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Duncan's model for $\overline{X}$-control charts: sensitivity analysis to input parameters

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Keywords: ANOVA, Economic Design, Resolution V Fractional Factorial Design.

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1 Introduction

The economic design of $\overline{X}$-control charts is an approach for setting width of control limits, size and frequency of samples of a traditional Shewhart chart according to costs and features of the specific process under study. This methodology allows to customize the standard procedure, where the chart parameters are set only on statistical grounds, making the chart more suitable to the characteristics of the production process to be monitored.

A rich literature exists for this topic covering contributions starting essentially in 1956 with the original paper of Duncan until the most recent papers. In Section 2 a bibliographic review of this topic is proposed.

In Section 3 the model is described in detail both for the simpler continuous case and the discontinuous case.

Section 4 presents the main contribution of this paper: a sensitivity analysis is performed for the optimal solutions of the model with respect to incorrect estimates for inputs (costs associated to wrong decisions, non conforming production, or time required to inspect and repair the system after an out-of-control shift, etc.). In many situations inputs can not be specified without uncertainty by the operators and it is thus essential that the statistician is aware of consequences of wrong or inaccurate inputs specifications. As underlined at the end of the bibliographic review, similar contributions existing in literature analyze the effect of changes in input parameters with a "one-at-time" scheme: all inputs are kept fixed except for one which is in turn modified. In this paper, along with the method proposed by Panagos et al. (1985), the sensitivity analysis is performed through an analysis of variance applied to a two-level fractional factorial design: each input represents a factor with two possible levels; a factorial experiment is performed in order to simultaneously vary *all* inputs and correctly study the effect on a response variable which is in turn represented by the optimal value of the criterion function and the optimal values for each of the chart parameters. In Panagos et al. (1985), however, the ANOVA is performed neglecting interactions among factors, due to constrains in the alias structure of the design there used. In this paper, data from a much larger experimental design are used to highlight the role of two-factor interactions. Some brief concluding remarks are proposed in Section 5.

2 A review

Detailed reviews for economic design of control charts are Montgomery (1980), Vance (1983), Svoboda (1991) and Ho and Case (1994a).

In Duncan (1956) the original model is proposed together with the cost function associated to the statistical control process and the minimization procedure for this cost function. Nowadays, modern calculation techniques make approximated procedures negligible, but in the past feasibility reasons drew the attention on simplified versions of the original model (see, among the others, e.g. Chiu and Wetherill, 1974, where a constrained optimization is proposed). Montgomery (1982) reports a FORTRAN program to solve the optimization problem.

Duncan (1956) model assumes that the process continues in operation while searches for the assignable cause are made (*continuous process*). This assumption arises in order to keep the analysis as simple as possible, thanks to the exclusion of repair costs and recalibration to in-control state costs. Panagos et al. (1985) propose a new model in order to deal with the case of a *discontinuous process*, i.e. when the process must be shut down during the search of a cause and the authors compare the two approaches in order to state an optimal rule for the choice between them.

Saniga (1977) has presented a model for the joint economic design of $\overline{X}$ and R control charts. The same argument is treated in Saniga (1989) and Rahim and Costa (2000). A statistical model for the joint economic statistical design of $\overline{x}$ and S^2 control charts is proposed in Yang and Rahim (2000), where the problem is tackled through a Markov Chain approach.

Most studies regarding the economic design of control charts take samples of fixed size within a fixed time interval (i.e., the fixed-sampling interval, FSI). In a sample statistic with some indications of process variations, the variable-sampling-interval (VSI) control charts perform more effectively than the FSI ones due to a higher sampling rate. Therefore the VSI model has been suggested to improve the conventional FSI policy (Reynolds et al. (1988), Cui and Reynolds (1988), Reynolds (1989), Reynolds (1996), Runger and Pignatiello (1991), Baxley (1995)). Note that of course this is a response-adaptive procedure. A survey about statistical and economic design of adaptive control charts is Tagaras (1998).

Varying the sample size is another way of varying the sampling rate as a function of process data. Variable sample size (VSS) charts have also been proposed in the papers of Prabhu et al. (1993) and Reynolds (1996). Park and Reynolds (1999) combined the features of VSI and VSS charts and proposed an economic design for variable sampling rate (VSR) $\overline{X}$ chart with multiple assignable causes. In their study, two sample sizes (n_1, n_2), two sampling intervals (h_1, h_2) and three limits (C_s, C_l, C) are used for design parameters.

Costa and Rahim (2000) considers the problem of a continuous production process where both the mean and variance are simultaneously monitored by an $\overline{X}$ and R chart respectively. The process may be spoiled by two independent assignable causes (one cause changes the process mean and the other changes the process variance). However, the occurrence of one kind of assignable cause does not preclude the occurrence of the other. It is also assumed that the occurrence times of the assignable causes are described by Weibull distributions with increasing failure rates. A VSI scheme is adopted. A two-step search procedure is employed to determine the optimal design parameters. The economic design model is then extended to an economic-statistical design model for achieving desired levels of statistical performance while minimizing the expected cost.

Ohta et al. (2002) propose an economic model for the selection of time-varying control chart parameters for monitoring on-line the mean and variance of a normally distributed quality characteristic. As in Costa and Rahim (2000), the process is subject both to an assignable cause affecting the process mean and to an independent cause affecting the process variance (with Weibull occurrence times). The paper combines two existing models: the model of Costa and Rahim (2000) and the model of Ohta and Rahim (1997) for a dynamic economic design of control charts, where a single assignable cause occurs according to a Weibull distribution and all design parameters are time varying. The advantages of the proposed model over traditional $\overline{X}$ and R control charts with fixed parameters are presented.

The original Duncan's model assumes an exponential distribution for the length of the in-control period. Since this assumption entails a "memoryless" process, both Banerjee and Rahim (1988) and McWilliams (1989) extend the model to a Weibull distribution. The extension is performed through the setting of Lorenzen and Vance (1986) model. The latter model was proposed in order to extend economic design to any Shewart-type control chart. McWilliams (1989), in particular, evaluates the cost penalty due to wrongly assuming an exponential distribution instead of a Weibull one in four different situations: the comparison shows that cost loss due to the simpler exponential distribution is negligible. The analysis of Banerjee and Rahim (1988) is more complex. This paper too ends up proving that no significant difference exist between exponential and Weibull assumptions for a fixed sampling interval. That result looks however secondary with respect to the rest of their work where a great evidence is given to proving that the variable-sampling-interval strategy provides a great advantage over fixed under the Weibull assumption. It has to be remarked that although the length of each sampling interval is nonconstant, Banerjee and Rahim (1988) did not outline an adaptive control chart, since changes in the chart parameters do not depend upon previous response values. Further Rahim (1993) extended Banerjee and Rahim's model to allow the possibility of age-dependent replacement before failure when such a replacement yields economic benefits. Rahim and Banerjee (1993) is an extension of the work of Banerjee and Rahim (1988), where a general distribution of in-control periods having an increasing failure rate is assumed and the possibility of age-dependent repair before failure is considered. Several different truncated and non truncated probability models are chosen. It is proposed that economic benefits can be achieved by adopting a non uniform inspection scheme and by truncating a production cycle when it attains a certain age. A Weibull failure model is assumed also in Zhang and Berardi (1997).

In Duncan (1971) a generalization of the original model is suggested in order to remove the hypothesis of a unique assignable cause for out-of-control process state. A different approach for the same problem is shown in Knappenberger and Grandage (1969).

Chen and Yang (2000) proposed and constructed an economic design of $\overline{X}$-control chart for a multiplicity-cause model when the assignable causes follow Weibull distributions.

The economic VSI designs for a general $\overline{X}$ control chart and a continuous $\overline{X}$ control chart have been proposed by Bai and Lee (1998) and Yu and Chen (2005) respectively. With regard to the process data that are not normally distributed, Chen (2003) provided an economic-statistical design for the VSI $\overline{X}$ control chart. Previously cited works deal with a single assignable cause. Yu and Hou (2006) develop an economic model for the VSI control chart with multiple assignable causes.

Taguchi's quadratic loss function has been quite recently embedded in the economic design of control charts in Alexander et al. (1995), Chou et al. (2000) (where the Burr distribution is also assumed in order to replace the usual assumption of gaussian data), Liu et al. (2002) (where correlated measurements within a sample are also assumed), and Ben-Daya and Duffuaa (2003).

Many industrial products and processes are characterized by more than one measurable quantity, and their joint effect describes product quality. The economic design of T^2 control charts, the multivariate analogue of $\overline{X}$ control charts, is analyzed in Montgomery and Klatt (1972). Lowry and Montgomery (1995) is a review of multivariate control charts. Kapur and Cho (1996) derived the multivariate loss function to describe the relationship of m (for $m \geq 2$) correlated quality characteristics and the associated quality losses. Chou et al. (2002), combining the cost function developed by Montgomery and Klatt (1972) and the multivariate quality loss function presented by Kapur and Cho (1996), outline the procedure to carry out the design of multivariate charts with quality loss function.

Aparisi and Haro (2003) develop a T^2 control chart with variable sample size and variable sampling interval.

Parkhideh and Parkhideh (1996) presents a general version of Duncan's model for charts where supplementary runs rules are used to increase the sensitivity of the chart to small shifts. Moreover, a flexible-zone methodology in that the zone widths are not fixed a priori but are determined by economic optimization on the basis of the particular situation at work.

The economic design of CUSUM control charts is analyzed in Taylor (1968), Chiu (1974), Goel and Wu (1973) and Goel (1968).

Ho and Case (1994b) proposed the economically based EWMA control charts, while Montgomery et al. (1995) proposed, for the same model, additional statistical constrains on ARL or ATS.

Chen and Yang (2002) proposed a model for economic design of a moving average control chart with a Weibull failure mechanism.

Rahim (1994) presents an integrated cost model (with production setup cost, inventory holding cost and control chart implementation cost) for simultaneous determination of production quantity, inspection schedule and control chart design (when the in-control period follows a general probability distribution with increasing failure rate). Rahim and Ben-Daya (1998) generalizes Rahim's model by introducing a more realistic assumption concerning the stoppage of the machine during a false alarm.

Koo and Case (1990) first proposed an economic design for $\overline{X}$-control charts for use in monitoring a continuous flow process, i.e. when there is no well-defined production unit and each sample is not formed by units pulled all at one time (this would result in subgroup ranges of near zero and hence units are taken at regular intervals and their analytical results are combined into subgroups to be averaged and plotted on the chart).

Del Castillo and Montgomery (1996) present a new model for the optimal design of $\overline{X}$ charts utilized for the statistical monitoring of processes where production runs have a finite duration. The proposed model considers the effect of the setup operation on the chart design. The model contains both Duncan's model and Ladany's model (Ladany, 1973) as particular cases.

An optimal design for the control chart may be detected only if process parameters and single cost functions are known. Since in practical situations their estimates can not fully trusted, it is important to make evidence about the effect of misspecified inputs on the performance of the control charts. This is essentially the aim of Chiu (1976). This paper compares the optimal design with each input correctly estimated with suboptimal designs obtained when, in turn, one of the inputs is over- or under-specified. The author ends up pointing to δ, μ_o and σ as crucial parameters (see Table 1, p. 6 for nomenclature here used), since errors in their specification lead to noteworthy loss of efficiency. Montgomery and Storer (1986) approach the same problem of uncertainty of input quantities in a completely different way, i.e. by essentially reducing their number, through a model simplification (semi-economic model).

Parkhideh and Case (1989) developed a six decision variables in their economic model, while Ohta and Rahim (1997) proposed an alternative and simplified design methodology that reduces the number of six design variables to three.

In Yu and Hou (2006) a sensitivity analysis is proposed with respect to changes of time and cost required to repair the system, costs related to producing non conforming products, sampling costs, size of the mean shift leading to an out-of-control state and failure rate. Note that in this case, the sensitivity analysis is performed with a one-factor-at-time scheme. The same type of one-factor-at-time sensitivity analysis is performed also in Zhang and Berardi (1997) (w.r.t. both type I error and tipe II error probability, ATS, size of the mean shift leading to an out-of-control state and to parameters of the Weibull distribution).

3 Definition and assumptions

3.1 The continuous process

Duncan (1956) assumes that the process is characterized by an in-control state μ_o and that a single assignable cause, which occurs at random, results in a shift in the mean from μ_o to either $\mu_o + \delta\sigma$ or $\mu_o - \delta\sigma$ (in the following, refer also to Table 1 for symbol definitions). The shift is assumed to occur after a period whose length is exponentially distributed with mean $1/\lambda$ (time units, say hours). Since samples

Table 1: Nomenclature used in this paper.

n	sample size
h	sampling interval
k	coefficient of the control limits
$\overline{x}$	sample mean
μ_o	mean of the process characteristic in in-control state
σ	standard deviation of the process characteristic
α	Type I error probability of the chart
β	Type II error probability of the chart
$(1-\beta)$	power of the chart
UCL	upper control limit
LCL	lower control limit
b	the fixed sampling cost
c	the variable sampling cost
W	the cost of finding and repairing an assignable cause
V_o	the net hourly income for operating in in-control state
V_1	the net hourly income when operating in out-of-control state
$M = V_o - V_1$	the hourly penalty cost of operating out of control
T	the cost of investigating a false alarm
λ	reciprocal of the average process in-control time
δ	magnitude of the process mean shift
g	the average sampling, inspecting, evaluating and plotting time for each sample
D	the time required to find and repair the assignable cause
D_1	the time required to inspect the system due to a false alarm
S_1	the setup time to restart the process after an out-of-control signal
S_c	the setup cost
E(Inc)	the expected net income per cycle
E(A)	the expected net hourly income (E(Inc)/E(T))
E(L)	the expected total loss per hour

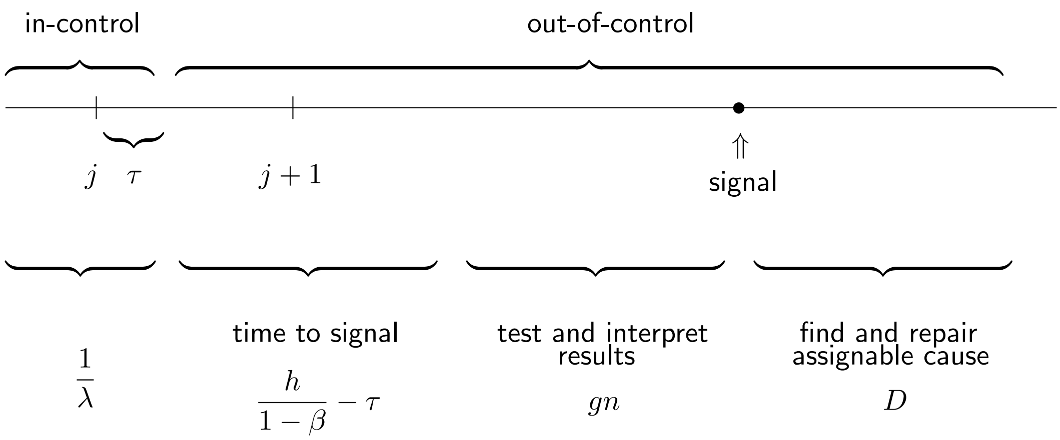


Figure 1: Production cycle for the continuous process.

of size n are taken at intervals of h hours, we can denote by

$$\tau = \frac{\displaystyle\int_{jh}^{(j+1)h} e^{-\lambda t}\lambda(t - jh)dt}{\displaystyle\int_{jh}^{(j+1)h} e^{-\lambda t}\lambda dt} = \frac{1 - (1 + \lambda h)e^{-\lambda h}}{\lambda(1 - e^{-\lambda h})} \tag{1}$$

the expected time of a shift occurrence, given that the shift arose between sample j and sample $j + 1$.

It is assumed that the process is monitored by an $\overline{X}$ chart with center line at μ_o and limits $\mu_o \pm k\sigma/\sqrt{n}$. This entails that the probability of a false alarm is

$$\alpha = 2\int_{k}^{\infty} \phi(z)dz, \tag{2}$$

where $\phi(\cdot)$ is the standard normal density function; moreover, after the mean shift, the probability of a signal in any subsequent sample is

$$1 - \beta = \int_{-\infty}^{-k-\delta\sqrt{n}} \phi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \phi(z)dz. \tag{3}$$

A production cycle is defined as the interval of time from the start of production, in a in-control state, until the detection and repair of an assignable cause. A schematic diagram of the cycle is proposed in Panagos et al. (1985) and a slightly modified version is here shown in Fig. 1.

Observe that what is commonly named "time to signal", i.e. $\frac{h}{1-\beta} - \tau$, should be named instead "time till extraction of the sample that will call a signal": a signal is effectively given after a further time period of length gn, required for testing and interpreting results. In this original model proposed by Duncan (1956) (and subsequently applied by many other contributions), the process is allowed to continue in operation even after an out-of-control signal during the search of an assignable cause. Panagos et al. (1985) refer to this kind of situation as *continuous process.* Given this assumption, the expected length of a production cycle is

$$\mathrm{E}(T) = \frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D, \tag{4}$$

where D denotes the time required to find and repair the assignable cause.

The expected net income per cycle, E(Inc), is obtained as the difference between the income both in the in-control and out-of-control periods and the process costs for the cycle. In detail, the hourly in-control income, V_o, has to be multiplied by the expected time the process is in control, $1/\lambda$; conversely, the hourly out-of-control income, V_1, has to be multiplied by the expected time the process is out-of-control,

$$\frac{h}{1-\beta} - \tau + gn + D \left(= \mathrm{E}(T) - \frac{1}{\lambda} \right). \tag{5}$$

The expected costs for each cycle include: (1) the sampling cost (average number of samples taken in the period, $\mathrm{E}(T)/h$, per cost of each sample, $b + cn$); (2) the cost for false alarms (average number of samples taken before the shift, $e^{-\lambda h}/(1 - e^{-\lambda h})$, per probability of a false alarm in a single sample, α, per cost of each false alarm, T); (3) the cost for searching and repairing the assignable cause, W. The formula for the expected net income *per cycle* is then,

$$\mathrm{E}(\mathrm{Inc}) = V_o \frac{1}{\lambda} + V_1 \left(\frac{h}{1-\beta} - \tau + gn + D \right) - W - \frac{T\alpha e^{-\lambda h}}{1 - e^{-\lambda h}} - (b + cn) \frac{\mathrm{E}(T)}{h}. \tag{6}$$

In order to obtain a measure which is independent from the length of the cycle, we can calculate the expected net *hourly* income as the ratio between expression (6) and expression (4):

$$\begin{aligned}
\mathrm{E}(A) &= \frac{\mathrm{E}(\mathrm{Inc})}{\mathrm{E}(T)} \\
&= \frac{1}{\mathrm{E}(T)} \left\{ V_o \frac{1}{\lambda} + V_1 \left(\frac{h}{1-\beta} - \tau + gn + D \right) - W - \frac{T\alpha e^{-\lambda h}}{1 - e^{-\lambda h}} - (b + cn) \frac{\mathrm{E}(T)}{h} \right\} \\
&= \frac{1}{\mathrm{E}(T)} \left\{ V_o \left[\mathrm{E}(T) - \left(\frac{h}{1-\beta} - \tau + gn + D \right) \right] + V_1 \left(\frac{h}{1-\beta} - \tau + gn + D \right) + \right. \\
&\qquad \left. - W - \frac{T\alpha e^{-\lambda h}}{1 - e^{-\lambda h}} - (b + cn) \frac{\mathrm{E}(T)}{h} \right\} \\
&= \frac{1}{\mathrm{E}(T)} \left\{ \left(\frac{h}{1-\beta} - \tau + gn + D \right) (V_1 - V_o) + V_o \mathrm{E}(T) - W - \frac{T\alpha e^{-\lambda h}}{1 - e^{-\lambda h}} + \right. \\
&\qquad \left. - (b + cn) \frac{\mathrm{E}(T)}{h} \right\}
\end{aligned}$$

$$= \frac{1}{\mathrm{E}(T)}\left\{\left(\frac{h}{1-\beta}-\tau+gn+D\right)(V_1-V_o)-W-\frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}}\right\}+V_o-\frac{(b+cn)}{h}. \tag{7}$$

But expected hourly income could be imagined as hourly expected loss, $\mathrm{E}(L)$, subtracted to hourly in-control income, V_o :

$$\mathrm{E}(A) = V_o - \mathrm{E}(L),$$

where

$$\begin{aligned}\mathrm{E}(L) &= V_o - \mathrm{E}(A)\\ &= \frac{1}{\mathrm{E}(T)}\left\{\left(\frac{h}{1-\beta}-\tau+gn+D\right)(V_o-V_1)+W+\frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}}\right\}+\frac{(b+cn)}{h}\\ &= \frac{b+cn}{h}+\frac{M[\frac{h}{1-\beta}-\tau+gn+D]+W+T\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda}+\frac{h}{1-\beta}-\tau+gn+D}. \end{aligned} \tag{8}$$

Obviously, minimizing expected loss (8), or maximizing (7) with respect to (n, h, k) is totally equivalent. Observe that "loss" is here intended as cost and should not be confused with "loss" in the decision theory sense (which is instead used in Alexander et al. (1995) and in the other papers mentioned in the review at the beginning of this paper [introduction], where Taguchi's quadratic loss function is applied to control charts).

3.2 The discontinuous process

Panagos et al. (1985) propose a new model in order to deal with the case of a *discontinuous process*, i.e. when the process must be shut down during the search of a cause. In this situation the production cycle is quite different with respect to the continuous process. In Fig. 2 the schematic diagram of Panagos et al. (1985) is shown. Observe that the position in the diagram of the period connected to production stop due to a false alarm may be misleading, since, obviously, it does not occur after the out-of-control period (it represents a break - or a sequence of breaks - during the in-control period). On the whole, the expected length of the complete production cycle is

$$\mathrm{E}(T_1) = \frac{1}{\lambda}+\frac{h}{1-\beta}-\tau+gn+D+S_1+\frac{(D_1+S_1)\alpha e^{-\lambda h}}{(1-e^{-\lambda h})}, \tag{9}$$

where D_1 is the time to search a non-existent assignable cause (it may be different from D, the time to search a true assignable cause), S_1 is the setup time to restart the process after the production stop (either after removing an assignable cause or after experiencing a false alarm).

For the discontinuous process too, the expected net income *per cycle,* $\mathrm{E}(\mathrm{Inc}_1)$, is obtained as the difference between the income both in the in-control and out-of-control periods and the process costs for the cycle. Here the hourly out-of-control

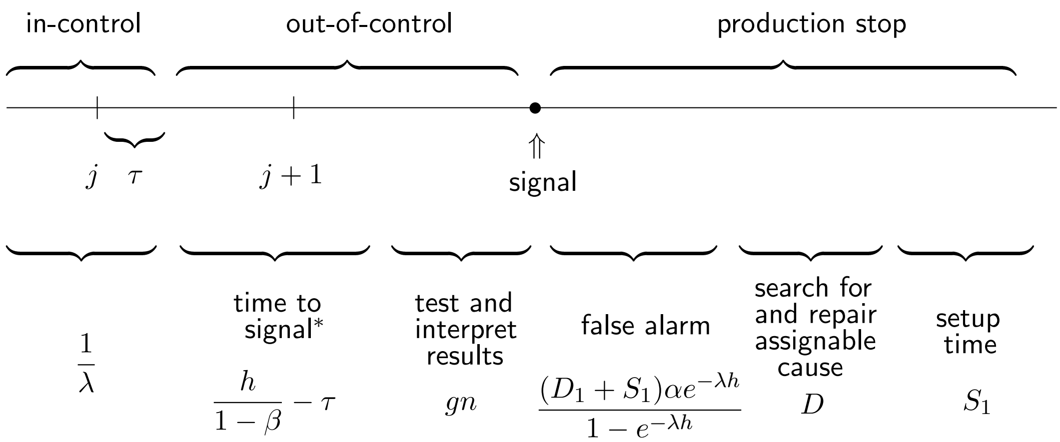


Figure 2: Production cycle for the discontinuous process.

income, V_1 has to be multiplied by the expected time the process is out-of-control,

$$\frac{h}{1-\beta} - \tau + gn. \tag{10}$$

With respect to the continuous process, here we have some differences in expected costs: (1) the sampling cost does not apply during the process stop; (2) besides the cost for searching and repairing the assignable cause, W, there is also a setup cost, S_c.

The expected net income per cycle is

$$\begin{aligned} \mathrm{E}(\mathrm{Inc}_1) &= V_o \frac{1}{\lambda} + V_1 \left(\frac{h}{1-\beta} - \tau + gn \right) + \\ & \quad -W - S_c - \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} - (b+cn)\frac{1}{h}\left(\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn \right). \end{aligned} \tag{11}$$

Observe that previous formula given in Panagos et al. (1985) does not include the setup cost in the cost component due to false alarm (while the setup time, S_1, following a false alarm is included in the expected length of the cycle). This approach is correct only if T, the cost of searching a false alarm, includes in the discontinuous case also the setup cost.

As we did for the continuous process, we can calculate the expected net *hourly* income as the ratio between expression (11) and expression (9):

$$\begin{aligned} \mathrm{E}(A_1) &= \frac{\mathrm{E}(I_1)}{\mathrm{E}(T_1)} = \frac{1}{\mathrm{E}(T_1)} \left\{ V_o \frac{1}{\lambda} + V_1 \left(\frac{h}{1-\beta} - \tau + gn \right) - W - S_c - \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} + \right. \\ & \quad \left. -(b+cn)\frac{1}{h}\left(\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn \right) \right\} \end{aligned} \tag{12}$$

Let

$$B' = \frac{h}{1-\beta} - \tau + gn$$
$$C' = D + S_1 + \frac{(D_1 + S_1)\alpha e^{-\lambda h}}{(1 - e^{-\lambda h})}.$$

Then expression (12) simplifies to

$$\begin{aligned}
\mathrm{E}(A_1) &= \frac{\mathrm{E}(I_1)}{\mathrm{E}(T_1)} \\
&= \frac{1}{\mathrm{E}(T_1)} \left\{ V_o \frac{1}{\lambda} + V_1 B' + \right. \\
&\qquad \left. -W - S_c - \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} - (b+cn)\frac{1}{h}\left(\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn\right)\right\} \\
&= \frac{1}{\mathrm{E}(T_1)} \left\{ V_o \left[\mathrm{E}(T_1) - B' - C'\right] + V_1 B' + \right. \\
&\qquad \left. -W - S_c - \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} - (b+cn)\frac{1}{h}\left[\mathrm{E}(T_1) - C'\right]\right\} \\
&= \frac{1}{\mathrm{E}(T_1)} \left\{ V_o \mathrm{E}(T_1) - V_o B' - V_o C' + V_1 B' + \right. \\
&\qquad \left. -W - S_c - \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} - \frac{(b+cn)}{h}\mathrm{E}(T_1) + \frac{(b+cn)}{h} C'\right\} \\
&= V_o - \frac{(b+cn)}{h} + \\
&\quad + \frac{1}{\mathrm{E}(T_1)} \left\{ -V_o B' - V_o C' + V_1 B' - W - S_c - \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} + \frac{(b+cn)}{h} C'\right\} \\
&= V_o - \frac{(b+cn)}{h} + \\
&\quad + \frac{1}{\mathrm{E}(T_1)} \left\{ (V_1 - V_o) B' - V_o C' - W - S_c - \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} + \frac{(b+cn)}{h} C'\right\} \qquad (13)
\end{aligned}$$

It follows that the hourly expected loss can be written as

$$\begin{aligned}
\mathrm{E}(L_1) &= V_o - \mathrm{E}(A_1) \\
&= \frac{(b+cn)}{h} + \\
&\quad \frac{1}{\mathrm{E}(T_1)} \left\{ (V_o - V_1) B' + W + S_c + \frac{T\alpha e^{-\lambda h}}{1-e^{-\lambda h}} + V_o C' - \frac{(b+cn)}{h} C'\right\} \\
&= \frac{b+cn}{h}\left(1 - \frac{C'}{\mathrm{E}(T_1)}\right) + \frac{MB' + W + S_c + T\alpha e^{-\lambda h}/(1-e^{-\lambda h}) + V_o C'}{\mathrm{E}(T_1)}. \qquad (14)
\end{aligned}$$

Observe that Duncan (1956) and Panagos et al. (1985) optimize a simplified version of expected loss (8) and (14), obtained through the following approximations:

$$\tau = \frac{1 - (1+\lambda h)e^{-\lambda h}}{\lambda(1 - e^{-\lambda h})} \simeq \frac{h}{2} - \lambda \frac{h^2}{12}, \qquad (15)$$

and, for the expected number of samples before the shift,

$$\frac{e^{-\lambda h}}{(1-e^{-\lambda h})} \simeq \frac{1}{\lambda h}. \tag{16}$$

In Duncan (1956) details about the accuracy of both approximations are given. Since the aim of this paper is to deepen the work of Panagos et al. (1985), the same approximations will be used here. Accordingly, formulas (8) and (14) are modified, respectively, to

$$\begin{aligned}
\mathrm{E}(L) &= V_o - \mathrm{E}(A) \\
&= \frac{b+cn}{h} + \frac{M[\frac{h}{1-\beta} - \tau + gn + D] + W + T\alpha e^{-\lambda h}/(1-e^{-\lambda h})}{\frac{1}{\lambda} + \frac{h}{1-\beta} - \tau + gn + D} \\
&\simeq \frac{b+cn}{h} + \frac{M[\frac{h}{1-\beta} - (\frac{h}{2} - \lambda\frac{h^2}{12}) + gn + D] + W + \frac{T\alpha}{\lambda h}}{\frac{1}{\lambda} + \frac{h}{1-\beta} - (\frac{h}{2} - \lambda\frac{h^2}{12}) + gn + D} \\
&= \frac{b+cn}{h} + \frac{M[h(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}) + gn + D] + W + \frac{T\alpha}{\lambda h}}{\frac{1}{\lambda} + h(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}) + gn + D}
\end{aligned} \tag{17}$$

and

$$\begin{aligned}
\mathrm{E}(L_1) &= \frac{b+cn}{h}\left(1 - \frac{C'}{\mathrm{E}(T_1)}\right) + \frac{MB' + W + S_c + T\alpha e^{-\lambda h}/(1-e^{-\lambda h}) + V_o C'}{\mathrm{E}(T_1)} \\
&\simeq \frac{b+cn}{h}\left(1 - \frac{D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}}{\frac{1}{\lambda} + \frac{h}{1-\beta} - (\frac{h}{2} - \lambda\frac{h^2}{12}) + gn + D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}}\right) + \\
&\quad + \frac{M\left[\frac{h}{1-\beta} - (\frac{h}{2} - \lambda\frac{h^2}{12}) + gn\right] + W + S_c + \frac{T\alpha}{\lambda h} + V_o\left[D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}\right]}{\frac{1}{\lambda} + \frac{h}{1-\beta} - (\frac{h}{2} - \lambda\frac{h^2}{12}) + gn + D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}}. \\
&= \frac{b+cn}{h}\left(1 - \frac{D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}}{\frac{1}{\lambda} + h(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}) + gn + D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}}\right) + \\
&\quad + \frac{M\left[h(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}) + gn\right] + W + S_c + \frac{T\alpha}{\lambda h} + V_o\left[D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}\right]}{\frac{1}{\lambda} + h(\frac{1}{1-\beta} - \frac{1}{2} + \frac{\lambda h}{12}) + gn + D + S_1 + \frac{(D_1+S_1)\alpha}{\lambda h}}.
\end{aligned} \tag{18}$$

4 Sensitivity analysis

The main purpose of Panagos et al. (1985) paper was to highlight the cost penalty due to wrong model selection: the authors of that paper, through the analysis of 32 situations with different parameters' configurations, concluded that there is a serious penalty when a continuous process model design is used for a discontinuous manufacturing process (which is most likely to occur in practice, since Duncan's original model is more commonly adopted). Lower penalties are instead associated to selection of the optimal chart with a discontinuous process model design for a continuous process.

Besides previous contribution, Panagos et al. (1985) paper describes a sensitivity analysis of the performance of the optimal control charts with respect to each of the parameters in the target function. This is done through an experimental design where cost and process parameters play the role of factors while optimal values for the loss function ((17) and (18)), n, k or h represent alternatively the univariate response variable. In the continuous case there are 9 factors while in the discontinuous case there are 13 factors. The experimental design schedules two levels for each factor.

A single experimental run consists of the minimization of $\mathrm{E}(L)$ (or $\mathrm{E}(L_1)$) with respect to n, k, and h when cost and process parameters are fixed according to the corresponding factors' levels profile. The minimization is performed through the evaluation of the function on a three-dimensional search grid and may require a non negligible computing time.

Remark that from the statistical point of view this might appear as a deterministic process without natural variability. Actually this is not exactly true, since, given the levels' profile, the observed responses (i.e. the optimal values for $\mathrm{E}(L)$, n, k or h arising from the optimization procedure above described) do not perfectly equal the true optimal values: the reason is the discrete search grid. We may thus imagine that differences between true and observed response mimic the presence of a stochastic term.

Panagos et al. (1985), maybe due to computational burden (even heavier more than twenty years ago), implemented a 2^{9-4} fractional factorial design for the continuous case and a 2^{13-8} fractional factorial design for the discontinuous case. In both situations the design used is a 32 runs resolution IV design (a detailed presentation of fractional factorial designs could be found, e.g., in Montgomery (2004)). This means that only main effects could be isolated, while aliases were created among the two-factor interactions. For this reason, the authors state explicitly that F-ratios in ANOVA tables should not be interpreted as significance measures of the effects of each factor (because of the potential inflating effect of neglected interactions on the estimate of the experimental error). Panagos et al. (1985), instead, intended to use their analysis to see whether each factor has a positive or a negative effect on the response variable.

We argue however that, if non negligible two-factor interactions affect the response variable, main effects are meaningless (both with respect to their magnitude and with respect to their sign). The aim of this paper is thus to analyze the data collected through a much larger experimental design, namely a resolution V design with 256 runs, from which two-factor interactions may be estimated.

4.1 The designs used

In this work levels are chosen exactly as in Panagos et al. (1985). A summary of factors and levels used is proposed in Table 2. Since the 9 factors of the continuous case are a proper subset of the 13 factors of the discontinuous case, it was decided

Table 2: Factors and levels used in the experimental design.

DISCONT. PROCESS (factors A–M); CONT. PROCESS (factors A–I)

Factors	Levels	
	Low	High
A = M	50.00	100.00
B = δ	1.00	2.00
C = λ	0.01	0.05
D = g	0.05	0.50
E = D	3.00	20.00
F = b	0.50	5.00
G = c	0.10	1.00
H = W	35.00	250.00
I = T	50.00	500.00
J = V_o	50.00	150.00
K = S	10.00	100.00
L = S_c	0.05	1.00
M = D_1	4.00	40.00

to use "paired " designs, i.e. designs for which the first 9 factor levels for the discontinuous case are identical to the corresponding factor level in the continuous case.

For the continuous case, a 2^{9-1} design was used with defining contrast given by

$$\tilde{I} = \text{ABCDEFGHI}, \tag{19}$$

where $\tilde{I}$, denotes the identity column and should not be confused with I, which is one of the factors used, namely the factor describing the effect of T, the cost of investigating a false alarm.

For the discontinuous case, a 2^{13-5} design was used with defining contrasts given by

$$\begin{array}{lll} \tilde{I} = \text{ABCDEFGHI} & \tilde{I} = \text{BCDEJ} & \tilde{I} = \text{ACDFK} \\ \tilde{I} = \text{BEFHL} & \tilde{I} = \text{ABCGM}. & \end{array} \tag{20}$$

As previously mentioned, for each level factors' configuration, the minimization of E(L) or E(L_1) was performed. For each of the two designs, four ANOVA were performed by assigning to the response values either the minimum value of the target function, or the optimal values for n, k or h.

4.2 Results for the continuous case

In Table 3 a summary of results obtained by Panagos et al. (1985) is proposed. In particular, that table focuses on significant main effects on responses arising from the ANOVA tables for the 32 runs experiments (where obviously interactions had to be neglected).

Results from the 256 runs experiments describe a different situation with respect to $E(L)$ (see Table 4): 15 two-factor interactions are significant and *all* factors

Table 3: Continuous case: significant main effects according to Panagos et al. (1985).

Response variable	Significant effects (+: positive effect) (–: negative effect)						
$E(L)$	A (M) +	C (λ) +	E (D) +				
n	B (δ) –	D (g) –	F (b) +	G (c) –	I (T) +		
h	A (M) –	B (δ) –	C (λ) –	F (b) +	G (c) +		
k	B (δ) +	C (λ) –	D (g) –	E (D) –	F (b) –	G (c) –	I (T) +

are involved with the exception of F (b) (fixed sampling cost), significantly acting however as a main effect.

Plots of interesting significant effects on $E(L)$ are illustrated in Fig. 3 (the complete set of main effect plots and interaction plots can be found in the Appendix). In particular, A (M), the penalty cost due to out-of-control, interacts with C (λ), D (g) and E (D) : the effect on $E(L)$ of a greater cost due to out-of-control is larger whenever assignable causes arise more frequently, time to test and interpret results or time to find and repair the assignable cause are larger. Moreover B (δ) interacts with C (λ) and I (T) : quite intuitively, a greater precision – lower values for δ – entails greater costs further increased for a less stable process or when experiencing expensive false alarms. Interaction CE shows that larger values for λ increase costs with a larger effect when D, time to find and repair the assignable cause are longer. Finally we underline interaction CH: the effect of a less stable process is increased whenever finding and repairing an assignable cause is more expensive.

The sample size, n (see Table 5 and Fig. 4), is significantly affected by interactions involving all effects with the exception of H (W) (whose main effect is also negligible). B (δ) interacts both with D (g) and I (T) : greater power (lower values for δ) can be achieved obviously with larger sample size, but the effect on size is weaker when g is large (essentially g counterbalance power) and larger for more expensive false alarms. D (g) interacts also with G (c) : a shorter time to interpret results encourages an increase in sample size, especially with a low variable sampling cost. A peculiar behaviour relates to interaction between E (D) and G (c) : with a low variable sampling cost, a longer time to repair the assignable cause increases the sample size; conversely, when c is large, D affects negatively the sample size (maybe because when c is large, the effect of a larger D is faced through smaller and more frequent samples). Interaction between F (b) and G (c) is unsurprising: an increase in fixed sampling costs produces larger samples, even larger when variable sampling costs are low.

The ANOVA table for the sampling interval, h, highlights the presence of several interactions since only 9 out of 36 are non significant (see Table 6 and Fig. 5). In particular, A (M) and C (λ) interact with almost any other factor. For example, interaction AB shows that higher penalty costs due to out-of-control entail more frequent samples, with larger effect when δ is smaller. The interaction between B (δ) and G (c) depicts an interesting situation: samples are more frequent when less

Table 4: Continuous Process: Analysis of Variance for $E(L)$.

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	10775.7	1	10775.7	4515.94	0.0000	12.9758	0.19309
B	449.282	1	449.282	188.29	0.0000	-2.64953	0.19309
C	33369.9	1	33369.9	13984.81	0.0000	22.8343	0.19309
D	341.724	1	341.724	143.21	0.0000	2.31072	0.19309
E	13938.1	1	13938.1	5841.26	0.0000	14.7575	0.19309
F	157.576	1	157.576	66.04	0.0000	1.56911	0.19309
G	205.564	1	205.564	86.15	0.0000	1.79219	0.19309
H	1142.72	1	1142.72	478.90	0.0000	4.22552	0.19309
I	85.2141	1	85.2141	35.71	0.0000	1.15389	0.19309
AB	28.4573	1	28.4573	11.93	0.0007	-0.666818	0.19309
AC	2522.83	1	2522.83	1057.28	0.0000	6.27847	0.19309
AD	35.5315	1	35.5315	14.89	0.0002	0.745103	0.19309
AE	2015.07	1	2015.07	844.49	0.0000	5.61119	0.19309
AF	7.34391	1	7.34391	3.08	0.0808	0.338746	0.19309
AG	6.13566	1	6.13566	2.57	0.1103	0.309628	0.19309
AH	0.167716	1	0.167716	0.07	0.7912	0.0511914	0.19309
AI	7.28824	1	7.28824	3.05	0.0820	0.337459	0.19309
BC	23.8228	1	23.8228	9.98	0.0018	-0.610107	0.19309
BD	34.1407	1	34.1407	14.31	0.0002	-0.730376	0.19309
BE	32.2383	1	32.2383	13.51	0.0003	0.709735	0.19309
BF	0.549817	1	0.549817	0.23	0.6317	0.092687	0.19309
BG	16.4576	1	16.4576	6.90	0.0093	-0.5071	0.19309
BH	0.749973	1	0.749973	0.31	0.5756	0.108251	0.19309
BI	18.7938	1	18.7938	7.88	0.0055	-0.541897	0.19309
CD	32.003	1	32.003	13.41	0.0003	0.70714	0.19309
CE	1906.17	1	1906.17	798.85	0.0000	5.45746	0.19309
CF	8.96238	1	8.96238	3.76	0.0540	0.374216	0.19309
CG	5.07951	1	5.07951	2.13	0.1461	0.281722	0.19309
CH	357.916	1	357.916	150.00	0.0000	2.36483	0.19309
CI	7.64161	1	7.64161	3.20	0.0750	0.345543	0.19309
DE	40.123	1	40.123	16.81	0.0001	-0.791784	0.19309
DF	1.68298	1	1.68298	0.71	0.4020	0.162162	0.19309
DG	6.64798	1	6.64798	2.79	0.0966	-0.322296	0.19309
DH	0.933037	1	0.933037	0.39	0.5324	-0.120742	0.19309
DI	16.1519	1	16.1519	6.77	0.0099	0.502368	0.19309
EF	6.68101	1	6.68101	2.80	0.0958	-0.323096	0.19309
EG	7.27502	1	7.27502	3.05	0.0823	-0.337153	0.19309
EH	54.4633	1	54.4633	22.82	0.0000	-0.922491	0.19309
EI	6.18402	1	6.18402	2.59	0.1089	-0.310846	0.19309
FG	9.17141	1	9.17141	3.84	0.0513	-0.378554	0.19309
FH	0.181026	1	0.181026	0.08	0.7833	-0.0531839	0.19309
FI	0.100774	1	0.100774	0.04	0.8374	-0.0396812	0.19309
GH	0.150124	1	0.150124	0.06	0.8022	-0.0484323	0.19309
GI	6.65657	1	6.65657	2.79	0.0964	0.322504	0.19309
HI	0.182204	1	0.182204	0.08	0.7826	-0.0533566	0.19309
Error	501.091	210	2.38615				
Total (corr)	68190.8	255					

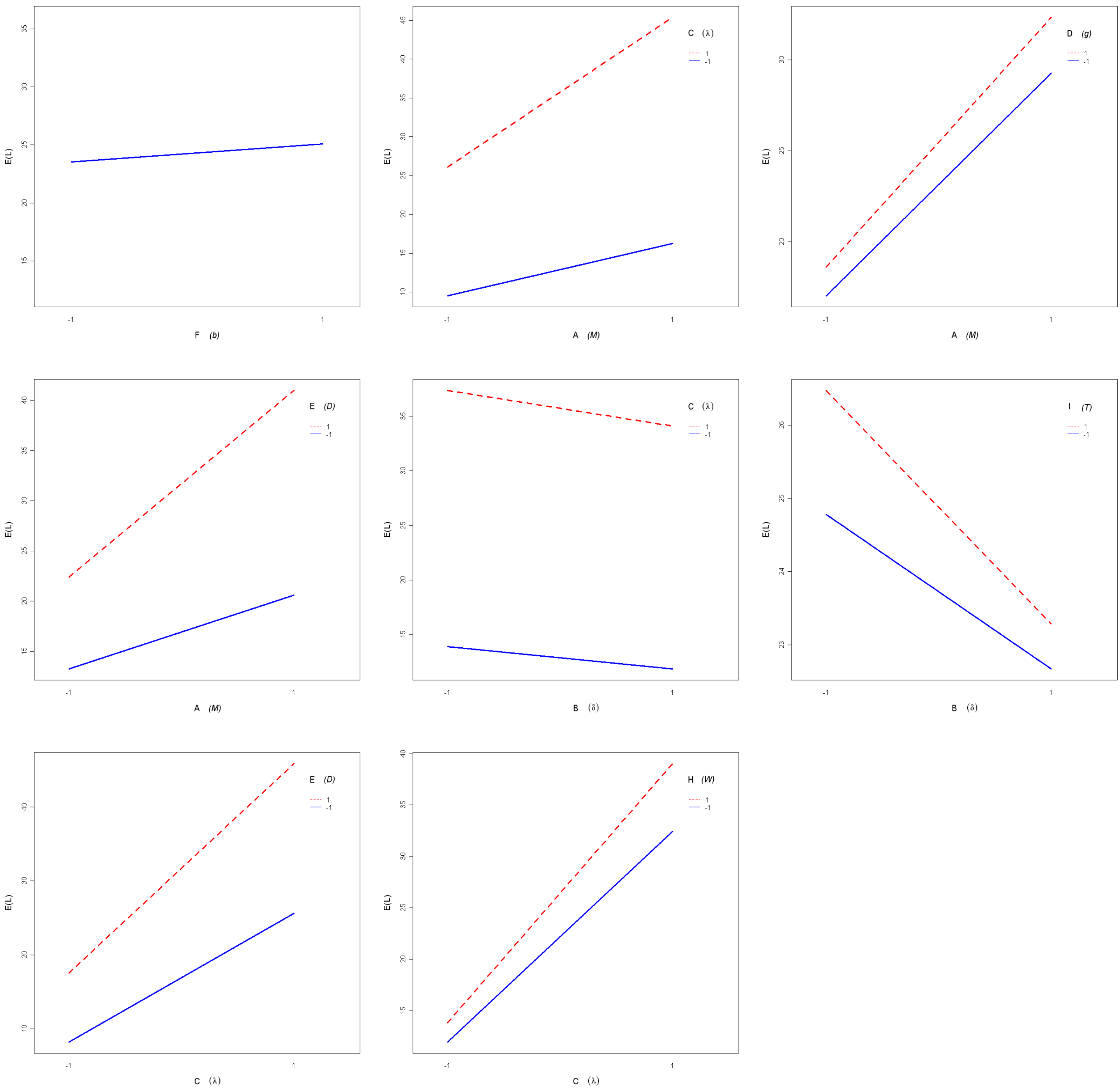


Figure 3: Continuous Process: primary effects plots for $E(L)$.

Table 5: Continuous Process: Analysis of Variance for n.

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	32.3477	1	32.3477	86.53	0.0000	-0.710938	0.0764288
B	1843.63	1	1843.63	4931.51	0.0000	-5.36719	0.0764288
C	98.7539	1	98.7539	264.16	0.0000	-1.24219	0.0764288
D	1282.54	1	1282.54	3430.64	0.0000	-4.47656	0.0764288
E	0.878906	1	0.878906	2.35	0.1267	0.117188	0.0764288
F	91.4414	1	91.4414	244.60	0.0000	1.19531	0.0764288
G	308.441	1	308.441	825.05	0.0000	-2.19531	0.0764288
H	0.660156	1	0.660156	1.77	0.1853	0.101563	0.0764288
I	585.035	1	585.035	1564.91	0.0000	3.02344	0.0764288
AB	10.9727	1	10.9727	29.35	0.0000	0.414063	0.0764288
AC	0.0351563	1	0.0351563	0.09	0.7594	-0.0234375	0.0764288
AD	1.12891	1	1.12891	3.02	0.0837	-0.132813	0.0764288
AE	0.191406	1	0.191406	0.51	0.4751	0.0546875	0.0764288
AF	0.191406	1	0.191406	0.51	0.4751	-0.0546875	0.0764288
AG	2.06641	1	2.06641	5.53	0.0196	0.179688	0.0764288
AH	0.878906	1	0.878906	2.35	0.1267	-0.117188	0.0764288
AI	0.660156	1	0.660156	1.77	0.1853	-0.101563	0.0764288
BC	28.2227	1	28.2227	75.49	0.0000	0.664063	0.0764288
BD	397.504	1	397.504	1063.28	0.0000	2.49219	0.0764288
BE	0.0976563	1	0.0976563	0.26	0.6098	-0.0390625	0.0764288
BF	15.5039	1	15.5039	41.47	0.0000	-0.492188	0.0764288
BG	67.0352	1	67.0352	179.31	0.0000	1.02344	0.0764288
BH	0.0351563	1	0.0351563	0.09	0.7594	-0.0234375	0.0764288
BI	177.223	1	177.223	474.05	0.0000	-1.66406	0.0764288
CD	0.472656	1	0.472656	1.26	0.2621	0.0859375	0.0764288
CE	0.00390625	1	0.00390625	0.01	0.9187	-0.0078125	0.0764288
CF	0.878906	1	0.878906	2.35	0.1267	-0.117188	0.0764288
CG	2.06641	1	2.06641	5.53	0.0196	0.179688	0.0764288
CH	0.0976563	1	0.0976563	0.26	0.6098	0.0390625	0.0764288
CI	1.12891	1	1.12891	3.02	0.0837	-0.132813	0.0764288
DE	3.28516	1	3.28516	8.79	0.0034	0.226563	0.0764288
DF	9.37891	1	9.37891	25.09	0.0000	-0.382813	0.0764288
DG	139.535	1	139.535	373.24	0.0000	1.47656	0.0764288
DH	0.0351563	1	0.0351563	0.09	0.7594	0.0234375	0.0764288
DI	21.9727	1	21.9727	58.77	0.0000	-0.585938	0.0764288
EF	0.00390625	1	0.00390625	0.01	0.9187	-0.0078125	0.0764288
EG	4.78516	1	4.78516	12.80	0.0004	-0.273438	0.0764288
EH	0.660156	1	0.660156	1.77	0.1853	-0.101563	0.0764288
EI	0.316406	1	0.316406	0.85	0.3586	0.0703125	0.0764288
FG	9.37891	1	9.37891	25.09	0.0000	-0.382813	0.0764288
FH	0.0351563	1	0.0351563	0.09	0.7594	-0.0234375	0.0764288
FI	1.41016	1	1.41016	3.77	0.0535	0.148438	0.0764288
GH	0.316406	1	0.316406	0.85	0.3586	-0.0703125	0.0764288
GI	1.72266	1	1.72266	4.61	0.0330	0.164063	0.0764288
HI	0.00390625	1	0.00390625	0.01	0.9187	-0.0078125	0.0764288
Error	78.5078	210	0.373847				
Total (corr.)	5221.46	255					

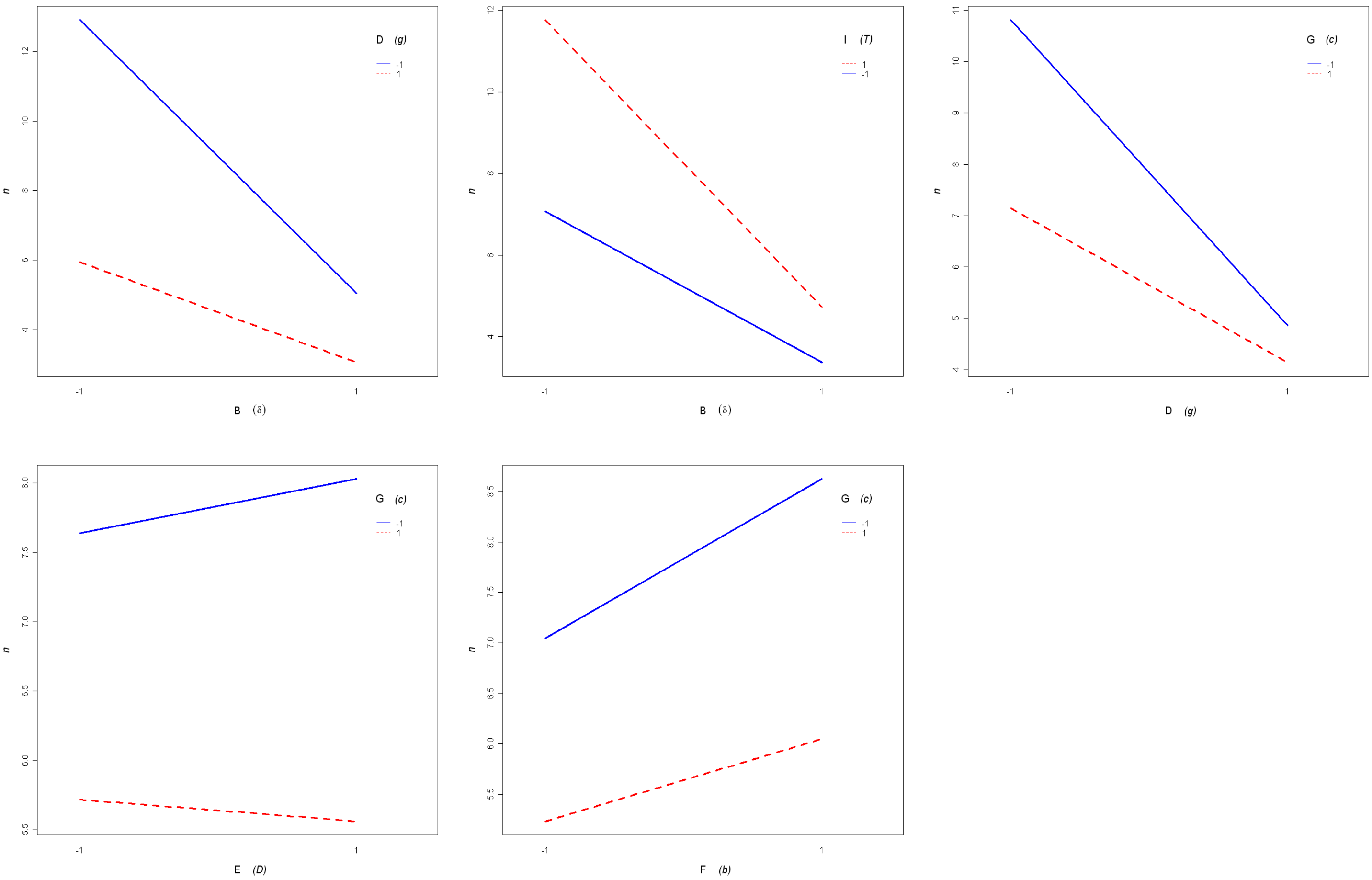


Figure 4: Continuous Process: primary interaction effects plots for n.

power is required (a lower probability *in a single sample* to detect a shift produces smaller samples which, in the light of a global optimality criterion, have to be offset through lower sampling intervals), and this effect is stronger when c is high (since this entails a further sample size reduction). F (b) interacts with G (c) : higher fixed sampling costs produce larger sampling intervals and the effect is stronger when c is low (since samples can be greater). Panagos et al. (1985) results did not make evidence on the effect of D (g) on sampling interval. This factor however proved here to be effective in more than one interaction. Interactions between D (g) and both F (b) and G (c) can be easily interpreted: longer time to interpret each sampled unit entails shorter sampling intervals (again as an indirect effect of consequent decrease in sample size) and the shortening is stronger when b is low or c is high.

The ANOVA table for the control limits, k, excludes only H (W) from significant factors (see Table 7 and Fig. 6). Factor A (M) interacts only with F (b) : limits are narrower for high penalty costs in out-of-control state and the effect is stronger when high fixed sampling costs are experienced (maybe because these costs produce

Table 6: Continuous Process: Analysis of Variance for h.

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	143.655	1	143.655	1153.69	0.0000	-1.4982	0.0441089
B	35.9775	1	35.9775	288.93	0.0000	-0.749766	0.0441089
C	126.408	1	126.408	1015.18	0.0000	-1.40539	0.0441089
D	27.7005	1	27.7005	222.46	0.0000	-0.657891	0.0441089
E	83.7339	1	83.7339	672.46	0.0000	1.14383	0.0441089
F	292.517	1	292.517	2349.20	0.0000	2.13789	0.0441089
G	209.689	1	209.689	1684.00	0.0000	1.81008	0.0441089
H	3.70803	1	3.70803	29.78	0.0000	0.240703	0.0441089
I	8.36294	1	8.36294	67.16	0.0000	0.361484	0.0441089
AB	2.58205	1	2.58205	20.74	0.0000	0.200859	0.0441089
AC	3.75148	1	3.75148	30.13	0.0000	0.242109	0.0441089
AD	0.321347	1	0.321347	2.58	0.1097	0.0708594	0.0441089
AE	2.16642	1	2.16642	17.40	0.0000	-0.183984	0.0441089
AF	8.33405	1	8.33405	66.93	0.0000	-0.360859	0.0441089
AG	8.92142	1	8.92142	71.65	0.0000	-0.373359	0.0441089
AH	0.607425	1	0.607425	4.88	0.0283	-0.0974219	0.0441089
AI	1.60814	1	1.60814	12.91	0.0004	-0.158516	0.0441089
BC	0.729957	1	0.729957	5.86	0.0163	0.106797	0.0441089
BD	5.57845	1	5.57845	44.80	0.0000	0.295234	0.0441089
BE	1.40867	1	1.40867	11.31	0.0009	-0.148359	0.0441089
BF	0.0151598	1	0.0151598	0.12	0.7275	0.0153906	0.0441089
BG	19.2886	1	19.2886	154.91	0.0000	-0.548984	0.0441089
BH	0.220313	1	0.220313	1.77	0.1849	-0.0586719	0.0441089
BI	1.2502	1	1.2502	10.04	0.0018	-0.139766	0.0441089
CD	3.58866	1	3.58866	28.82	0.0000	0.236797	0.0441089
CE	12.5537	1	12.5537	100.82	0.0000	0.442891	0.0441089
CF	5.70911	1	5.70911	45.85	0.0000	-0.298672	0.0441089
CG	8.36294	1	8.36294	67.16	0.0000	-0.361484	0.0441089
CH	1.40571	1	1.40571	11.29	0.0009	0.148203	0.0441089
CI	0.677535	1	0.677535	5.44	0.0206	-0.102891	0.0441089
DE	0.000141016	1	0.000141016	0.00	0.9732	-0.00148438	0.0441089
DF	2.33517	1	2.33517	18.75	0.0000	0.191016	0.0441089
DG	2.18116	1	2.18116	17.52	0.0000	-0.184609	0.0441089
DH	0.0372973	1	0.0372973	0.30	0.5848	0.0241406	0.0441089
DI	1.24462	1	1.24462	10.00	0.0018	-0.139453	0.0441089
EF	6.84803	1	6.84803	55.00	0.0000	0.327109	0.0441089
EG	4.46002	1	4.46002	35.82	0.0000	0.263984	0.0441089
EH	0.10041	1	0.10041	0.81	0.3702	0.0396094	0.0441089
EI	0.122938	1	0.122938	0.99	0.3215	0.0438281	0.0441089
FG	3.6888	1	3.6888	29.62	0.0000	-0.240078	0.0441089
FH	0.350316	1	0.350316	2.81	0.0950	0.0739844	0.0441089
FI	1.29248	1	1.29248	10.38	0.0015	-0.142109	0.0441089
GH	0.383625	1	0.383625	3.08	0.0807	0.0774219	0.0441089
GI	6.68546	1	6.68546	53.69	0.0000	0.323203	0.0441089
HI	0.0254004	1	0.0254004	0.20	0.6520	-0.0199219	0.0441089
Error	26.1488	210	0.124518				
Total (corr.)	1076.74	255					

Table 7: Continuous Process: Analysis of Variance for k.

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	0.142979	1	0.142979	19.42	0.0000	-0.0472656	0.0107253
B	14.2553	1	14.2553	1936.31	0.0000	0.471953	0.0107253
C	2.19966	1	2.19966	298.78	0.0000	-0.185391	0.0107253
D	5.45514	1	5.45514	740.97	0.0000	-0.291953	0.0107253
E	0.524719	1	0.524719	71.27	0.0000	-0.0905469	0.0107253
F	3.32834	1	3.32834	452.09	0.0000	-0.228047	0.0107253
G	15.1954	1	15.1954	2064.00	0.0000	-0.487266	0.0107253
H	0.000172266	1	0.000172266	0.02	0.8786	0.00164062	0.0107253
I	42.0471	1	42.0471	5711.28	0.0000	0.810547	0.0107253
AB	0.00256289	1	0.00256289	0.35	0.5558	0.00632813	0.0107253
AC	0.00914414	1	0.00914414	1.24	0.2663	-0.0119531	0.0107253
AD	0.0183941	1	0.0183941	2.50	0.1155	-0.0169531	0.0107253
AE	0.00175352	1	0.00175352	0.24	0.6260	-0.00523438	0.0107253
AF	0.0339941	1	0.0339941	4.62	0.0328	-0.0230469	0.0107253
AG	0.00447227	1	0.00447227	0.61	0.4366	0.00835937	0.0107253
AH	0.0201285	1	0.0201285	2.73	0.0997	-0.0177344	0.0107253
AI	0.0130816	1	0.0130816	1.78	0.1840	0.0142969	0.0107253
BC	0.0190785	1	0.0190785	2.59	0.1089	0.0172656	0.0107253
BD	0.00109727	1	0.00109727	0.15	0.6998	0.00414062	0.0107253
BE	0.0270191	1	0.0270191	3.67	0.0568	0.0205469	0.0107253
BF	0.0187348	1	0.0187348	2.54	0.1122	0.0171094	0.0107253
BG	0.27366	1	0.27366	37.17	0.0000	0.0653906	0.0107253
BH	0.012516	1	0.012516	1.70	0.1937	0.0139844	0.0107253
BI	0.262016	1	0.262016	35.59	0.0000	-0.0639844	0.0107253
CD	0.0422816	1	0.0422816	5.74	0.0174	-0.0257031	0.0107253
CE	0.217972	1	0.217972	29.61	0.0000	-0.0583594	0.0107253
CF	0.102	1	0.102	13.85	0.0003	-0.0399219	0.0107253
CG	0.00145352	1	0.00145352	0.20	0.6573	-0.00476562	0.0107253
CH	0.00118164	1	0.00118164	0.16	0.6891	-0.00429687	0.0107253
CI	0.0573004	1	0.0573004	7.78	0.0058	0.0299219	0.0107253
DE	0.0000878906	1	0.0000878906	0.01	0.9131	-0.00117188	0.0107253
DF	0.663207	1	0.663207	90.08	0.0000	-0.101797	0.0107253
DG	0.506766	1	0.506766	68.83	0.0000	0.0889844	0.0107253
DH	0.00196914	1	0.00196914	0.27	0.6056	-0.00554687	0.0107253
DI	0.102	1	0.102	13.85	0.0003	0.0399219	0.0107253
EF	0.000328516	1	0.000328516	0.04	0.8329	-0.00226563	0.0107253
EG	0.0837379	1	0.0837379	11.37	0.0009	-0.0361719	0.0107253
EH	0.0177223	1	0.0177223	2.41	0.1223	-0.0166406	0.0107253
EI	0.0145504	1	0.0145504	1.98	0.1612	0.0150781	0.0107253
FG	0.426572	1	0.426572	57.94	0.0000	0.0816406	0.0107253
FH	0.000425391	1	0.000425391	0.06	0.8103	-0.00257813	0.0107253
FI	0.205322	1	0.205322	27.89	0.0000	0.0566406	0.0107253
GH	0.0119629	1	0.0119629	1.62	0.2038	-0.0136719	0.0107253
GI	0.175875	1	0.175875	23.89	0.0000	0.0524219	0.0107253
HI	0.000206641	1	0.000206641	0.03	0.8671	-0.00179687	0.0107253
Error	1.54604	210	0.00736212				
Total (corr.)	88.0455	255					

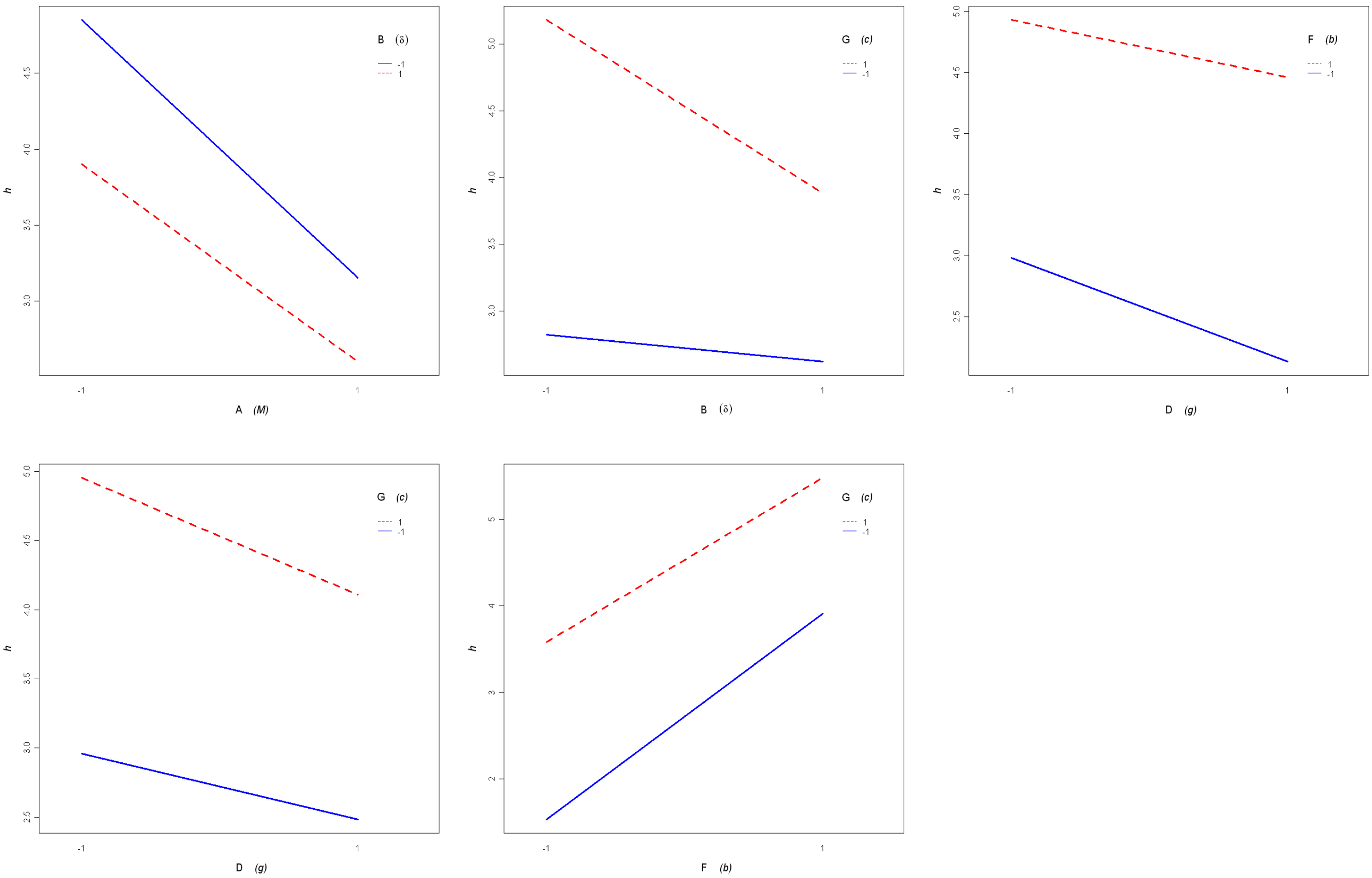


Figure 5: Continuous Process: primary interaction effects plots for h.

less frequent samples which have to be offset by a tightened control). Factor I (T) is involved in many interactions: it acts modifying the effect of B (δ), C (λ) and D (g). In particular, factor B (δ) acts positively on k (larger shifts can be detected also with larger limits) and the effect is stronger when false alarm cost are higher. Both factors C (λ) and D (g) have a negative effect on k : more frequent mean shifts require narrower limits and longer time to interpret results produces smaller samples (with the compensating effect of narrower limits). In both cases the effect is stronger when T is smaller. The interaction between C (λ) and E (D) suggests that the negative effect of λ on k is stronger when a longer time is required to repair an assignable cause (the time "lost" for repairing the system has to be compensated by a powerful setting of control limits). Finally, factor D (g) interacts also with C (λ), F (b) and G (c). In particular, the negative effect of D (g) is stronger when b has high value (high fixed sampling costs are associated to small frequent samples with narrow limits).

If we consider globally the four responses we can summarize our results for the

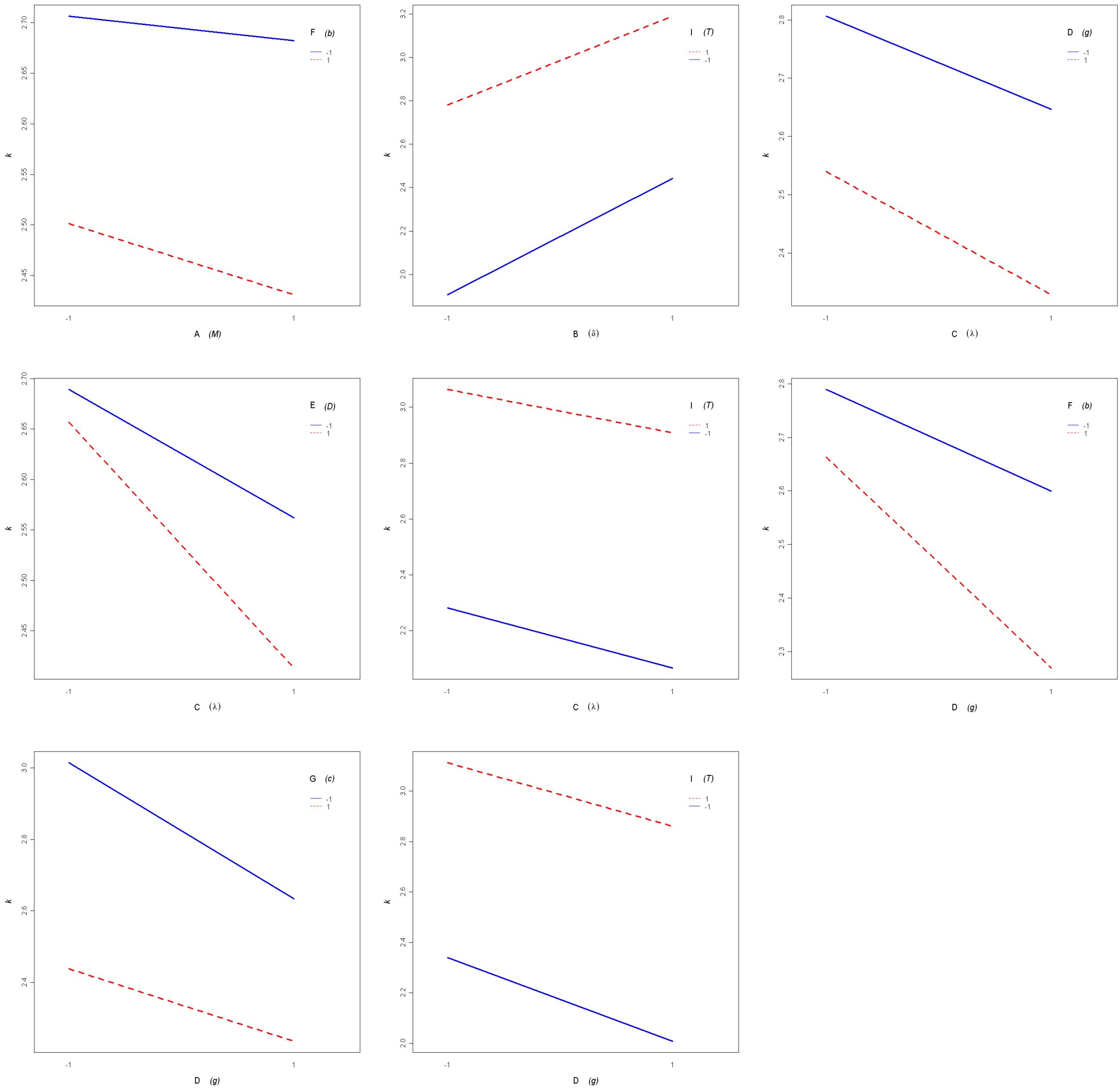


Figure 6: Continuous Process: primary interaction effects plots for k.

continuous case underlying that Panagos et al. (1985) results are, on one hand, not confirmed since almost all factors proved here to significantly affect responses (while the cited paper focused only on smaller subsets of factors), but, on the other hand, the signs of the effects here described are the same described in that paper (and none of the interactions highlighted resulted in effects of opposite sign for the first factor at different levels of the second one).

4.3 Results for the discontinuos case

The analysis of significant interaction effects for the 256 run experiment highlights the role played on loss, $E(L_1)$, by B (δ), D (g) and H (W) (besides the factors already listed in Panagos et al. (1985) (see Table 8). Moreover, factors F (b), G (c) and K (S) prove to have a significant main effect. (see Tables 9 and 10 and Figg. 7 and 8).

Factor A (M) interacts with B (δ), C (λ), D (g), E (D), and J (V_o). The positive effect on $E(L_1)$ of the penalty cost of operating in out-of-control state is stronger when δ is smaller or λ, g, D or V_o have an higher value; this happens when the process is less stable, the time required to interpret results or repair an assignable cause are longer or when the income in in-control state is high (because of the connection among M, V_o and V_1, given M, an high value for V_o corresponds to low values for V_1 identifying a process with low returns in out-of-control state).

Factor C (λ) interacts with E (D), H (W) and J (V_o). A less stable process produces higher costs especially when the production stop due to repairing the assignable cause is longer or this step has higher costs or V_o is high.

Finally interactions EH and EJ show intuitive links among time and cost to repair an assignable cause and hourly in-control income.

The ANOVA for sample size, n, highlights the presence of many interactions – not involving H (W), I (T) and L (S_c), with I (T) significant as a main effect (see Tables 11 and 12). The main result comes however from the inspection of Fig. 9), where a subset of relevant interaction plots is shown. The sign of the effect of one factor is in many cases reversed by the level of the second factor: this result obviously makes the analysis of main effects completely meaningless.

Factor A (M) is involved in many interactions of this type. When C (λ) is high (less stable process), increasing penalty costs for out-of-control state increase sample size, but for more stable processes the increase of M causes a reduction of n (maybe because for stable processes, the optimal solution try to cope the increase in M with a less expensive procedure). If we consider interaction AD, we observe that higher values for M entail a sample size increase only if g, the time to interpret a

Table 8: Discontinuous case: significant main effects according to Panagos et al. (1985).

Response variable	Significant effects (+: positive effect) (–: negative effect)							
$E(L_1)$	A (M) +	C (λ) +	E (D) +	J (V_o) +				
n	A (M) –	B (δ) –	D (g) –	G (c) –	J (V_o) +	M (D_1) +		
h	A (M) –	B (δ) –	C (λ) –	F (b) +	G (c) +			
k	B (δ) –	C (λ) –	D (g) –	F (b) –	G (c) –	I (T) +	J (V_o) +	M (D_1) +

Table 9: Discontinuous Process: Analysis of Variance for $E(L_1)$ (first part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	3688.54	1	3688.54	186.77	0.0000	7.59167	0.555495
B	593.787	1	593.787	30.07	0.0000	-3.04597	0.555495
C	41618.1	1	41618.1	2107.37	0.0000	25.5007	0.555495
D	504.399	1	504.399	25.54	0.0000	2.80735	0.555495
E	14858.7	1	14858.7	752.38	0.0000	15.237	0.555495
F	99.8604	1	99.8604	5.06	0.0259	1.24913	0.555495
G	213.674	1	213.674	10.82	0.0012	1.8272	0.555495
H	897.212	1	897.212	45.43	0.0000	3.74419	0.555495
I	5.06668	1	5.06668	0.26	0.6132	0.281366	0.555495
J	17596.6	1	17596.6	891.02	0.0000	16.5815	0.555495
K	164.429	1	164.429	8.33	0.0044	1.60287	0.555495
L	76.0714	1	76.0714	3.85	0.0514	1.09024	0.555495
M	20.2884	1	20.2884	1.03	0.3123	0.563033	0.555495
AB	91.7227	1	91.7227	4.64	0.0326	-1.19715	0.555495
AC	1691.05	1	1691.05	85.63	0.0000	5.1403	0.555495
AD	100.282	1	100.282	5.08	0.0256	1.25176	0.555495
AE	584.744	1	584.744	29.61	0.0000	3.02269	0.555495
AF	6.09288	1	6.09288	0.31	0.5793	0.308547	0.555495
AG	12.2931	1	12.2931	0.62	0.4313	0.43827	0.555495
AH	8.27803	1	8.27803	0.42	0.5183	0.359645	0.555495
AI	1.05552	1	1.05552	0.05	0.8175	0.128423	0.555495
AJ	1083.77	1	1083.77	54.88	0.0000	4.11508	0.555495
AK	2.03235	1	2.03235	0.10	0.7488	0.1782	0.555495
AL	0.58975	1	0.58975	0.03	0.8630	0.095994	0.555495
AM	4.1181	1	4.1181	0.21	0.6485	0.253664	0.555495
BC	21.4931	1	21.4931	1.09	0.2984	-0.579508	0.555495
BD	11.0867	1	11.0867	0.56	0.4548	0.416208	0.555495
BE	31.0476	1	31.0476	1.57	0.2117	0.696504	0.555495
BF	2.99141	1	2.99141	0.15	0.6976	0.216196	0.555495
BG	16.2485	1	16.2485	0.82	0.3657	-0.503868	0.555495
BH	2.02168	1	2.02168	0.10	0.7494	0.177732	0.555495
BI	1.48901	1	1.48901	0.08	0.7840	-0.152531	0.555495
BJ	8.6386	1	8.6386	0.44	0.5093	-0.367394	0.555495
BK	0.0308346	1	0.0308346	0.00	0.9685	0.0219497	0.555495
BL	0.059505	1	0.059505	0.00	0.9563	0.0304921	0.555495
BM	1.54784	1	1.54784	0.08	0.7799	-0.155515	0.555495
CD	43.4513	1	43.4513	2.20	0.1399	0.82397	0.555495
CE	672.087	1	672.087	34.03	0.0000	3.24058	0.555495
CF	2.73475	1	2.73475	0.14	0.7103	0.206713	0.555495
CG	0.126159	1	0.126159	0.01	0.9364	-0.0443986	0.555495
CH	234.684	1	234.684	11.88	0.0007	1.91493	0.555495
CI	0.985967	1	0.985967	0.05	0.8235	0.12412	0.555495
CJ	2881.4	1	2881.4	145.90	0.0000	6.70984	0.555495
CK	50.4147	1	50.4147	2.55	0.1120	0.887541	0.555495
CL	8.67695	1	8.67695	0.44	0.5084	0.368208	0.555495
CM	0.241996	1	0.241996	0.01	0.9120	-0.0614914	0.555495

Table 10: Discontinuous Process: Analysis of Variance for $E(L_1)$ (second part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
DE	34.1717	1	34.1717	1.73	0.1902	-0.730707	0.555495
DF	6.09745	1	6.09745	0.31	0.5792	0.308663	0.555495
DG	4.38056	1	4.38056	0.22	0.6383	-0.261622	0.555495
DH	2.32987	1	2.32987	0.12	0.7317	-0.190799	0.555495
DI	1.30161	1	1.30161	0.07	0.7977	0.14261	0.555495
DJ	5.3757	1	5.3757	0.27	0.6026	0.289819	0.555495
DK	0.107165	1	0.107165	0.01	0.9414	0.0409201	0.555495
DL	0.00020598	1	0.00020598	0.00	0.9974	0.001794	0.555495
DM	5.22823	1	5.22823	0.26	0.6076	0.285816	0.555495
EF	8.08324	1	8.08324	0.41	0.5232	-0.355388	0.555495
EG	17.3121	1	17.3121	0.88	0.3505	-0.520098	0.555495
EH	80.5725	1	80.5725	4.08	0.0450	-1.12203	0.555495
EI	0.115264	1	0.115264	0.01	0.9392	-0.0424382	0.555495
EJ	3799.98	1	3799.98	192.42	0.0000	7.7055	0.555495
EK	15.3391	1	15.3391	0.78	0.3794	-0.489564	0.555495
EL	14.2662	1	14.2662	0.72	0.3966	-0.472133	0.555495
EM	2.44065	1	2.44065	0.12	0.7256	-0.195282	0.555495
FG	8.13649	1	8.13649	0.41	0.5219	-0.356557	0.555495
FH	0.806329	1	0.806329	0.04	0.8401	-0.112245	0.555495
FI	0.000735833	1	0.000735833	0.00	0.9951	0.00339078	0.555495
FJ	1.46575	1	1.46575	0.07	0.7856	-0.151335	0.555495
FK	15.0143	1	15.0143	0.76	0.3845	0.484353	0.555495
FL	0.544407	1	0.544407	0.03	0.8683	-0.0922299	0.555495
FM	0.00464657	1	0.00464657	0.00	0.9878	-0.00852072	0.555495
GH	0.286589	1	0.286589	0.01	0.9043	-0.0669175	0.555495
GI	0.389898	1	0.389898	0.02	0.8884	0.0780523	0.555495
GJ	2.45381	1	2.45381	0.12	0.7249	-0.195808	0.555495
GK	0.0778577	1	0.0778577	0.00	0.9500	-0.0348788	0.555495
GL	0.0123752	1	0.0123752	0.00	0.9801	0.0139055	0.555495
GM	7.0941	1	7.0941	0.36	0.5498	-0.332934	0.555495
HI	0.0180341	1	0.0180341	0.00	0.9759	-0.0167864	0.555495
HJ	8.10748	1	8.10748	0.41	0.5226	-0.35592	0.555495
HK	0.734537	1	0.734537	0.04	0.8473	0.107131	0.555495
HL	0.0442906	1	0.0442906	0.00	0.9623	0.0263067	0.555495
HM	0.114768	1	0.114768	0.01	0.9393	-0.0423468	0.555495
IJ	1.0933	1	1.0933	0.06	0.8143	-0.130701	0.555495
IK	0.00134997	1	0.00134997	0.00	0.9934	-0.00459275	0.555495
IL	2.03826	1	2.03826	0.10	0.7484	-0.17846	0.555495
IM	12.1847	1	12.1847	0.62	0.4333	-0.436333	0.555495
JK	1.41483	1	1.41483	0.07	0.7893	-0.148683	0.555495
JL	22.2903	1	22.2903	1.13	0.2896	0.590158	0.555495
JM	0.00227935	1	0.00227935	0.00	0.9914	-0.00596781	0.555495
KL	5.27796	1	5.27796	0.27	0.6059	0.287173	0.555495
KM	0.0403537	1	0.0403537	0.00	0.9640	0.0251103	0.555495
LM	0.467997	1	0.467997	0.02	0.8778	-0.0855129	0.555495
Error	3238.8	164	19.7488				
Total (corr.)	95240.3	255					

Table 11: Discontinuous Process: Analysis of Variance for n (first part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	4.25391	1	4.25391	0.76	0.3841	0.257813	0.29543
B	3652.69	1	3652.69	653.92	0.0000	-7.55469	0.29543
C	603.316	1	603.316	108.01	0.0000	-3.07031	0.29543
D	1477.44	1	1477.44	264.50	0.0000	-4.80469	0.29543
E	93.8477	1	93.8477	16.80	0.0001	-1.21094	0.29543
F	98.7539	1	98.7539	17.68	0.0000	1.24219	0.29543
G	226.879	1	226.879	40.62	0.0000	-1.88281	0.29543
H	0.0351563	1	0.0351563	0.01	0.9369	0.0234375	0.29543
I	30.9414	1	30.9414	5.54	0.0198	0.695313	0.29543
J	15.5039	1	15.5039	2.78	0.0976	0.492188	0.29543
K	0.191406	1	0.191406	0.03	0.8534	-0.0546875	0.29543
L	0.0976563	1	0.0976563	0.02	0.8950	0.0390625	0.29543
M	299.723	1	299.723	53.66	0.0000	2.16406	0.29543
AB	2.84766	1	2.84766	0.51	0.4762	-0.210938	0.29543
AC	84.4102	1	84.4102	15.11	0.0001	1.14844	0.29543
AD	98.7539	1	98.7539	17.68	0.0000	-1.24219	0.29543
AE	133.691	1	133.691	23.93	0.0000	1.44531	0.29543
AF	0.191406	1	0.191406	0.03	0.8534	0.0546875	0.29543
AG	4.78516	1	4.78516	0.86	0.3560	0.273438	0.29543
AH	0.0976563	1	0.0976563	0.02	0.8950	-0.0390625	0.29543
AI	0.00390625	1	0.00390625	0.00	0.9789	0.0078125	0.29543
AJ	119.629	1	119.629	21.42	0.0000	1.36719	0.29543
AK	0.0351563	1	0.0351563	0.01	0.9369	-0.0234375	0.29543
AL	0.191406	1	0.191406	0.03	0.8534	-0.0546875	0.29543
AM	0.878906	1	0.878906	0.16	0.6921	-0.117188	0.29543
BC	254.004	1	254.004	45.47	0.0000	1.99219	0.29543
BD	242.191	1	242.191	43.36	0.0000	1.94531	0.29543
BE	46.4102	1	46.4102	8.31	0.0045	0.851563	0.29543
BF	12.6914	1	12.6914	2.27	0.1336	-0.445313	0.29543
BG	48.1289	1	48.1289	8.62	0.0038	0.867188	0.29543
BH	0.00390625	1	0.00390625	0.00	0.9789	-0.0078125	0.29543
BI	8.62891	1	8.62891	1.54	0.2157	-0.367188	0.29543
BJ	1.12891	1	1.12891	0.20	0.6536	-0.132813	0.29543
BK	0.316406	1	0.316406	0.06	0.8122	0.0703125	0.29543
BL	0.00390625	1	0.00390625	0.00	0.9789	0.0078125	0.29543
BM	98.7539	1	98.7539	17.68	0.0000	-1.24219	0.29543
CD	38.2852	1	38.2852	6.85	0.0097	0.773438	0.29543
CE	96.2852	1	96.2852	17.24	0.0001	-1.22656	0.29543
CF	3.75391	1	3.75391	0.67	0.4135	-0.242188	0.29543
CG	5.34766	1	5.34766	0.96	0.3293	0.289063	0.29543
CH	0.0351563	1	0.0351563	0.01	0.9369	-0.0234375	0.29543
CI	0.472656	1	0.472656	0.08	0.7715	0.0859375	0.29543
CJ	75.4727	1	75.4727	13.51	0.0003	-1.08594	0.29543
CK	1.12891	1	1.12891	0.20	0.6536	-0.132813	0.29543
CL	0.00390625	1	0.00390625	0.00	0.9789	-0.0078125	0.29543
CM	5.34766	1	5.34766	0.96	0.3293	-0.289063	0.29543

Table 12: Discontinuous Process: Analysis of Variance for n (second part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
DE	49.8789	1	49.8789	8.93	0.0032	0.882813	0.29543
DF	1.72266	1	1.72266	0.31	0.5794	-0.164063	0.29543
DG	142.504	1	142.504	25.51	0.0000	1.49219	0.29543
DH	0.0351563	1	0.0351563	0.01	0.9369	0.0234375	0.29543
DI	0.472656	1	0.472656	0.08	0.7715	-0.0859375	0.29543
DJ	48.1289	1	48.1289	8.62	0.0038	0.867188	0.29543
DK	1.12891	1	1.12891	0.20	0.6536	0.132813	0.29543
DL	4.25391	1	4.25391	0.76	0.3841	0.257813	0.29543
DM	17.5352	1	17.5352	3.14	0.0783	-0.523438	0.29543
EF	0.660156	1	0.660156	0.12	0.7315	-0.101563	0.29543
EG	1.12891	1	1.12891	0.20	0.6536	-0.132813	0.29543
EH	0.0976563	1	0.0976563	0.02	0.8950	-0.0390625	0.29543
EI	0.0351563	1	0.0351563	0.01	0.9369	-0.0234375	0.29543
EJ	111.566	1	111.566	19.97	0.0000	-1.32031	0.29543
EK	0.191406	1	0.191406	0.03	0.8534	-0.0546875	0.29543
EL	0.0351563	1	0.0351563	0.01	0.9369	-0.0234375	0.29543
EM	0.0351563	1	0.0351563	0.01	0.9369	-0.0234375	0.29543
FG	13.5977	1	13.5977	2.43	0.1206	-0.460938	0.29543
FH	0.0976563	1	0.0976563	0.02	0.8950	0.0390625	0.29543
FI	0.00390625	1	0.00390625	0.00	0.9789	-0.0078125	0.29543
FJ	0.00390625	1	0.00390625	0.00	0.9789	0.0078125	0.29543
FK	61.0352	1	61.0352	10.93	0.0012	-0.976563	0.29543
FL	0.0351563	1	0.0351563	0.01	0.9369	0.0234375	0.29543
FM	0.472656	1	0.472656	0.08	0.7715	0.0859375	0.29543
GH	0.878906	1	0.878906	0.16	0.6921	-0.117188	0.29543
GI	0.0351563	1	0.0351563	0.01	0.9369	0.0234375	0.29543
GJ	0.0976563	1	0.0976563	0.02	0.8950	0.0390625	0.29543
GK	0.191406	1	0.191406	0.03	0.8534	0.0546875	0.29543
GL	0.660156	1	0.660156	0.12	0.7315	-0.101563	0.29543
GM	33.7852	1	33.7852	6.05	0.0150	-0.726563	0.29543
HI	0.00390625	1	0.00390625	0.00	0.9789	-0.0078125	0.29543
HJ	0.316406	1	0.316406	0.06	0.8122	0.0703125	0.29543
HK	0.316406	1	0.316406	0.06	0.8122	-0.0703125	0.29543
HL	0.191406	1	0.191406	0.03	0.8534	0.0546875	0.29543
HM	0.0351563	1	0.0351563	0.01	0.9369	0.0234375	0.29543
IJ	7.91016	1	7.91016	1.42	0.2358	-0.351563	0.29543
IK	0.0351563	1	0.0351563	0.01	0.9369	-0.0234375	0.29543
IL	5.34766	1	5.34766	0.96	0.3293	0.289063	0.29543
IM	11.8164	1	11.8164	2.12	0.1477	-0.429688	0.29543
JK	0.00390625	1	0.00390625	0.00	0.9789	-0.0078125	0.29543
JL	0.191406	1	0.191406	0.03	0.8534	0.0546875	0.29543
JM	15.5039	1	15.5039	2.78	0.0976	0.492188	0.29543
KL	0.0976563	1	0.0976563	0.02	0.8950	0.0390625	0.29543
KM	0.472656	1	0.472656	0.08	0.7715	-0.0859375	0.29543
LM	0.00390625	1	0.00390625	0.00	0.9789	0.0078125	0.29543
Error	916.078	164	5.58584				
Total (corr.)	9336.21	255					

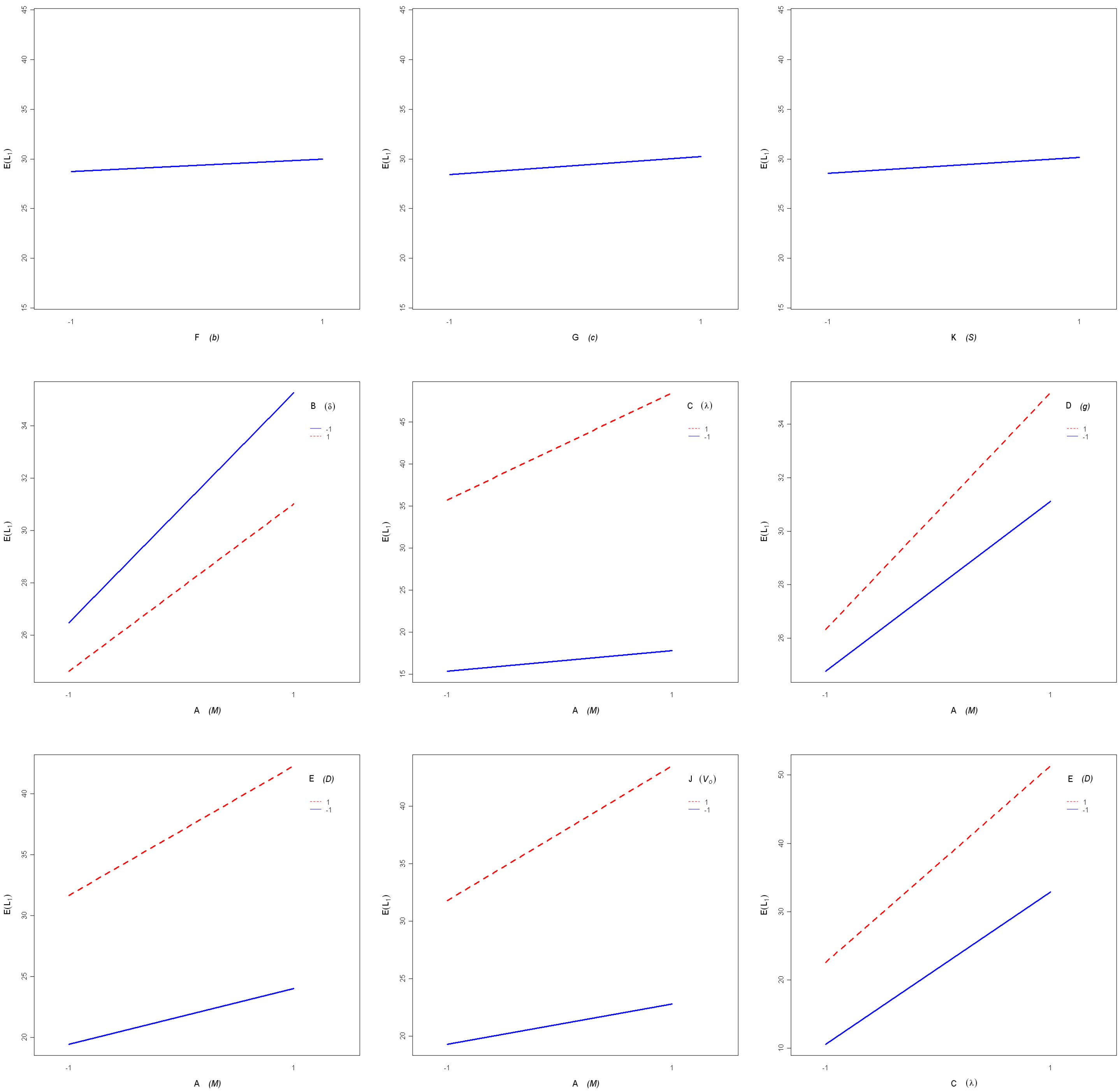


Figure 7: Discontinuous Process: primary effects plots for $E(L_1)(1/2)$.

single data unit, has a small value. Conversely, when g is high, M decreases sample size. Interaction between A (M) and E (D) denotes that, for high D values, an increase in M increases n (when long breaks in production for repairing the system are

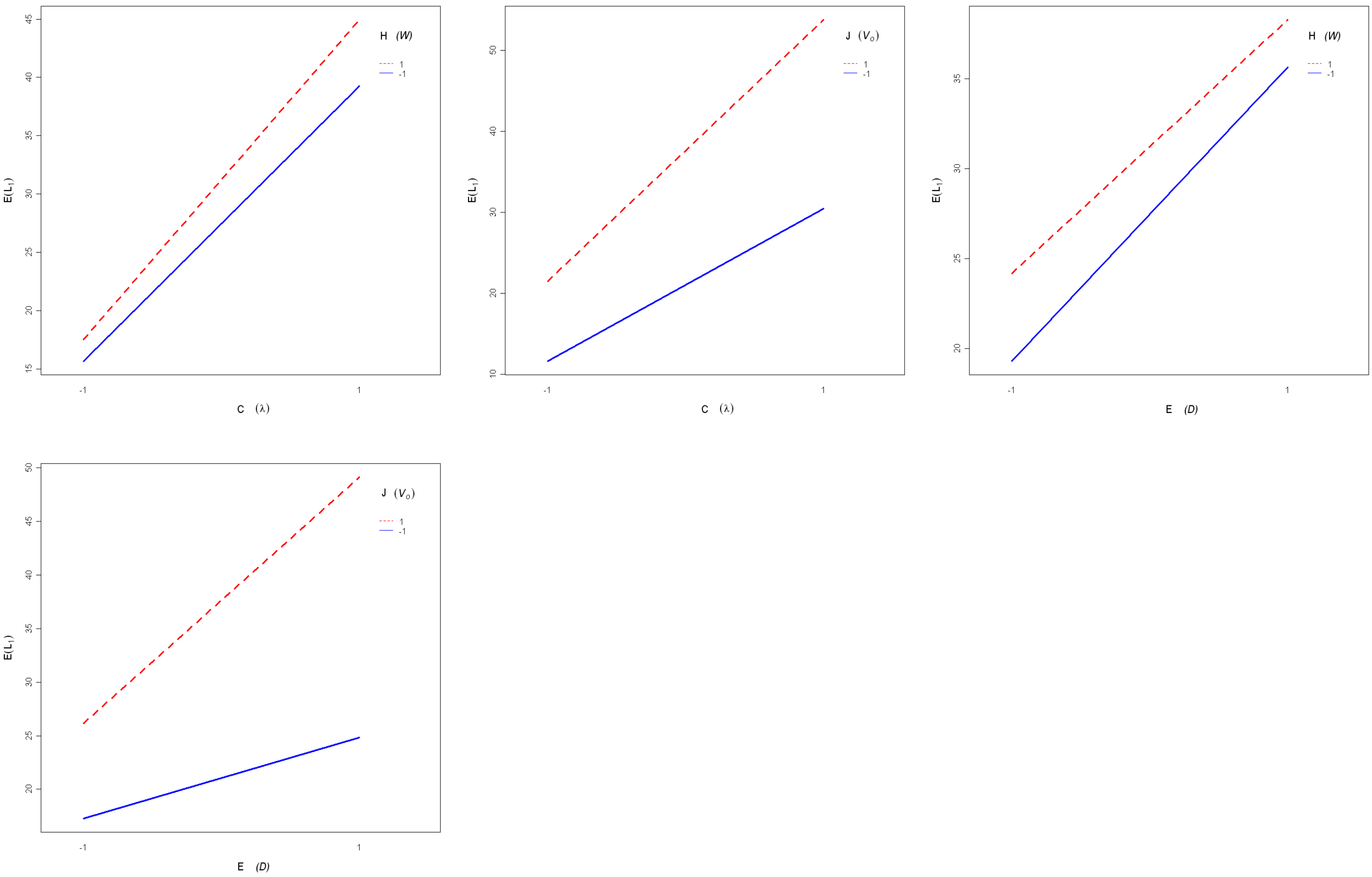


Figure 8: Discontinuous Process: primary effects plots for $E(L_1)(2/2)$.

needed, an increase in loss is fronted with bigger samples), while for small D values (quick repair) the cost of a bigger M is compensated with a small reduction for n. A similar behaviour is observed when D is replaced by V_o. Finally, interaction EJ between D and V_o says that for processes with a low in-control income, increases in D produce bigger samples, while for high in-control income, increases in D produce a great damage because of a long stop in production and that damage is fronted with smaller (but more frequent) samples.

The ANOVA for h (see Tables 13, 14 and Fig. 10) proves that factors H (W), I (T), K (S), L (S_c) and M (D_1) do not affect the optimal sampling interval (neither in interactions nor as main effects). All other factors are conversely involved in interactions, sometimes with opposite effect of one factor at different levels of the other one. For example, interaction BD shows that δ has a positive effect on h when g is high, while for low g the effect of δ is negative (increasing δ means a lower precision required to the chart and thus a likely reduction in sample size with a consequent raise in sampling frequency; this behaviour is confirmed when g is low, but

Table 13: Discontinuous Process: Analysis of Variance for h (first part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	582.136	1	582.136	452.62	0.0000	-3.01594	0.14176
B	52.3814	1	52.3814	40.73	0.0000	-0.904687	0.14176
C	36.0	1	36.0	27.99	0.0000	-0.75	0.14176
D	27.2484	1	27.2484	21.19	0.0000	-0.6525	0.14176
E	204.026	1	204.026	158.63	0.0000	1.78547	0.14176
F	202.528	1	202.528	157.47	0.0000	1.77891	0.14176
G	298.598	1	298.598	232.17	0.0000	2.16	0.14176
H	3.93526	1	3.93526	3.06	0.0821	0.247969	0.14176
I	0.226814	1	0.226814	0.18	0.6751	0.0595312	0.14176
J	289.723	1	289.723	225.27	0.0000	2.12766	0.14176
K	0.816764	1	0.816764	0.64	0.4267	0.112969	0.14176
L	0.303877	1	0.303877	0.24	0.6276	0.0689062	0.14176
M	1.45806	1	1.45806	1.13	0.2886	0.150938	0.14176
AB	2.3409	1	2.3409	1.82	0.1792	0.19125	0.14176
AC	32.5756	1	32.5756	25.33	0.0000	-0.713438	0.14176
AD	2.27256	1	2.27256	1.77	0.1856	-0.188437	0.14176
AE	80.3936	1	80.3936	62.51	0.0000	-1.12078	0.14176
AF	5.99638	1	5.99638	4.66	0.0323	-0.306094	0.14176
AG	12.0583	1	12.0583	9.38	0.0026	-0.434063	0.14176
AH	0.957952	1	0.957952	0.74	0.3894	-0.122344	0.14176
AI	0.0135141	1	0.0135141	0.01	0.9185	-0.0145312	0.14176
AJ	108.134	1	108.134	84.08	0.0000	-1.29984	0.14176
AK	0.137827	1	0.137827	0.11	0.7438	-0.0464063	0.14176
AL	0.0558141	1	0.0558141	0.04	0.8352	-0.0295312	0.14176
AM	2.29522	1	2.29522	1.78	0.1834	0.189375	0.14176
BC	5.30151	1	5.30151	4.12	0.0439	0.287812	0.14176
BD	106.502	1	106.502	82.81	0.0000	1.29	0.14176
BE	0.329189	1	0.329189	0.26	0.6136	0.0717187	0.14176
BF	1.0025	1	1.0025	0.78	0.3786	0.125156	0.14176
BG	30.1401	1	30.1401	23.43	0.0000	-0.68625	0.14176
BH	0.185977	1	0.185977	0.14	0.7042	-0.0539063	0.14176
BI	0.00620156	1	0.00620156	0.00	0.9447	-0.00984375	0.14176
BJ	0.857939	1	0.857939	0.67	0.4153	-0.115781	0.14176
BK	1.2572	1	1.2572	0.98	0.3243	0.140156	0.14176
BL	0.00878906	1	0.00878906	0.01	0.9342	-0.0117187	0.14176
BM	2.31801	1	2.31801	1.80	0.1813	0.190313	0.14176
CD	7.91016	1	7.91016	6.15	0.0141	0.351563	0.14176
CE	67.0147	1	67.0147	52.11	0.0000	1.02328	0.14176
CF	16.677	1	16.677	12.97	0.0004	-0.510469	0.14176
CG	32.3192	1	32.3192	25.13	0.0000	-0.710625	0.14176
CH	1.27408	1	1.27408	0.99	0.3211	0.141094	0.14176
CI	0.00113906	1	0.00113906	0.00	0.9763	-0.00421875	0.14176
CJ	101.229	1	101.229	78.71	0.0000	1.25766	0.14176
CK	0.0000140625	1	0.0000140625	0.00	0.9974	-0.00046875	0.14176
CL	0.167077	1	0.167077	0.13	0.7190	0.0510938	0.14176
CM	0.718256	1	0.718256	0.56	0.4560	0.105938	0.14176

Table 14: Discontinuous Process: Analysis of Variance for h (second part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
DE	0.192502	1	0.192502	0.15	0.6993	0.0548437	0.14176
DF	1.69325	1	1.69325	1.32	0.2529	0.162656	0.14176
DG	0.9801	1	0.9801	0.76	0.3840	-0.12375	0.14176
DH	0.0102516	1	0.0102516	0.01	0.9290	0.0126563	0.14176
DI	0.0791016	1	0.0791016	0.06	0.8044	-0.0351563	0.14176
DJ	2.06281	1	2.06281	1.60	0.2071	0.179531	0.14176
DK	2.66098	1	2.66098	2.07	0.1522	0.203906	0.14176
DL	0.0489516	1	0.0489516	0.04	0.8456	0.0276563	0.14176
DM	2.55201	1	2.55201	1.98	0.1608	0.199687	0.14176
EF	0.140625	1	0.140625	0.11	0.7413	0.046875	0.14176
EG	0.0192516	1	0.0192516	0.01	0.9028	-0.0173437	0.14176
EH	0.0729	1	0.0729	0.06	0.8121	-0.03375	0.14176
EI	0.0081	1	0.0081	0.01	0.9368	-0.01125	0.14176
EJ	109.464	1	109.464	85.11	0.0000	1.30781	0.14176
EK	0.0689062	1	0.0689062	0.05	0.8172	0.0328125	0.14176
EL	0.104006	1	0.104006	0.08	0.7765	-0.0403125	0.14176
EM	1.67379	1	1.67379	1.30	0.2556	-0.161719	0.14176
FG	5.42308	1	5.42308	4.22	0.0416	-0.291094	0.14176
FH	0.6084	1	0.6084	0.47	0.4926	0.0975	0.14176
FI	0.0144	1	0.0144	0.01	0.9159	-0.015	0.14176
FJ	0.140625	1	0.140625	0.11	0.7413	0.046875	0.14176
FK	2.29522	1	2.29522	1.78	0.1834	-0.189375	0.14176
FL	0.0855562	1	0.0855562	0.07	0.7968	0.0365625	0.14176
FM	0.0791016	1	0.0791016	0.06	0.8044	-0.0351562	0.14176
GH	0.185977	1	0.185977	0.14	0.7042	0.0539062	0.14176
GI	0.143452	1	0.143452	0.11	0.7388	0.0473438	0.14176
GJ	3.78789	1	3.78789	2.95	0.0880	0.243281	0.14176
GK	0.167077	1	0.167077	0.13	0.7190	0.0510937	0.14176
GL	0.000126562	1	0.000126562	0.00	0.9921	-0.00140625	0.14176
GM	0.0441	1	0.0441	0.03	0.8533	0.02625	0.14176
HI	0.00140625	1	0.00140625	0.00	0.9737	-0.0046875	0.14176
HJ	0.333506	1	0.333506	0.26	0.6113	0.0721875	0.14176
HK	0.065025	1	0.065025	0.05	0.8224	-0.031875	0.14176
HL	0.0162562	1	0.0162562	0.01	0.9106	0.0159375	0.14176
HM	0.0877641	1	0.0877641	0.07	0.7942	0.0370313	0.14176
IJ	0.124256	1	0.124256	0.10	0.7563	-0.0440625	0.14176
IK	0.0081	1	0.0081	0.01	0.9368	0.01125	0.14176
IL	0.632025	1	0.632025	0.49	0.4843	-0.099375	0.14176
IM	0.337852	1	0.337852	0.26	0.6090	-0.0726563	0.14176
JK	0.1296	1	0.1296	0.10	0.7513	0.045	0.14176
JL	0.216225	1	0.216225	0.17	0.6823	0.058125	0.14176
JM	0.0000140625	1	0.0000140625	0.00	0.9974	-0.00046875	0.14176
KL	0.0126562	1	0.0126562	0.01	0.9211	0.0140625	0.14176
KM	0.0425391	1	0.0425391	0.03	0.8559	0.0257813	0.14176
LM	0.0213891	1	0.0213891	0.02	0.8975	0.0182813	0.14176
Error	210.927	164	1.28614				
Total (corr.)	2669.6	255					

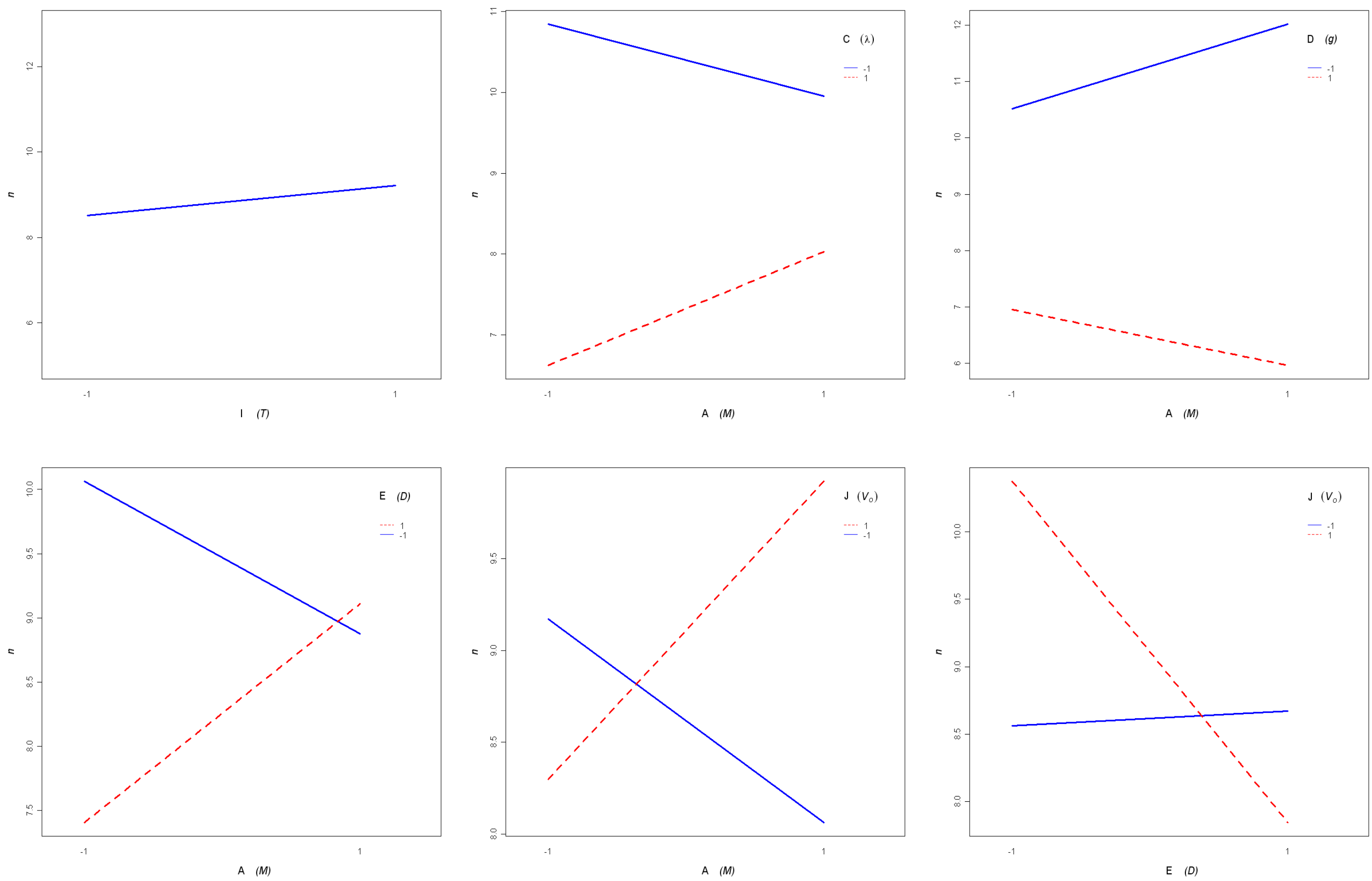


Figure 9: Discontinuous Process: primary effects plots for n.

disclaimed for high values in g). The interaction between C (λ) and E (D) shows a peculiar effect on h : when a short time, D, is required to repair an assignable cause, a more unstable process should be monitored with higher frequency; but a longer D entails that more unstable process require a small increase in sampling interval. Factor C (λ) interacts also with G (c) and J (V_o). A more unstable process leaves the optimal sampling interval unaffected for small values of c, variable sampling costs (maybe because in that case the best reaction is an increase in sample size); conversely, when bigger sample size are very expensive, a more unstable process is should be fronted with (small) more frequent samples. Interaction CJ shows that more frequent samples are suggested to front a more unstable process only when the income in in-control state is low; conversely, when V_o is high, a more unstable process should be monitored slightly less frequently (but most likely with bigger samples). Finally interaction FG shows that an increase in fixed sampling costs leads obviously to less frequent samples, but this effect is stronger when variable sampling costs are low.

Table 15: Discontinuous Process: Analysis of Variance for k (first part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
A	46.9653	1	46.9653	45.83	0.0000	-0.856641	0.126533
B	13.2997	1	13.2997	12.98	0.0004	0.455859	0.126533
C	24.7817	1	24.7817	24.18	0.0000	0.622266	0.126533
D	2.75353	1	2.75353	2.69	0.1031	-0.207422	0.126533
E	36.8373	1	36.8373	35.95	0.0000	0.758672	0.126533
F	1.56406	1	1.56406	1.53	0.2184	-0.156328	0.126533
G	9.91069	1	9.91069	9.67	0.0022	-0.393516	0.126533
H	0.0242191	1	0.0242191	0.02	0.8780	-0.0194531	0.126533
I	1.83772	1	1.83772	1.79	0.1824	0.169453	0.126533
J	78.4221	1	78.4221	76.53	0.0000	1.10695	0.126533
K	0.0130816	1	0.0130816	0.01	0.9102	-0.0142969	0.126533
L	0.0291129	1	0.0291129	0.03	0.8664	0.0213281	0.126533
M	12.1235	1	12.1235	11.83	0.0007	0.435234	0.126533
AB	0.177979	1	0.177979	0.17	0.6774	-0.0527344	0.126533
AC	43.1403	1	43.1403	42.10	0.0000	-0.821016	0.126533
AD	0.3143	1	0.3143	0.31	0.5804	-0.0700781	0.126533
AE	41.8205	1	41.8205	40.81	0.0000	-0.808359	0.126533
AF	0.0358629	1	0.0358629	0.03	0.8518	-0.0236719	0.126533
AG	0.00844102	1	0.00844102	0.01	0.9278	-0.0114844	0.126533
AH	0.00155039	1	0.00155039	0.00	0.9690	-0.00492187	0.126533
AI	0.0114223	1	0.0114223	0.01	0.9160	0.0133594	0.126533
AJ	40.2352	1	40.2352	39.27	0.0000	-0.792891	0.126533
AK	0.00256289	1	0.00256289	0.00	0.9602	-0.00632813	0.126533
AL	0.00534727	1	0.00534727	0.01	0.9425	-0.00914062	0.126533
AM	0.0372973	1	0.0372973	0.04	0.8489	0.0241406	0.126533
BC	0.353282	1	0.353282	0.34	0.5579	0.0742969	0.126533
BD	41.2405	1	41.2405	40.25	0.0000	0.802734	0.126533
BE	0.289579	1	0.289579	0.28	0.5957	0.0672656	0.126533
BF	0.000425391	1	0.000425391	0.00	0.9838	0.00257813	0.126533
BG	0.305947	1	0.305947	0.30	0.5855	0.0691406	0.126533
BH	0.000425391	1	0.000425391	0.00	0.9838	0.00257812	0.126533
BI	0.0106348	1	0.0106348	0.01	0.9190	-0.0128906	0.126533
BJ	0.0911285	1	0.0911285	0.09	0.7659	0.0377344	0.126533
BK	0.00256289	1	0.00256289	0.00	0.9602	-0.00632812	0.126533
BL	0.00185977	1	0.00185977	0.00	0.9661	-0.00539062	0.126533
BM	0.0780504	1	0.0780504	0.08	0.7829	-0.0349219	0.126533
CD	0.0372973	1	0.0372973	0.04	0.8489	0.0241406	0.126533
CE	38.6806	1	38.6806	37.75	0.0000	0.777422	0.126533
CF	0.0317285	1	0.0317285	0.03	0.8605	-0.0222656	0.126533
CG	0.0481254	1	0.0481254	0.05	0.8287	0.0274219	0.126533
CH	0.0157816	1	0.0157816	0.02	0.9014	-0.0157031	0.126533
CI	0.0402504	1	0.0402504	0.04	0.8431	0.0250781	0.126533
CJ	45.6385	1	45.6385	44.54	0.0000	0.844453	0.126533
CK	0.0304066	1	0.0304066	0.03	0.8634	-0.0217969	0.126533
CL	0.0000878906	1	0.0000878906	0.00	0.9926	-0.00117188	0.126533
CM	0.0980473	1	0.0980473	0.10	0.7575	-0.0391406	0.126533

Table 16: Discontinuous Process: Analysis of Variance for k (second part).

Source	Sum of Squares	Df	Mean Square	F-Ratio	P-Value	Effect Estimate	Stnd Error
DE	0.197469	1	0.197469	0.19	0.6612	0.0555469	0.126533
DF	0.250625	1	0.250625	0.24	0.6216	-0.0625781	0.126533
DG	0.521104	1	0.521104	0.51	0.4768	0.0902344	0.126533
DH	0.000791016	1	0.000791016	0.00	0.9779	-0.00351562	0.126533
DI	0.00430664	1	0.00430664	0.00	0.9484	0.00820313	0.126533
DJ	0.404019	1	0.404019	0.39	0.5309	0.0794531	0.126533
DK	0.00590977	1	0.00590977	0.01	0.9396	-0.00960937	0.126533
DL	0.00650039	1	0.00650039	0.01	0.9366	0.0100781	0.126533
DM	0.0317285	1	0.0317285	0.03	0.8605	-0.0222656	0.126533
EF	0.00534727	1	0.00534727	0.01	0.9425	0.00914063	0.126533
EG	0.0139535	1	0.0139535	0.01	0.9072	0.0147656	0.126533
EH	0.00101602	1	0.00101602	0.00	0.9749	-0.00398438	0.126533
EI	0.00219727	1	0.00219727	0.00	0.9631	0.00585937	0.126533
EJ	41.6267	1	41.6267	40.62	0.0000	0.806484	0.126533
EK	0.00185977	1	0.00185977	0.00	0.9661	-0.00539062	0.126533
EL	0.00481289	1	0.00481289	0.00	0.9454	0.00867187	0.126533
EM	0.0822973	1	0.0822973	0.08	0.7772	-0.0358594	0.126533
FG	0.385175	1	0.385175	0.38	0.5407	0.0775781	0.126533
FH	0.0000316406	1	0.0000316406	0.00	0.9956	0.000703125	0.126533
FI	0.00481289	1	0.00481289	0.00	0.9454	0.00867187	0.126533
FJ	0.073916	1	0.073916	0.07	0.7886	0.0339844	0.126533
FK	0.200816	1	0.200816	0.20	0.6586	-0.0560156	0.126533
FL	0.000594141	1	0.000594141	0.00	0.9808	0.00304687	0.126533
FM	0.0177223	1	0.0177223	0.02	0.8955	0.0166406	0.126533
GH	0.00382852	1	0.00382852	0.00	0.9513	-0.00773438	0.126533
GI	0.00711914	1	0.00711914	0.01	0.9337	0.0105469	0.126533
GJ	0.0822973	1	0.0822973	0.08	0.7772	0.0358594	0.126533
GK	0.0000316406	1	0.0000316406	0.00	0.9956	-0.000703125	0.126533
GL	0.00126914	1	0.00126914	0.00	0.9720	-0.00445313	0.126533
GM	0.0866566	1	0.0866566	0.08	0.7716	-0.0367969	0.126533
HI	0.0114223	1	0.0114223	0.01	0.9160	0.0133594	0.126533
HJ	0.00844102	1	0.00844102	0.01	0.9278	0.0114844	0.126533
HK	0.00650039	1	0.00650039	0.01	0.9366	-0.0100781	0.126533
HL	0.00481289	1	0.00481289	0.00	0.9454	0.00867187	0.126533
HM	0.00000351562	1	0.00000351562	0.00	0.9985	-0.000234375	0.126533
IJ	0.438079	1	0.438079	0.43	0.5141	-0.0827344	0.126533
IK	0.0000316406	1	0.0000316406	0.00	0.9956	0.000703125	0.126533
IL	0.0387598	1	0.0387598	0.04	0.8460	-0.0246094	0.126533
IM	0.740675	1	0.740675	0.72	0.3965	-0.107578	0.126533
JK	0.000172266	1	0.000172266	0.00	0.9897	0.00164062	0.126533
JL	0.000425391	1	0.000425391	0.00	0.9838	0.00257813	0.126533
JM	0.0387598	1	0.0387598	0.04	0.8460	0.0246094	0.126533
KL	0.00126914	1	0.00126914	0.00	0.9720	0.00445313	0.126533
KM	0.00650039	1	0.00650039	0.01	0.9366	-0.0100781	0.126533
LM	0.0464941	1	0.0464941	0.05	0.8316	-0.0269531	0.126533
Error	168.048	164	1.02468				
Total (corr.)	694.763	255					

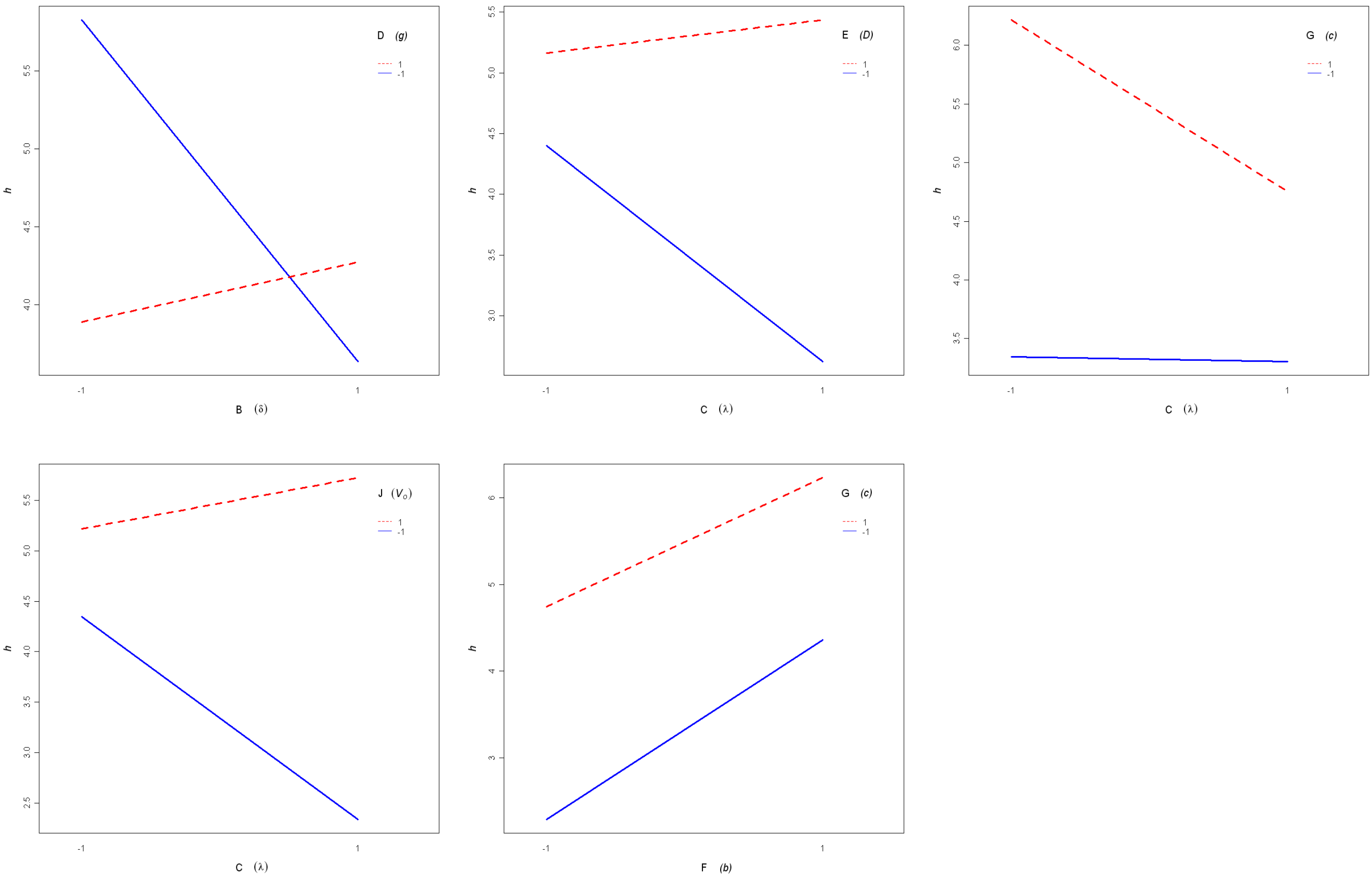


Figure 10: Discontinuous Process: primary effects plots for h.

The ANOVA for the control limits, k, is summarized in Tables 15, 16 and Fig. 11. A much lower number of significant interactions is observed (if we compare it with previous cases) and factors F (b), H (W), I (T), K (S) and L (S_c) prove to have a negligible effect on k. Factors G (c) and M (D_1) act only as main effects. Interactions AC, AE and AJ have a common interpretation: a growth in penalty costs due to out-of-control, M, reduces k in a consistent way only if λ, D or V_o have high values; otherwise (when the process is stable or assignable causes can be quickly repaired or are not too expensive), the reduction of k is very small. Interaction BD deny that an increase in δ always expands k : when g is small, k is reduced. Factor C (λ) interacts both with E (D) and J (V_o) : a less stable process reduces k when D or V_o are low; conversely when D or V_o have high values, limits are strongly increased by λ (maybe because, in that case an increase in λ is better fronted with bigger and more frequent samples). A similar explanation holds for interaction EJ.

In general, for the discontinuous case, interactions do not confirm the analysis proposed in Panagos et al. (1985). The links among factors are quite strong and

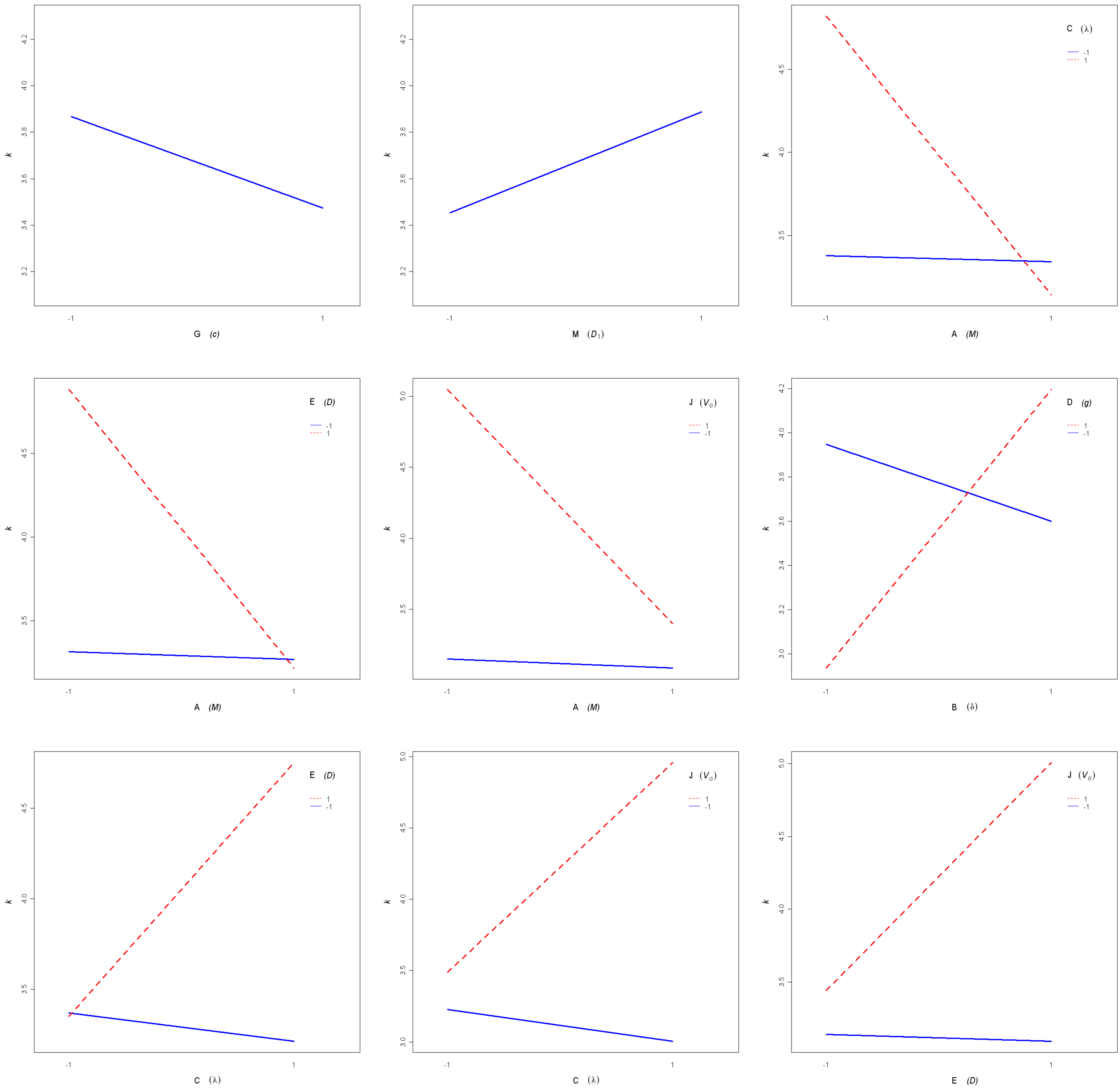
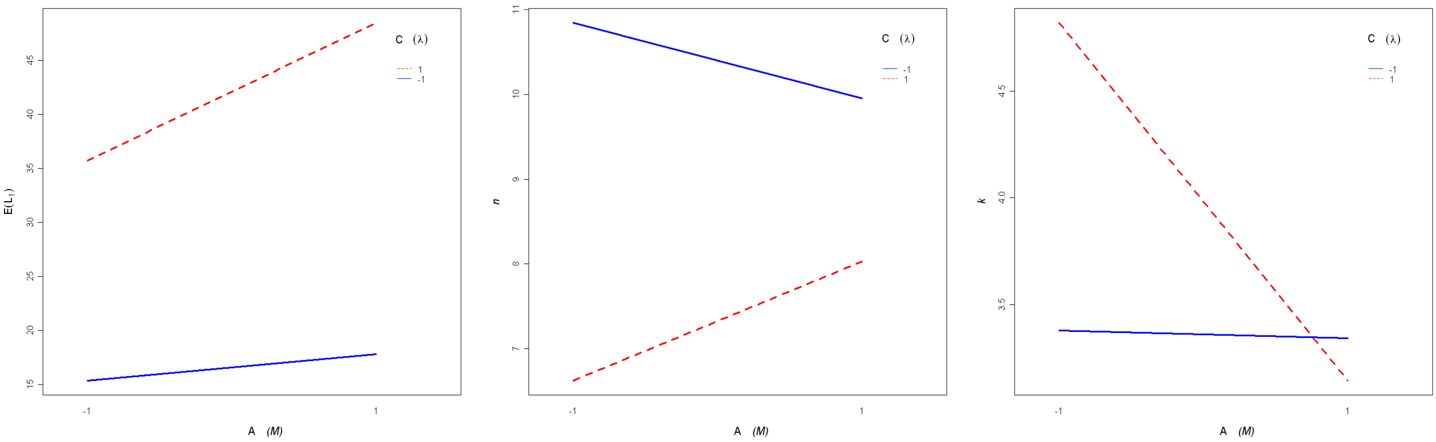


Figure 11: Discontinuous Process: primary effects plots for k.

complicated but their effects cannot be neglected.

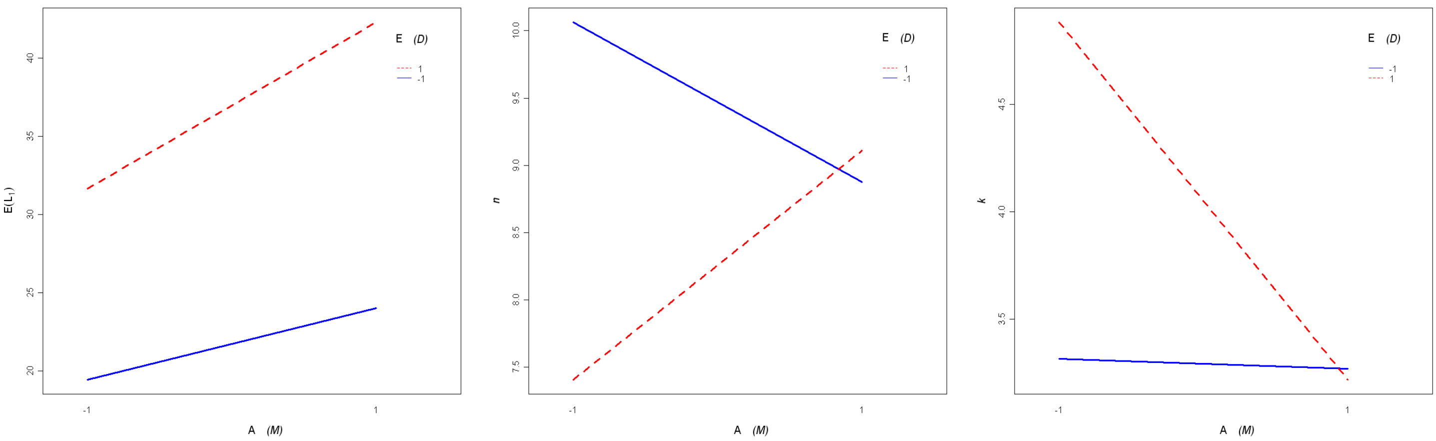
In order to explain better the effect of some interactions which proved to significantly affect many response variables, the following summary is proposed.

▶ Interaction AC:



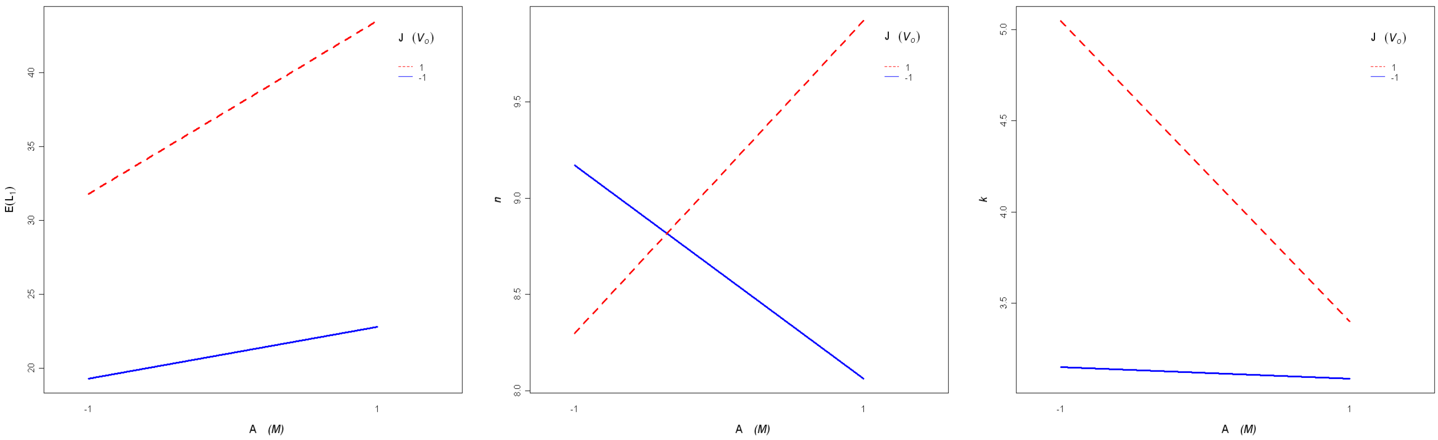
When λ is high (less stable process), an increase in M (penalty costs associated to out-of-control state) produces an increase in loss function, and bigger samples with narrower limits. Conversely, when λ is low, an increase in M, produces a small increase in loss function, smaller samples with a small reduction in control limits.

▶ Interaction AE:



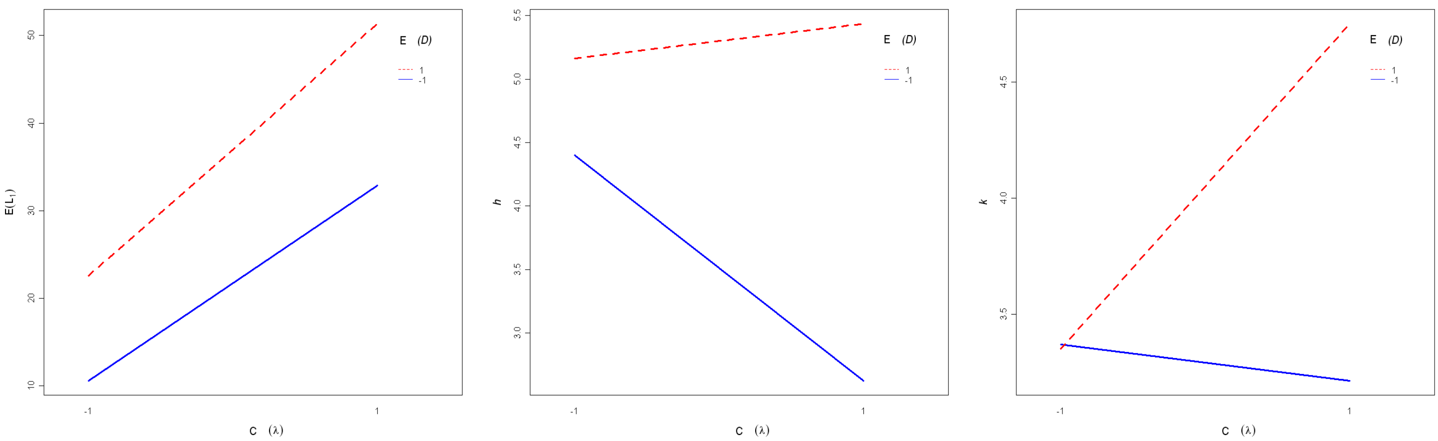
When D is high (longer time required to find and repair an assignable cause), an increase in M (penalty costs associated to out-of-control state) – during time D no income is available due to production stop – produces an increase in loss function, and bigger samples with narrower limits. Conversely, when D is low, an increase in M produces a small increase in loss function, smaller samples with a small reduction in control limits.

▶ Interaction AJ:



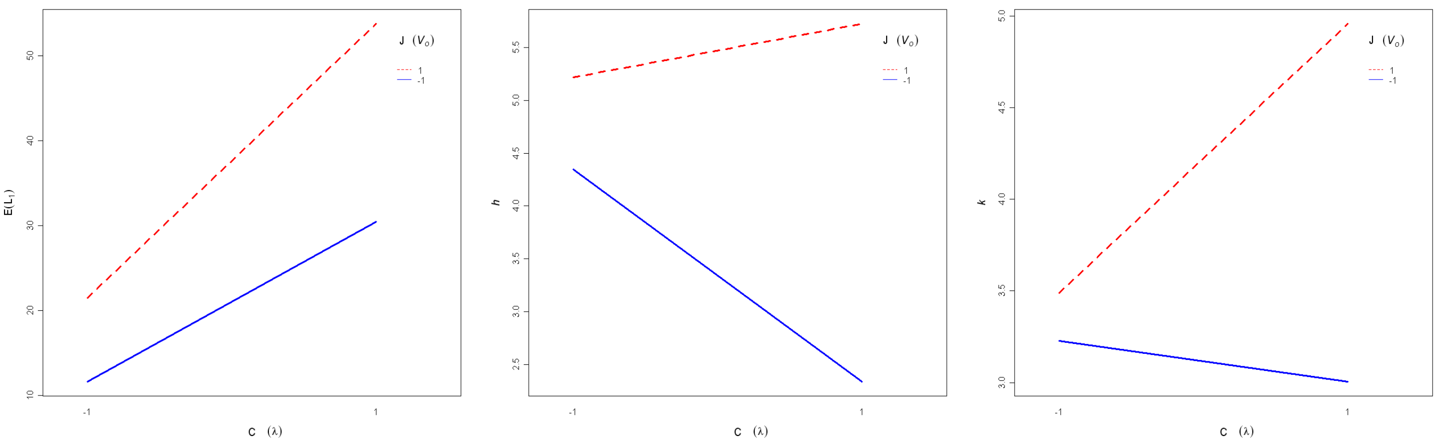
A very similar behaviour is observed for the interaction between $M(= V_o - V_1)$ and V_o. Obviously, when V_o is high, an increase in M corresponds to low income in out-of-control, V_1 and hence, both undetected out-of-control and production stop periods entail a great income loss. In order to minimize this loss, it is thus essential a prompt alarm signal (high power) which is achieved through much bigger samples and narrower control limits.

▶ Interaction CE:



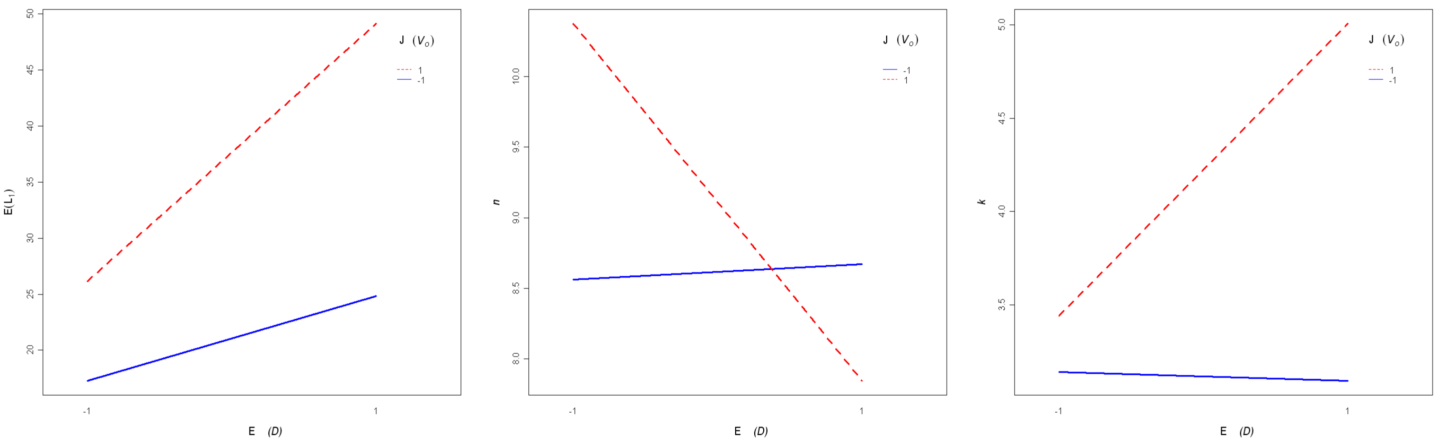
When D is high (longer time required to find and repair an assignable cause), an increase in λ (more unstable process) produces an increase in loss function and, surprisingly, slightly less frequent samples with much larger control limits. Conversely, when D is low, an increase in λ produces more frequent samples with narrower limits.

▶ Interaction CJ:



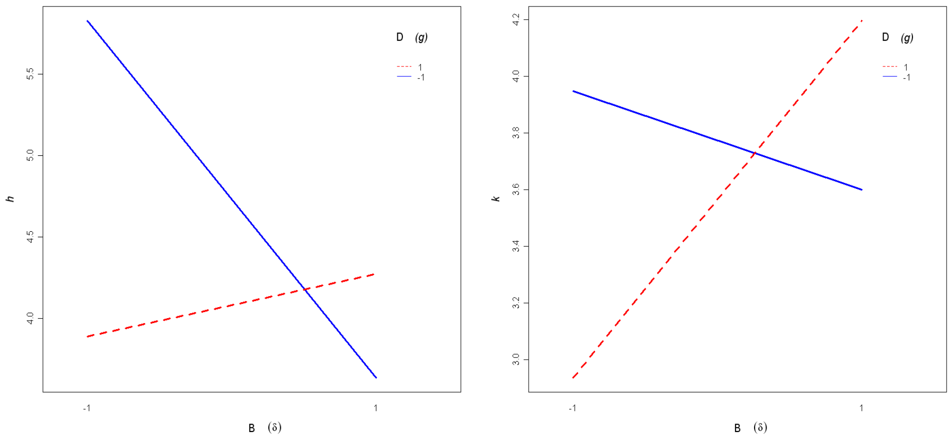
The same behaviour linking changes in λ with changes in D, is observed when D is replaced by V_o. We have to observe that, for high V_o, an increase in λ produces slightly less frequent samples with much larger control limits.

▶ Interaction EJ:



When V_o is high, an increase in D produces smaller samples with larger limits. Conversely, when V_o is low, an increase in D produces slightly bigger samples with smaller limits.

► **Interaction BD:**



When g is high, an increase in δ (larger shift in the process mean) can be detected with less frequent samples and larger limits. The surprising result is that when g is low, an increase in δ produces more frequent samples with narrower limits.

5 Conclusions

Results from ANOVA applied to data from the large experimental design here performed highlight a complex interaction structure among factors both in the continuous and the discontinuos case. While, however, in the former case, interactions do not contradict the sign of factors effects (although a significant effect is observed here for more factors than stated in Panagos et al. (1985)), the discontinuous case shows many contradictions to commonly believed effects. We argue that this work may represent a preliminary step because interactions with surprising effects should be embedded in even higher order interactions and these could be estimated only with a bigger experimental design.

Appendix

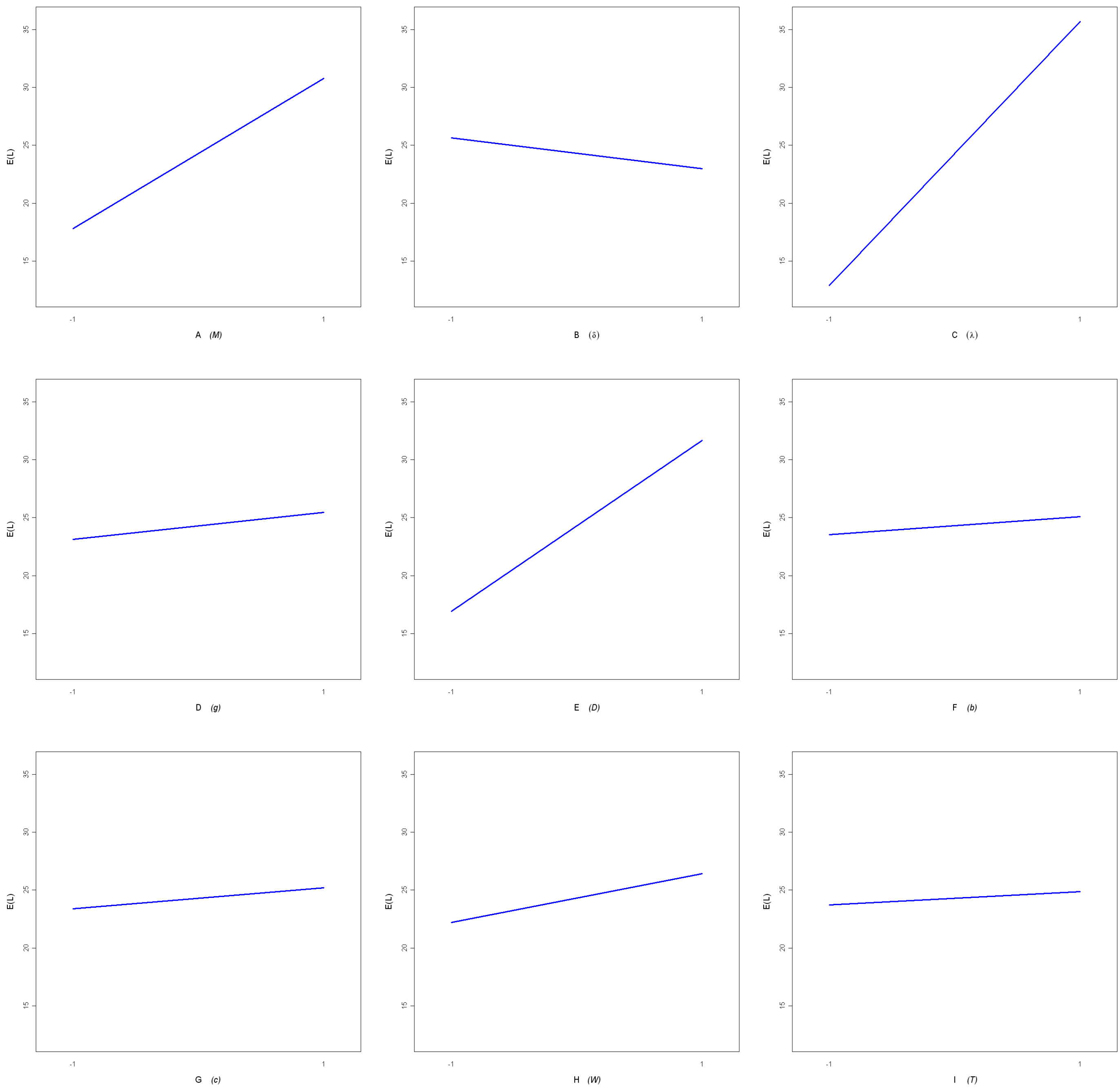


Figure 12: Continuous Process: main effects plots for $E(L)$.

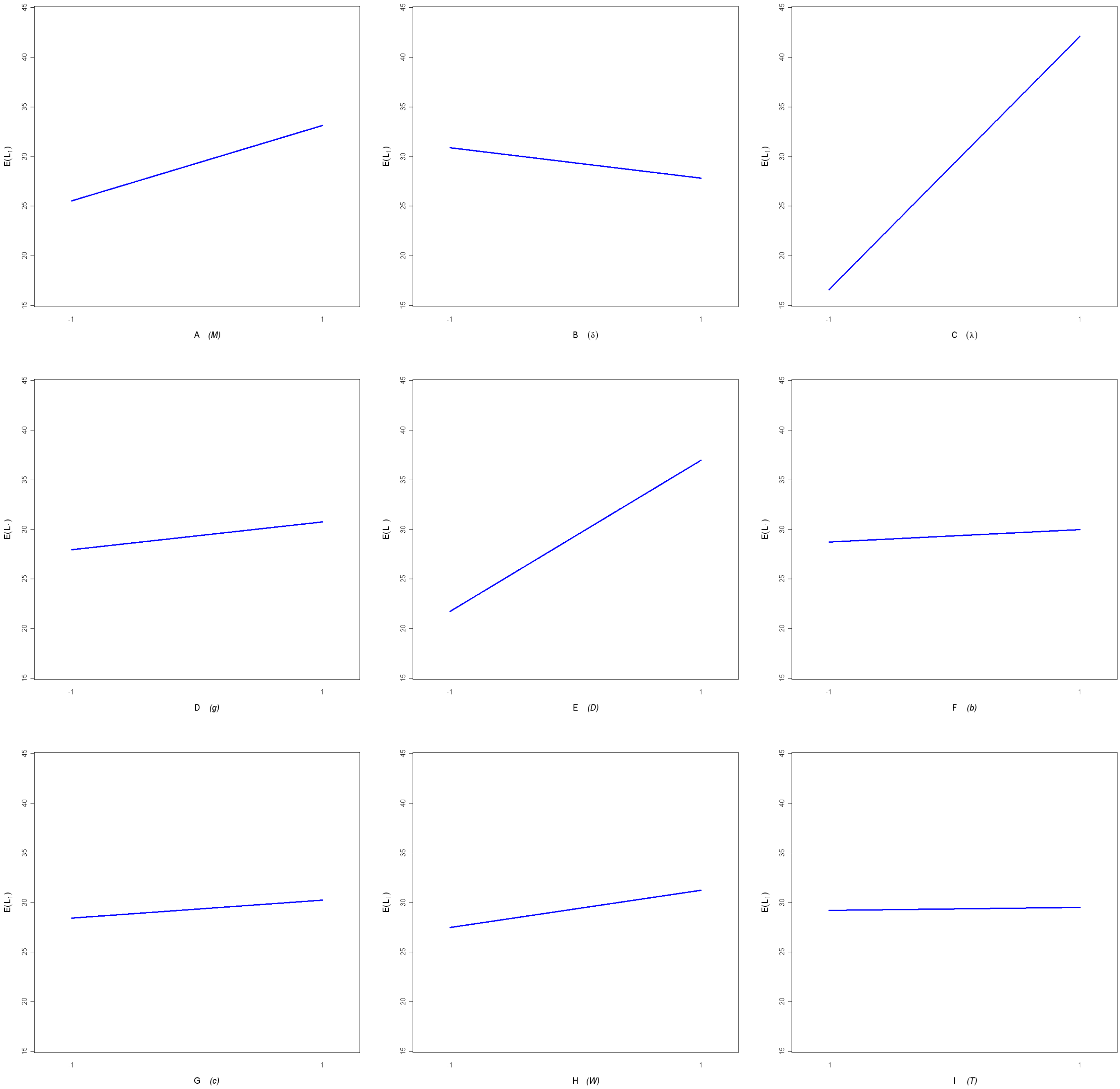


Figure 13: Discontinuous Process: main effects plots for $E(L_1)$ (1/2).

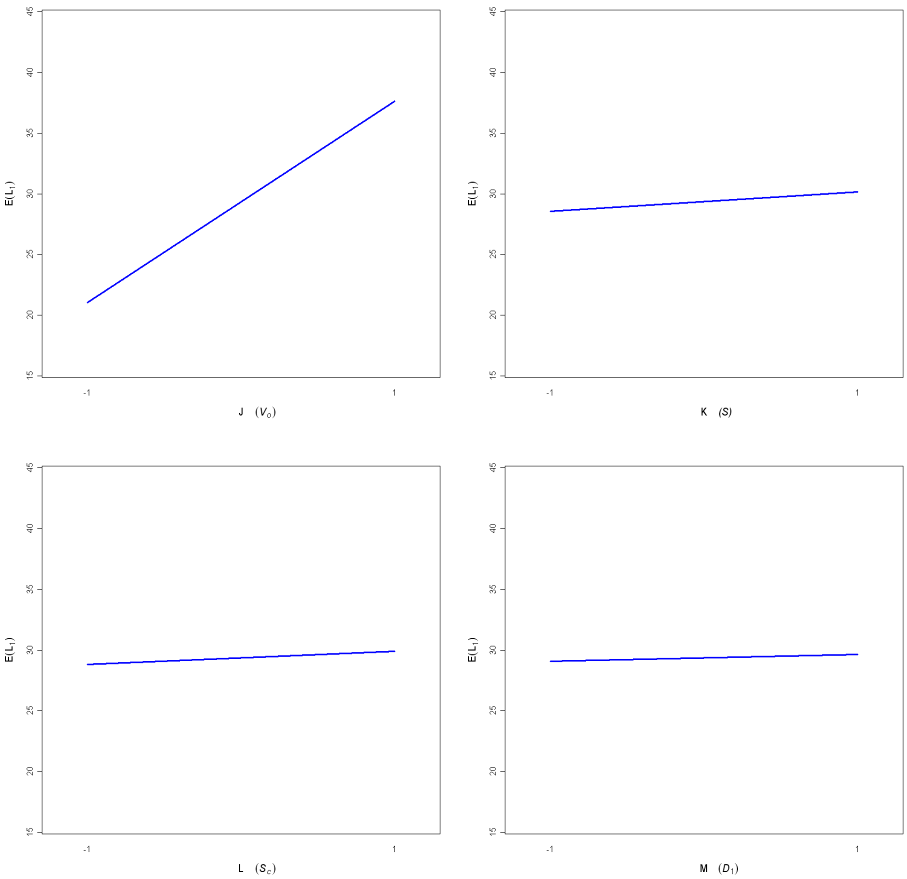


Figure 14: Discontinuous Process: main effects plots for $E(L_1)$ (2/2).

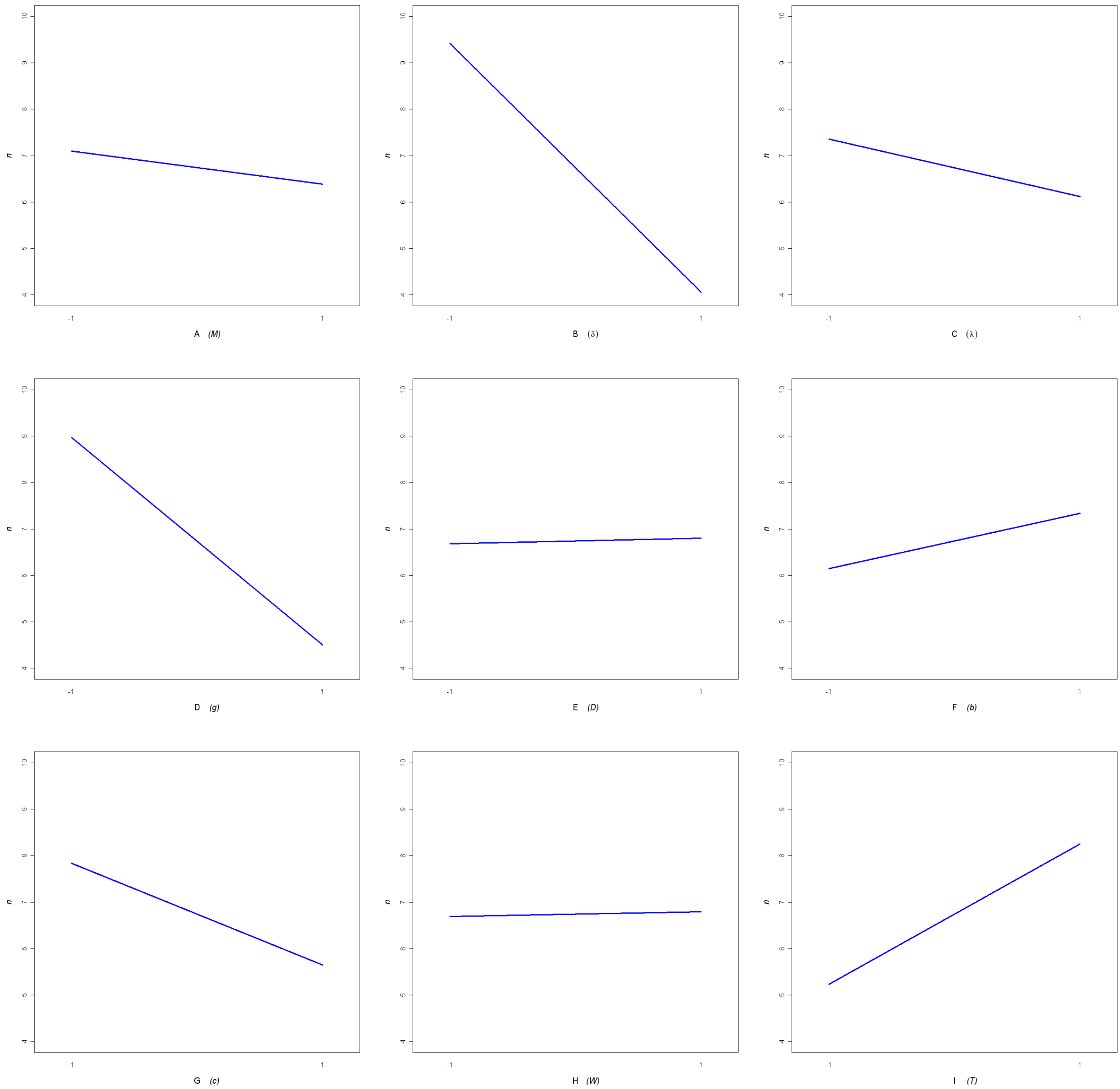


Figure 15: Continuous Process: main effects plots for n.

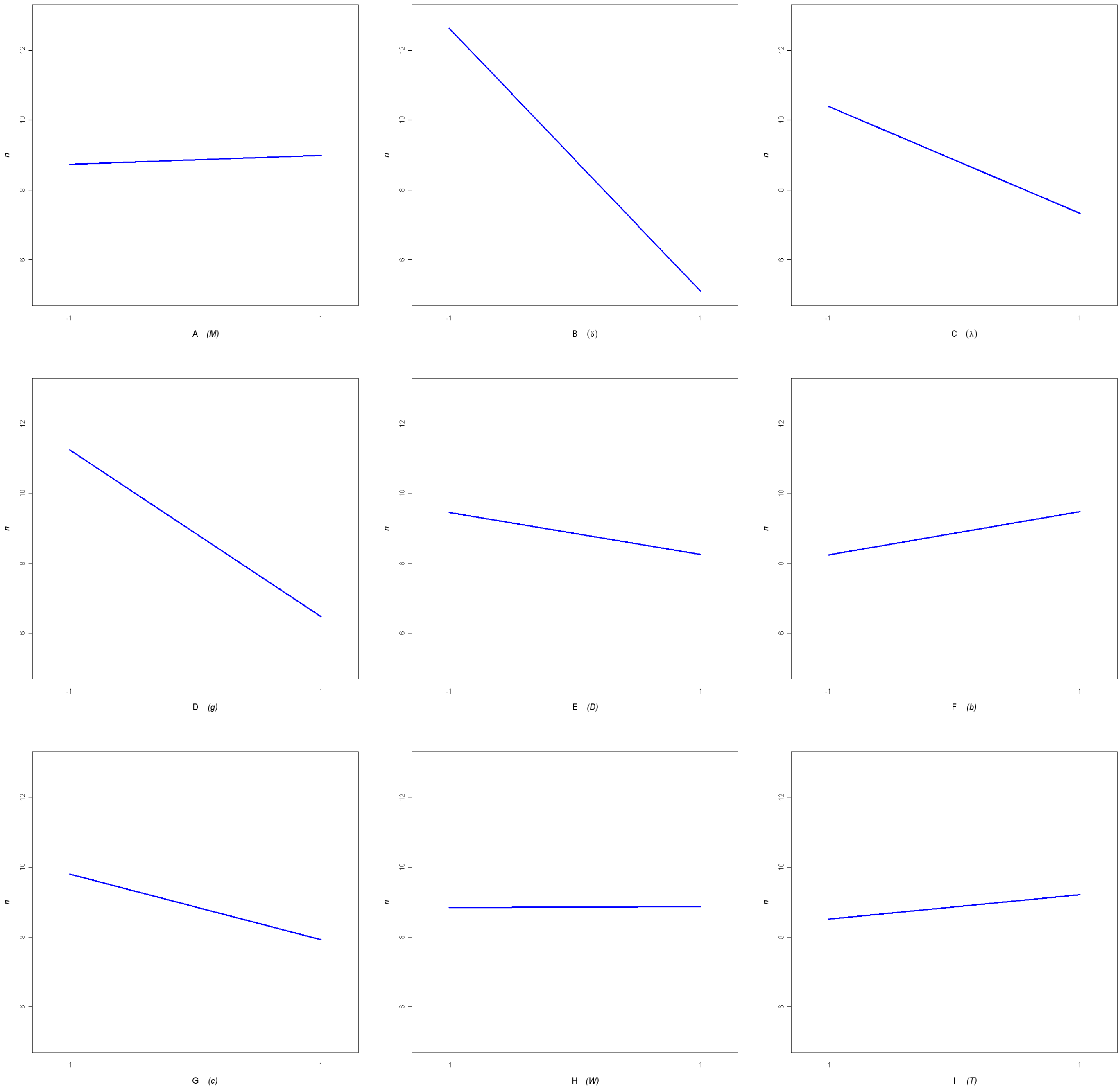


Figure 16: Discontinuous Process: main effects plots for n (1/2).

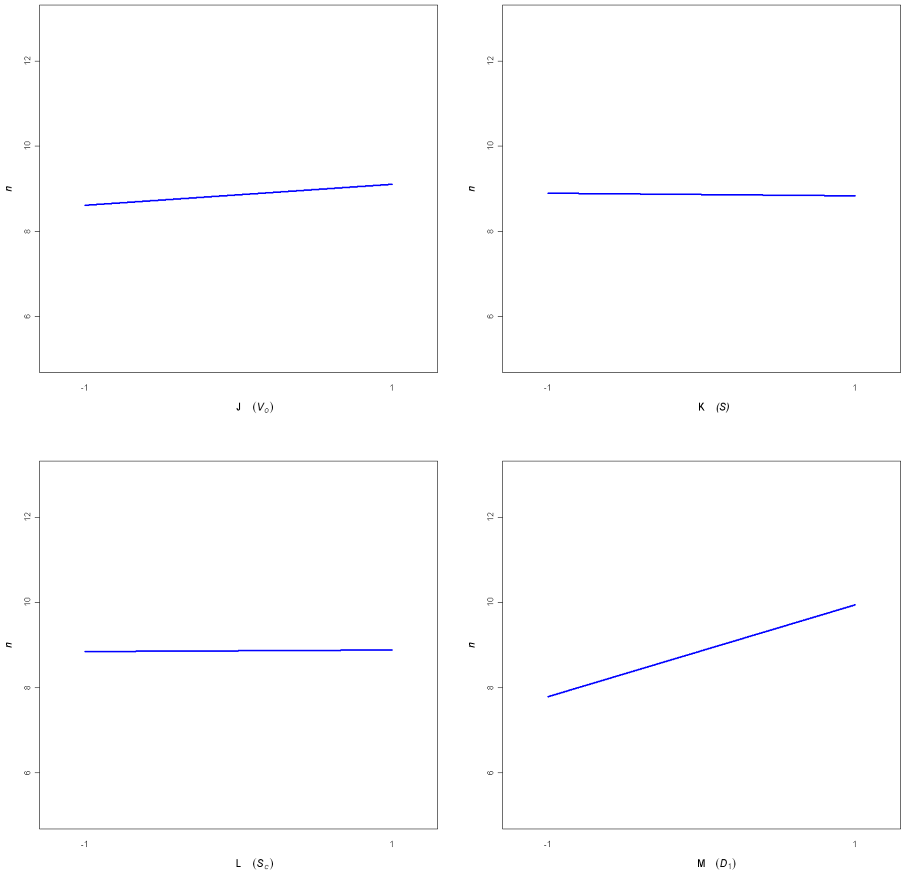


Figure 17: Discontinuous Process: main effects plots for n (2/2).

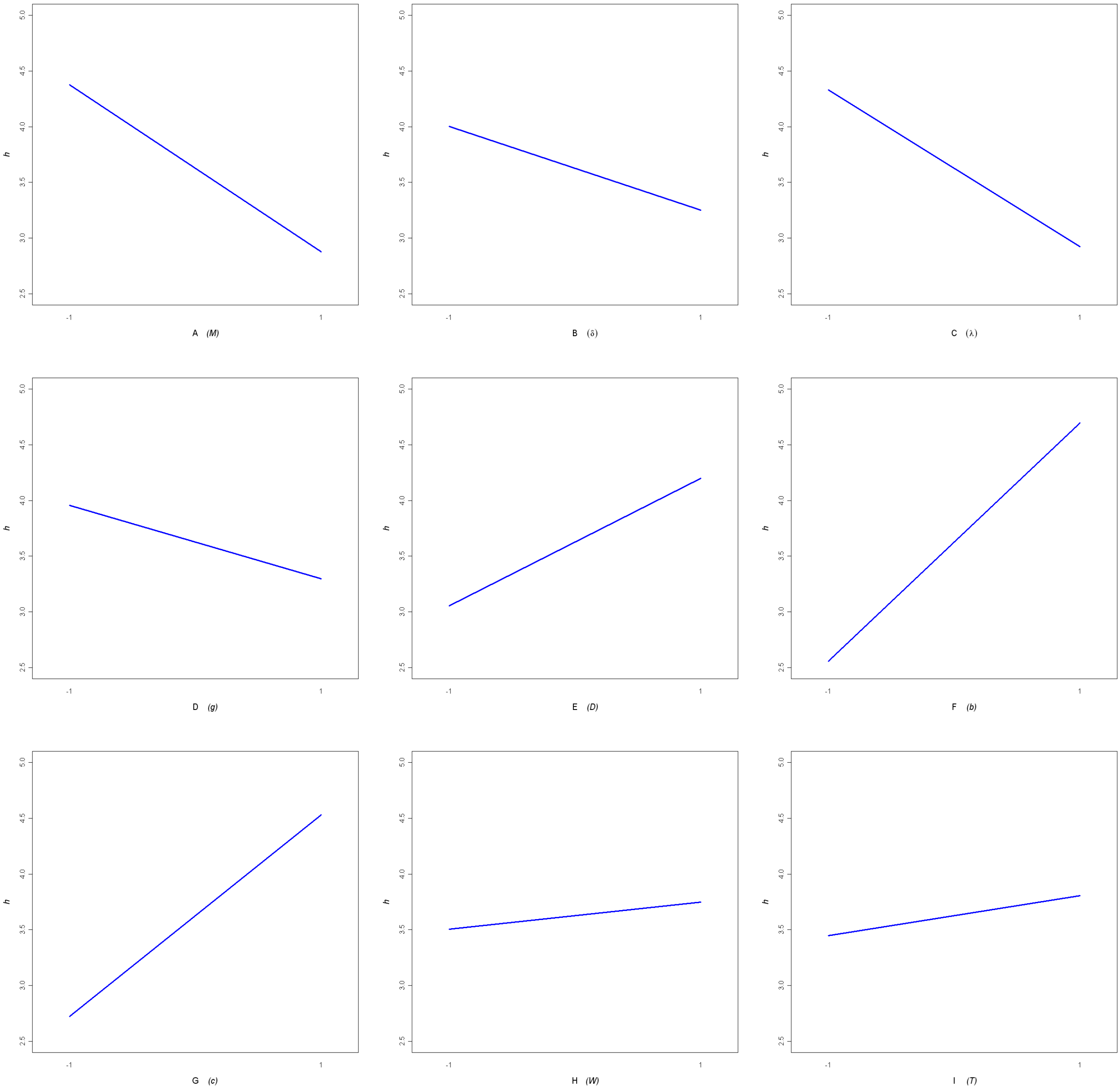


Figure 18: Continuous Process: main effects plots for h.

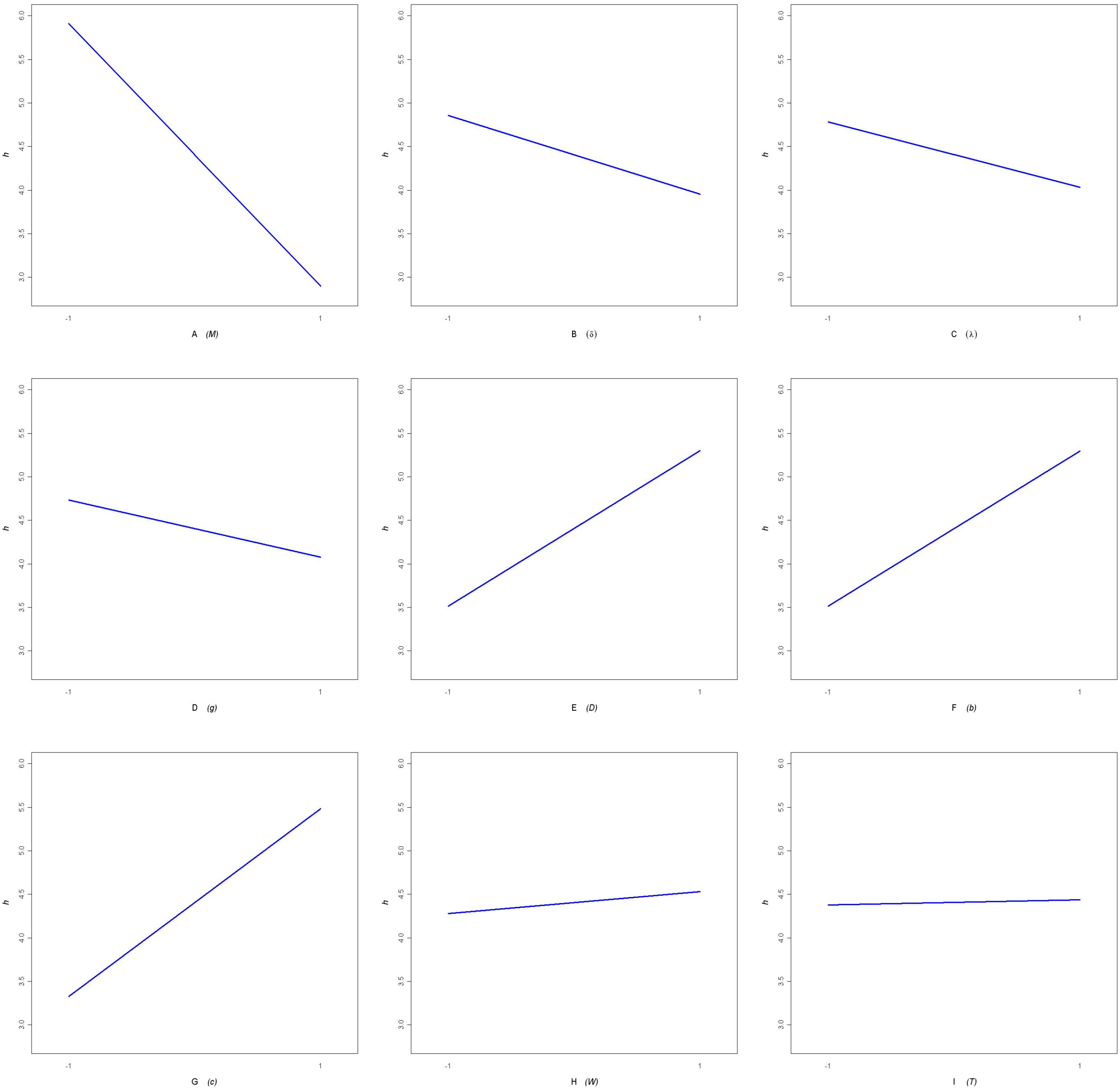


Figure 19: Discontinuous Process: main effects plots for h (1/2).

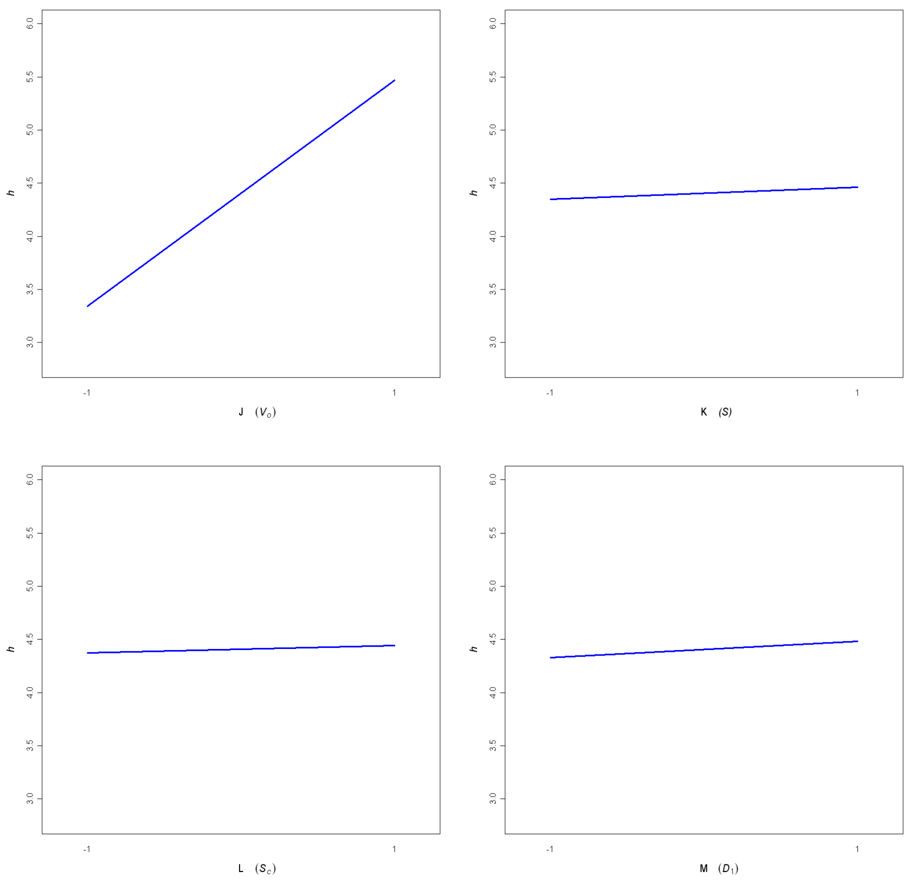


Figure 20: Discontinuous Process: main effects plots for h (2/2).

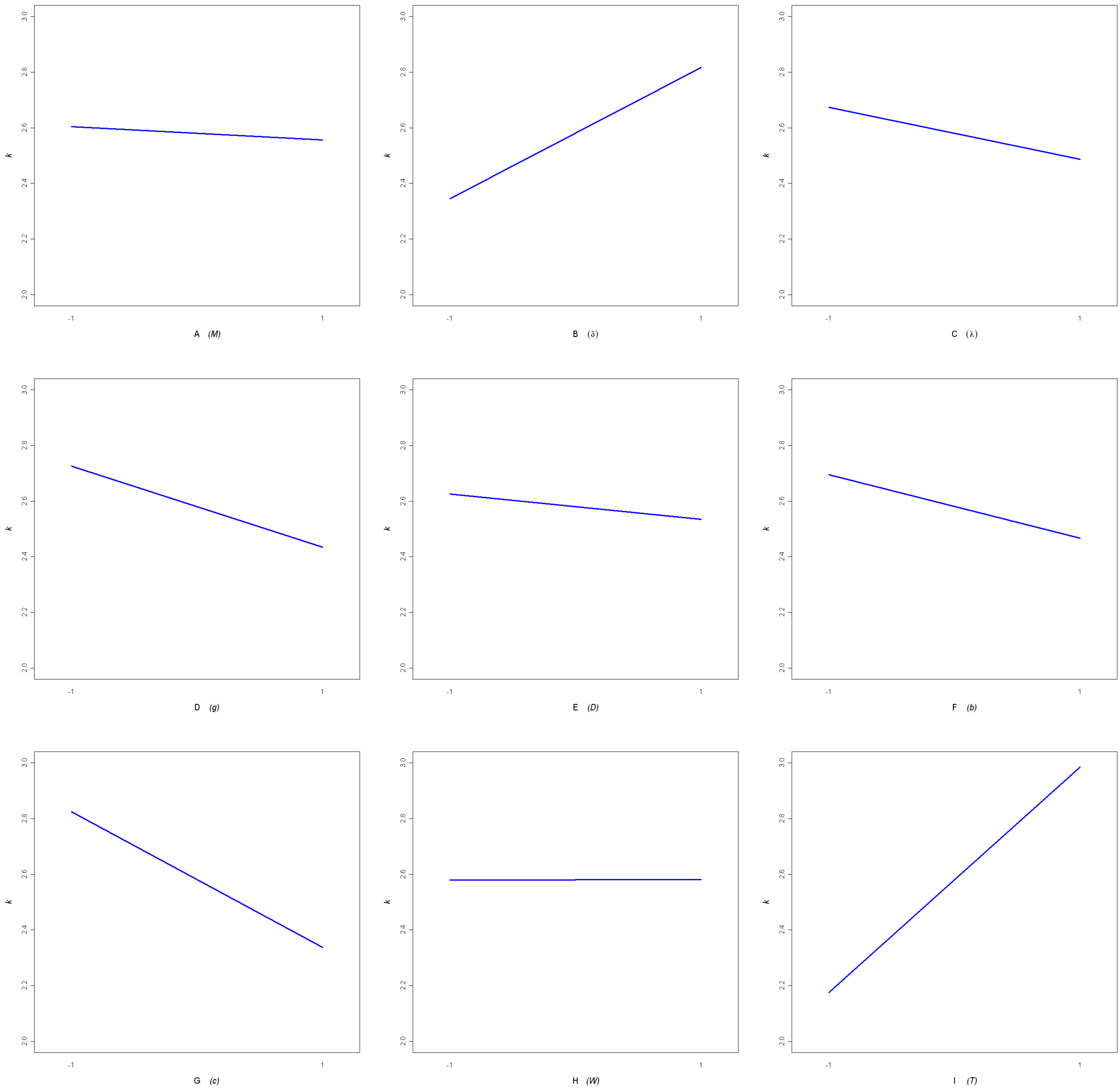


Figure 21: Continuous Process: main effects plots for k.

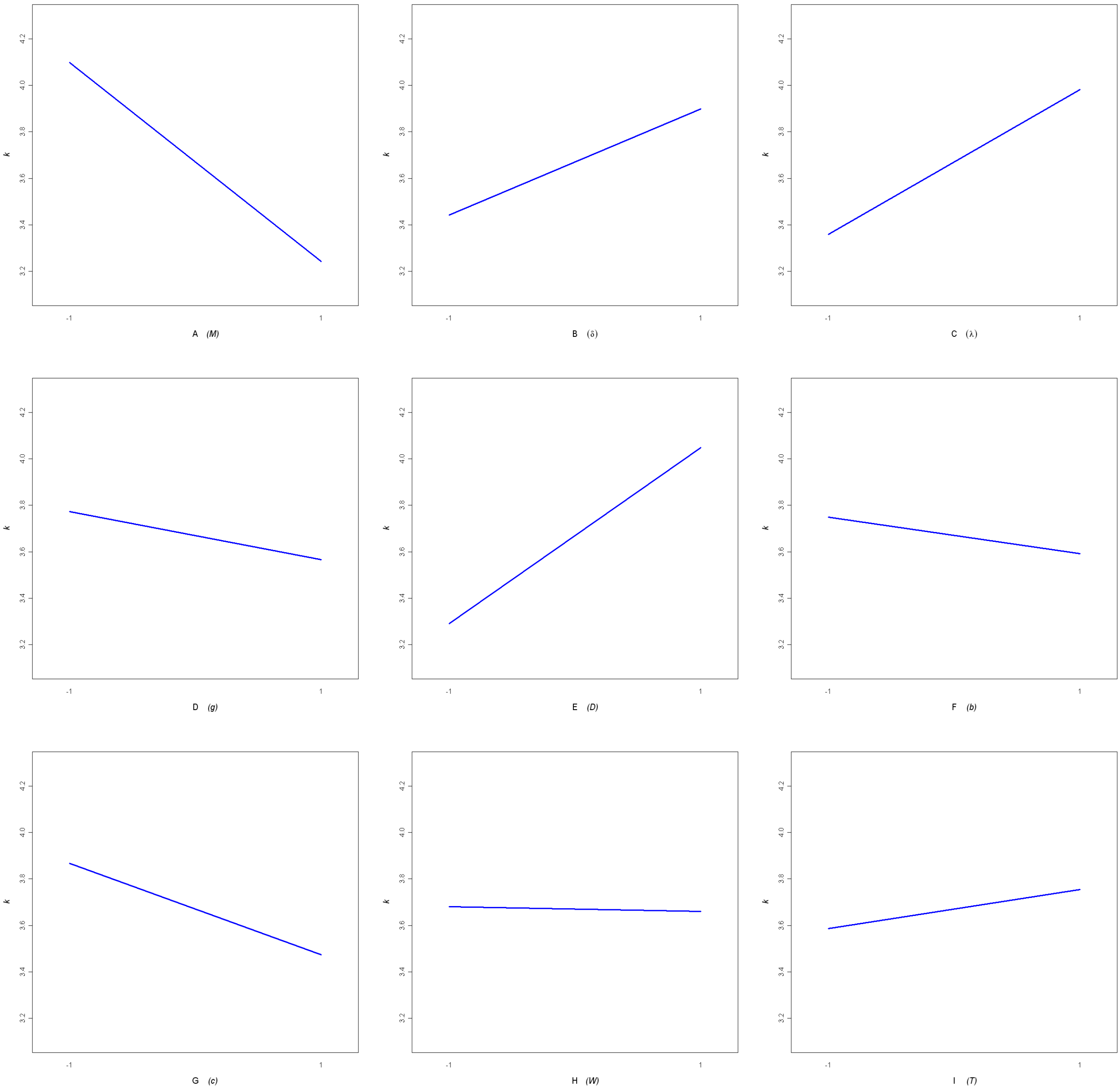


Figure 22: Discontinuous Process: main effects plots for k (1/2).

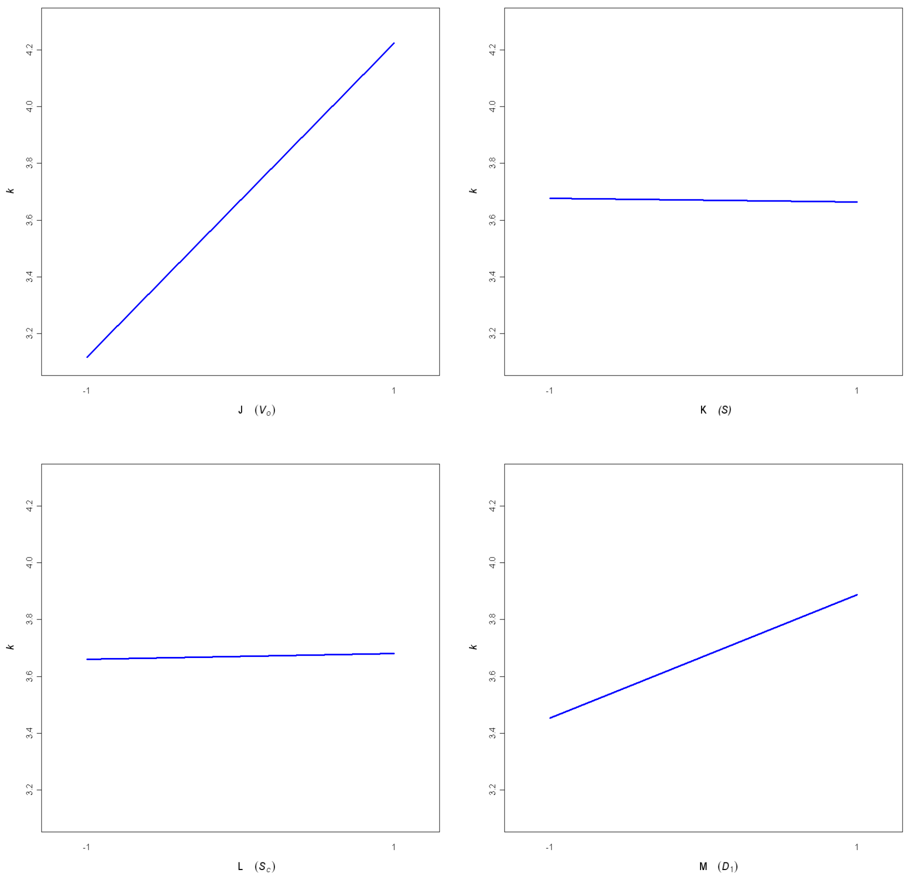


Figure 23: Discontinuous Process: main effects plots for k (2/2).

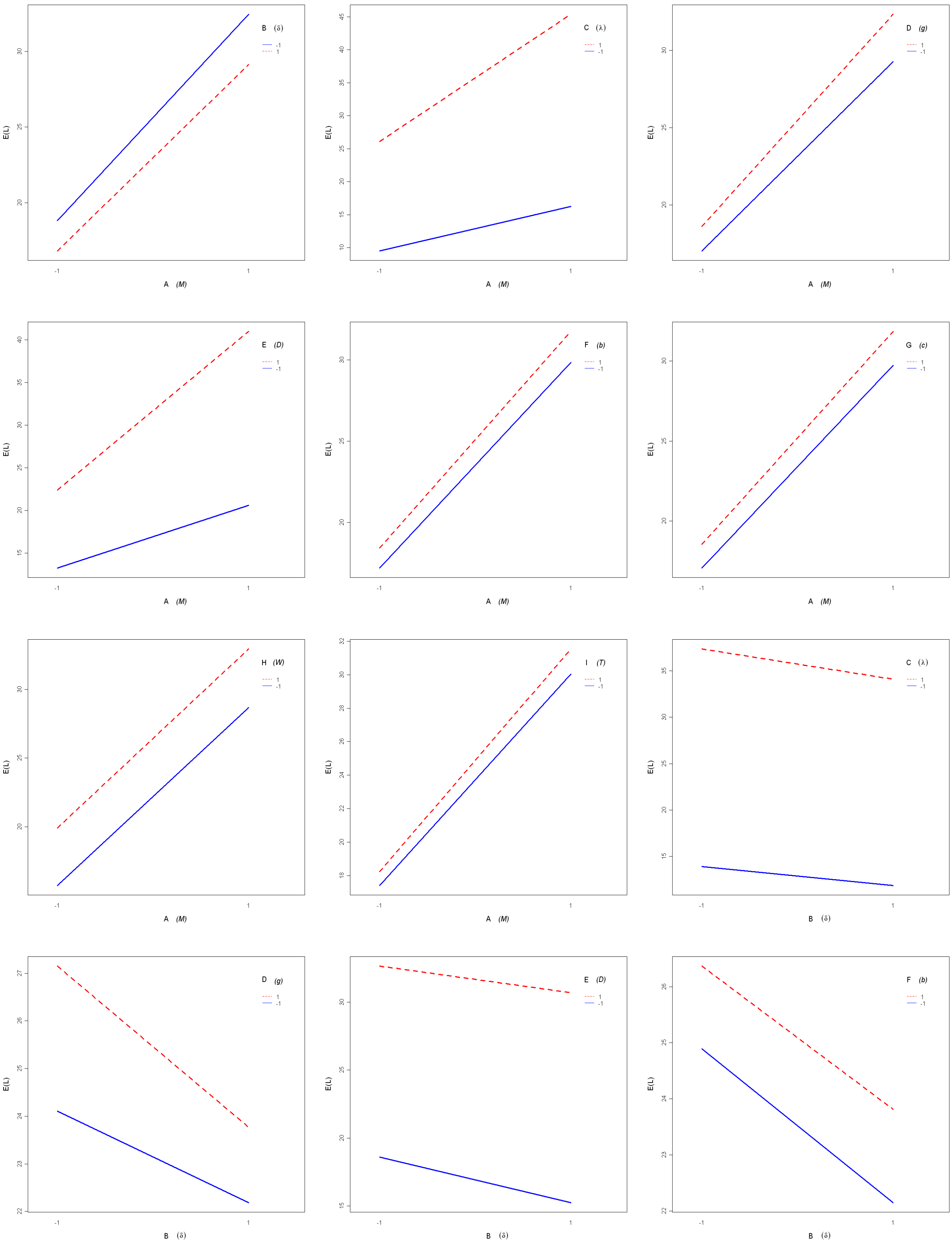


Figure 24: Continuous Process: interaction plots for $E(L)$ (1/3).

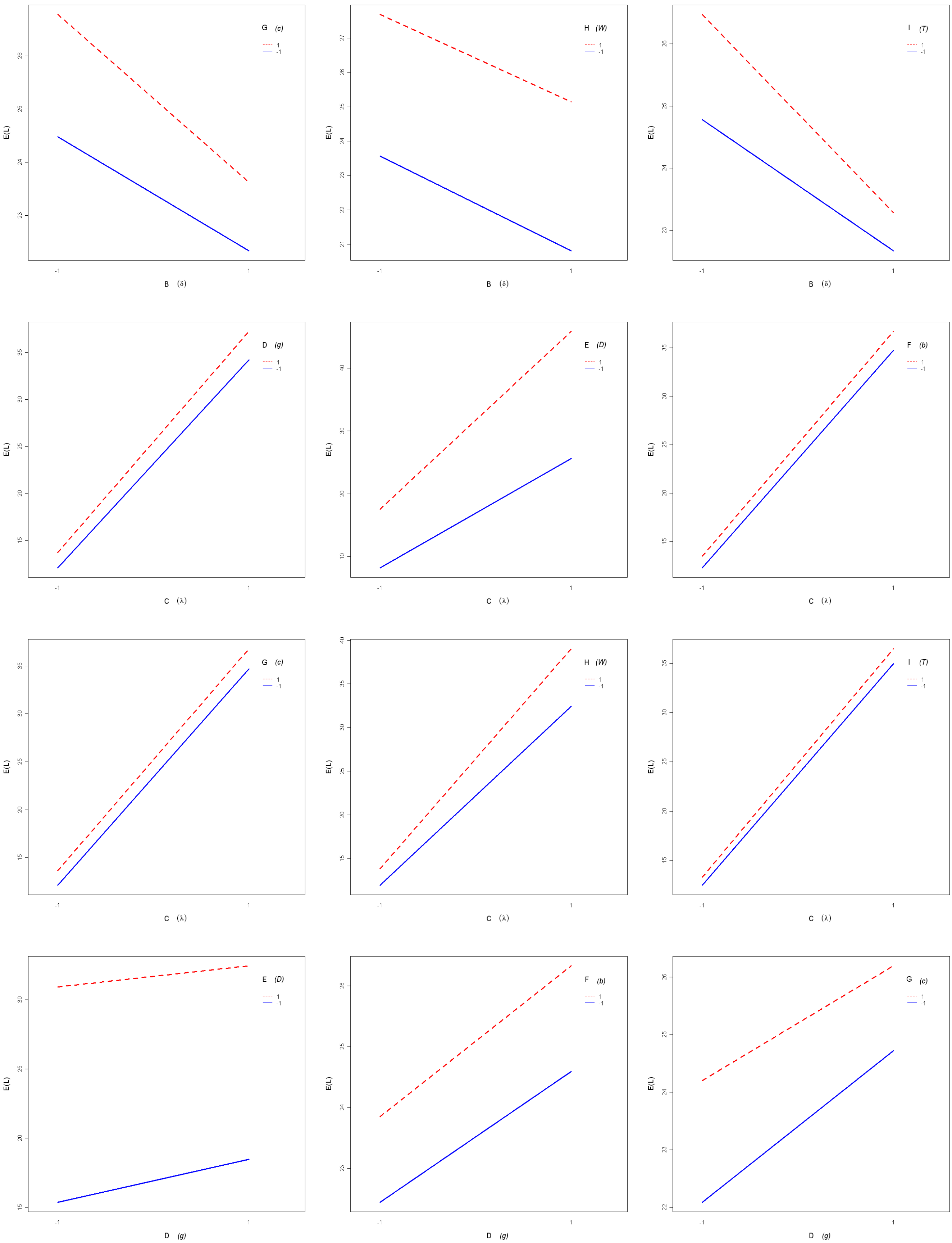


Figure 25: Continuous Process: interaction plots for $E(L)$ (2/3).

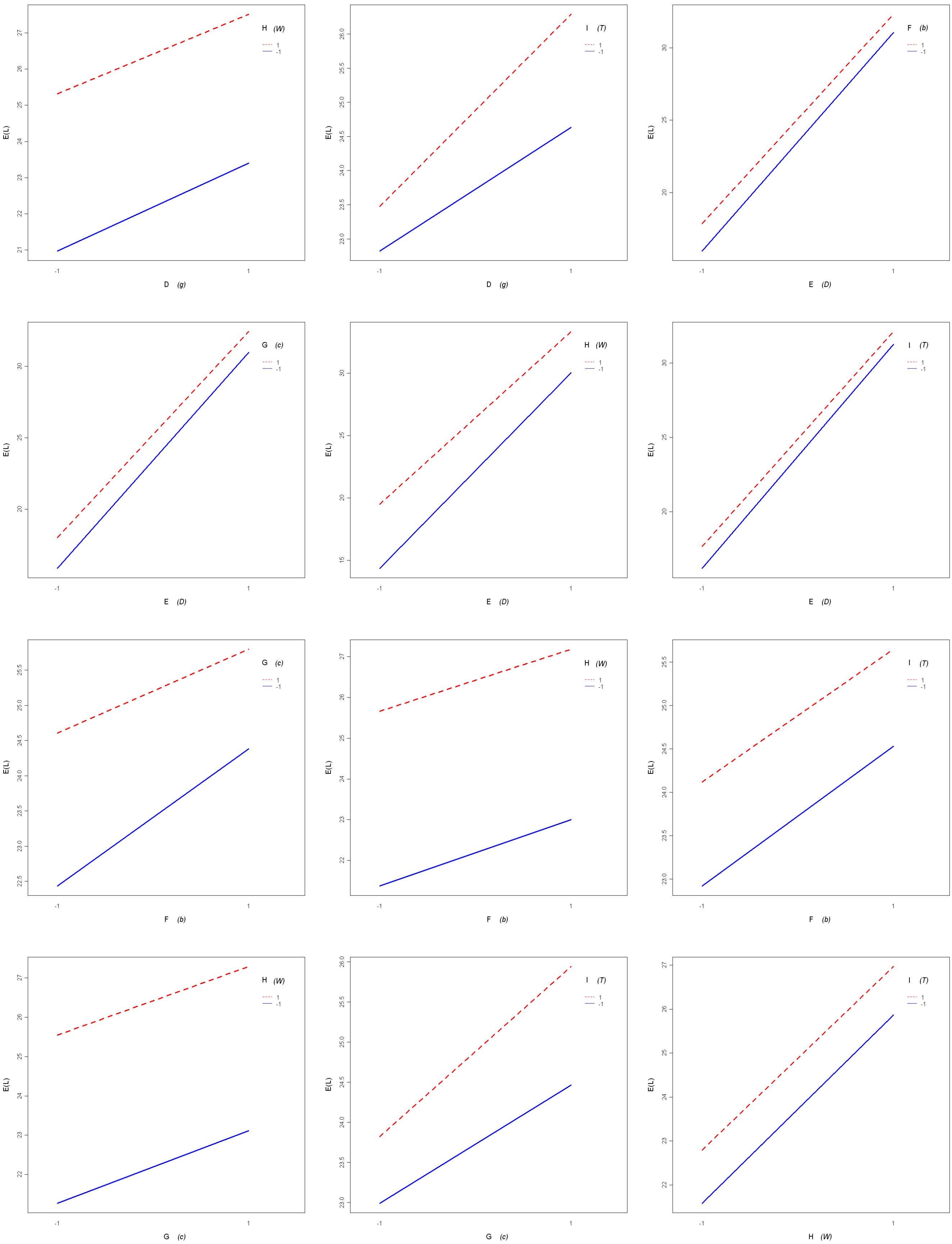


Figure 26: Continuous Process: interaction plots for $E(L)$ (3/3).

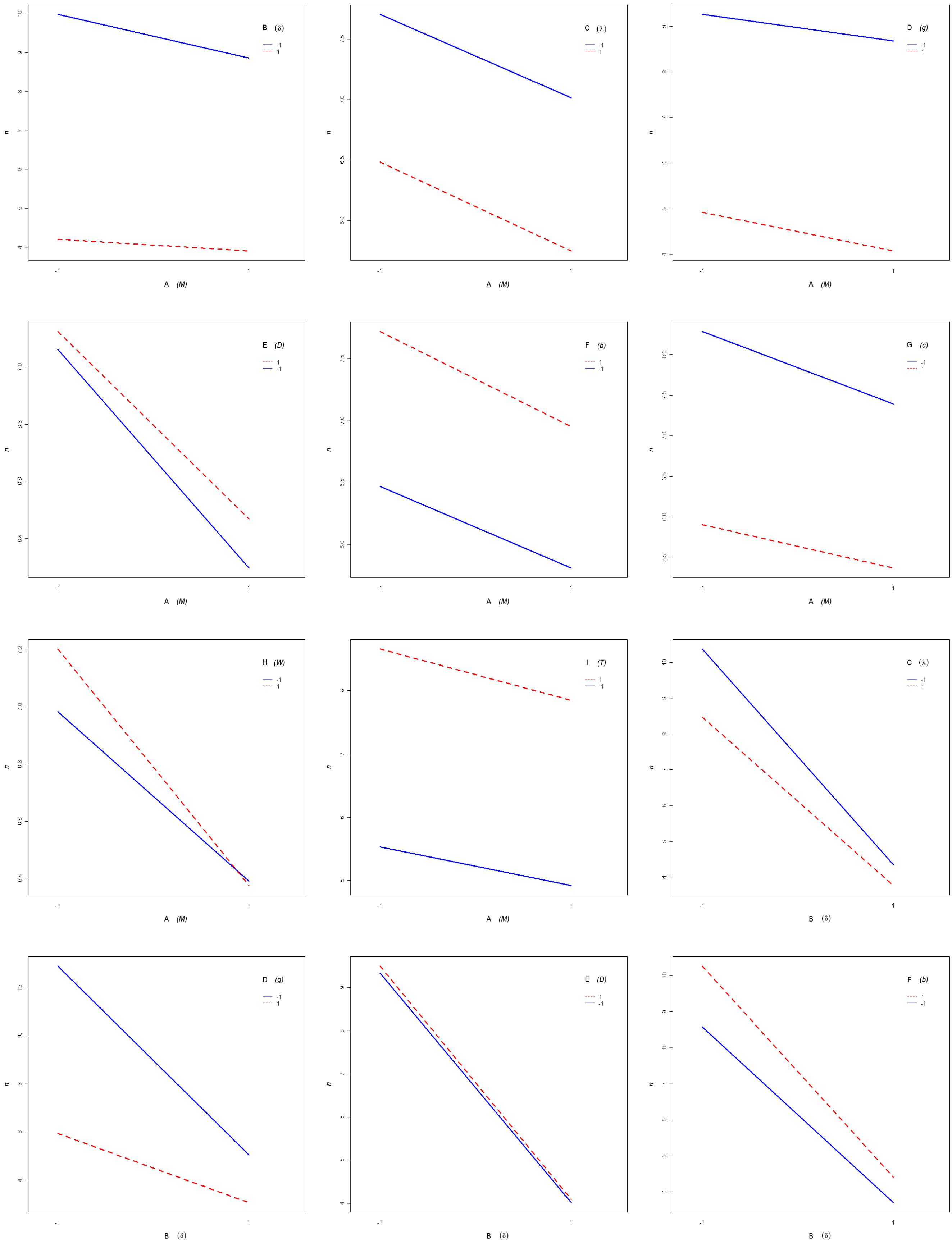


Figure 27: Continuous Process: interaction plots for n (1/3).

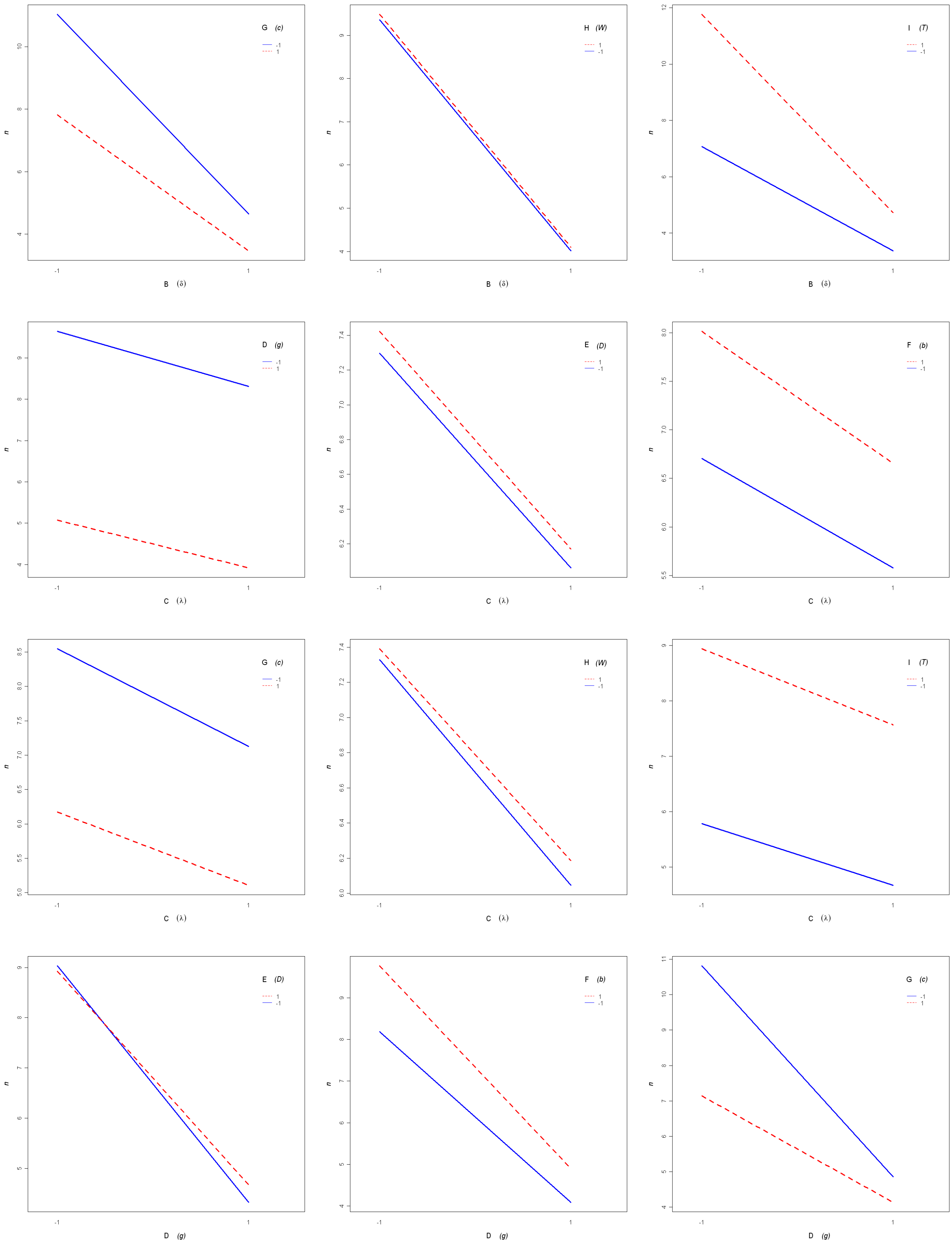


Figure 28: Continuous Process: interaction plots for n (2/3).

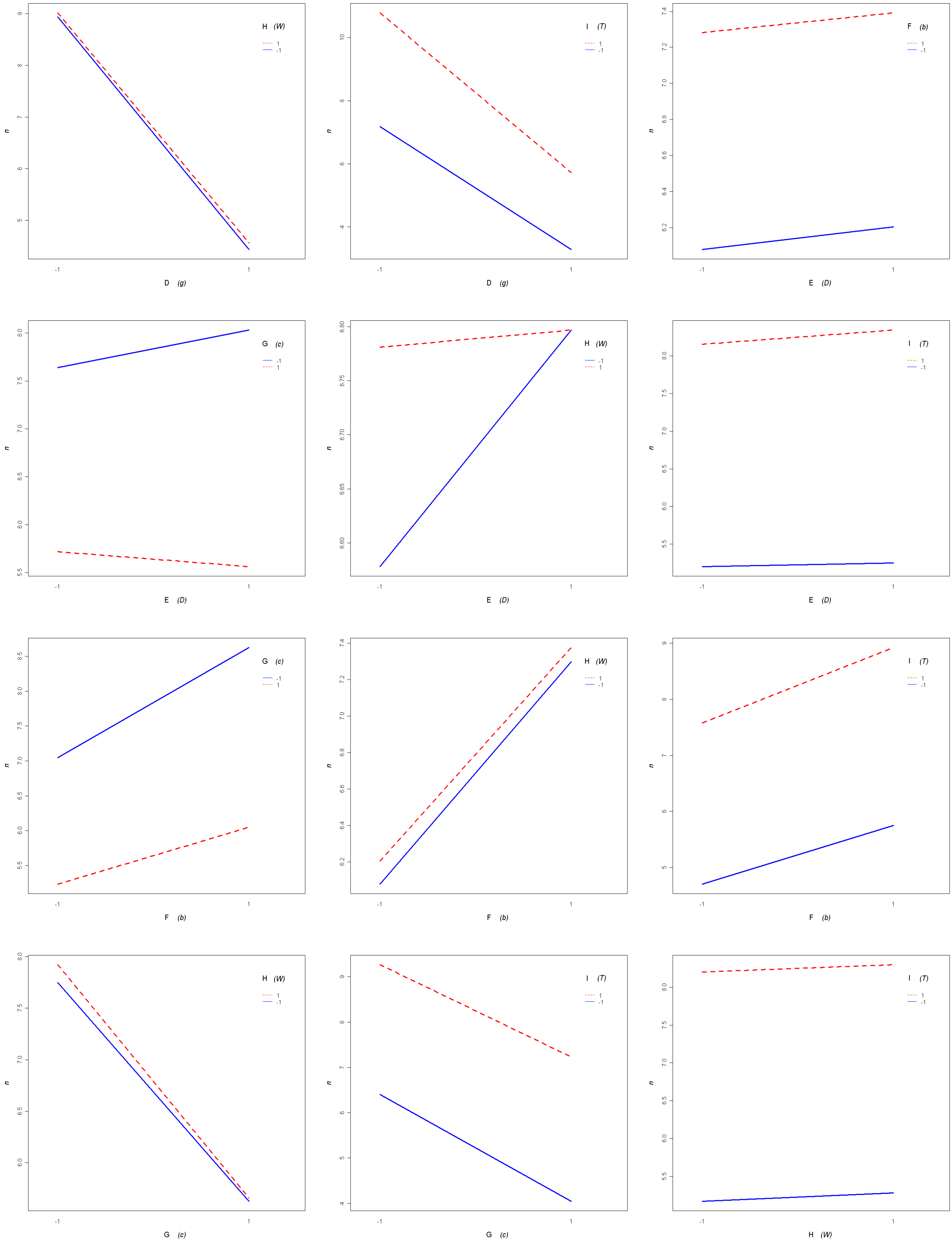


Figure 29: Continuous Process: interaction plots for n (3/3).

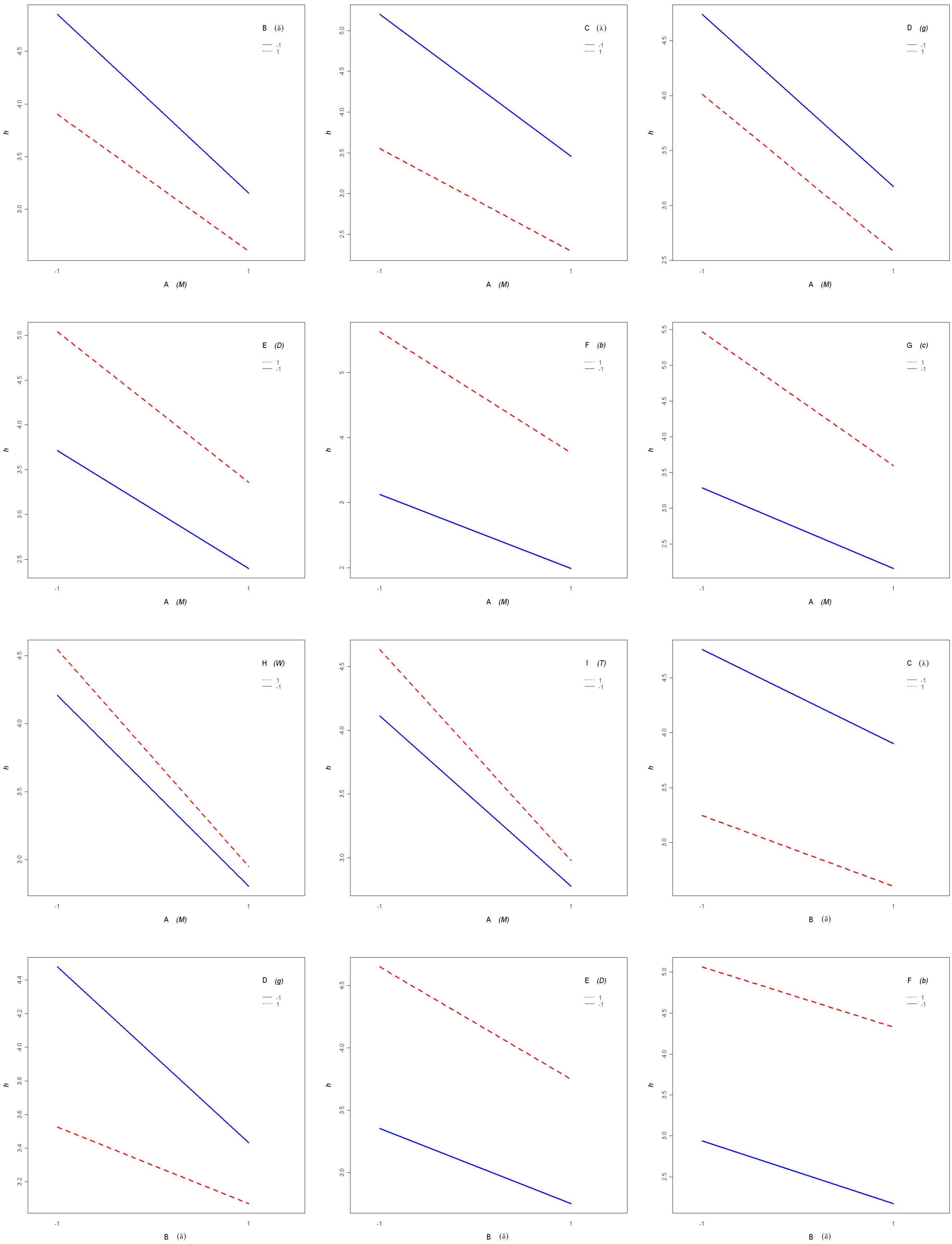


Figure 30: Continuous Process: interaction plots for h (1/3).

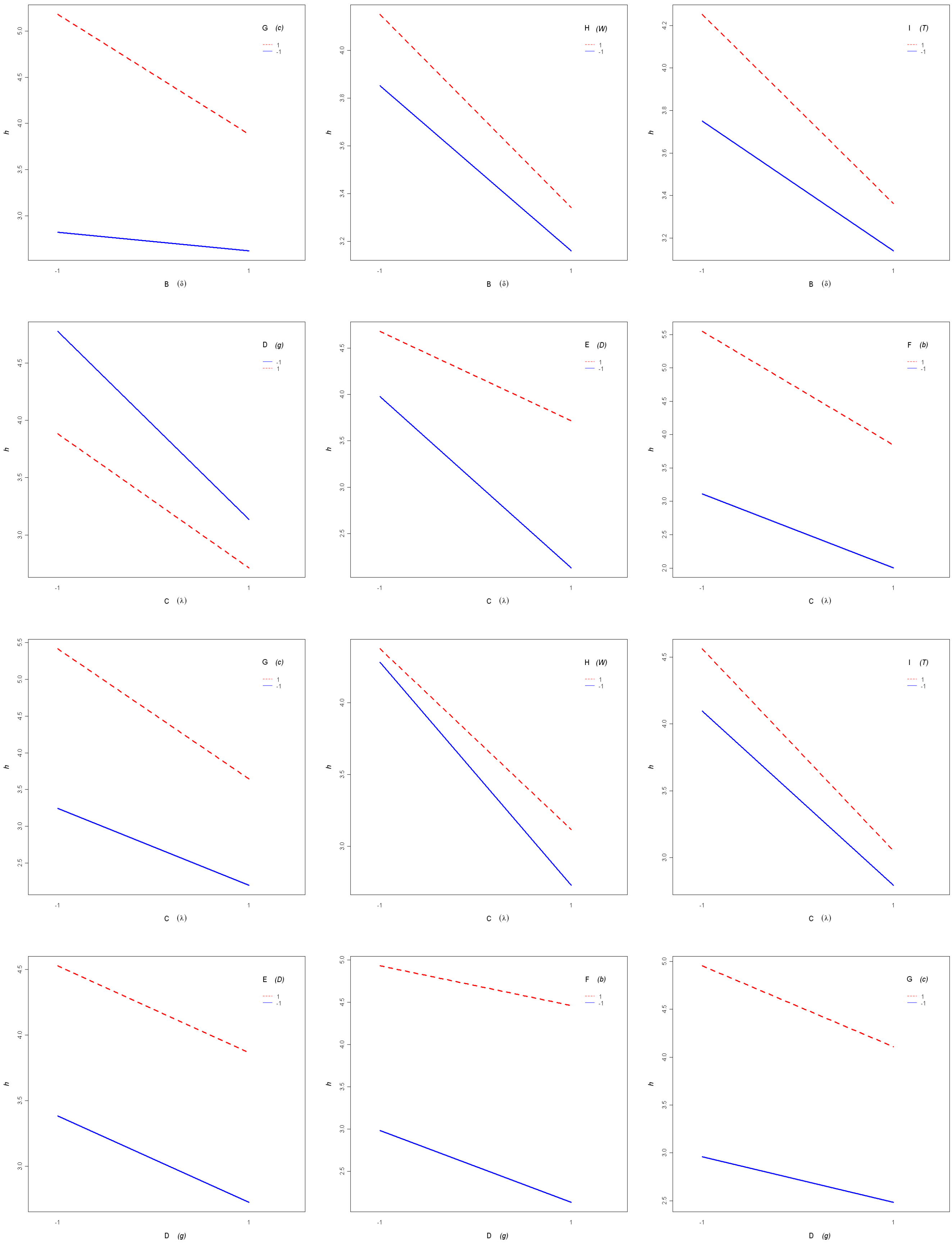


Figure 31: Continuous Process: interaction plots for h (2/3).

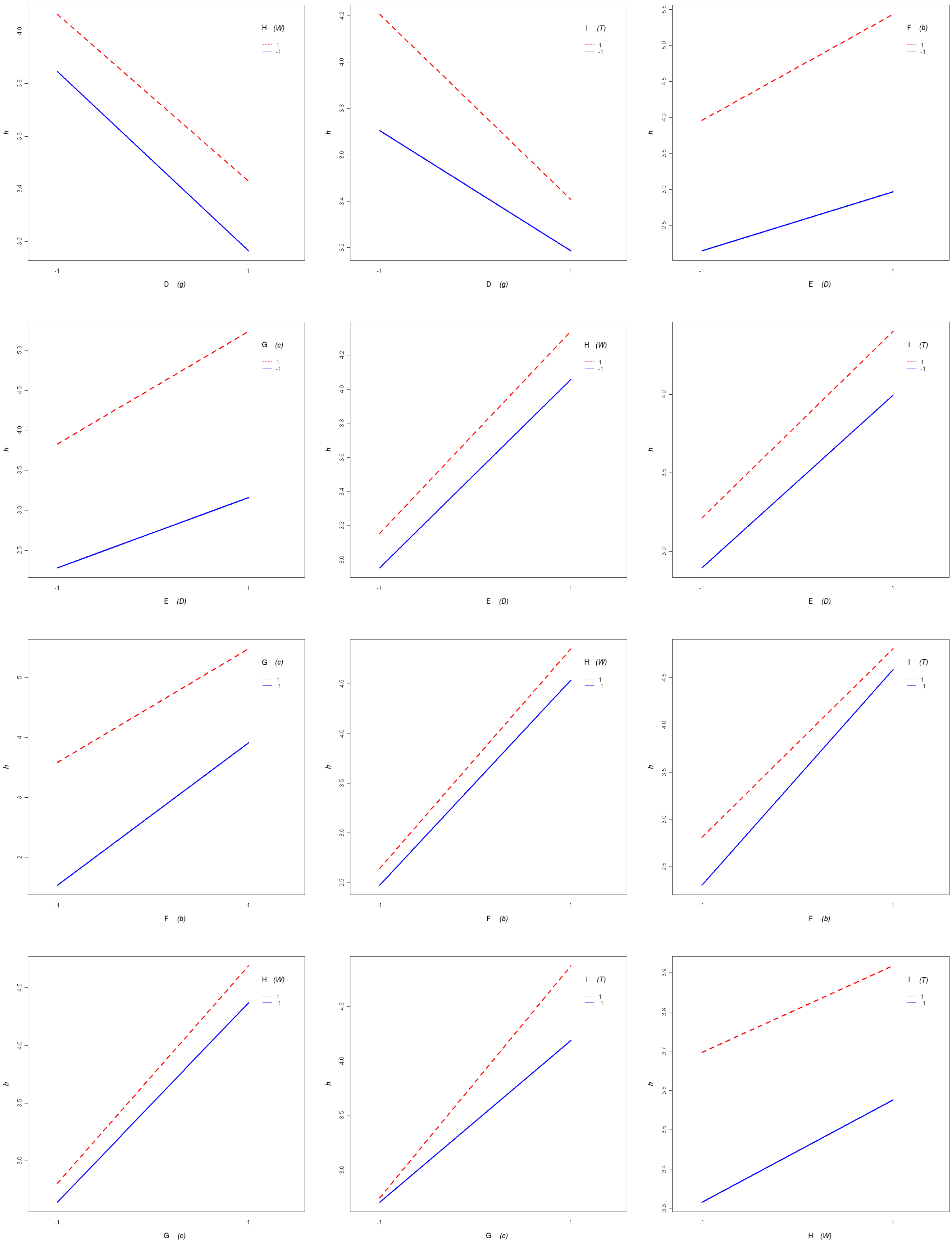


Figure 32: Continuous Process: interaction plots for h (3/3).

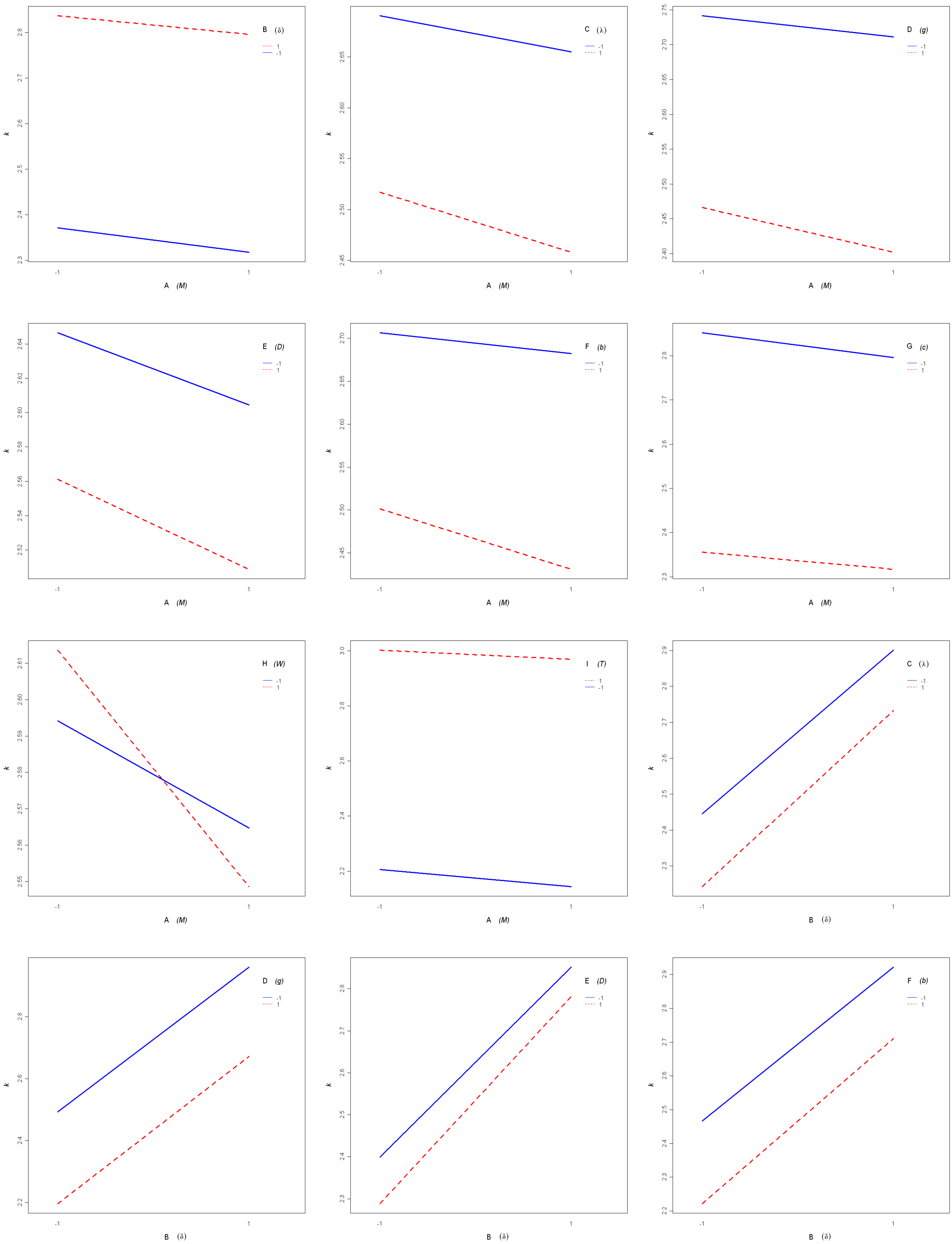


Figure 33: Continuous Process: interaction plots for k (1/3).

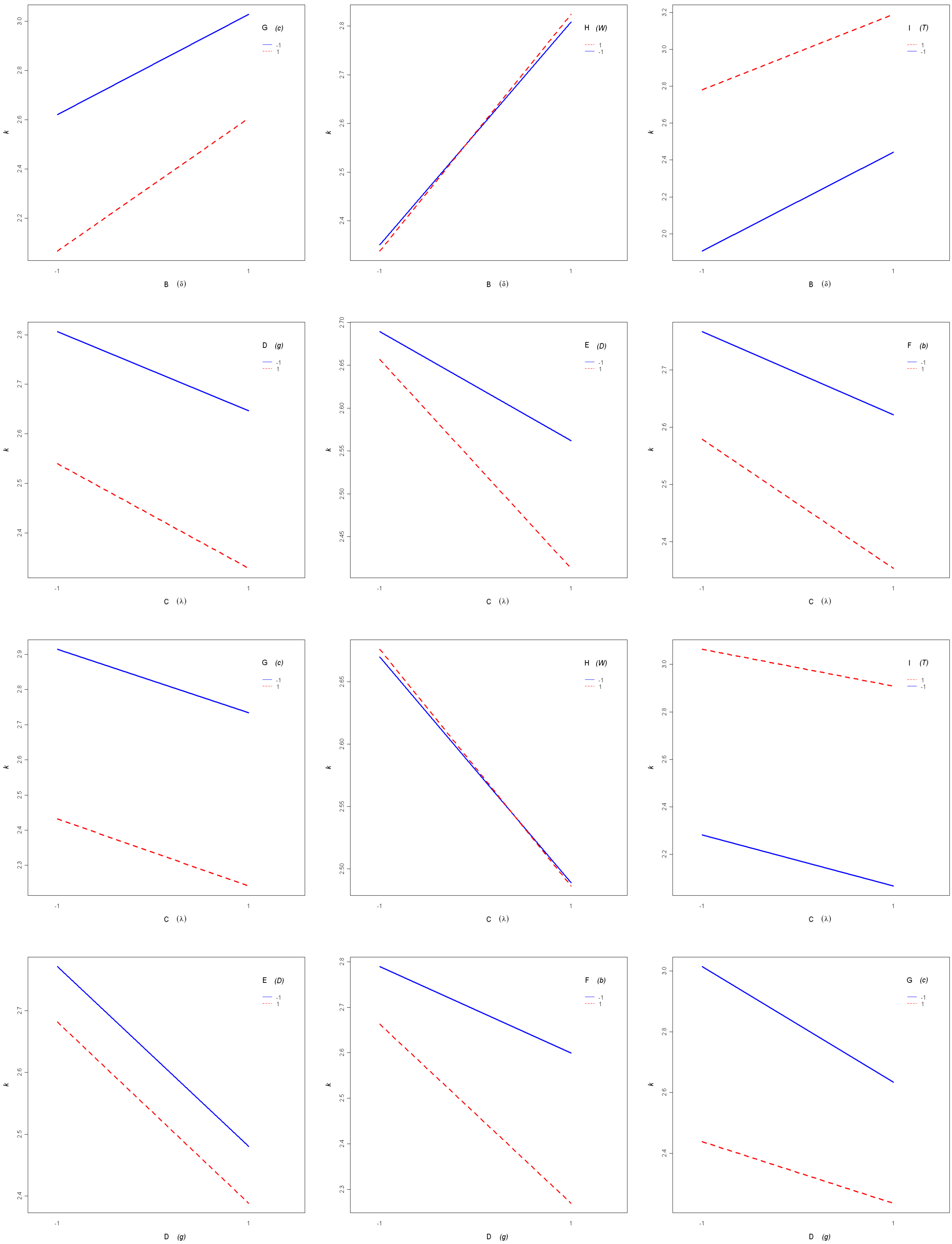


Figure 34: Continuous Process: interaction plots for k (2/3).

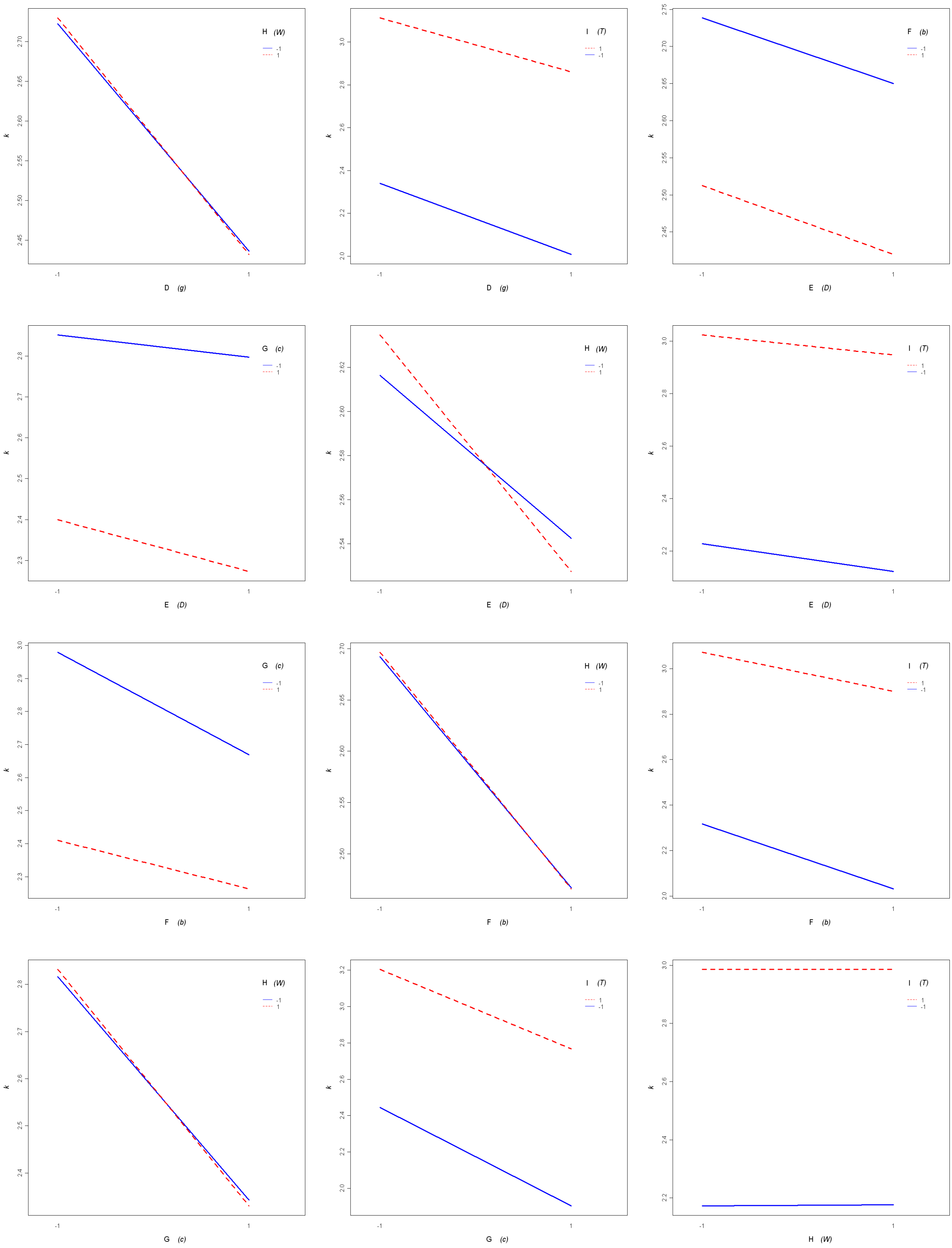


Figure 35: Continuous Process: interaction plots for k (3/3).

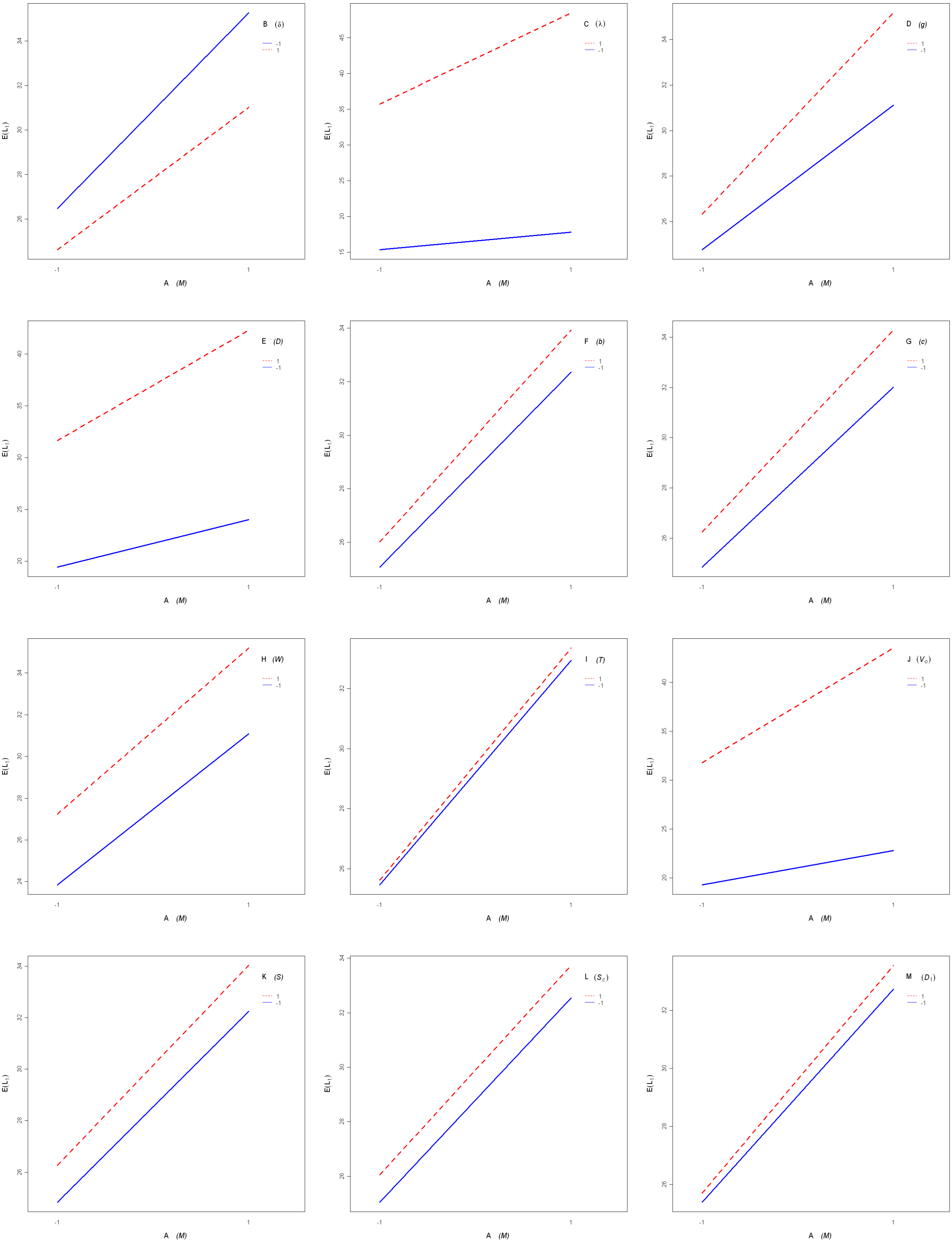


Figure 36: Discontinuous Process: interaction plots for $E(L_1)$ (1/7).

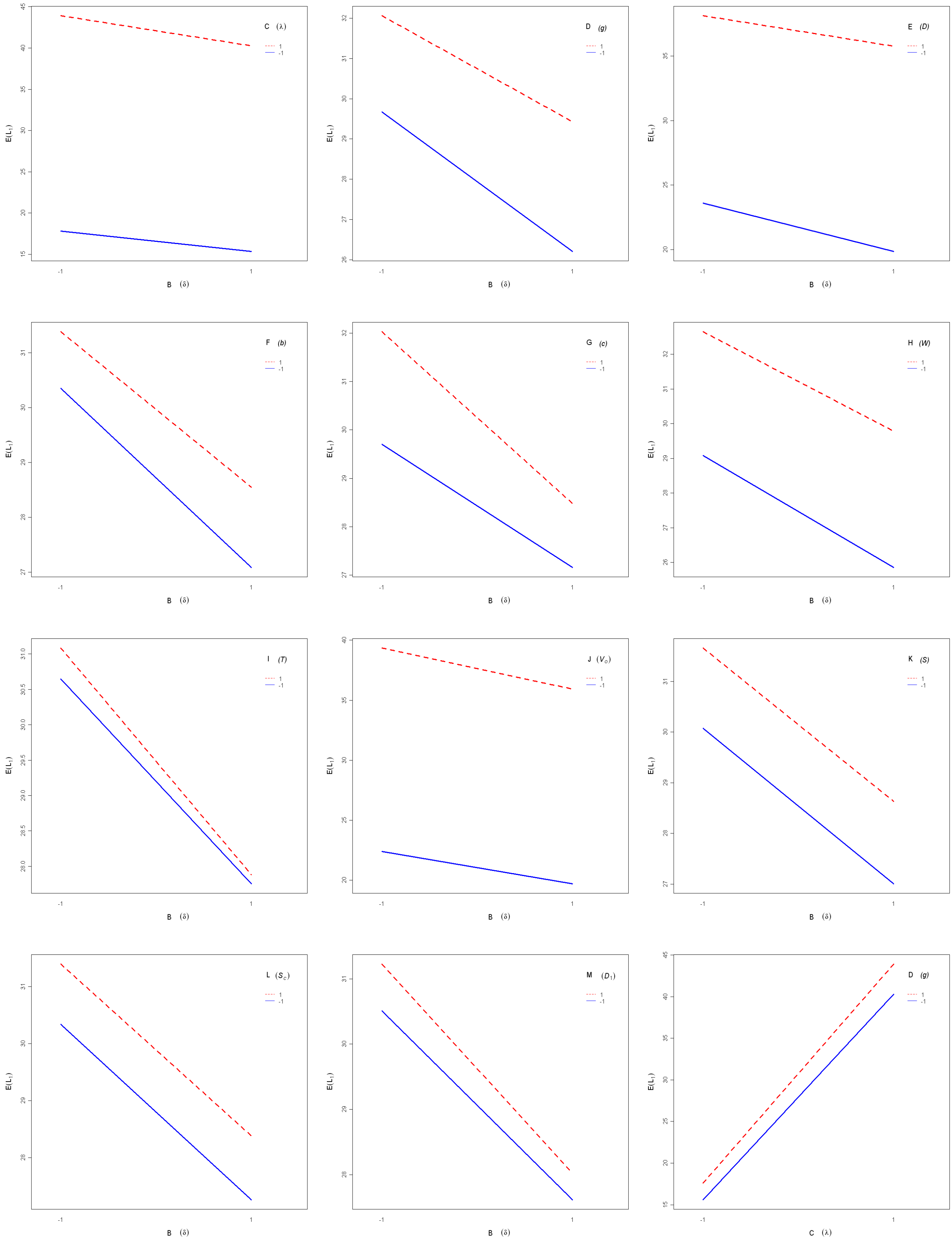


Figure 37: Discontinuous Process: interaction plots for $E(L_1)$ (2/7).

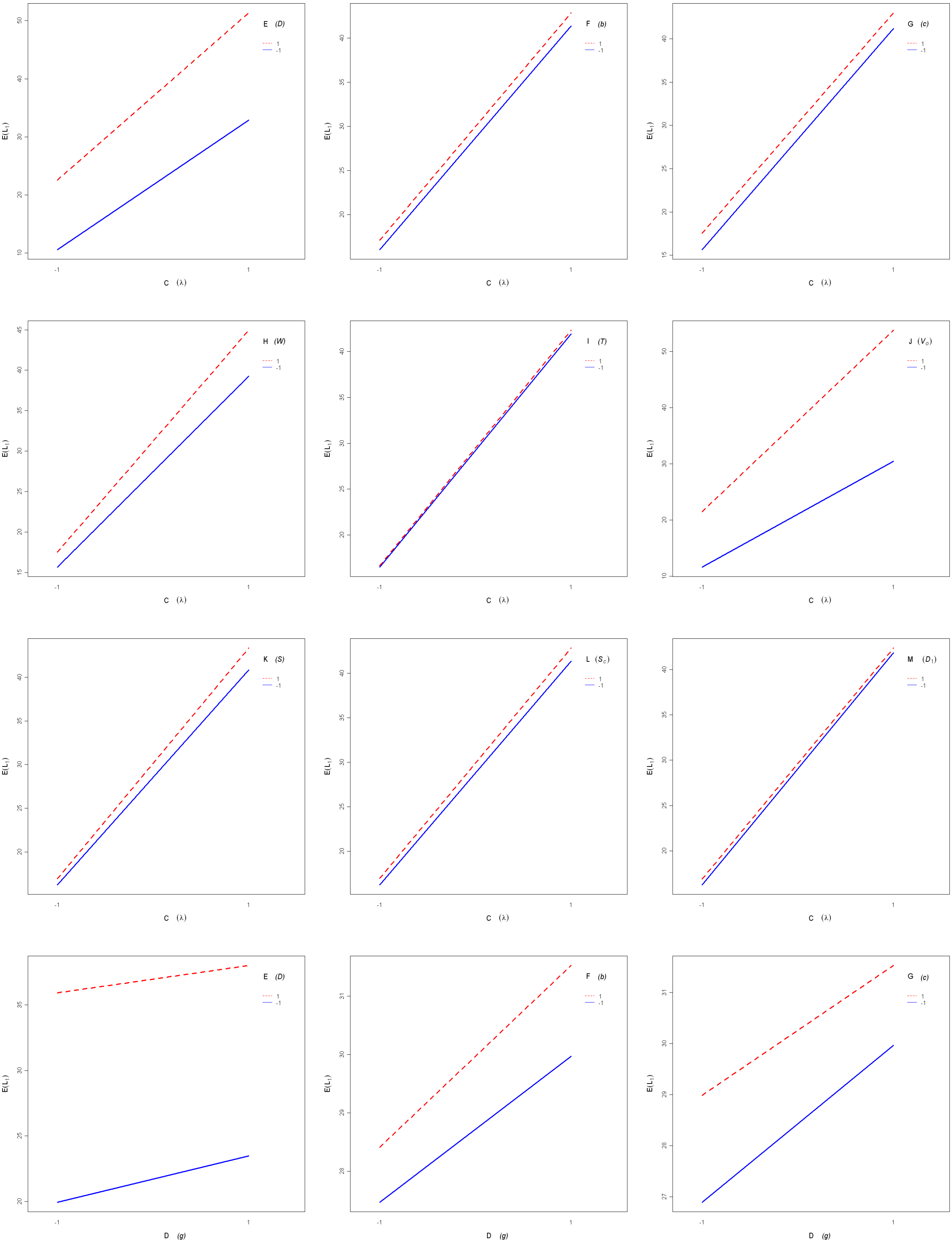


Figure 38: Discontinuous Process: interaction plots for $E(L_1)$ (3/7).

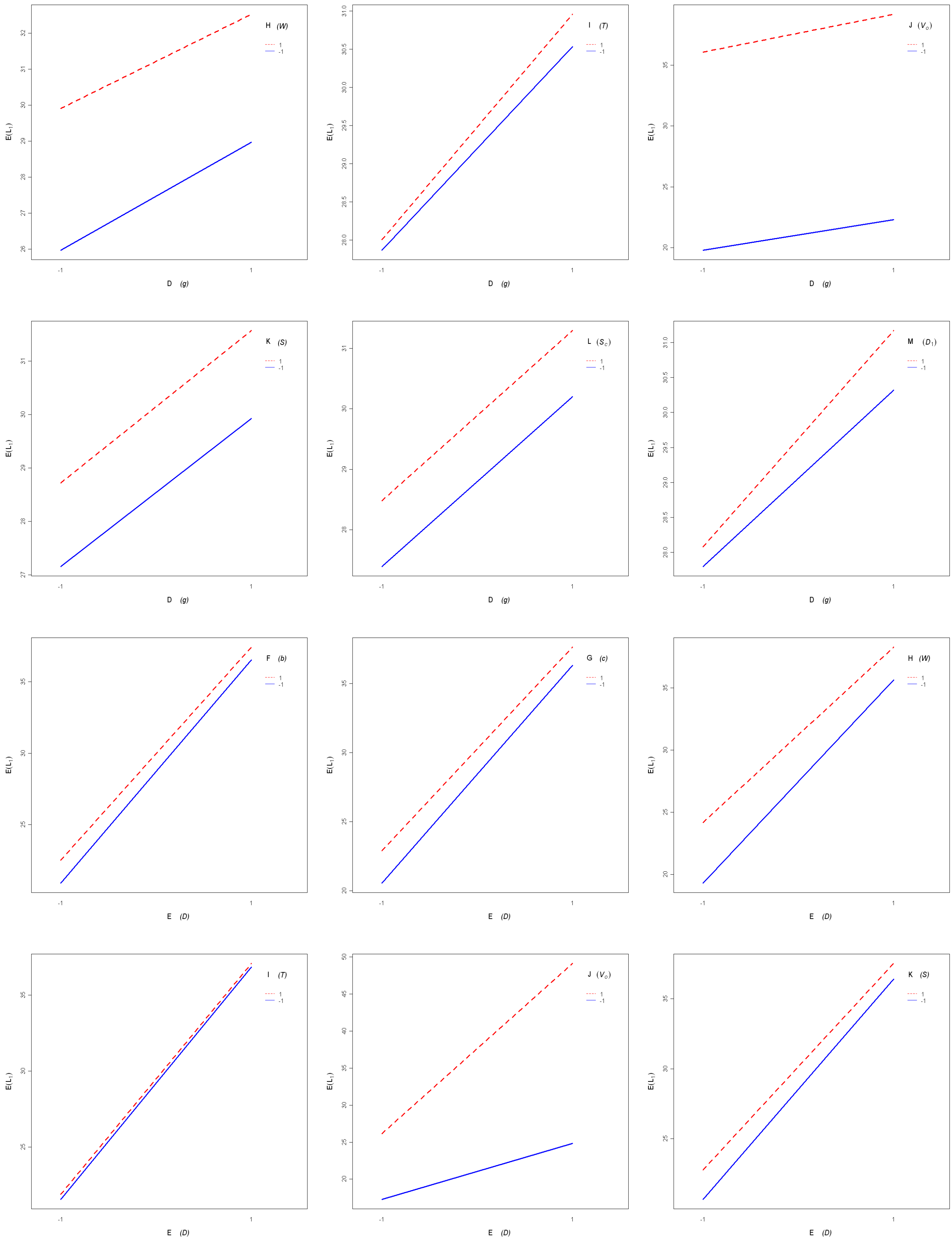


Figure 39: Discontinuous Process: interaction plots for $E(L_1)$ (4/7).

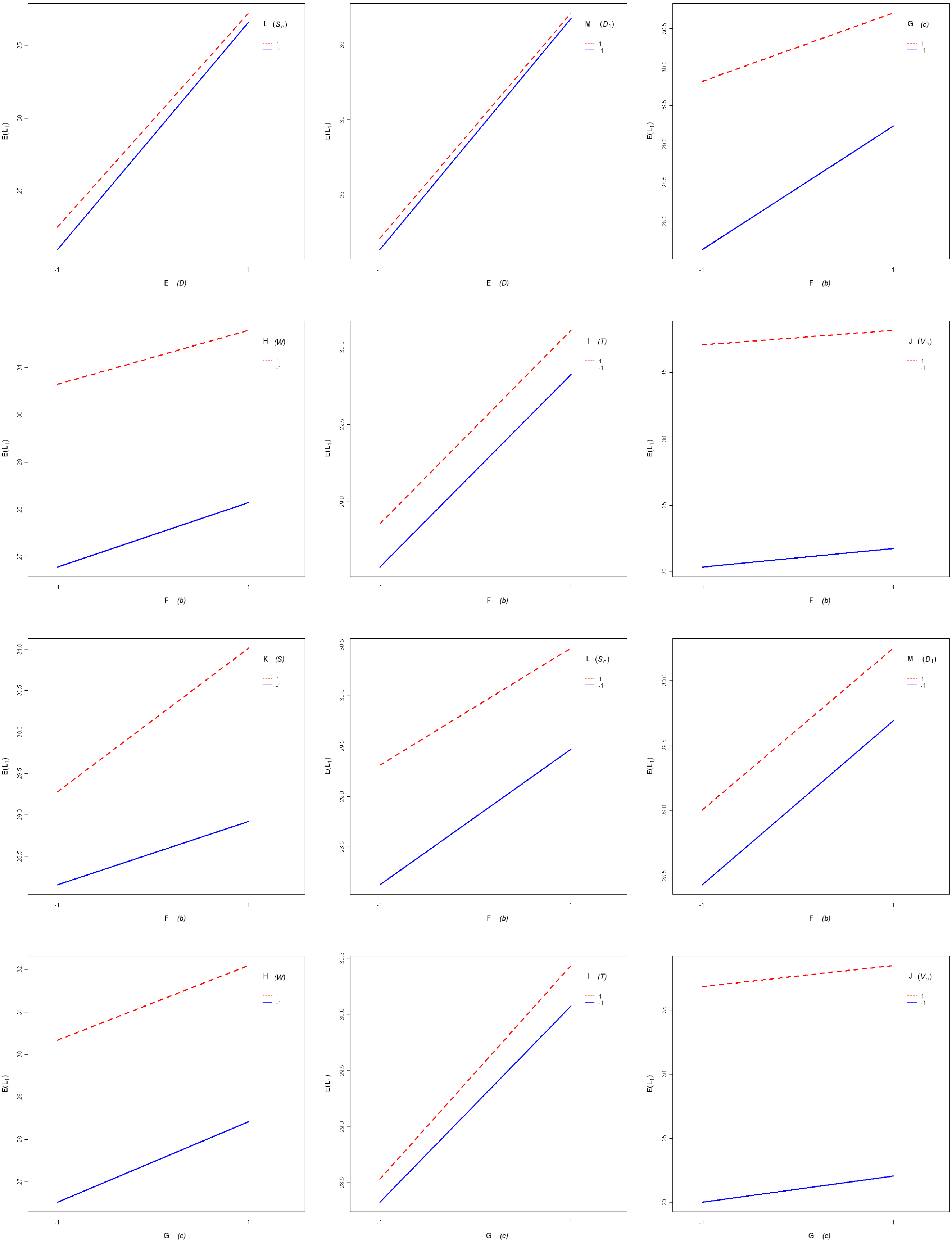


Figure 40: Discontinuous Process: interaction plots for $E(L_1)$ (5/7).

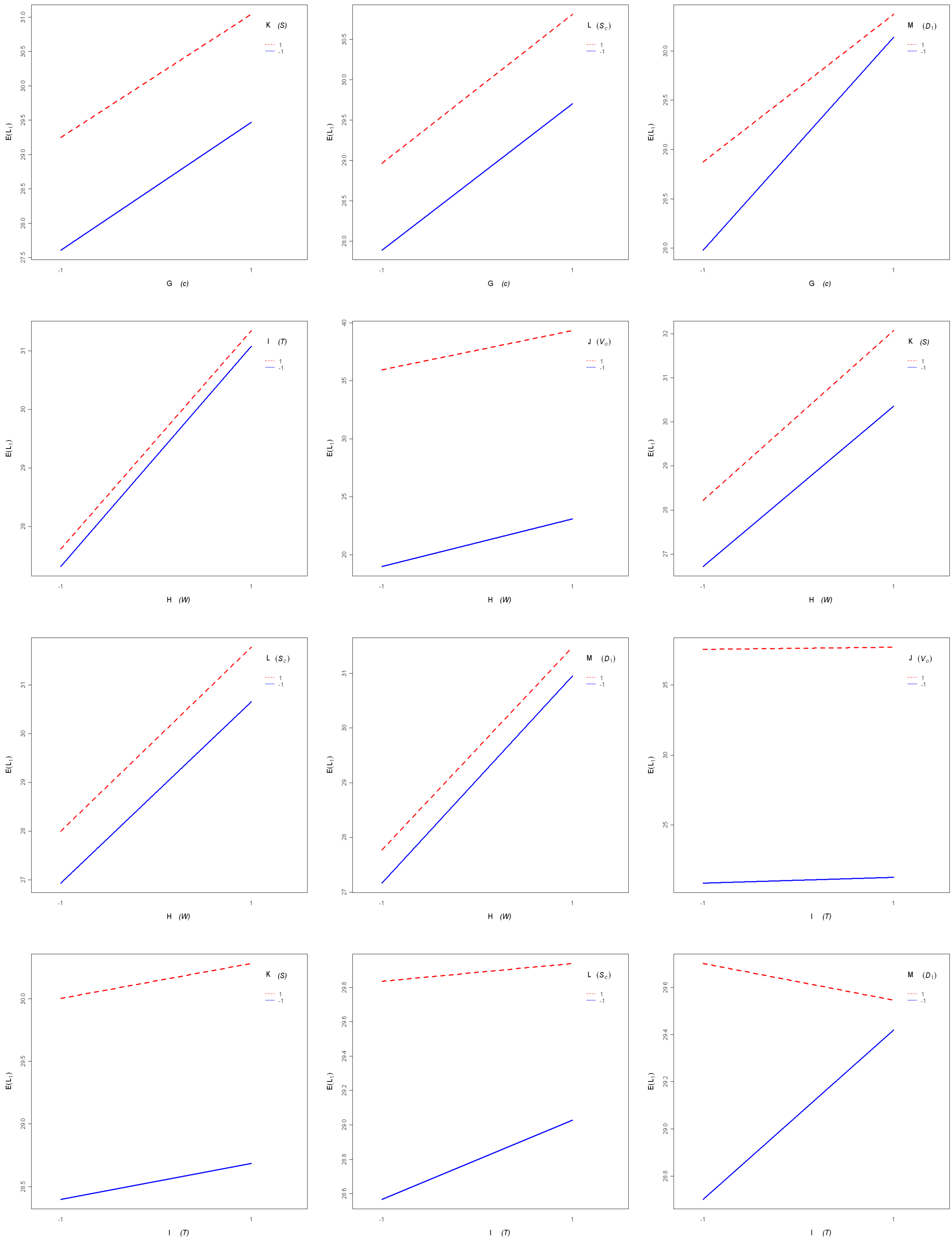


Figure 41: Discontinuous Process: interaction plots for $E(L_1)$ (6/7).

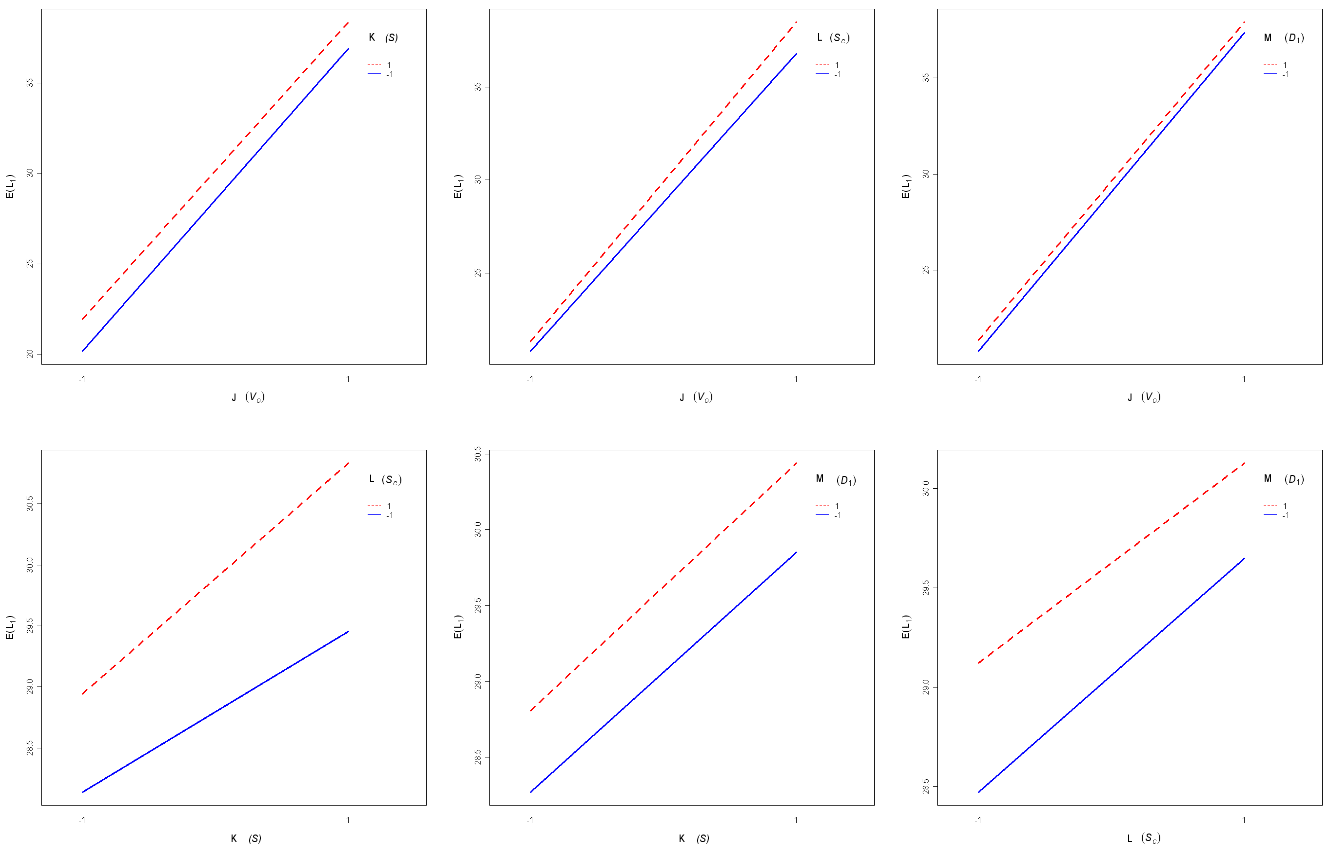


Figure 42: Discontinuous Process: interaction plots for $E(L_1)$ (7/7).

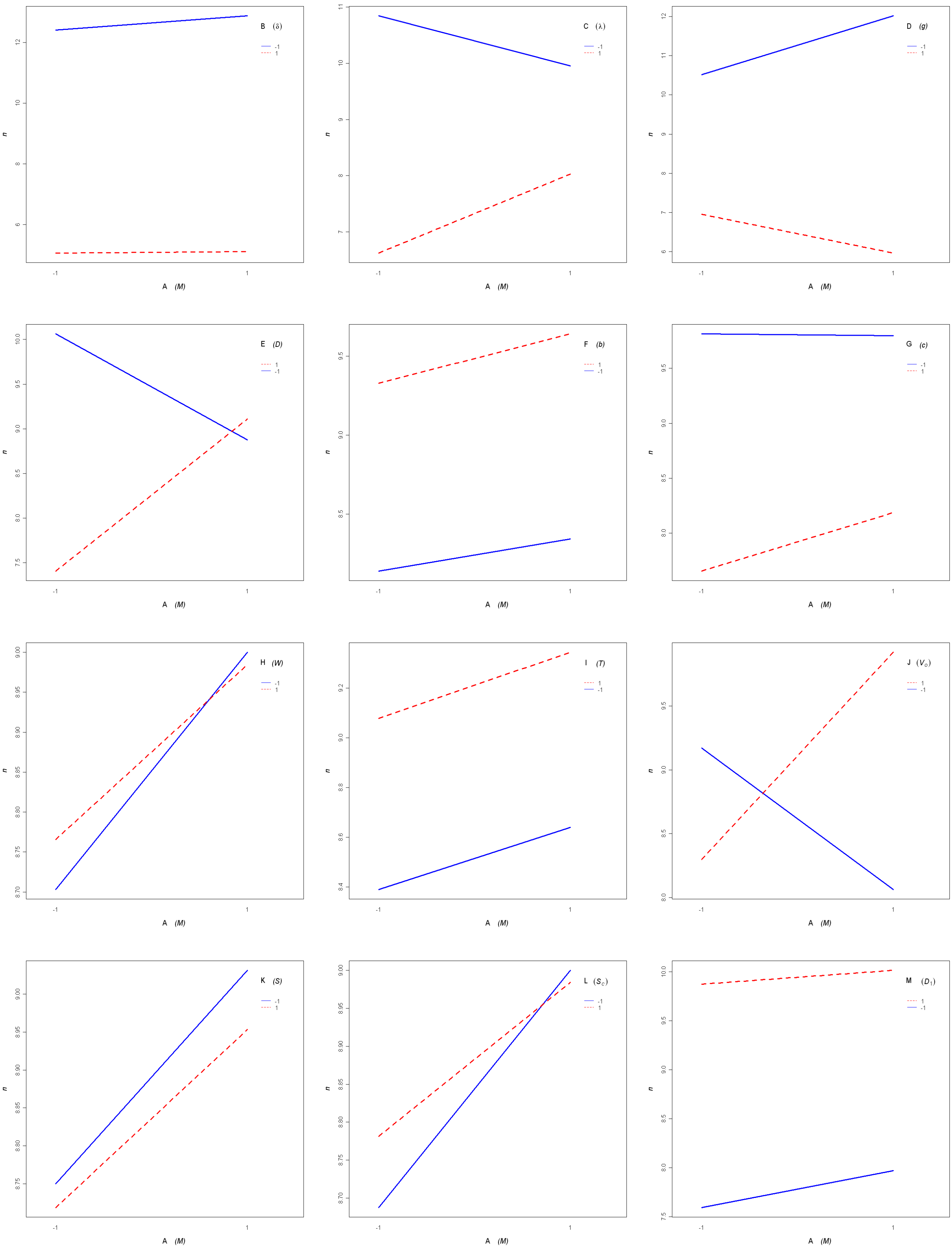


Figure 43: Discontinuous Process: interaction plots for n (1/7).

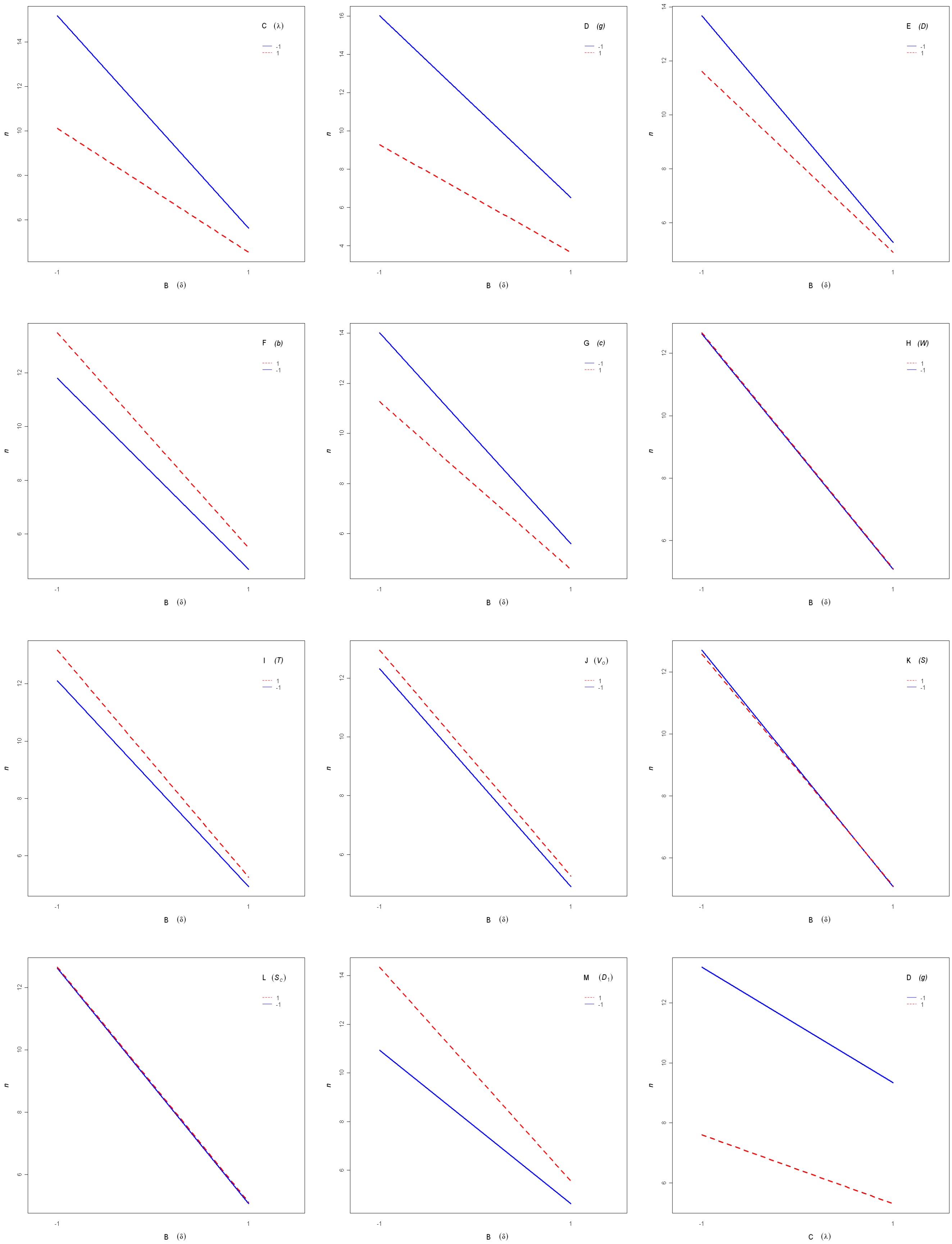


Figure 44: Discontinuous Process: interaction plots for n (2/7).

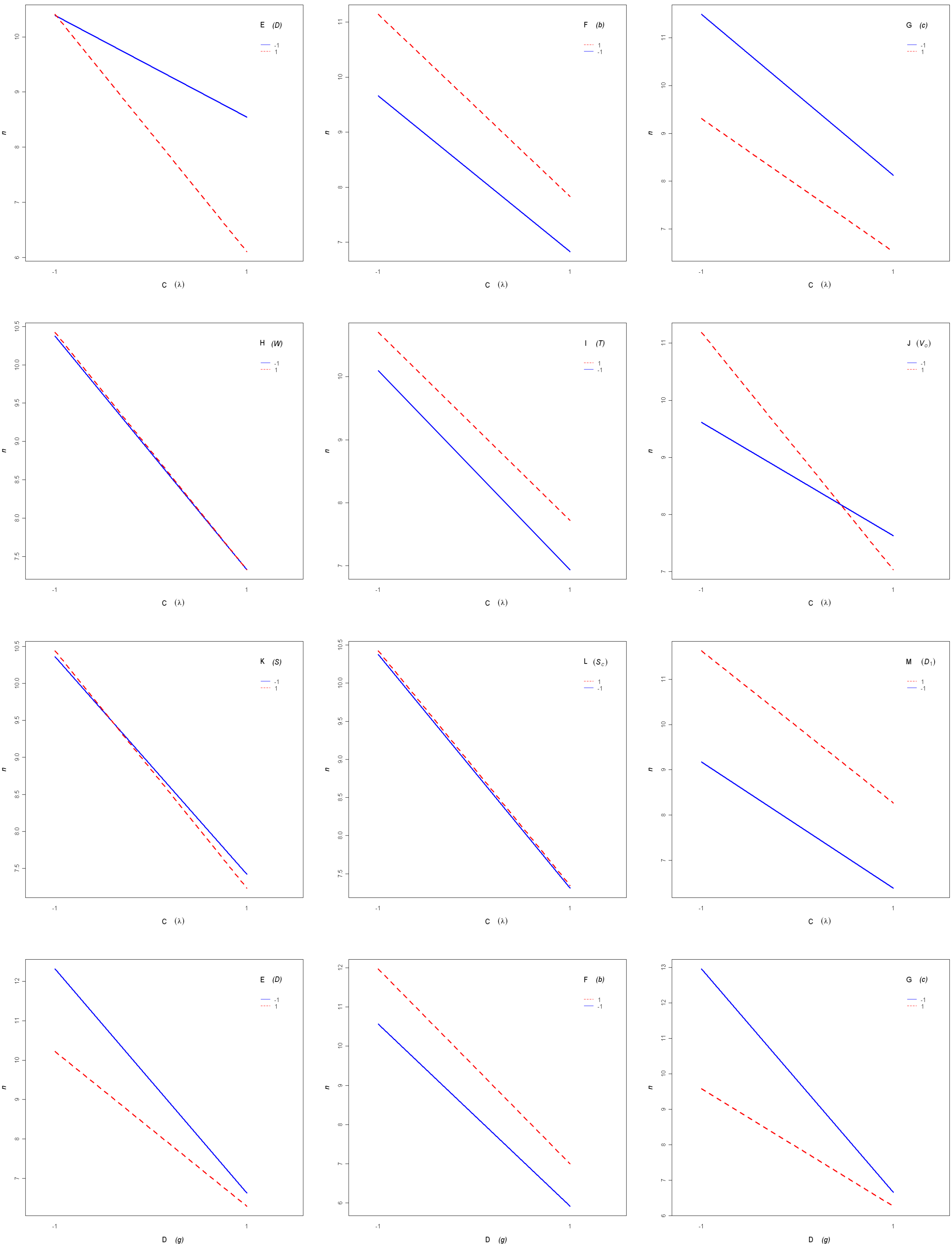


Figure 45: Discontinuous Process: interaction plots for n (3/7).

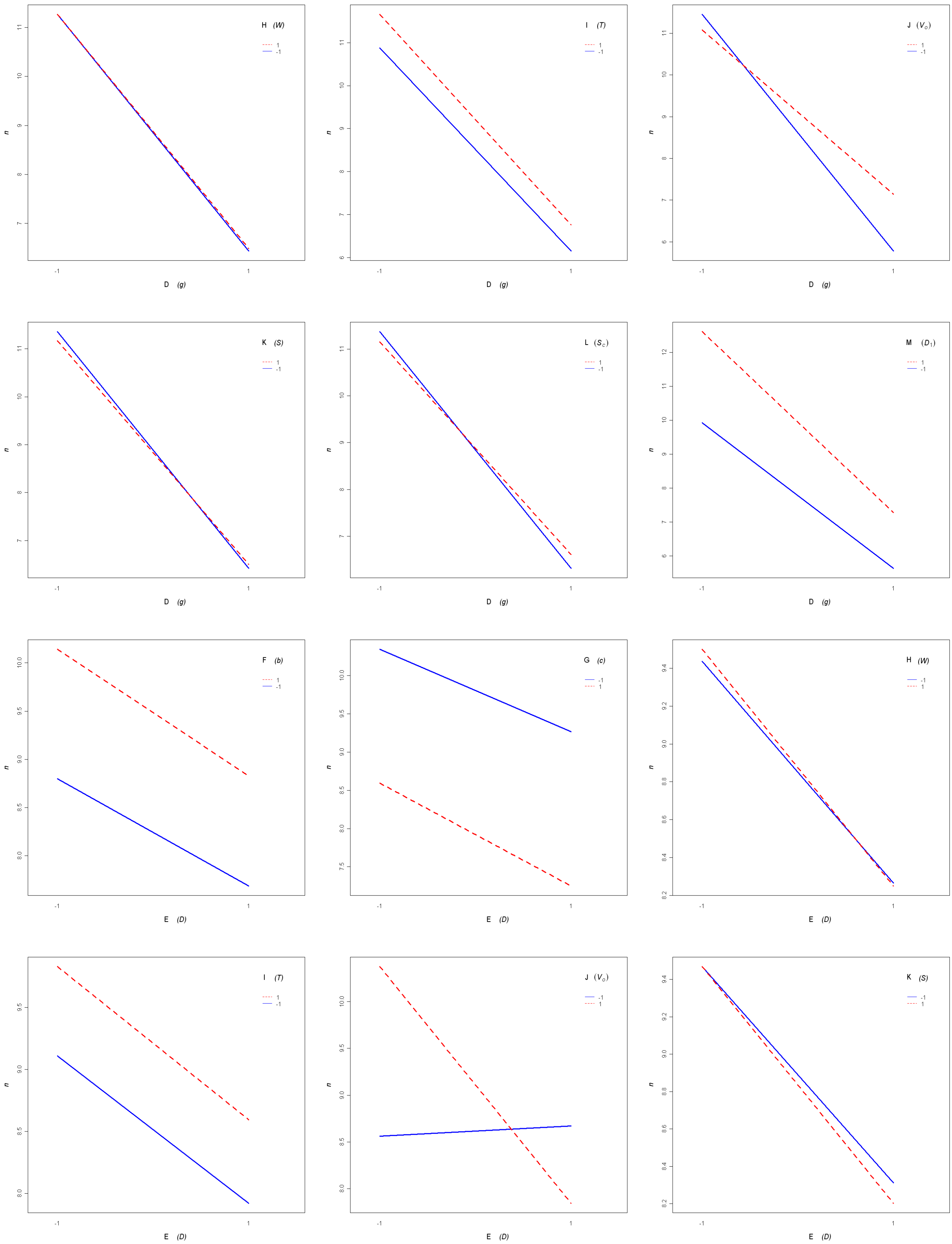


Figure 46: Discontinuous Process: interaction plots for n (4/7).

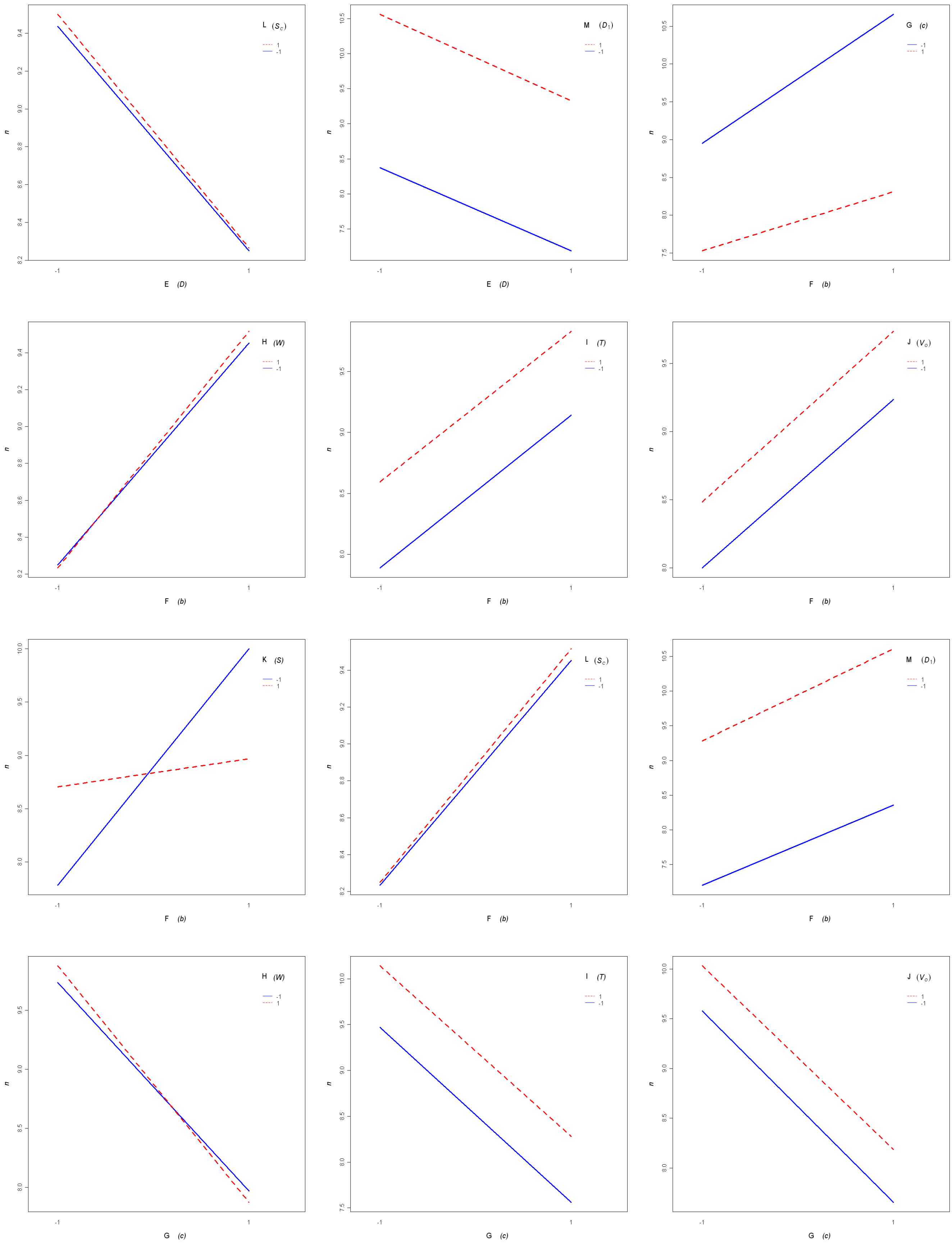


Figure 47: Discontinuous Process: interaction plots for n (5/7).

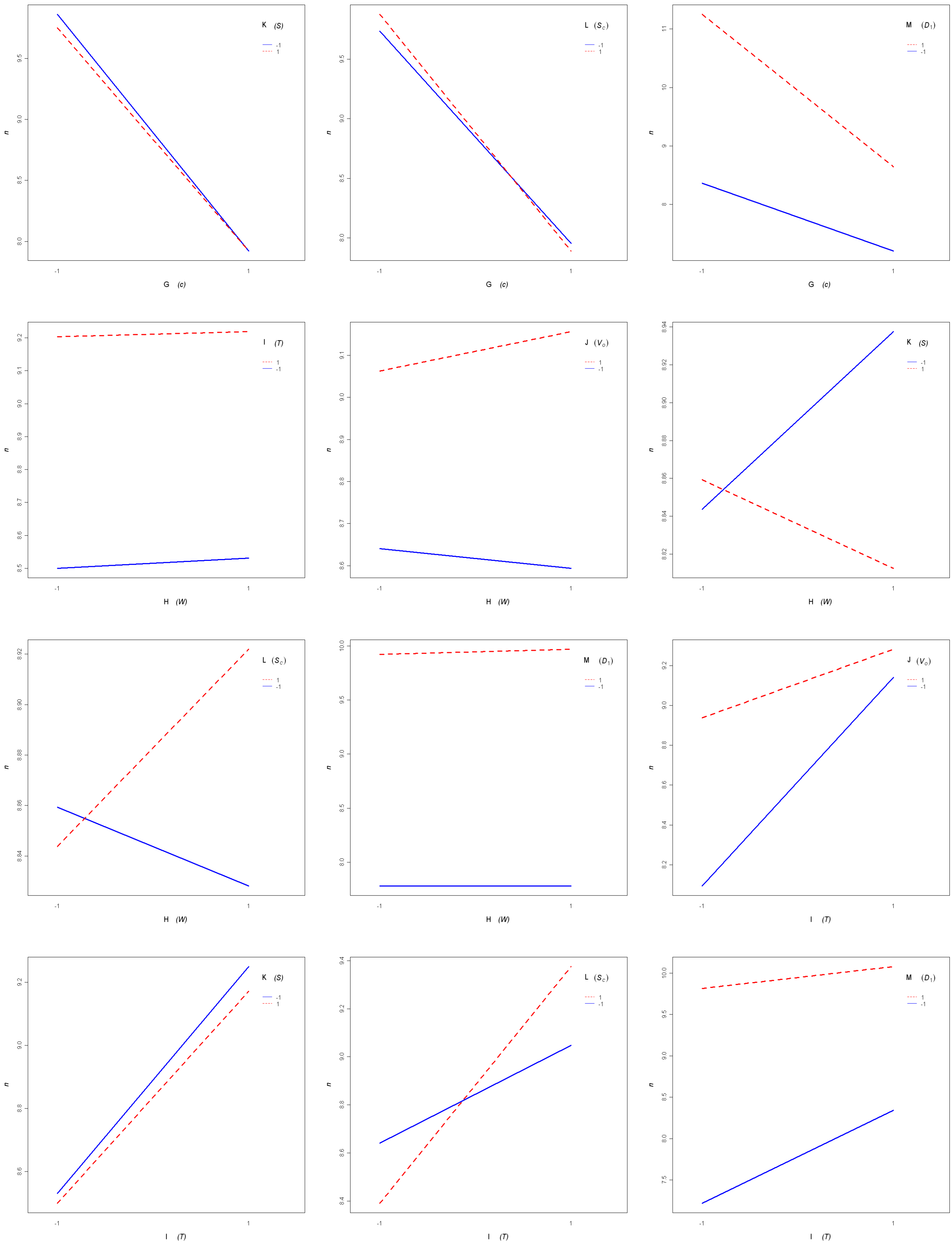


Figure 48: Discontinuous Process: interaction plots for n (6/7).

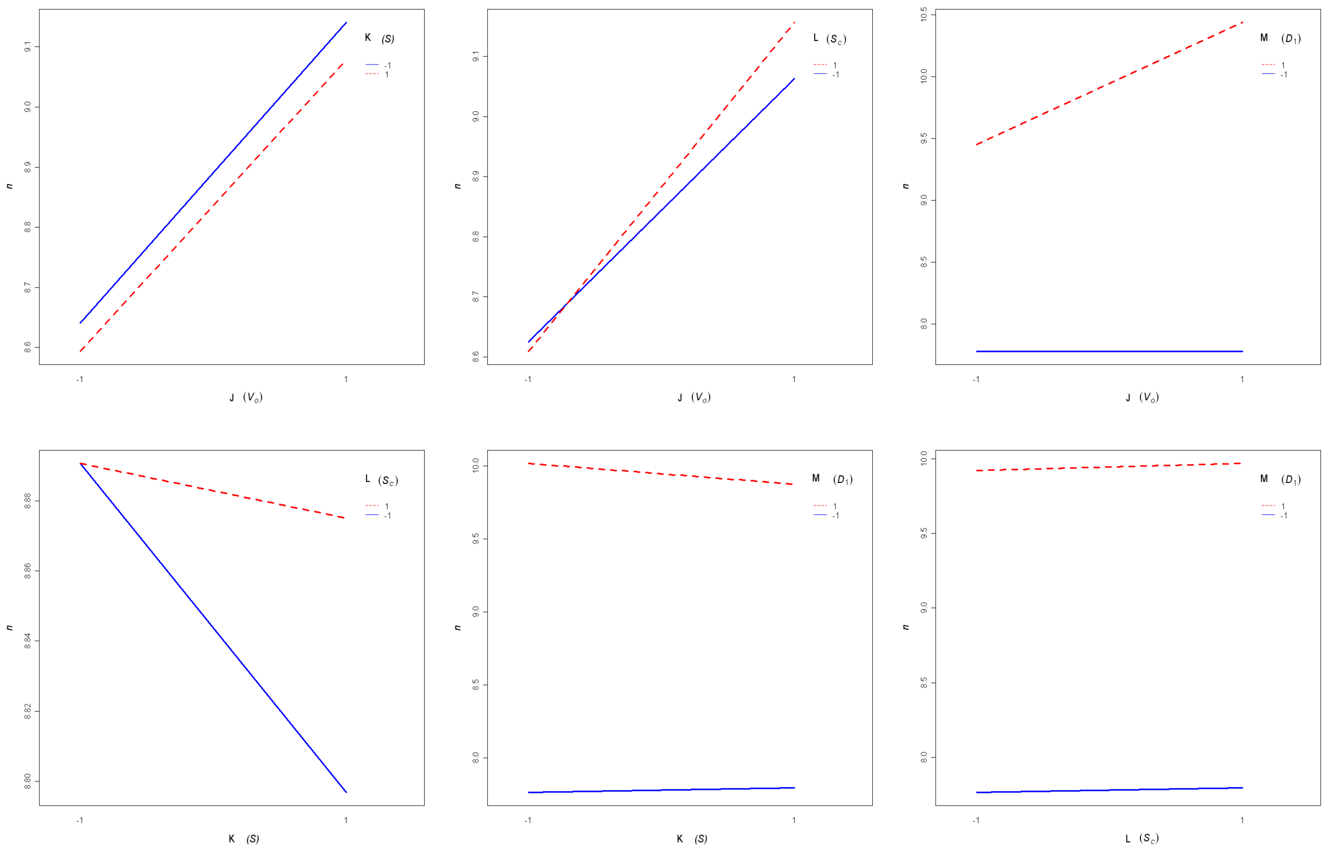


Figure 49: Discontinuous Process: interaction plots for n (7/7).

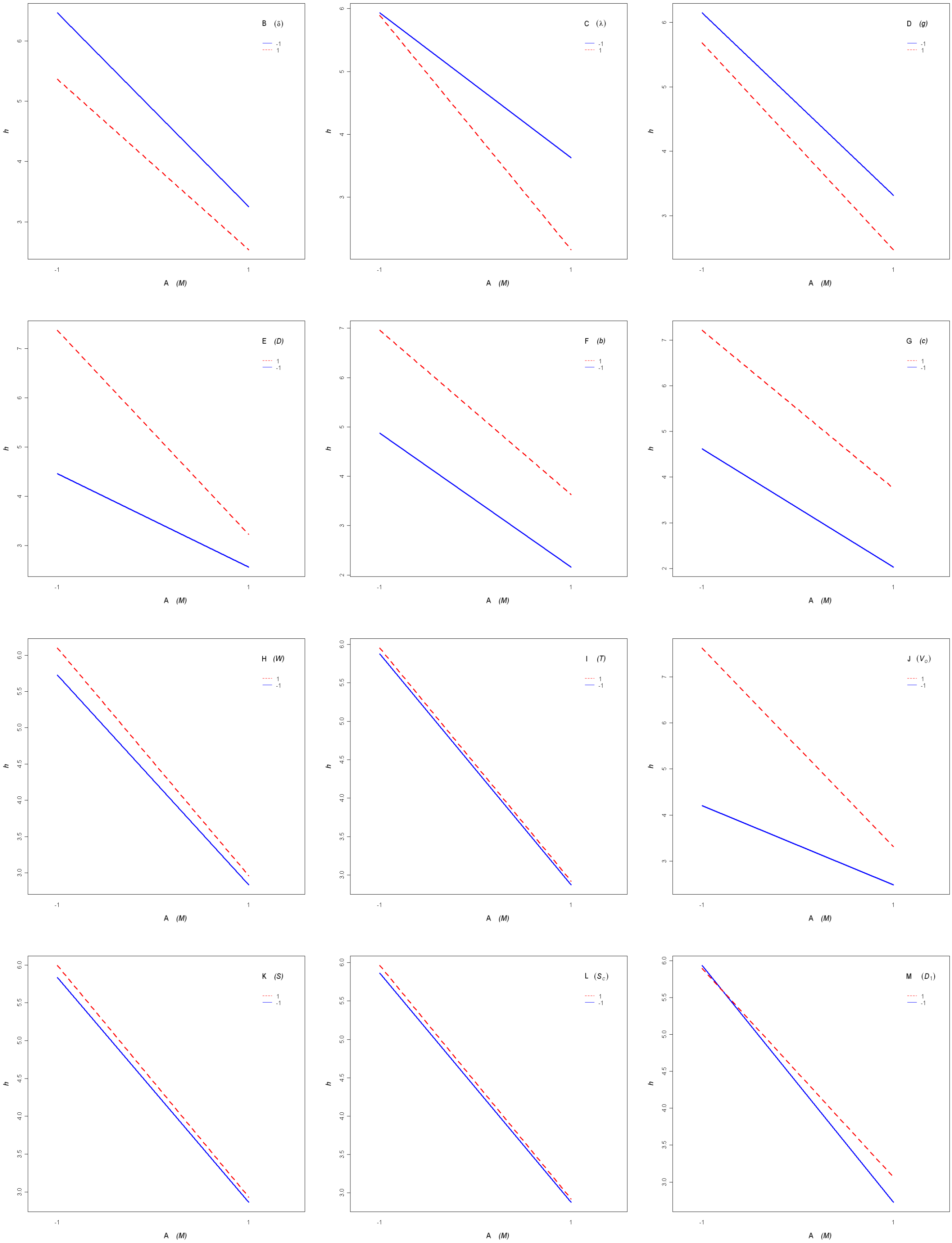


Figure 50: Discontinuous Process: interaction plots for h (1/7).

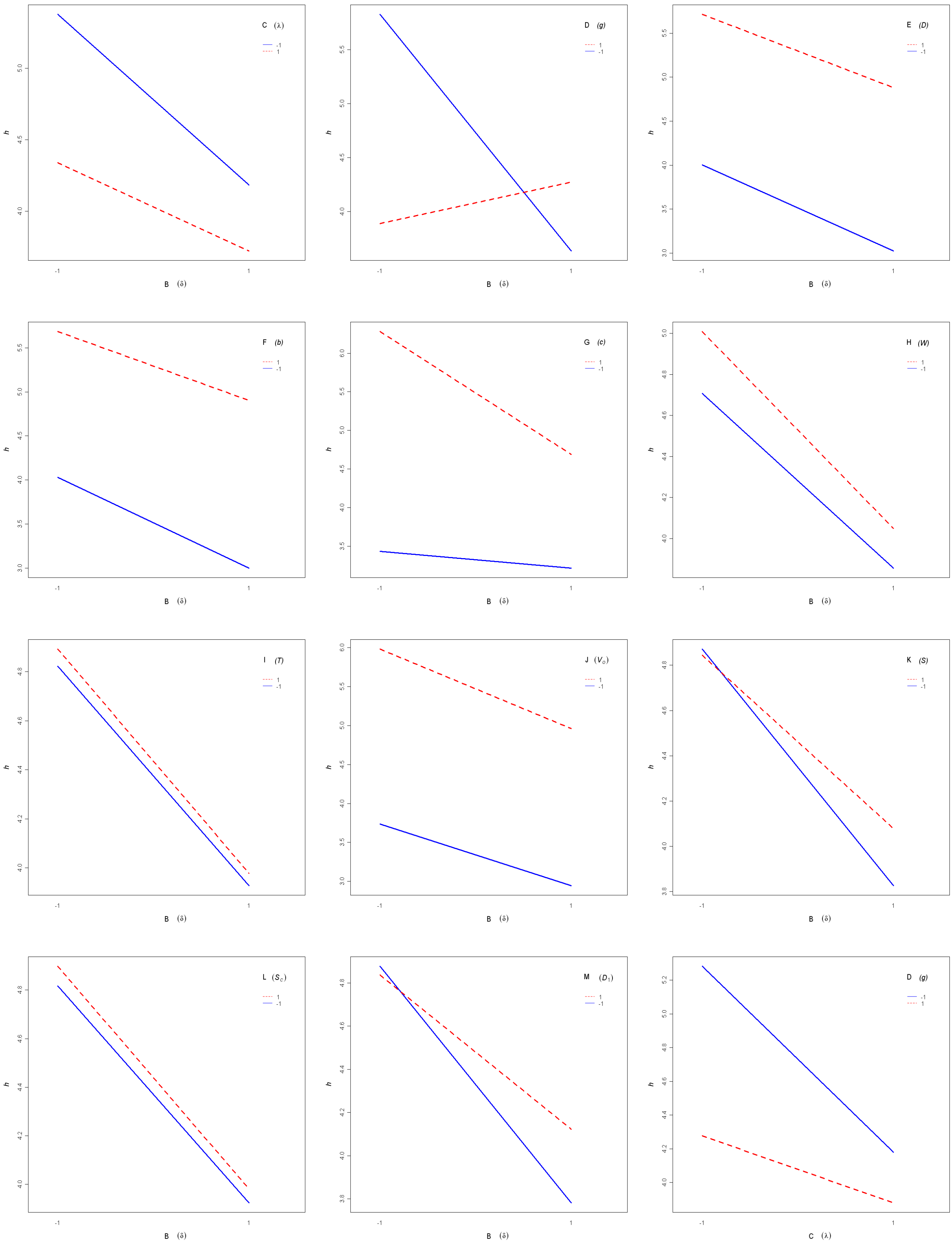


Figure 51: Discontinuous Process: interaction plots for h (2/7).

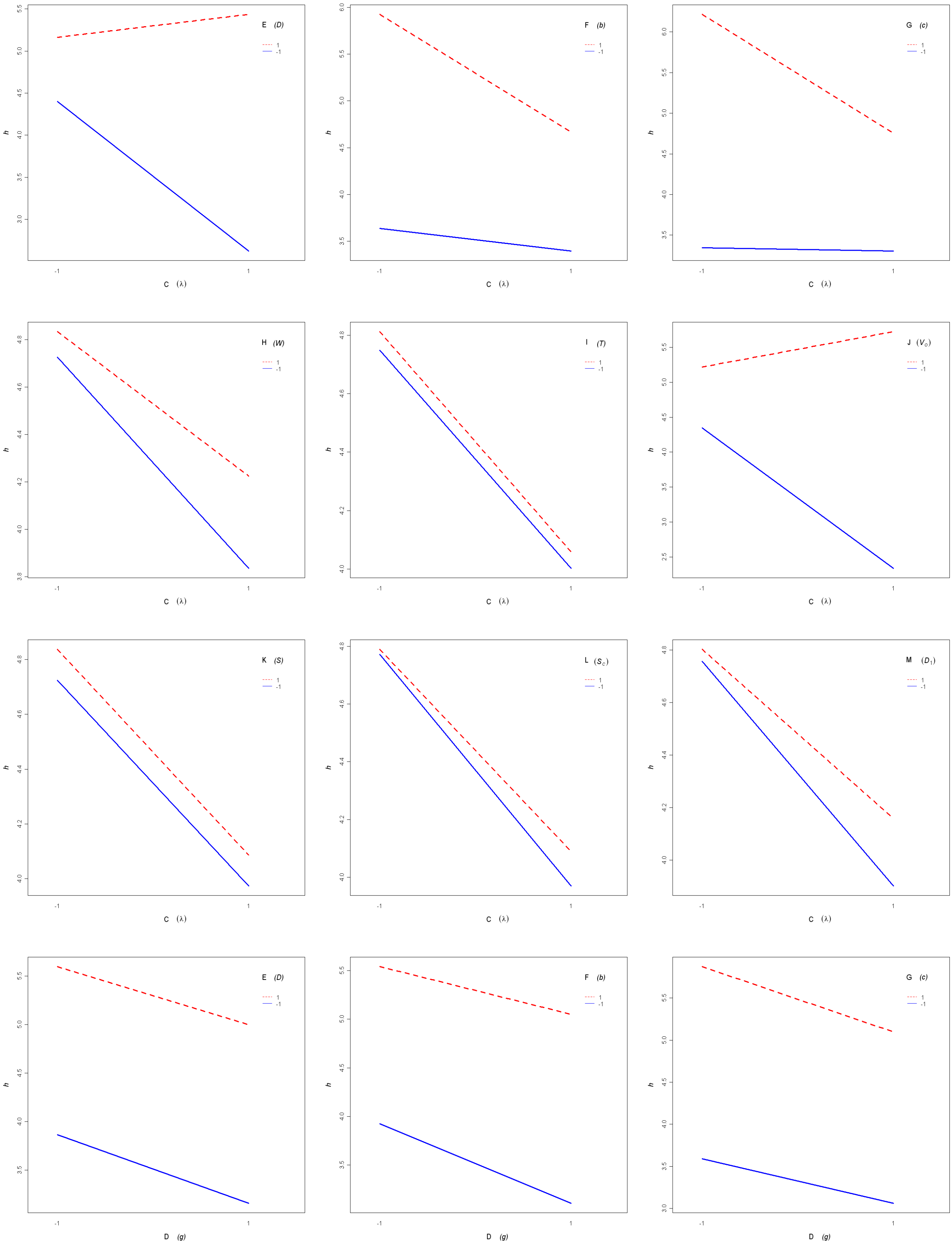


Figure 52: Discontinuous Process: interaction plots for h (3/7).

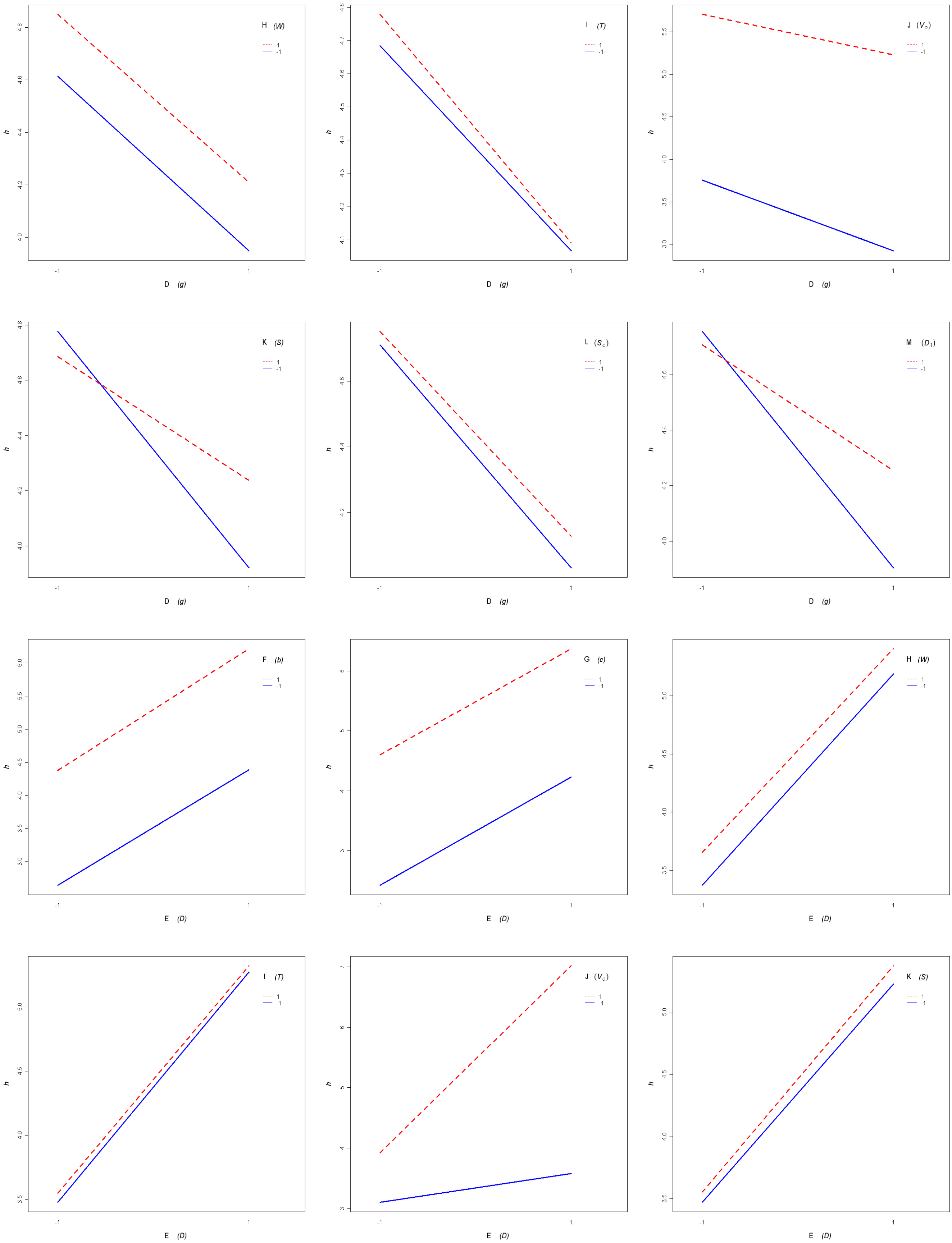


Figure 53: Discontinuous Process: interaction plots for h (4/7).

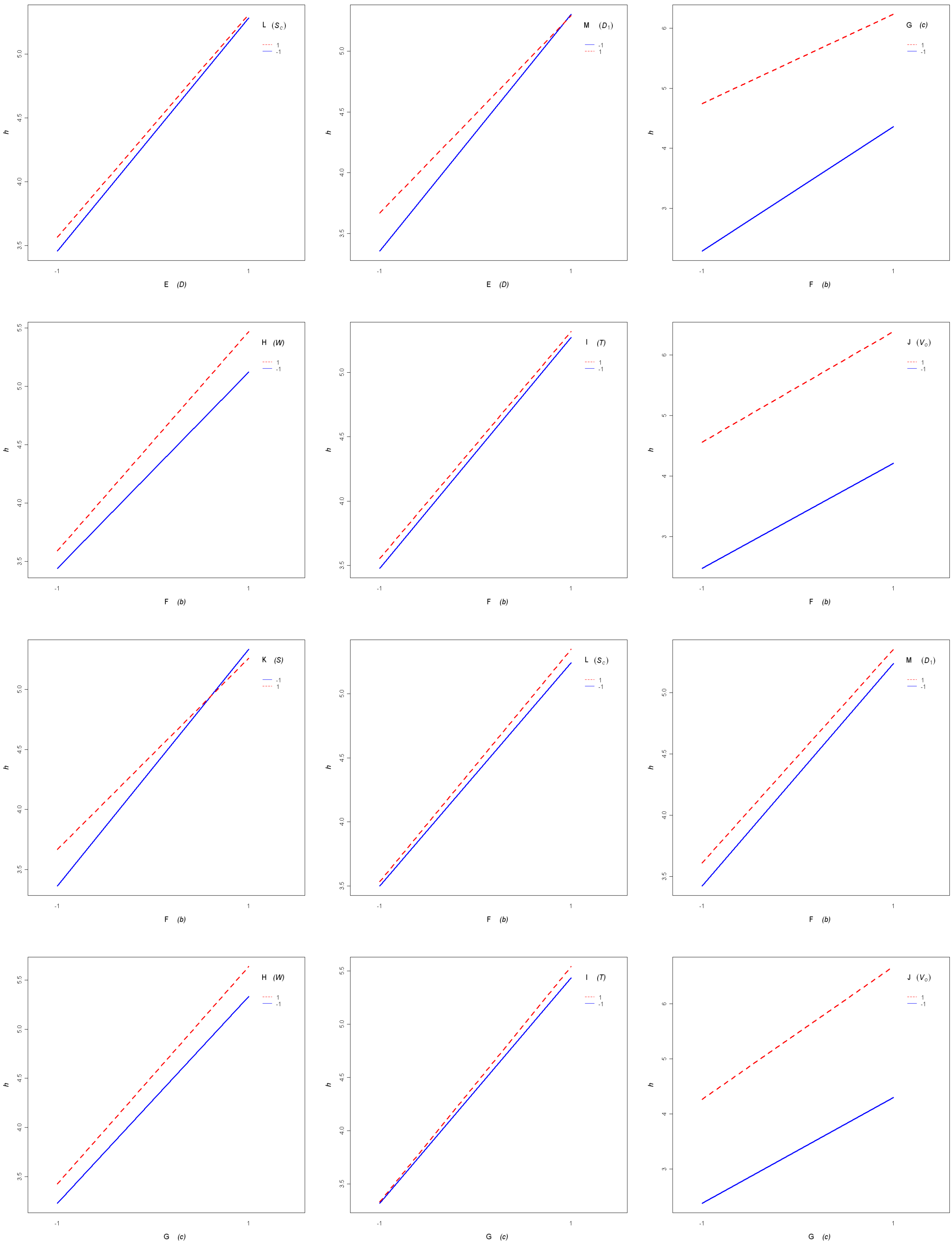


Figure 54: Discontinuous Process: interaction plots for h (5/7).

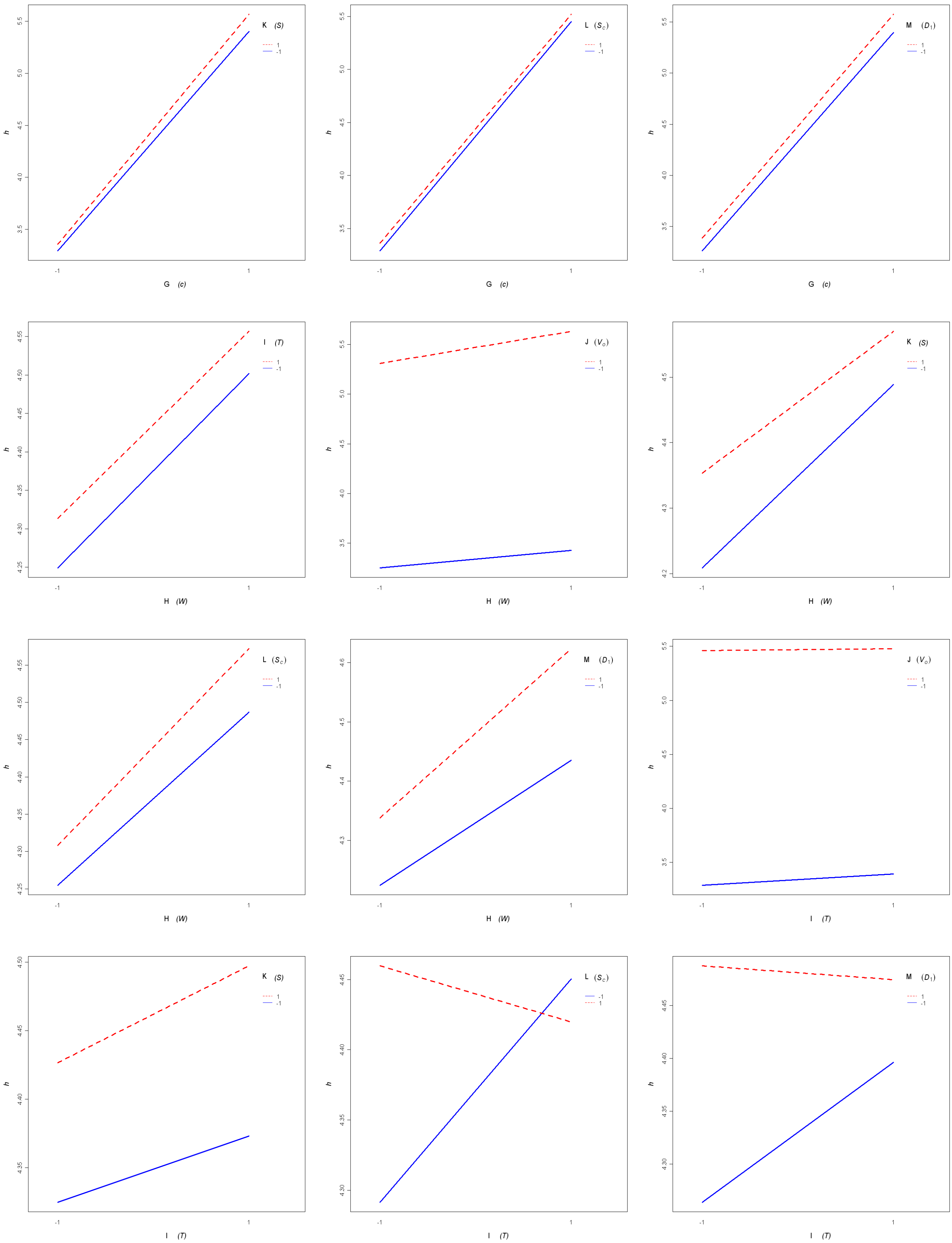


Figure 55: Discontinuous Process: interaction plots for h (6/7).

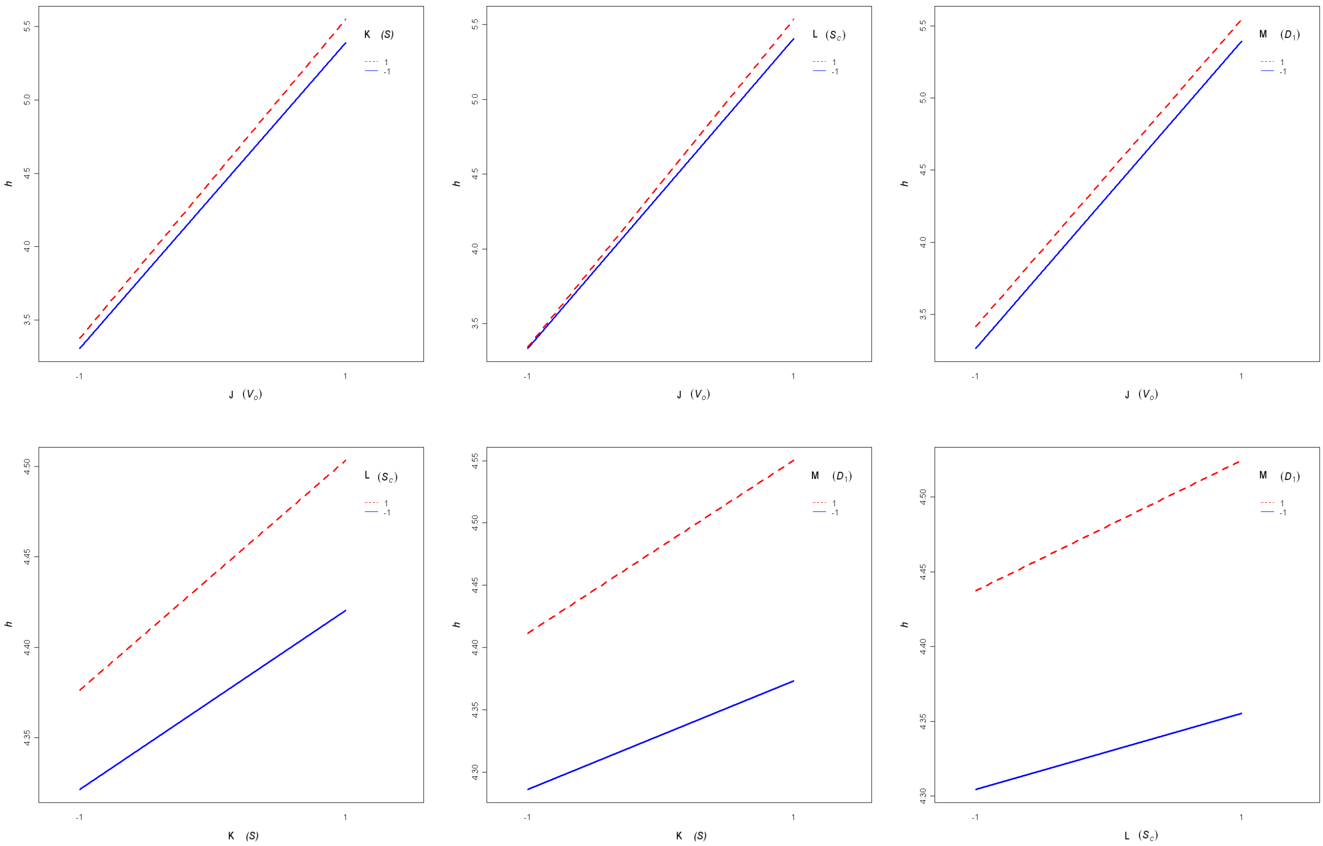


Figure 56: Discontinuous Process: interaction plots for h (7/7).

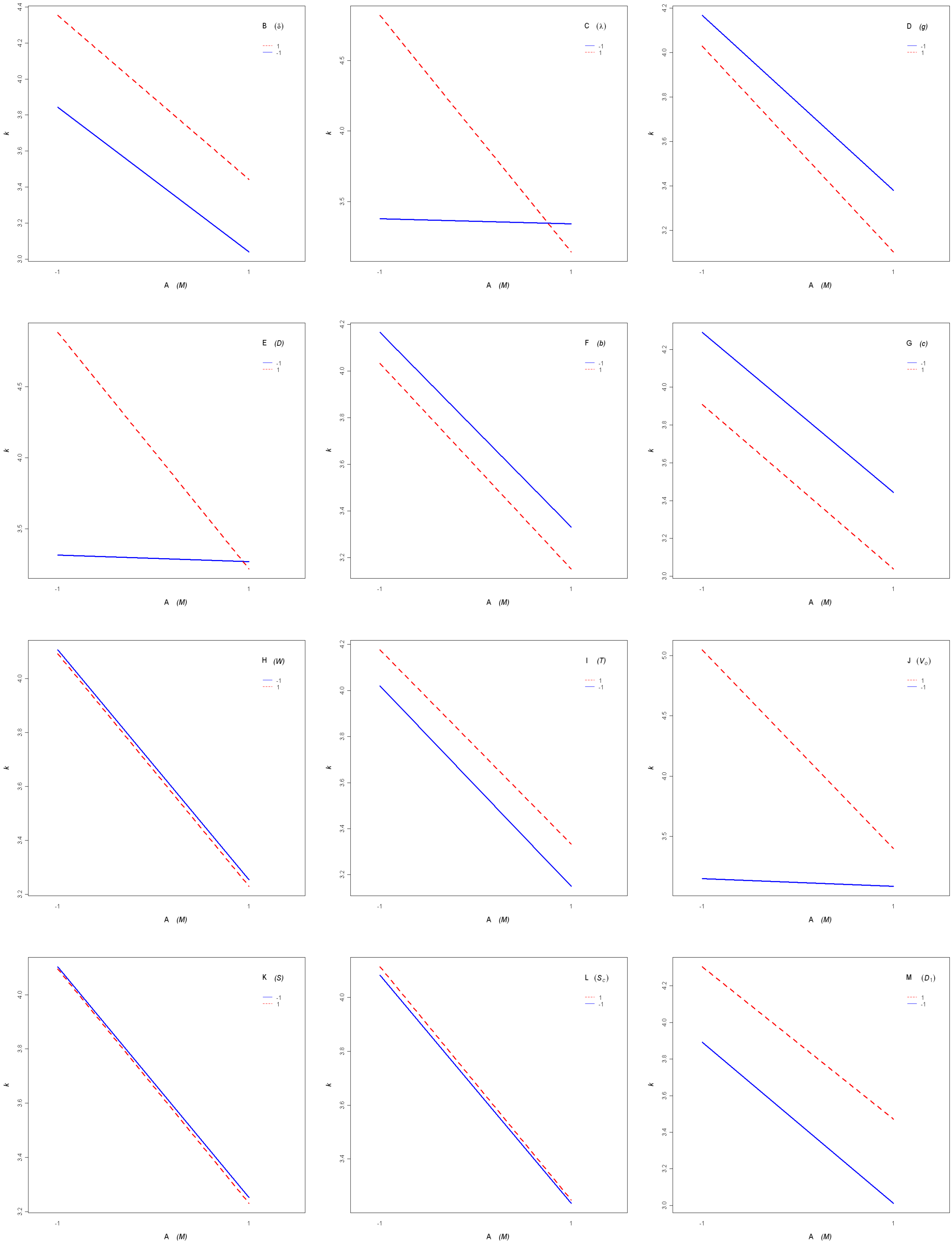


Figure 57: Discontinuous Process: interaction plots for k (1/7).

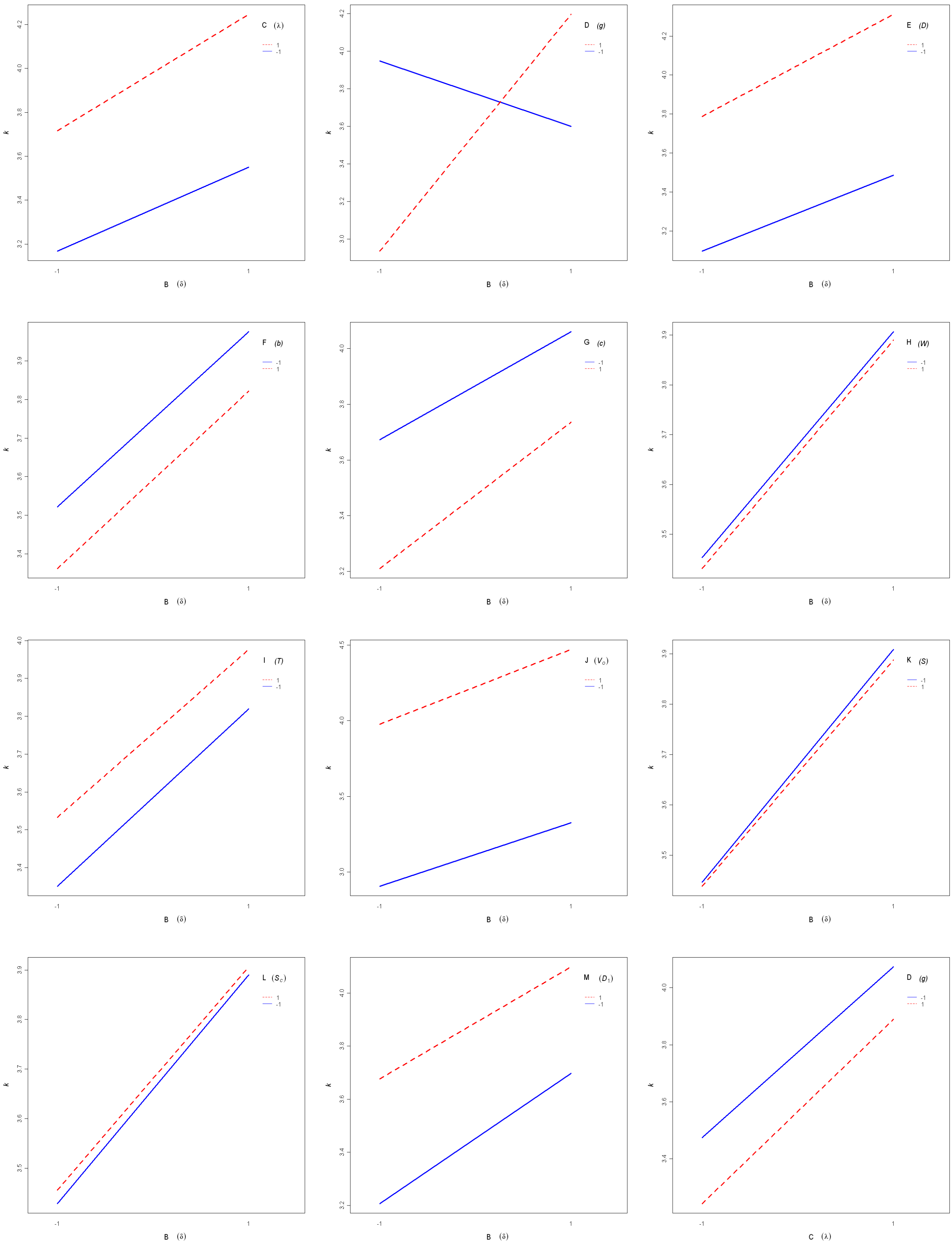


Figure 58: Discontinuous Process: interaction plots for k (2/7).

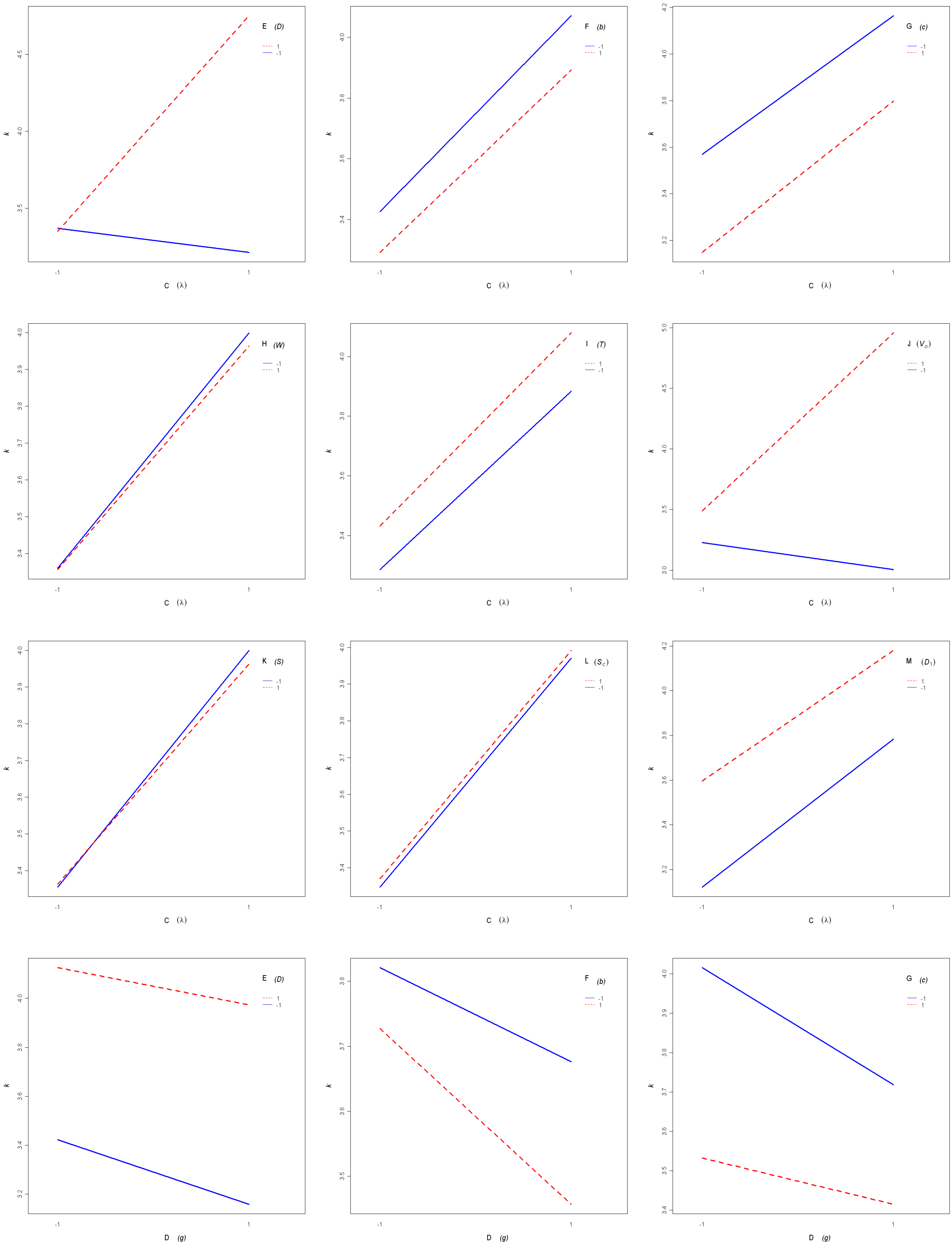


Figure 59: Discontinuous Process: interaction plots for k (3/7).

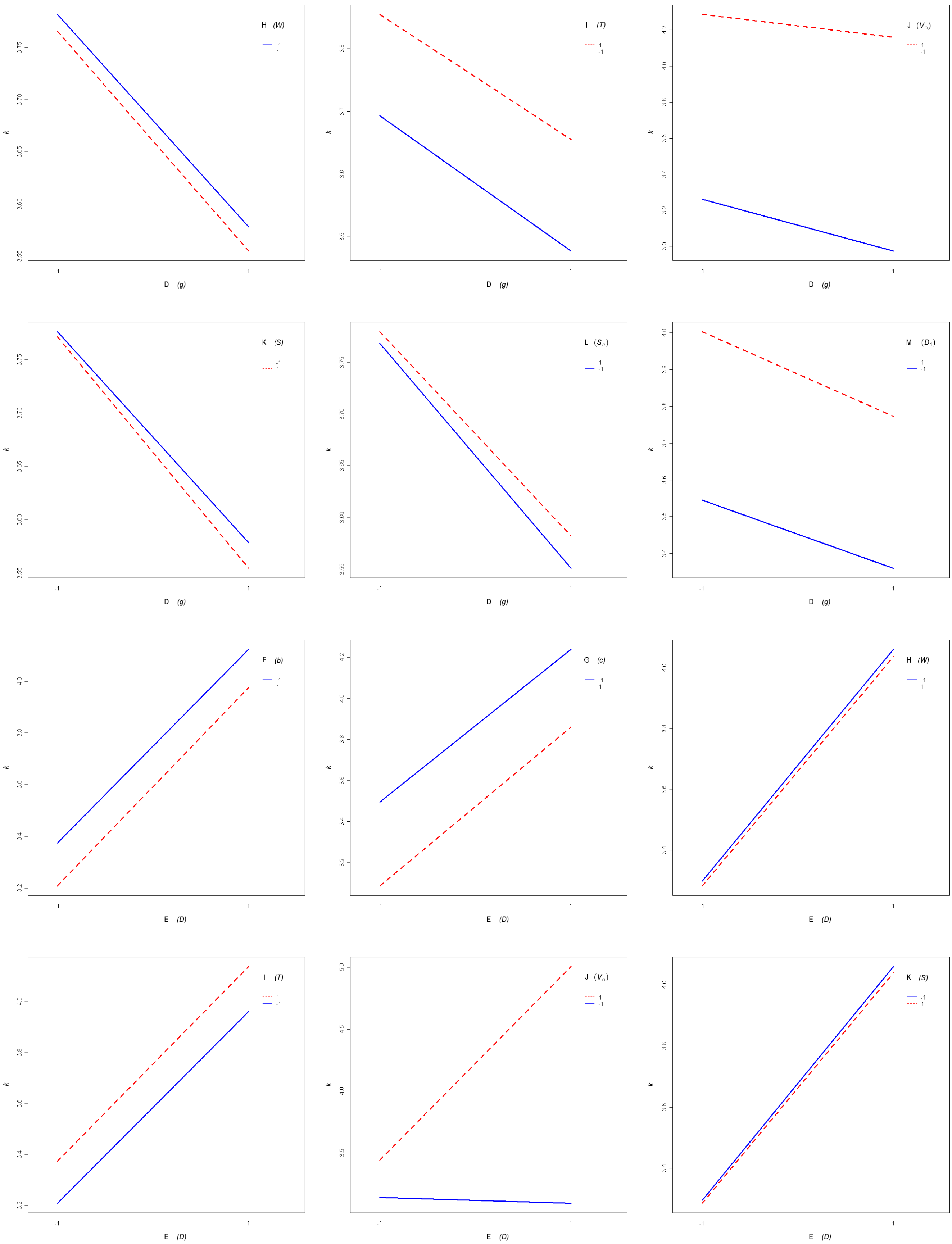


Figure 60: Discontinuous Process: interaction plots for k (4/7).

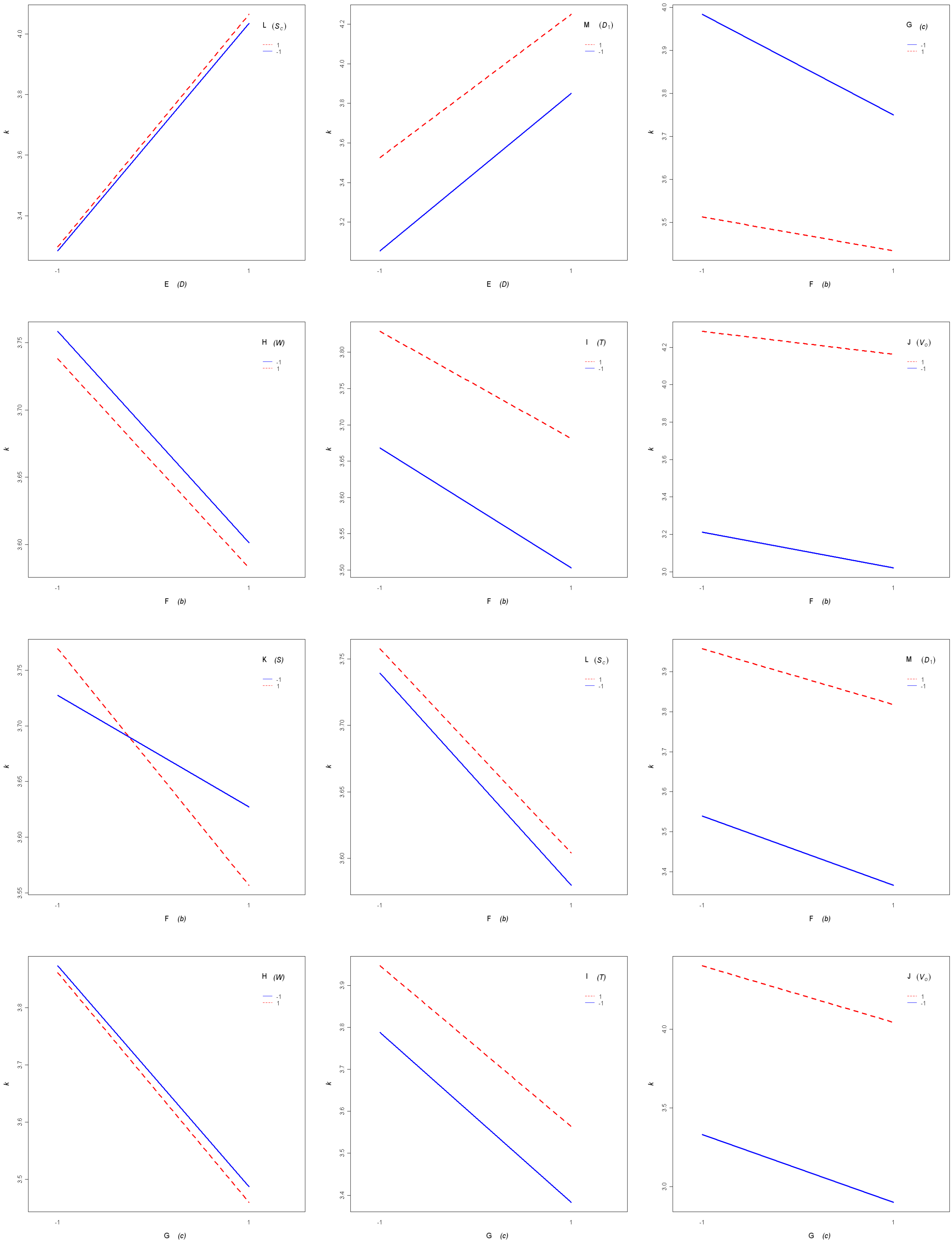


Figure 61: Discontinuous Process: interaction plots for k (5/7).

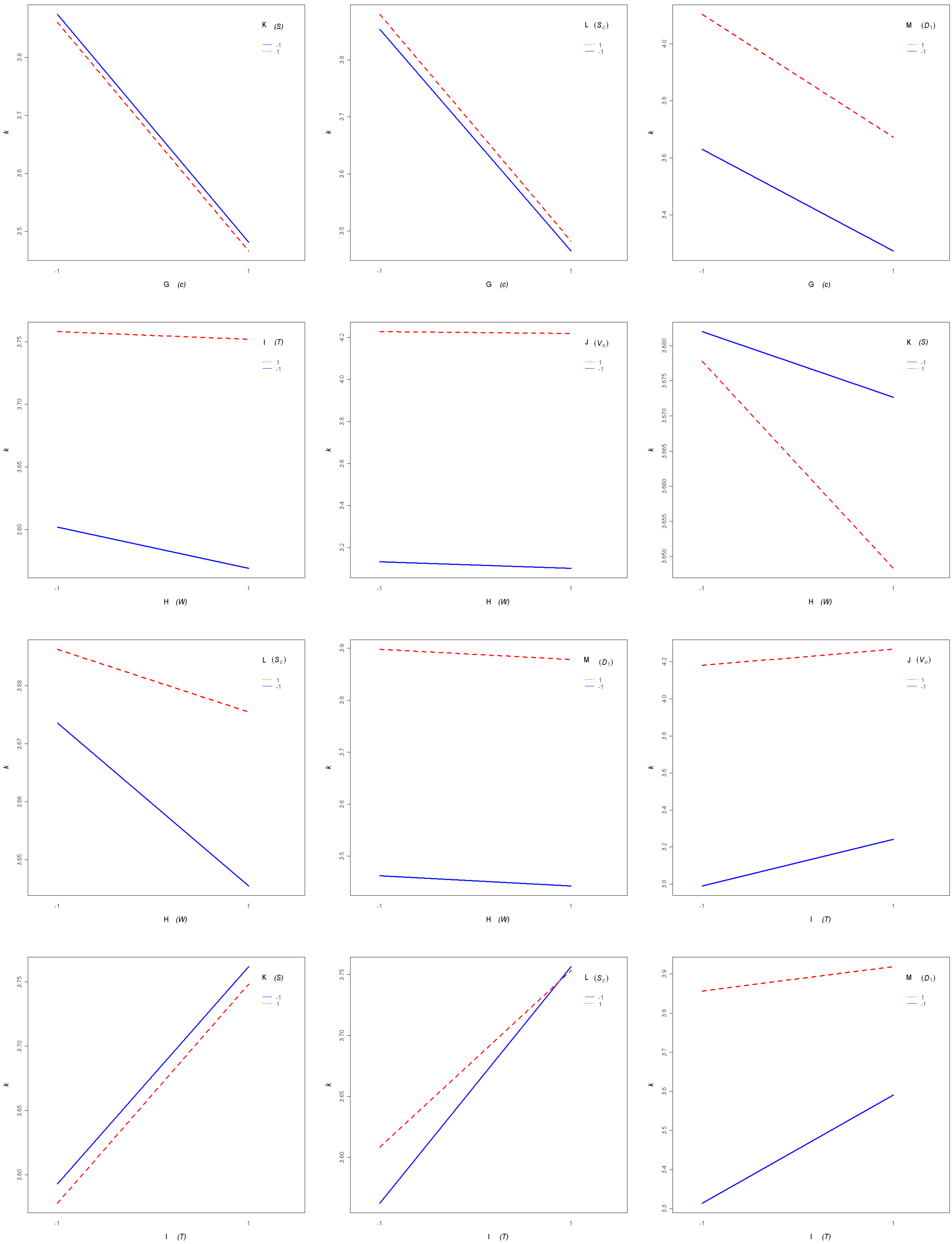


Figure 62: Discontinuous Process: interaction plots for k (6/7).

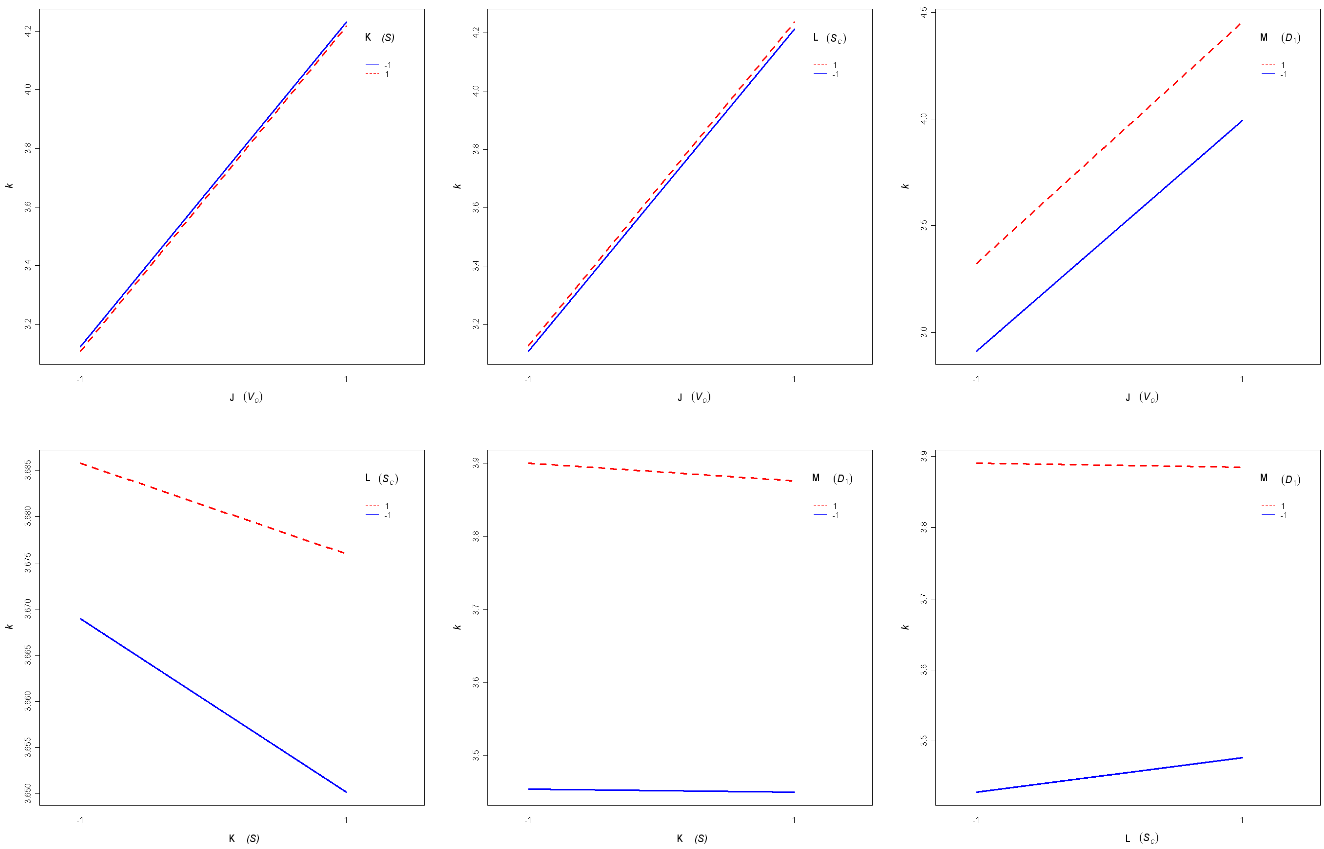


Figure 63: Discontinuous Process: interaction plots for k (7/7).

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