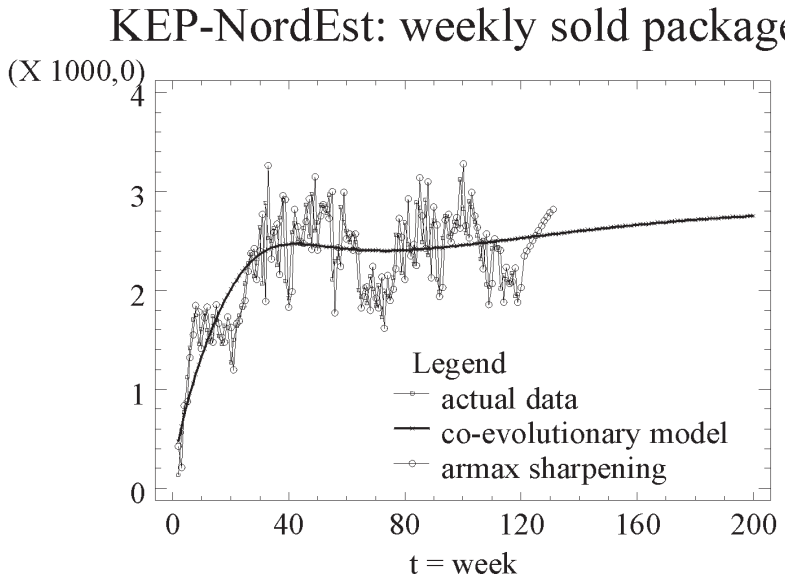

Figure 7: "KEP-NordEst", Italy: Co–evolutionary non–cumulative model with no exit rule, ARMAX sharpening, and actual weekly "KEP" packages sales data.

Table 7: "LYR-Italy", Italy: Co-evolutionary cumulative model with no exit rule and ARMAX(2,0,2) sharpening. ( ) t–statistic; [ ] p–values

$AR(1)$	$AR(2)$	$MA(1)$	$MA(2)$	$lyrcevCM$	$mean$	SSE	indexes
1.8721	-0.9392	0.7180	0.2906	1.00071	-1384.72	1.8585021E9	$R^2_2 = 0.9999904$
(40.19)	(-20.343)	(8.6091)	(3.3500)	(2205.34)	(-1.1176)	$\{d.f.\ 91\}$	$P^2 = 0.8951793$
[0.0000]	[0.0000]	[0.0000]	[0.0012]	[0.0000]	[0.2667]		$F \simeq 188$

zation for adoption and communication components,

$$\begin{aligned} a &= p_s + q_s; \qquad b = q_s/p_s; \qquad c = p_c + q_c; \qquad d = q_c/p_c, \\ p_s &= a/(1+b); \quad q_s = ab/(1+b); \quad p_c = c/(1+d); \quad q_c = cd/(1+d). \end{aligned} \tag{18}$$

The corresponding co–evolutionary model is

$$\begin{aligned} w(t) &= K \cdot \sqrt{\frac{1-e^{-ct}}{1+de^{-ct}}}\ \frac{1-e^{-at}}{1+be^{-at}} + \varepsilon(t) \\ &= K \cdot K(t,a,b,c,d) + \varepsilon(t) \\ &= K \cdot \sqrt{F(t,c,d)}\ G(t,a,b) + \varepsilon(t) \\ &= K \cdot \sqrt{F(t)}\ G(t) + \varepsilon(t), \end{aligned} \tag{19}$$

where $F(t) = F(t,c,d) = (1-e^{-ct})/(1+de^{-ct})$ and $G(t) = G(t,a,b) = (1-e^{-at})/(1+be^{-at})$. Under the usual internal unimodality assumptions on diffusion parameters,

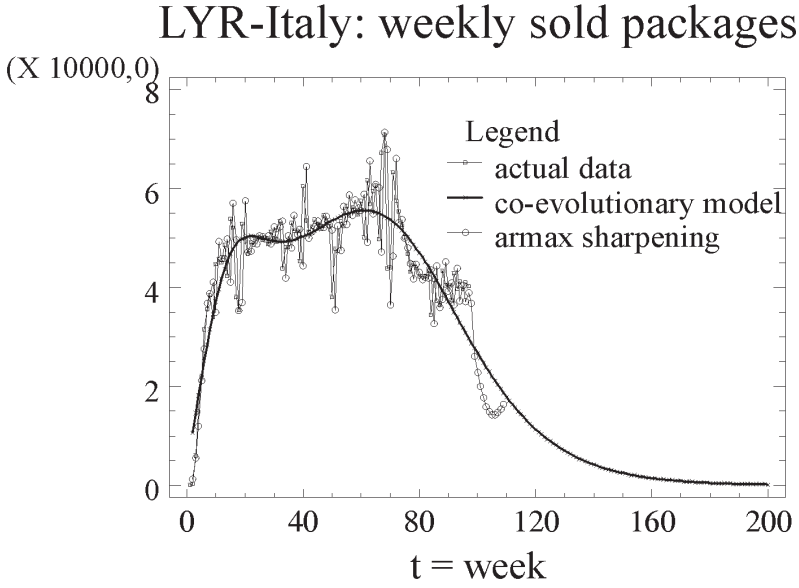

Figure 8: "LYR-Italy", Italy: Co–evolutionary non–cumulative model with no exit rule, ARMAX sharpening, and actual weekly "LYR" packages sales data.

$0 < p_s < q_s$ and $0 < p_c < q_c$ or, simply under the non–negativity of parameters p_c, q_c, p_s and q_s, we highlight that $K(t) = \sqrt{F(t)}\, G(t) = K(t, a, b, c, d)$ is a probability distribution function.

Let us consider a representation of its density $k(t) = \partial K(t)/\partial t$ with a reduced notation, i.e.,

$$k(t) = \frac{1}{2} F(t)^{-1/2} G(t) f(t) + F(t)^{1/2} g(t) = k_1(t) + k_2(t), \qquad t > 0, \tag{20}$$

where $f(t) = \partial F(t)/\partial t$ and $g(t) = \partial G(t)/\partial t$.

We may consider a normalization of non–negative functions $k_1(t)$ and $k_2(t)$ deriving two corresponding densities $\tilde{k}_i(t) = k_i(t)/K_i$ with $K_i = \int_0^\infty k_i(t)dt$, $i = 1, 2$. Suppose that a random variable X is associated to $\tilde{k}_1(t)$ and, similarly, a random variable Y to $\tilde{k}_2(t)$.

Definition. We say that Y is *larger* than X in *likelihood ratio order*, $X \leq_{lr} Y$, if X and Y have densities such that, for all $s \leq t$,

$$\tilde{k}_1(t) \cdot \tilde{k}_2(s) \leq \tilde{k}_1(s) \cdot \tilde{k}_2(t). \tag{21}$$

Notice that the inequality based on $\tilde{k}_i(t)$, $i = 1, 2$ densities does not depend on the quantities K_1 or K_2 or their ratio, so that we can directly compare $k_i(t)$, $i = 1, 2$. Equation (21) states that $\tilde{k}_2(t)/\tilde{k}_1(t)$ or $k_2(t)/k_1(t)$ is increasing, avoiding the special cases with vanishing denominators.

It is well–known that the *likelihood ratio order* is stronger than the *usual stochastic order*, i.e., if $X \leq_{lr} Y$ then $X \leq_{st} Y$ (see, e.g., theorem 1.4.4 in Müller and Stoyan

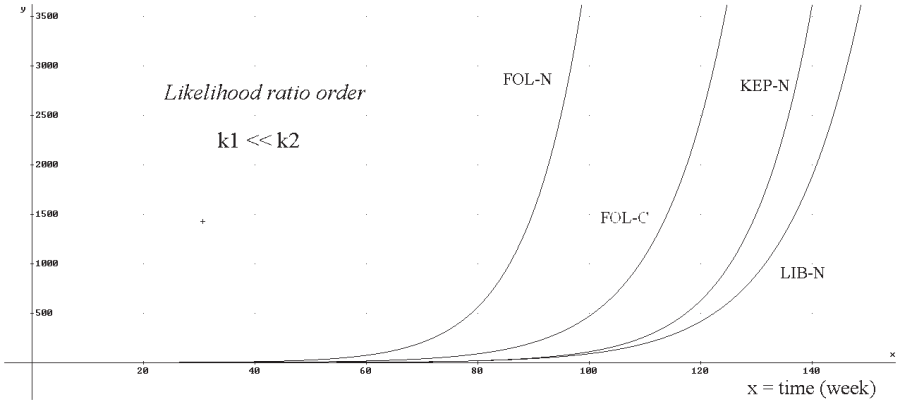

Figure 9: Likelihood Ratio Order between $k_2(t)$ and $k_1(t)$ for pharmaceutical drugs "FOL-NordEst", "FOL-Centro", "LIB-NordEst" and "KEP-NordEst" in Italy. The increasing ratio denotes a (first order) stochastic dominance of $k_2(t)$ component or, equivalently, a driving role of communication, $k_1(t)$. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

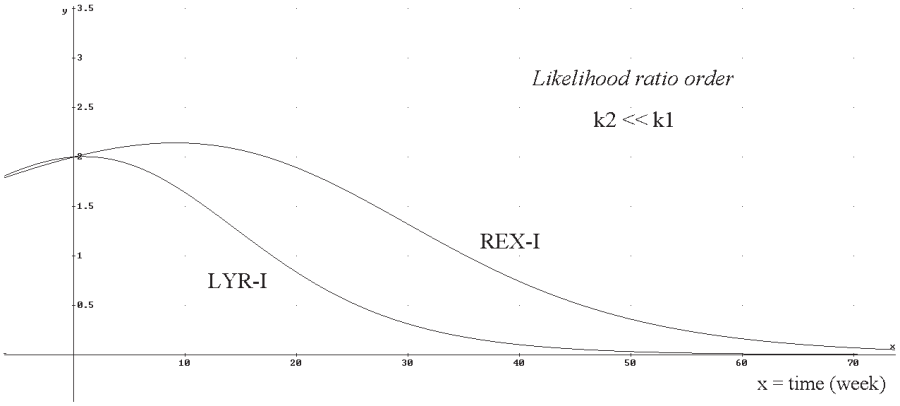

Figure 10: Likelihood Ratio Order between $k_2(t)$ and $k_1(t)$ for pharmaceutical drugs "REX-Italy" and "LYR-Italy" in Italy. The decreasing ratio denotes a (first order) stochastic dominance of $k_1(t)$ component or, equivalently, a driving role of adoption, $k_2(t)$. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

(2002)). Notice that a random variable X is smaller than a random variable Y under the *usual stochastic order*, $X \leq_{st} Y$, if the corresponding distribution functions satisfy, for all t, inequality $F_X(t) \geq F_Y(t)$.

As a direct control, we can compute the likelihood ratios $\tilde{k}_2(t)/\tilde{k}_1(t)$ or $k_2(t)/k_1(t)$ pertaining to the diffusions of the six pharmaceutical drugs, i.e., "FOL-NordEst", "FOL-Centro", "LIB-NordEst","REX-Italy", "KEP-NordEst" and "LYR-Italy".

We report the results concerning likelihood ratios in two separate plots. Figure 9 highlights a practically increasing behaviour of "FOL-NordEst", "FOL-Centro", "LIB-NordEst"and "KEP-NordEst". This allows a simple interpretation: the effect associated to $k_1(t)$, i.e., the communication effect, has an earlier dominance in the evolution of these drugs.

Vice versa, Figure 10 depicts the opposite diffusion structure of "REX-Italy" and "LYR-Italy" with an earlier driving role pertaining to adoption forces. The

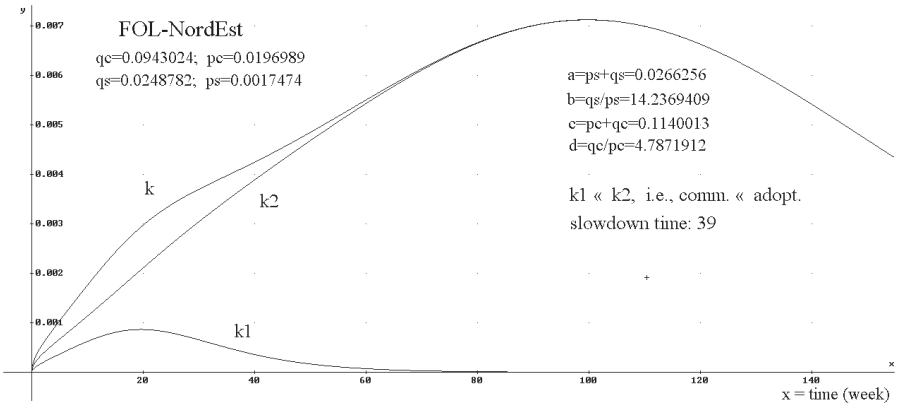

Figure 11: "FOL-NordEst": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "FOL" in the "NordEst" area of Italy. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

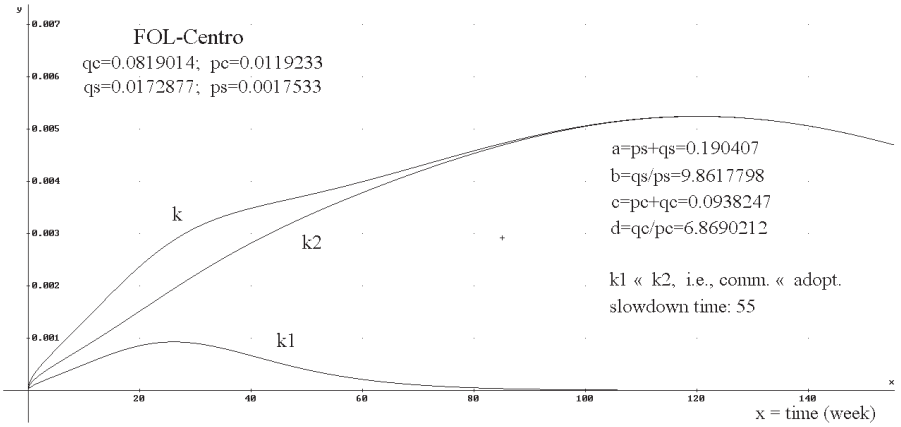

Figure 12: "FOL-Centro": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "FOL" in the "Centro" area of Italy. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

direct use of the *likelihood ratio criterion* gives a strong result for the order of basic components $k_1(t)$ and $k_2(t)$. Nevertheless, this ratio requires plotting features to recognize dominance between adoption and communication effects. We propose, as an alternative procedure, some location indexes for detecting such a dominance, in a *weak order* sense.

Equation (20) highlights that density $k(t)$ is the sum of two non negative components, $k_1(t)$ and $k_2(t)$ which, in turn, are special transformations of the basic Bass densities $f(t)$ and, respectively, $g(t)$.

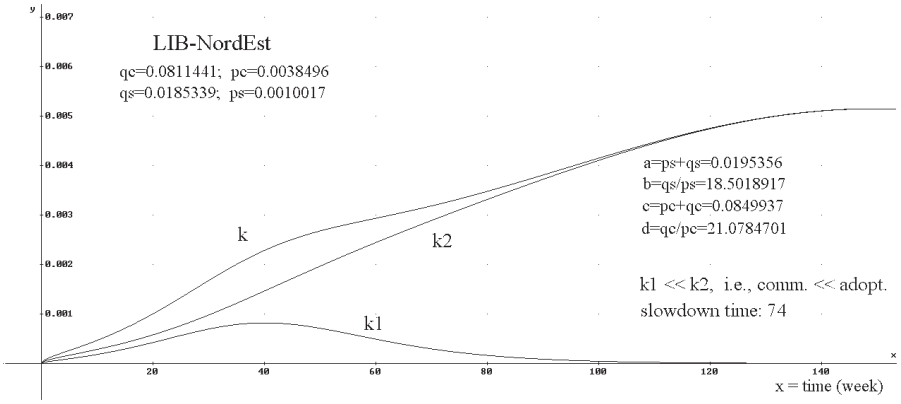

Figure 13: "LIB-NordEst": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "LIB" in the "NordEst" area of Italy. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

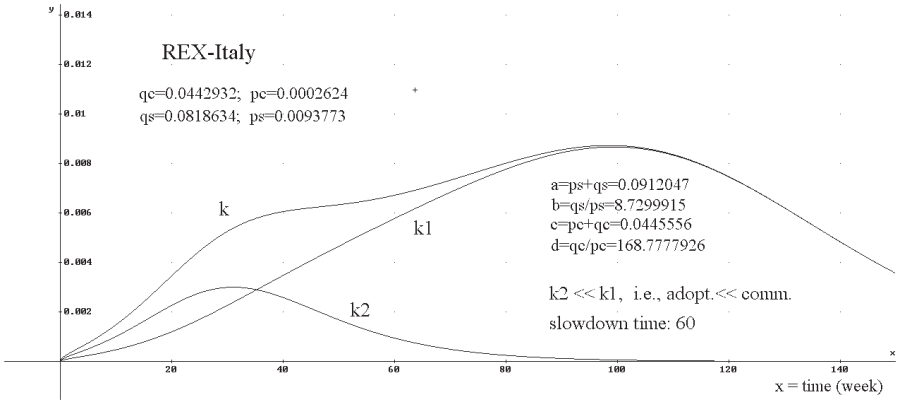

Figure 14: "REX-Italy": two synergistic components. Adoption (k_2) is a precursor of communication (k_1) for "REX" in Italy. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

We can emphasize this aspect with an equivalent form of Equation (20), i.e.,

$$\begin{aligned} k(t) &= \frac{1}{2}F(t)^{1/2}\left(\frac{G(t)}{F(t)}f(t)+2g(t)\right) \\ &= \frac{(G(t)+2F(t))}{2F(t)^{1/2}}\left(\frac{G(t)f(t)+2F(t)g(t)}{G(t)+2F(t)}\right). \end{aligned} \tag{22}$$

The second factor is a special normalized convex adaptive mixture of the two basic densities, where adaptive weighting depends upon t level. If we consider two random variables in time domain, T_F, and T_G, with distribution functions $F(t)$ and $G(t)$, respectively, then we say, following the *usual stochastic order*, that T_F is *stochastically inferior* to T_G, $T_F \leq_{st} T_G$, if and only if $F(t) \geq G(t)$ for all $t \in R$.

Let us consider the second factor in Equation (22),

$$h(t) = \left(\frac{G(t)f(t) + 2F(t)g(t)}{G(t) + 2F(t)} \right). \tag{23}$$

We may neglect the first factor in Equation (22) because it is an increasing monotone function of the variable t, and this omission does not alter the modal value t^+, which is the same for $k(t)$ or $h(t)$.

Let us consider, for clarity, a strong departure between random variables T_F and T_G. If $T_F \leq_{st} T_G$ then $h(t) \simeq g(t)$, and vice versa, for $T_G \leq_{st} T_F$ we attain that $h(t) \simeq f(t)$. Notice that the *usual stochastic order* between T_F and T_G, e.g. $\leq_{st}$, implies an analogous (total) *weak order* of corresponding mean values, $E(T_F) \leq E(T_G)$, associated to $f(t)$ and $g(t)$ respectively (see, e.g., theorem 1.2.9 in Müller and Stoyan (2002)). We may use different location indexes related to the random variables T_F and T_G, the mode, t^+, the median, $t_{0.5}$, and the mean value, $\bar{t}$, i.e.,

$$\begin{aligned} t^+_{com} &= \frac{\ln d}{c}; \qquad {}_F t_{0.5} = \frac{1}{c}\ln(2+d); \qquad E(T_F) = \bar{t}_F = \frac{1}{q_c}\ln(1+d), \\ t^+_{ado} &= \frac{\ln b}{a}; \qquad {}_G t_{0.5} = \frac{1}{a}\ln(2+b); \qquad E(T_G) = \bar{t}_G = \frac{1}{q_s}\ln(1+b). \end{aligned} \tag{24}$$

For $0 < p_c < q_c$ and $0 < p_s < q_s$ the location indexes within T_F and T_G – mode, median and mean value – are increasing values, $t^+ < t_{0.5} < \bar{t}$. Moreover, we can prove that the *likelihood ratio order* or the *usual stochastic order* imply an equivalent *weak order* in terms of medians: $T_F \leq_{st} T_G \rightarrow {}_F t_{0.5} \leq {}_G t_{0.5}$.

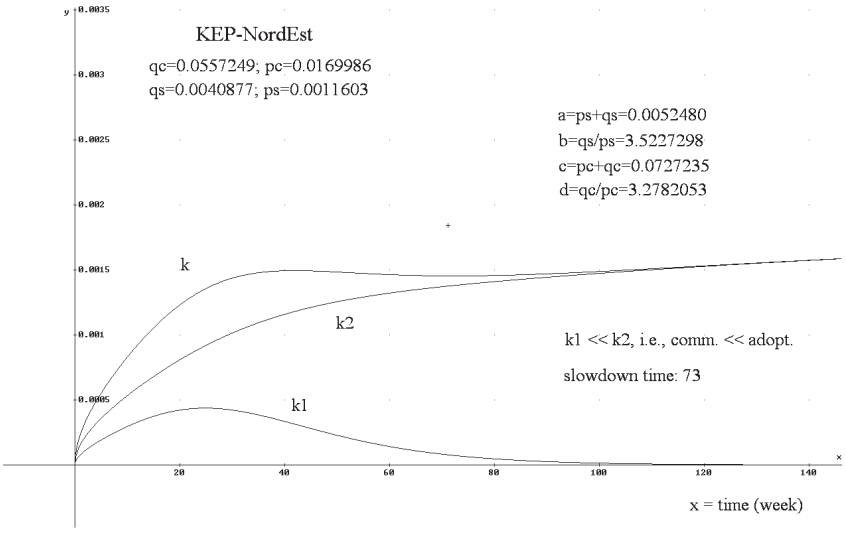

Figure 15: "KEP-NordEst": two synergistic components. Communication (k_1) is a precursor of adoption (k_2) for "KEP" in the "NordEst" area of Italy. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

The reverse version of the above mentioned theorem 1.2.9, or an equivalent form for medians, is not true in general. Nevertheless, the total *weak order* based on

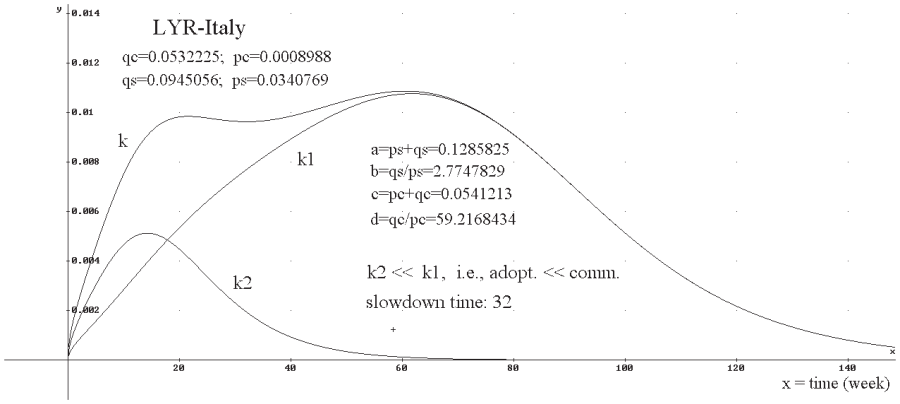

Figure 16: "LYR-Italy": two synergistic components. Adoption (k_2) is a precursor of communication (k_1) for "LYR" in Italy. Data source: IMS-Health, Italy. Normalized weekly packages sold; period: 8/2005 – 7/2007.

mean values or medians or modes (24) may be a strong symptom of the existence of an analogous *usual stochastic order*. Therefore, we can obtain some information on the relative time position of components $k_1(t)$ and $k_2(t)$ by diagnosing it via couples of corresponding location indexes. In particular, observing that $t^+_{com} < t^+_{ado}$, ${}_Ft_{0.5} < {}_Gt_{0.5}$ and $\bar{t}_F < \bar{t}_G$, we may argue that we are facing the most common situation where the communication process is the main driver of a possible take–off in diffusion, so that we have T_F *smaller* than T_G, following a location *weak order* denoted by the symbol $<<$, $k_1(t) << k_2(t)$. The actual order may be a stronger one, i.e., $\leq_{lr}$ or $\leq_{st}$.

Vice versa, for $t^+_{com} > t^+_{ado}$, ${}_Ft_{0.5} > {}_Gt_{0.5}$ and $\bar{t}_F > \bar{t}_G$ we expect a reverse order in time domain, i.e., in this case the adoption component is the main driver, and communication gives rise to a maintenance effect.

5 Pharmaceutical drugs in Italy: driving forces effects

In this Section we examine, with some detail, the six pharmaceutical drugs reported in previous Sections with the aim to detect and interpret the relative positioning of the two synergistic forces, communication and adoption, in the corresponding co–evolutionary diffusion process through a *weak order* based on simple location indexes.

These effects are not directly observable, but the multiplicative co-evolutionary model allows conceptual and statistical identification just using adoption data.

We underline that neither components $k_1(t)$ and $k_2(t)$ in Equation (20) represent pure effects: they are functions of the four parameters p_c, q_c, p_s and q_s. However, interpretation may establish reasonable associations, $k_1(t) \leftrightsquigarrow f(t)$ and $k_2(t) \leftrightsquigarrow g(t)$.

In Table 8 we summarize the basic estimates, the modal time values, t^+_{com}, t^+_{ado}, the median time values, ${}_Ft_{0.5}$, ${}_Gt_{0.5}$, and the mean values, $\bar{t}_F$, $\bar{t}_G$, referred to communication and adoption components, respectively.

Table 8: Pharmaceutical drugs' diffusion. Parameters' estimates of co–evolutionary models for "FOL", "LIB", "REX", "KEP" and "LYR" in some areas of Italy with no exit rule. t^{+}_{com}, ${}_{F}t_{0.5}$, $\bar{t}_F$ and t^{+}_{ado}, ${}_{G}t_{0.5}$, $\bar{t}_G$ define the modal, median and mean times of communication and adoption components; the weak order between communication component, $k_1(t)$, and adoption component, $k_2(t)$, is denoted by symbol "<<".

drug-area	q_c	p_c	q_s	p_s	R^2	t^{+}_{com}	t^{+}_{ado}
FOL-NordEst	0.0943024	0.0196989	0.0248782	0.0017474	0.999961	13.7	99.7
FOL-Centro	0.0819014	0.0119233	0.0172877	0.0017533	0.999967	20.5	120.2
LIB-NordEst	0.0811441	0.0038496	0.0185339	0.0010017	0.999939	35.9	149.4
REX-Italy	0.0442932	0.0002624	0.0818634	0.0093773	0.999957	115.1	23.7
KEP-NordEst	0.0557249	0.0169986	0.0040877	0.0011603	0.999640	16.3	239.9
LYR-Italy	0.0532225	0.0008988	0.0945056	0.0340769	0.999531	75.4	7.9

drug-area	t^{+}_{com}	${}_{F}t_{0.5}$	$\bar{t}_F$	order	t^{+}_{ado}	${}_{G}t_{0.5}$	$\bar{t}_G$
FOL-NordEst	13.7	16.8	18.6	<<	99.7	104.7	109.5
FOL-Centro	20.5	23.3	25.2	<<	120.2	129.9	138.0
LIB-NordEst	35.9	36.9	38.1	<<	149.4	154.6	160.3
REX-Italy	115.1	115.4	115.9	>>	23.7	26.0	27.8
KEP-NordEst	16.3	22.9	26.1	<<	239.9	325.6	369.2
LYR-Italy	75.4	76.0	77.0	>>	7.9	12.2	14.1

Observing Figures 11, 12, 13, 15, we see that in four cases, namely "FOL–NordEst", "FOL–Centro", "LIB–NordEst", "KEP–NordEst", the weak order based on location indexes confirms a predictable behaviour, according to which communication has a driving role, preceding and pulling adoptions. Instead, in the cases of "REX–Italy" and "LYR–Italy", depicted in Figures 14 and 16, we observe an explicit inversion, so that the adoption component appears to dominate the first part of diffusion. We believe that this difference in behaviour may be related to the nature of the drugs considered.

As we anticipated in Section 3, drugs developed for treating severe pathologies typically present an accumulated demand at the time of their launch into market, so that an early growth of adoptions is to be expected: this may be exactly the case for both "LYR" and "REX".

"LYR" was originally developed for neuropathic pain, a symptom common to various pathologies that are extremely difficult to understand and to treat. The painful condition of patients affected by this problem makes their continuous search for every possible solution unsurprising. The use of antiepileptic drugs for neuropathic pain management began with two active principles, *Carbamazepine* and *Gabapentin*: however, as reported in Gawande (2002), these drugs did not always have the expected results, so that physicians and patients were waiting for the announced new generation of "neurostabilizers" drugs. Thus, when "LYR" was put into commerce there probably was an accumulation of demand for it: consistently with this fact, we have seen that adoption dynamics have a driving role in the first part of its diffusion process. Moreover, "LYR" exhibits a saturating life cycle, probably due to its special formulation, which is based on a cumulative concentration with a natural delayed response, and to the cost of a prolonged therapy. These aspects may explain the reduction in adoptions and a possible reversion to *Gabapentin*.

A similar perspective may be adopted in analyzing the case of "REX", which

is based on a new statin for the treatment of hypercholesterolemia: as reported by several studies, statins are the most effective forms of treatment of high cholesterol, when dietary advices prove insufficient. Some studies have shown that diet can reduce cholesterol levels by 15%. That is, a modest decrease in cholesterol levels suggests that diet may be sufficient in the treatment of mildly elevated levels. Although hypercholesterolemia is not a disease *per se*, its correlation with cardiovascular diseases has been widely observed. The fact that "REX" has experienced an early dominance of adoptions would confirm that for those patients affected by severe hypercholesterolemia, pharmaceutical treatment is a reasonable measure to prevent hard cardiovascular outcomes, such as death or myocardial infarction. This prompt response of patients to the new statin (for the Italian market) is arguably due to previous negative interactions of *Cerivastatin*.

On the other side, "FOL", "LIB" and "KEP" are drugs that do not treat specific pathologies but are assumed as a precautionary measure to avoid more serious consequences ("FOL" and "LIB") or as a treatment for minor ailments ("KEP"). In these cases we may conclude that communication, both institutional and informal, has exerted its natural effect of stimulating adoptions through the generation of market potential.

In particular, we have seen that "FOL" is prescribed to expectant mothers to prevent fetal malformations and that the assumption of Folic Acid has been recommended through various informative campaigns, involving both physicians and patients, to create a wide awareness about the importance of a precautionary behaviour against neural tube defects by women intending to become pregnant. "LIB" and "KEP" do not seem to have the characteristics of really innovative products, so that in these cases we argue that the pattern "first communication, then adoption" is explained by the simple need for promoting the new product when put into commerce.

In Table 9 we report the corresponding reparameterization and some information regarding the presence of a slowdown. In two cases, "KEP-NordEst" and "LYR-Italy", we observe a deeper effect, a saddle, indicating a stronger separation between communication and adoption.

The slowdown observed in the analyzed cases does not emerge as a natural element of the model: as one may see in Figure 2, for certain parameter combinations, the model yields a classical bell–shaped curve, while for others the slowdown and the saddle appear. Consequently, the presence of slowdown and saddle may depend on managerial choices.

6 Final Remarks and Discussion

This paper examines theoretical, technical and applied aspects of a well–known diffusion of innovations class of effects: slowdown, dip, saddle or chasm. The proposed framework emphasizes a new interpretation of such a systematic depression in the early stages of the diffusion process. This effect may be described by a binary model for an adoption process nested in a communication network that evolves over time,

Table 9: Pharmaceutical drugs' diffusion. Parameters' estimates of co–evolutionary models for "FOL", "LIB", "REX", "KEP" and "LYR" in some areas of Italy with no exit rule. New parameterization: a and b refer to adoption, c and d to communication.

drug-area	$a = p_s + q_s$	$b = q_s/p_s$	$c = p_c + q_c$	$d = q_c/p_c$	$k_1 \sim k_2$	slowdown	saddle
FOL-NordEst	0.0266256	14.2369409	0.1140013	4.7871912	<<	yes	no
FOL-Centro	0.0190407	9.8617798	0.0938247	6.8690212	<<	yes	no
LIB-NordEst	0.0195356	18.5018917	0.0849937	21.0784701	<<	yes	no
REX-Italy	0.0912407	8.7299915	0.0445556	168.777793	>>	yes	no
KEP-NordEst	0.0052480	3.5227298	0.0727235	3.2782053	<<	yes	yes
LYR-Italy	0.1285825	2.7747829	0.0541213	59.2168434	>>	yes	yes

generating a basic precursor to the corresponding latent variable market potential (see Guseo and Guidolin (2009)).

As a general remark for innovation diffusion theory, we highlight that the "dual–effect" approach recognizes levels and locations of two fundamental forces in market expansion. Radical innovations, dominating their own market niche and often patented, may exhibit an interesting inversion –adoption/communication– that suggests oriented managerial actions towards specialized agents (hubs). Vice versa, incremental innovations, based on an already existing technology and competing in a mature marketplace, may require a more common managerial effort based on continuous and diffuse communication actions. The proposed "dual–effect" approach may assess, during the evolution of the co–evolving processes, the relative performances of competing products or services in different areas or markets. This normative indication will be confirmed or not by a direct check through the temporal distribution of sales' data.

In the sequel, we summarize some specific properties of the co–evolutionary approach with reference to alternatives:

a) There is incontrovertible evidence concerning the non-uniqueness of the causal forces generating a slowdown. Dual–market interpretations or correlated mixture modelling representations introduce a two–segment partition of adopters simply because a temporal decomposition of adoptions pertaining to rigid segments is assumed. We note that repeated adoptions due to the same adopter may be realized in different contexts and with different awareness levels;

b) The segmented sub–population approach does not explain why "two–level" is so special or characteristic. Why not three or four levels?

Karmeshu and Goswami's (2001) interpolatory approach is very flexible, but introduces seven parameters: one for the assumed fixed market potential and six for the "internal" and "external" dynamics with, in our opinion, some difficulties in interpreting results;

c) A slowdown effect may be originated by an external environmental effect, which is an approach totally different from internal heterogeneity of agents. If we can

absorb a local depression with a generalized Bass model (GBM) following, e.g., Guseo and Dalla Valle (2005) or Guseo et al. (2007), we have to adequately motivate or support such a modelling choice, which may be correct in some circumstances but not in others;

d) We believe that the proposed decomposition of density related to Guseo and Guidolin's model allows a simple interpretation of internal heterogeneity effects which are not imputed to different and stable characteristics of agents. This decomposition recognizes the existence of a self–reinforcing diffusion governed by two synergistic forces, communication and adoption, that are not ordered in a fixed way during time evolution;

e) The Guseo and Guidolin model allows an interchangeable allocation of the two driving forces. As we have recognized by examining six new pharmaceutical drugs introduced in Italy from August 2005 to July 2007, for two of them, "REX" and "LYR", there is an "inversion" in the role of adoption due to an interpretable context effect, namely the severity of disease and the accumulating demand effect in initial stages;

f) The recognition of the alternative order is simple to discover with a strong *likelihood ratio order* or, much more practically, through a simplified *weak order* based on easy to compute location indexes: mode, median, mean values;

g) The model only requires time series of adoptions. In particular, the dynamics of market potential, which take into account the evolution of the communication effort and related word–of–mouth, are estimated through the only observable adoption data;

h) From a computational and statistical point of view, the proposed framework is easy to implement with common commercial software.

APPENDIX A

A Riccati Equation

Let us consider the following special non–autonomous Riccati equation in (X,Y) real space

$$y'_x = a\frac{f(x)}{g(x)}y^2 + \left(bf(x) + \frac{g'(x)}{g(x)}\right)y + cf(x)g(x), \tag{25}$$

where $a, b, c \in R$, $D = \sqrt{b^2 - 4ac} > 0$ and $g(x) \neq 0$, $f(x)$ are real functions. Its general discussion may be found in Guseo and Guidolin (2009). Here we report the final results concerning its closed form solution.

Let us consider the real roots of equation $az^2+bz+c = 0$, i.e., $r_i = (-b \pm D)/2a \in R, i = 1, 2$, where $D = a(r_2 - r_1) = \sqrt{b^2 - 4ac} > 0$.

The general solution of Equation (25) is,

$$y(x) = g(x)\frac{r_1 r_2(1 - G(x)) - C(r_1 - r_2 G(x))}{r_2 - r_1 G(x) - C(1 - G(x))}, \tag{26}$$

where $G(x) = e^{D\int_0^x f(\tau)d\tau}$, and C is an arbitrary constant of integration.

If the initial condition is set to zero, $y(0) = 0$, we obtain $C = 0$ and, therefore,

$$y(x) = g(x)\frac{1 - e^{-D\int_0^x f(\tau)d\tau}}{\frac{1}{r_2} - \frac{1}{r_1}e^{-D\int_0^x f(\tau)d\tau}}. \tag{27}$$

If $\lim_{x\to\infty}\int_0^x f(\tau)d\tau = +\infty$, we obtain an interesting limiting behaviour of $y(x)$, i.e., $\lim_{x\to\infty} y(x) = r_2 \lim_{x\to\infty} g(x)$.

References

Bass, F., T. Krishnan, D. Jain (1994). Why the Bass model fits without decision variables. *Marketing Science 13*, 203–223.

Cohen, W., D. Levinthal (1990). A new Perspective on Learning and Innovation. *Administrative Science Quarterly 35*, 128–152.

Gawande, A. (2002). *Complications: A surgeon's notes on an imperfect science.* Metropolitan Books, Henry Holt and Co., New York.

Goldenberg, J., B. Libai, E. Muller (2002). Riding the Saddle: How Cross–Market Communications can create a Major Slump in Sales. *Journal of Marketing 66*(2), 1–16.

Guseo, R. (2004). Interventi strategici e aspetti competitivi nel ciclo di vita di innovazioni. Technical Report N. 11/2004, Department of Statistical Sciences, University of Padova. http://www.stat.unipd.it/ricerca/fulltext?wp=308.

Guseo, R., A. Dalla Valle (2005). Oil and Gas Depletion: Diffusion Models and Forecasting under Strategic Intervention. *Statistical Methods and Applications 14*, 375–387.

Guseo, R., A. Dalla Valle, and M. Guidolin (2007). World Oil Depletion Models: Price Effects Compared with Strategic or Technological Interventions. *Technological Forecasting and Social Change 74*(4), 452–469.

Guseo, R., M. Guidolin (2009). Modelling a dynamic market potential: A class of automata networks for diffusion of innovations. *Technological Forecasting and Social Change (in press).* doi:10.1016/j.techfore.2008.10.005.

Karmeshu, D. Goswami (2001). Stochastic evolution of innovation diffusion in heterogeneous groups: study of life cycle patterns. *IMA Journal of Management Mathematics 12*, 107–126.

Mahajan, V., E. Muller (1998). When is it Worthwhile Targeting the Majority Instead of the Innovators in a New Product Launch? *Journal of Marketing Research 35*, 488–495.

Moore, G. (1991). *Crossing the Chasm: Marketing and Selling Technology Products to Mainstream Customers*. Harper Collins Publishers, New York.

Müller, A., D. Stoyan (2002). *Comparison Methods for Stochastic Models and Risks*. Wiley, New York.

Muller, E., G. Yogev (2006). When does the majority become majority? Empirical analysis of the time at which main market adopters purchase the bulk of our sales. *Technological Forecasting and Social Change 73*(9), 1107–1120.

Rogers, E. (2003). *Diffusion of Innovations* (5th ed.). Free–Press, Simon and Shuster, New York.

Seber, G., C. Wild (1989). *Nonlinear Regression* (1st ed.). Wiley, New York.

Vakratsas, D., C. Kolsarici (2008). A dual–market diffusion model for a new prescription pharmaceutical. *International Journal of Research in Marketing 25*, 282–293.

Van den Bulte, C., Y. Joshi (2007). New Product Diffusion with Influentials and Imitators. *Marketing Science 26*(3), 400–421.

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