Statistical Reconciliation of Time Series
Movement Preservation *vs.* a Data Based Procedure

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Keywords: Benchmarking, Data reconciliation procedures, Temporal and contemporaneous constraints, Temporal disaggregation of systems of time series.

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1 Introduction

Most applications of economic models to real world issues must deal with the problem of extracting results from data, or economic relationships, with noise. One particular problem which arises from the presence of errors in variables, or due to the nature of the statistical procedures the data have passed through, is that of reconciling a data set of observed variables which should satisfy a number of linear[1] accounting restrictions, but, for any reason, these restrictions are not met.

This happens, for example, because economic data are frequently collected by different methods, using different sample surveys or different pieces of measuring equipment. Many applied fields are touched by this problem: during the complex

[1]Reconciliation of economic data may sometimes involve also non-linear accounting restrictions, for example when dealing with aggregates expressed in both current and constant prices (see Weale, 1988; Antonello, 1990).

production process of national accounts and balance of payments, data are often incomplete at some level of disaggregation. A consequence of this is that there could be different estimates of the same variable or, more generally, linear restrictions which the observations should satisfy but fail to (Weale, 1985, 1992). In such situations economic statisticians may choose to make indirect estimates of missing items. The question then arises how to use this information efficiently in order to produce fit-to-use estimates of the variables (Guerrero and Peña, 2000).

Two principles normally guide such estimation. First, estimates of the missing data are usually constrained so that they are consistent with observed and prior restrictions on their values. Secondly, values are preferred which more closely - in terms of a pre-specified distance metric - reflect prior estimates. These principles form the basis for the famous RAS method of Stone (1961)[2] and for a number of data reconciliation procedures, the most preminent of which is the least-squares adjustment method originally developed by Stone, Champernowne, and Meade (1942) for balancing national accounts.

For a long time the main emphasis of data reconciliation has been on the problem of interpolating input-output tables, for which the RAS method is computationally easier to use than the least-squares adjustment. But the times are somehow changing, and since Byron (1978) attention has reverted to least-squares reconciliation (van der Ploeg, 1982, 1984, 1985; Barker, van der Ploeg, and Weale, 1984; Weale, 1988, 1992; Solomou and Weale, 1993; Arkhipoff, 1995; Pedullà, 1995; Sefton and Weale, 1995; Smith, Weale, and Satchell, 1998).

In this paper we deal with the estimation of a system (expressed in form of a table) of high-frequency (say, quarterly) series subject to contemporaneous (*between* variables) and temporal (*within* variables) constraints. We assume that the available information is given by a table of high-frequency preliminary series to be reconciled in line with (i) known - assumed 'true' - temporal aggregates (say, the annual series of the variables of interest) and with (ii) a high frequency contemporaneously aggregated series (say, the quarterly sum of the various constitutive aggregates), either assumed known or not.

In the time series literature this problem, involving either one or many time series, is generally known as temporal disaggregation or benchmarking of time series[3], where the latter definition (benchmarking) naturally matches with the 'philosophy' of the data reconciliation procedures[4] introduced so far.

The links between least-squares reconciliation procedures and benchmarking are

[2]As far as the origin of RAS is concerned, "at least as early as the 1930s, researchers documented biproportional adjustment techniques - also known as 'iterative proportional fitting' or 'raking'. (...) The two first documented pieces were by Kruithof (1937) (...) and - according to Bregman (1967) - by Leningrad architect G.V. Sheleikhovskii" (Lahr and de Mesnard, 2004, p 115).

[3]Eurostat (1999) contains a survey, taxonomy and description of the main temporal disaggregation methods proposed by literature, and in Bloem, Dippelsman, and Mæhle (2001) two chapters analyze the relevant techniques to be used for benchmarking, extrapolation and related problems occurring in the compilation of quarterly national accounts. More complete references can be found in Di Fonzo and Marini (2006).

[4]Cholette (1987, p. 14) states: "Benchmarking is the process of optimally combining the original sub-annual series with the annual benchmarks and with the sub-annual benchmarks, in order to obtain a more reliable sub-annual series and a more reliable annual series".

indeed very strict, in the sense that practically any relevant benchmarking technique can be seen as a particular reconciliation procedure. The crucial point is whether (and how) the covariance matrix of the preliminary series is estimated or defined (and imposed) once and for all.

One reason that researchers seem to have moved away from RAS is that it was difficult to incorporate information (when available) on the relative reliability of the data. In fact, the classical formulation of RAS does not take into account any measure of preliminary estimates' accuracy[5]. Needless to say, balancing matrices composed of data with different data qualities can be an imposing challenge, especially for the national accounting agencies, which typically draw upon a variety of data sources in constructing the economic aggregates.

The distinctive feature of the least-squares reconciliation approach is instead that it can take into account the precision of the various constitutive aggregates of the accounting tables. However, for a long time these procedures have depended on the assumption that the covariance matrix (or any other indicators of the estimates' accuracy) of the figures to be reconciled was known. In this case, the data are adjusted in the light of their relative variances so as to satisfy the linear restrictions. But another - perhaps more delicate - challenge raises when any reliability measure, neither coming from a survey nor of subjective nature, is available: the solutions proposed in literature for this case are basically of two types, both of which are consistent with the least-squares approach of Stone *et al.* (1942):

1. mathematical/mechanical solutions: the data-set is balanced by minimizing a penalty criterion which 'induces' a covariance matrix (which is simply a statistical artifact);

2. data-based solutions: the variability of the data to be reconciled is estimated using the available observations[6].

In this paper we discuss two different procedures of benchmarking a system of high-frequency time series in line with a set of pre-specified accounting constraints, and precisely (i) an extension (Eurostat, 1999; Di Fonzo, 2002) of the univariate approach by Denton (1971), founded on a well known *movement preservation principle*, and (ii) a data-based benchmarking procedure (Guerrero and Nieto, 1999) which exploits the autoregressive properties of the preliminary series to be adjusted.

Both procedures take into account the fact that the data to be reconciled come from time series, and thus explicitely allows for temporal autocorrelation. But, as Solomou and Weale (1993, p. 90) point out, "there is (...) no theoretical problem in balancing estimates of national income in a manner which takes account of autocorrelation. The difficulties are practical. First, the structure of autocorrelation must be identified from information about the way in which the data were constructed.

[5]Robinson, Cattaneo, and El-Said (2001) briefly summarize the efforts undertaken by a few authors to handle differing data reliabilities in a RAS framework.

[6]Weale (1992) presents an estimator for use with a sequence of observations when the data variances are not known which is calculated from the time-series variances and covariances of the inconsistent observations. Solomou and Weale (1993) extend this result in order to account for the autocorrelation which is likely to characterize economic variables when measurement errors are present.

Secondly, because the data are all balanced in one step, $\mathbf{V}$ is typically a matrix of large proportions and so the problem must be solved in a way which exploits the sparsity of this matrix", where $\mathbf{V}$ denotes the covariance matrix of the preliminary data to be reconciled.

As we shall see, both procedures suffer from the problems due to the dimensions of the involved matrices[7]. However, while the extension of Denton's approach is a mathematical/mechanical device, working under a pre-specified condition according to which the temporal dynamics of the estimated series should be *as close as possible* to those of the preliminary counterparts, the data-based procedure by Guerrero and Nieto is intended to 'extract' information about the variability of the series making use of the observed data.

The rationale behind the latter way of recovering information on the reliabilities of the preliminary series seems in line with Weale (1992, p. 168): "It may seem strange that the variance of the measurement error can be inferred from the time-series variances. An immediate reaction might be that genuinely volatile data will be treated as unreliable. However, the reason that the time-series variance can be used is that the accounting constraint can be used to purge the genuine volatility, leaving only the noise". Moreover, when in a system of time series it is deemed unlikely that the measurement errors are independent of the true data, the data-based procedure by Guerrero and Nieto (1999) provides a means of identifying information about the data (un)reliability: by extending the approach to the benchmarking of a single time series proposed by Guerrero (1990), a VAR specification valid for the preliminary series is exploited in order to infer the covariance matrix of the disturbances to be used in the final reconciliation formula.

The paper is organized as follows. In section 2 we review the reconciliation problem and discuss it in the light of the classical matrix balancing solutions proposed in literature (RAS and the least-squares procedure by Stone *et al.*, 1942). Section 3 is devoted to the formulation of the specific problem tackled in this paper, the benchmarking of a system of time series subject to both contemporaneous and temporal constraints. In section 4 we present the generalization of Denton's approach to benchmark a table of time series, while the procedure by Guerrero and Nieto is described in section 5. In section 6 both procedures are applied to simulated and real world data, in order to appreciate the way the procedures work and their performances in practical situations. Section 7 presents some conclusive remarks.

2 Reconciliation of economic data as a matrix balancing problem

Matrix balancing is an important problem that has attracted attention in many different fields: the need of adjusting the entries of a large matrix to satisfy prior

[7] We agree with Cholette (1987, p. 45): "Practical experience with simultaneous benchmarking may show that very similar results can be achieved with some combination of individual benchmarking with raking (...) However, simultaneous benchmarking does provide a standard, i.e. a norm, against which alternative and simpler approaches may be assessed". Di Fonzo and Marini (2003) present simplified expressions of the extended Denton's procedure which save computation times and matrix storage space.

consistency requirements occurs frequently in economics, urban planning, statistics, demography, and stochastic modeling (Schneider and Zenios, 1990), and a large amount of both theoretical results and real-life-data applications of matrix balancing can be found in the specialized literature for all these fields.

In economics, data reconciliation might typically involve the updating and balancing of input-output or social and economic accounts matrices (see, *inter alia*, van der Ploeg, 1982; Barker, van der Ploeg, and Weale, 1984; Harrigan and McNicoll, 1986). In other words, many economic statistics, set in the form of tables spanning on one or several time periods, face this problem.

Denoting by $\mathbf{Y}$ the $(m \times n)$ matrix of entries to be reconciled, the above issue can be formalised as a *matrix balancing problem* (Bacharach, 1970). A well studied instance of this problem occurring in transportation planning and input-output analysis requires that $\mathbf{Y}$ be adjusted so that the row and column totals equal fixed positive values. A related problem occurring in developmental economics requires that the row and column totals (of a square matrix) be equal to each other, but not necessarily to prespecified values. As we shall see, *mutatis mutandis*, both cases[8] are of interest when a table of time series must be estimated in such a way as all accounting constraints are fulfilled.

A matrix balancing problem is typically posed as follows:

Given an $(m \times n)$ matrix $\mathbf{Y}$, determine an $(m \times n)$ matrix $\mathbf{Y}^*$ that is close to $\mathbf{Y}$ and satisfies a given set of linear restrictions on its entries.

There are several balancing problems, each with different consistency requirements, so the definition of a balanced matrix is problem dependent. In general, we can say that a matrix is defined to be balanced if it satisfies the given set of linear restrictions for the problem. As such, there are infinitely many matrices satisfying the consistency restrictions: the problem is to find a matrix that satisfies the restrictions and is related to the original matrix $\mathbf{Y}$ in a suitably defined manner.

The matrix balancing applications of interest for our work can be formulated in general as one of two problems.

Problem 1. Given an $(m \times n)$ matrix $\mathbf{Y}$ and two vectors $\mathbf{u}$ and $\mathbf{v}$ of dimensions m and n, respectively, determine a nearby $(m \times n)$ matrix $\mathbf{Y}^*$ such that $\sum_{j=1}^{n} y^*_{ij} = u_i$, for $i = 1, \ldots, m$, and $\sum_{i=1}^{m} y^*_{ij} = v_i$, for $j = 1, \ldots, n$.

Problem 2. Given an $(n \times n)$ matrix $\mathbf{Y}$, determine a nearby $(n \times n)$ matrix $\mathbf{Y}^*$ such that $\sum_{j=1}^{n} y^*_{ij} = \sum_{i=1}^{n} y^*_{ij}$, $i, j = 1, \ldots, n$.

Of course, for either problem to be well posed, some restrictions must be placed on the adjustments that can be made to $\mathbf{Y}$ so that the requirement of a nearby matrix is well defined. In input-output analysis, for example, if a direct coefficients matrix has to be updated/projected, matrices $\mathbf{Y}$ and $\mathbf{Y}^*$ and vectors $\mathbf{u}$ and $\mathbf{v}$ must be nonnegative and a sign-preservation-condition ($y^*_{ij} > 0$ only if $y_{ij} > 0$) is usually requested. This is not the case, however, for a table of national accounts, where negative entries can occur.

Different algorithmic approaches follow naturally from different types of restrictions. Moreover, it should be noted that, if we abstract from any specific application,

[8] In the former case the known marginal totals are *exogenous* constraints, while in the latter the constraints are *endogenous*.

a data reconciliation problem can be defined as a mathematical programming problem with a strictly convex minimand and linear constraints. The minimand reflects some informal measure of the 'distance'[9] between a set of prior and posterior estimates, and the constraints embody accounting and any other restrictions on the posterior estimates. Assuming the feasible set is non-empty, data reconciliation problems of this form yield a unique solution (Harrigan, 1990).

2.1 Procedures for matrix balancing: RAS and QPD optimization

Procedures for matrix balancing can be separated into two broad classes: biproportional, or scaling, algorithms (like RAS) and non biproportional algorithms.

Scaling algorithms are identified by the way they balance matrices: they iteratively multiply rows and columns of $\mathbf{Y}$ by positive constants to derive a series of candidate solution matrices until the matrix is balanced. However, despite their strong empirical characterization, biproportional procedures are also provided by a substantial theoretical basis. As it is well known (Uribe, de Leeuw, and Theil, 1965; Bacharach, 1970), the RAS procedure generates a solution equivalent to minimizing Kullback's information gain (Theil, 1967)[10], given by

$$\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij}^{*} \log \frac{y_{ij}^{*}}{y_{ij}}.$$

So the target matrix is as close as possible to the prior in the sense that "one estimates the unknown matrix as that value which, if realized, would occasion the least 'surprise' in view of the prior" (Bacharach, 1970, p. 84).

RAS algorithm can be used to estimate the 'interior' of the accounting matrix in the case formulated as Problem 1, that is when the row and columns totals of an accounting matrix are known and have been balanced so that the sum of the row sums equals the sum of the column sums. A classic example is where a prior benchmark input-output table is updated to be in line with national accounts of a later year. Since the national accounts have already been balanced, generally at a much higher level of aggregation, the condition concerning the row and column sums is satisfied. In fact, in such a case the row and column sums are exogenous variables, whereas they are endogenous variables in the balancing of the national accounts themselves where one starts out with a macro difference between total supply and total use of products. A similar situation occurs when a system of quarterly time series must be reconciled in line with a quarterly aggregate given by the sum of the component series and with the known annual series of each component variable. When the quarterly grand-total series is known, an exogenous constraint must be imposed in the estimation procedure, while when this is not the case, the constraint is endogenous.

[9]Here the term 'distance' is used in an informal sense. The minimands need not, in general, satisfy either the symmetry or triangle inequality properties of true distance measures (Harrigan, 1990).

[10]Moreover, "de Mesnard (1994) demonstrated that all algorithms with a biproportional form (...) do, in fact, yeld the same results" (Lahr and de Mesnard, 2004, p. 123).

Non bipoportional algorithms are all general constrained matrix problems that cannot be solved using the simpler set of scaling techniques[11]. They generally consist in optimization algorithms that minimize some measure of distance between all elements of a candidate balanced matrix from the original matrix $\mathbf{Y}$; the balance conditions are constraints in the optimization model so that the optimal solution is the balanced matrix closest to $\mathbf{Y}$ according to the metric induced by the chosen penalty function.

The objective function of these optimization problems can take many forms. Here we consider two simple criteria[12]: (i) that first presented to an input-output audience by Almon (1968), who proposed the minimization of the square of the Euclidean distance between $\mathbf{Y}^*$ (the target) and $\mathbf{Y}$ (the prior), and (ii) Pearson's χ^2 or the normalized squared differences used by Deming and Stephan (1940) and Friedlander (1961).

More precisely, Almon (1968) consider the objective function

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left(y_{ij}^* - y_{ij}\right)^2 ,$$

while the minimand of Deming and Stephan (1940) and Friedlander (1961) is

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\left(y_{ij}^* - y_{ij}\right)^2}{y_{ij}} .$$

It should be noted that, given the accounting constraints, this class of methods can generate negative-valued elements in $\mathbf{Y}^*$ even if there are none in $\mathbf{Y}$: that is, sign is not always preserved. Furthermore, it is immediately recognized that, provided $\mathbf{Y}$ is a nonnegative matrix[13], both the objective functions are Quadratic Positive Definite (QPD)[14] minimands (Harrigan, 1990) of the form

$$\left(\mathbf{y}^* - \mathbf{y}\right)' \mathbf{Q}^{-1} \left(\mathbf{y}^* - \mathbf{y}\right) \tag{1}$$

where $\mathbf{y}^*$ and $\mathbf{y}$ are both $(mn \times 1)$ vectors containing the elements of $\mathbf{Y}$ and $\mathbf{Y}^*$, respectively, $\mathbf{Q} = \mathbf{I}_{mn}$ for the Almon's formulation and $\mathbf{Q} = \hat{\mathbf{y}}$ for the latter, with $\hat{\mathbf{y}} = diag(\mathbf{y})$.

As these two constrained optimization problems can be seen as a particular case of the Stone *et al.* (1942) procedure, provided a specific choice of the covariance matrix of the data to be reconciled is made, the solution to these optimization problems will be presented in the next subsection.

[11] Non biproportional techniques involving nonlinear networks methods, conjugate gradient algorithms, Lagrangian relaxation, and successive overrelaxation, are not discussed here. The interested reader can refer to Schneider and Zenios (1990).

[12] For other objective functions proposed in literature, see Jackson and Murray (2004).

[13] Obviously, for the objective function to be QPD, nonnegativity is needed for Deming and Stephan (1940) and Friedlander (1961) minimand.

[14] Harrigan (1990) compares the use of a quadratic positive definite objective function with the Kullback-Leibler cross-entropy measure according to which RAS can be re-interpreted. He concludes that both "possess the desirable property that they give posterior estimates which better reflect the unknown, true values than do the associated prior estimates".

2.2 Least squares adjustment of economic data subject to linear restrictions

Let $\tilde{\mathbf{y}}$ be a $(mn \times 1)$ vector of unknown data which should satisfy the system of linear independent accounting constraints

$$\mathbf{A}\tilde{\mathbf{y}} = \mathbf{a} \tag{2}$$

where $\mathbf{A}$ is a $(k \times mn)$ matrix, with $rank(\mathbf{A}) = k < mn$, and $\mathbf{a}$ is a $(k \times 1)$ vector of known constants. Let $\mathbf{y}$ be a $(mn \times 1)$ vector of observed data, not fulfilling the linear constraint (2), related to $\tilde{\mathbf{y}}$ by the linear model

$$\mathbf{y} = \tilde{\mathbf{y}} + \mathbf{e} \tag{3}$$

where $\mathbf{e}$ is a $(mn \times 1)$ vector of disturbances with zero mean and known covariance matrix $\mathbf{V}$. Building on Stone *et al.* (1942), it has been demonstrated that an efficient estimator $\mathbf{y}^*$ of $\tilde{\mathbf{y}}$, which minimizes (1) for $\mathbf{Q} \equiv \mathbf{V}$ and satisfies constraints (2), is given by

$$\mathbf{y}^* = \mathbf{y} + \mathbf{V}\mathbf{A}' \left(\mathbf{A}\mathbf{V}\mathbf{A}'\right)^{-1} (\mathbf{a} - \mathbf{A}\mathbf{y}) \tag{4}$$

with

$$E\left(\mathbf{y}^* - \tilde{\mathbf{y}}\right) = \mathbf{V} - \mathbf{V}\mathbf{A}' \left(\mathbf{A}\mathbf{V}\mathbf{A}'\right)^{-1} \mathbf{V}. \tag{5}$$

Contrary to RAS, the Stone *et al.* (1942) procedure automatically balances when row and column totals are endogenous variables. It can therefore be applied to the problem of balancing the national accounts and to other situations of practical relevance, as those considered in this paper. Another advantage of this algorithm is that it is based on the least-squares principle and thus leans on a long and solid tradition in statistics. The fact that the data are adjusted in the light of their relative variances so as to satisfy the linear restrictions is certainly appreciable. Moreover, under normality assumptions the reconciled estimates are ML (Weale, 1985, 1988) and thus the standard tools of statistical inference can be applied.

A rather usual, simplified assumption on the disturbance terms, in line with Stone (1990), postulates they are independent. In this case $\mathbf{V}$ is diagonal. Taking the definition of probability as a "degree of reasonable belief", Stone proposes to determine $var(y_i)$ as $v_{ii} = (\theta y_i)^2$, $i = 1, \ldots, mn$, where θ_i is a subjectively determined reliability rating, expressing the percentage ratio of the standard error to the prior mean value of y_i.

The chief drawback from a conceptual point of view is that it does not guarantee preservation of sign of the variables, which sometimes can be problematic. Another possible weakness of the procedure depends on the assumption that the covariance matrix of the various estimates is known. In fact, in the past its application has been limited[15] by both its computational intensity and the possible lack of any good, objective basis for specifying the variance matrix, which is crucial in the minimization of least squares[16].

[15]Valuable exceptions are Byron (1978), van der Ploeg (1982, 1984, 1985), Barker, van der Ploeg, and Weale (1984), Weale (1992), Solomou and Weale (1993), Pedullà (1995), Smith, Weale, and Satchell (1998), Toh (1998), Gilchrist and St. Louis (1999).

[16]Practical applications of the Stone *et al.* (1942) algorithm have been relatively few and typ-

3 Benchmarking a system of time series with exogenous constraints

We deal with an indirect estimation problem involving a system of variables rather than a single one. More precisely, we wish to estimate M unknown $(n \times 1)$ vectors of high-frequency data, each pertaining to M basic (i.e., disaggregate) time series which have to satisfy both contemporaneous and temporal aggregation constraints.

The information basis is given by the following $M+1$ aggregated vectors:

1. $\mathbf{z}$, $(n \times 1)$ vector of contemporaneously aggregated data;
2. $\mathbf{y}_{0j}$, $j = 1, \ldots, M$, $(N \times 1)$ vectors of temporally aggregated data (say, annual).

We consider the case where M preliminary high-frequency (say, quarterly) time series, $\mathbf{y}_j$, $j = 1, \ldots, M$, are available[17], where $\sum_{j=1}^{M} \mathbf{y}_j \neq \mathbf{z}$ and/or $\mathbf{y}_j$ doesn't comply with $\mathbf{y}_{0j}$. Denoting by $\mathbf{y}_j^*$, $j = 1, \ldots, M$, the $(n \times 1)$ vectors of the reconciled data to be estimated, the following accounting constraints must hold:

$$\sum_{j=1}^{M} \mathbf{y}_j^* = \mathbf{z} \tag{6}$$

$$\mathbf{J}\mathbf{y}_j^* = \mathbf{y}_{0j}, \qquad j = 1, \ldots, M \tag{7}$$

where $\mathbf{J}$ is the $(N \times n)$ aggregation matrix converting high-frequency in low frequency data. Each element of $\mathbf{y}_{0j}$ can be viewed as a non overlapping linear combination of $\mathbf{y}_j^*$, with coefficients given by the $(s \times 1)$ vector $\mathbf{j}$, s being the temporal aggregation order. Thus, in general matrix $\mathbf{J}$ is equal to $\mathbf{J} = \left[\mathbf{I}_N \otimes \mathbf{j}' \;\vdots\; \mathbf{0}\right]$, where $\mathbf{0}$ is a null $(N \times (n - sN)M)$ matrix[18].

Constraint (6) can be re-written as $(\mathbf{1}_M' \otimes \mathbf{I}_n)\,\mathbf{y}^* = \mathbf{z}$, where $\mathbf{1}_M$ is an $(M \times 1)$ vector of ones and $\mathbf{y}^* = \left[\mathbf{y}_1^{*\prime} \cdots \mathbf{y}_j^{*\prime} \cdots \mathbf{y}_M^{*\prime}\right]'$. As far as temporal aggregation constraints (7) are concerned, we have $(\mathbf{I}_M \otimes \mathbf{J})\,\mathbf{y}^* = \mathbf{y}_0$, where $\mathbf{y}_0 = \left[\mathbf{y}_{01}' \cdots \mathbf{y}_{0j}' \cdots \mathbf{y}_{0M}'\right]'$. Let $\mathbf{H}$ be the $((n + NM \times nM) \times nM)$ aggregation matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{1}_M' \otimes \mathbf{I}_n \\ \mathbf{I}_M \otimes \mathbf{J} \end{bmatrix}$$

ically of an *ad hoc* nature. The privileged approach has been to specify some kind of quadratic loss function and assume information about the statistical properties of the error distributions. Harrigan and McNicoll (1986) argue persuasively for the advantages of a constrained maximization estimation approach in terms of flexibility, but are aware of the statistical problems. Byron (1978) and Schneider and Zenios (1990) also argue in favour of a constrained maximization approach, and are also skeptical of imposing strong statistical assumptions. Only a few authors (Weale, 1992; Solomou and Weale, 1993; Sefton and Weale, 1995; Smith *et al.* , 1998) take into account the autocorrelation which is likely to feature data subject to measurement errors.

[17] Rossi (1982), Di Fonzo (1990, 2002) and Cabrer and Pavía (1999) consider the more general case where a set of high-frequency related indicators is used to obtain indirect estimates of the M unknown time series. It should be noted that the distinction is not necessarily as strict as it seems, in that preliminary high-frequency series could have been individually obtained by using related indicators, as in Guerrero and Nieto (1999).

[18] The null matrix in $\mathbf{J}$ permits to deal with extrapolation. Obviously, when $n = sN$, $\mathbf{J} = [\mathbf{I}_N \otimes \mathbf{j}']$.

and $\mathbf{y}_a$ the $((n+NM)\times nM))$ vector $\mathbf{y}_a = [\mathbf{z}' \ \mathbf{y}_0']'$ containing both contemporaneous and temporal aggregates. The complete set of constraints between the reconciled values and the available aggregated information can be expressed in matrix form as

$$\mathbf{H}\mathbf{y}^* = \mathbf{y}_a. \tag{8}$$

Notice that the contemporaneous aggregation of temporally aggregated series implies

$$\sum_{j=1}^{M} y_{0j,T} = \sum_{h=1}^{s} z_{s(T-1)+h} = z_{0,T}, \qquad T = 1,\dots,N,$$

that is, in matrix form,

$$\mathbf{1}_M' \otimes \mathbf{I}_N = \mathbf{J}\mathbf{z}, \tag{9}$$

where $\mathbf{J}$ is the temporal aggregation matrix with $\mathbf{j} = \mathbf{1}_s$. ¿From relationship (9) it follows that only $r = n + N(M-1)$ aggregated observations are 'free', while N aggregated observations are redundant, thus matrix $\mathbf{H}$ has rank r. To clarify this fact, partition $\mathbf{H}$ in such a way as to distinguish the temporal aggregation constraints linking $\mathbf{y}_M^*$ to $\mathbf{y}_{0M}$ from the remainder:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_w \\ \dots \\ \mathbf{H}_M \end{bmatrix},$$

where $\mathbf{H}_w = \begin{bmatrix} \mathbf{1}_{M-1}' \otimes \mathbf{I}_n & \mathbf{I}_n \\ \mathbf{I}_{M-1} \otimes \mathbf{J} & \mathbf{0} \end{bmatrix}$ and $\mathbf{H}_M = \left[\mathbf{0} \vdots \mathbf{J}\right]$ are matrices $(r \times Mn)$ and $(N \times Mn)$, respectively. Denoting $\mathbf{W}$ the $(N \times r)$ matrix[19] $\mathbf{W} = \left[\mathbf{J} \vdots -(\mathbf{1}_{M-1}' \otimes \mathbf{I}_N)\right]$ and $\mathbf{R}$ the $((r+N)\times r)$ matrix $\mathbf{R} = [\mathbf{I}_r \ \mathbf{W}']$ we have $\mathbf{H}_M = \mathbf{W}\mathbf{H}_w$ and $\mathbf{H} = \mathbf{R}\mathbf{H}_w$.

Without loss of generality, consider the $(r \times 1)$ vector $\mathbf{w}$, which is simply the aggregated vector $\mathbf{y}_a$ bereft of its last M rows[20]: $\mathbf{w} = \left[\mathbf{z}' \ \mathbf{y}_{01}' \ \dots \ \mathbf{y}_{0M-1}'\right]'$. After some algebra (Di Fonzo and Marini, 2003), it can be demonstrated that the constraints operating on vector $\mathbf{y}^*$, expressed in (8) in terms of matrix $\mathbf{H}$ – which has not full row rank –, can now be expressed as a system of linearly independent constraints as in (2), with $\mathbf{A} \equiv \mathbf{H}_w$, instead of $\tilde{\mathbf{y}}$ and $\mathbf{a} \equiv \mathbf{w}$, that is

$$\mathbf{H}\mathbf{y}^* = \mathbf{w}.$$

3.1 Benchmarking as a least-squares reconciliation problem

Assume that the available data $\mathbf{y}_j$ are distributed without bias around the 'true' series $\tilde{\mathbf{y}}_j$ according to the model

$$\mathbf{y}_j = \tilde{\mathbf{y}}_j + \mathbf{e}_j, \qquad j = 1,\dots,M, \tag{10}$$

[19] Matrix $\mathbf{W}$ is very similar to matrix $\mathbf{B}$ defined by Chen and Dagum (1997).

[20] As shown by Di Fonzo and Marini (2006), contrary to what Chen and Dagum (1997) state, the results are invariant with respect to the choice of a particular sub-vector of $\mathbf{y}_a$, provided it has dimension $(N \times 1)$.

where $\mathbf{e}_j$ are $(n \times 1)$ zero-mean random disturbances, with $E\left(\mathbf{e}_i\mathbf{e}_j'\right) = \mathbf{V}_{ij}$, $i, j = 1, \dots, M$, and $\mathbf{V}_{ij}$ are $(n \times n)$ known matrices. Putting together the M relationships (10) we have the complete model $\mathbf{y} = \tilde{\mathbf{y}} + \mathbf{e}$, with $E(\mathbf{e}) = \mathbf{0}$ and $E(\mathbf{e}\mathbf{e}') = \mathbf{V}$, as in section 2.2. The simultaneously benchmarked series are the solution of the least squares problem

$$\min\left(\mathbf{y} - \mathbf{y}^*\right)' \mathbf{V}^{-1}\left(\mathbf{y} - \mathbf{y}^*\right) \qquad \text{subject to } \mathbf{H}\tilde{\mathbf{y}} = \mathbf{y}_a.$$

However, the constraints (8) being linearly dependent, an extension of the classical result by Stone *et al.* (1942) is needed due to the rank of the matrices involved in the procedure. It can be shown (Di Fonzo and Marini, 2003) that the benchmarked estimates can be expressed as

$$\mathbf{y}^* = \mathbf{y} + \mathbf{V}\mathbf{H}'(\mathbf{H}\mathbf{V}\mathbf{H}')^-(\mathbf{y}_a - \mathbf{H}\mathbf{y}), \tag{11}$$

where $(\mathbf{H}\mathbf{V}\mathbf{H}')^-$ denotes the Moore-Penrose generalized inverse of $\mathbf{V}_a = \mathbf{H}\mathbf{V}\mathbf{H}'$. A solution, equivalent to (11), and which does not involve singular matrices to be inverted, can be expressed in terms of the r 'free' constraints. In fact, the singular matrix $\mathbf{H}\mathbf{V}\mathbf{H}'$ can be written as $\mathbf{R}\mathbf{H}_w\mathbf{V}\mathbf{H}_w'\mathbf{R}' = \mathbf{R}\mathbf{V}_w\mathbf{R}'$, where $\mathbf{V}_w = \mathbf{H}_w\mathbf{V}\mathbf{H}_w'$ is a full rank $(r \times r)$ matrix. Furthermore, it can be readily checked (see Di Fonzo and Marini, 2006) that $\mathbf{V}_a^-$ is univocally given by

$$\mathbf{V}_a^- = (\mathbf{H}\mathbf{V}\mathbf{H}')^- = \mathbf{R}(\mathbf{R}'\mathbf{R})^{-1}\mathbf{V}_w^{-1}(\mathbf{R}'\mathbf{R})^{-1}\mathbf{R}'. \tag{12}$$

By substituting (12) into (11), and taking into account that $\mathbf{R}'\mathbf{H} = \mathbf{R}'\mathbf{R}\mathbf{H}_w$, after some algebra we find the more feasible benchmarking formula

$$\mathbf{y}^* = \mathbf{y} + \mathbf{V}\mathbf{H}_w'\mathbf{V}_w^{-1}(\mathbf{w} - \mathbf{H}_w\mathbf{y}). \tag{13}$$

The benchmarked estimates are thus obtained by distributing a linear combination of the discrepancies pertaining to r unconstrained observations of the aggregated vector $\mathbf{y}_a$ over the original unbenchmarked data. It should be noted that expression (13) involves the inversion only of full rank matrices and fulfils both temporal and contemporaneous constraints.

4 Reconciliation with an exogenous constraint according to Denton's multivariate benchmarking

The multivariate extension of Denton's benchmarking procedure can be seen as a simple generalization to $M > 1$ time series of the 'movement preservation principle' stated by Denton (1971)[21], according to which the temporal dynamics of the reconciled series should be as close as possible to those of the preliminary figures. As it is well known (Bloem *et al.*, 2001), this principle operates by focusing on, respectively, the simple period-to-period changes

$$(y_{j,t}^* - y_{j,t-1}^*) - (y_{j,t} - y_{j,t-1}) \equiv (y_{j,t}^* - y_{j,t}) - (y_{j,t-1}^* - y_{j,t-1}), \qquad j = 1, \dots, M,$$

[21] See also Cholette (1988).

or the proportional period-to-period changes[22],

$$\frac{y^*_{j,t} - y_{j,t}}{y_{j,t}} - \frac{y^*_{j,t-1} - y_{j,t-1}}{y_{j,t-1}} \equiv \frac{y^*_{j,t}}{y_{j,t}} - \frac{y^*_{j,t-1}}{y_{j,t-1}}, \qquad j = 1, \dots, M.$$

The objective functions to be minimized[23] are thus given by

$$\sum_{j=1}^{M}\sum_{t=2}^{n}\left[(y^*_{j,t} - y_{j,t}) - (y^*_{j,t-1} - y_{j,t-1})\right]^2 \qquad \text{Additive First Differences (AFD)}$$

and

$$\sum_{j=1}^{M}\sum_{t=2}^{n}\left(\frac{y^*_{j,t}}{y_{j,t}} - \frac{y^*_{j,t-1}}{y_{j,t-1}}\right)^2 \qquad \text{Proportional First Differences (PFD)}$$

respectively. Using matrix notation, let us consider the $(Mn \times Mn)$ matrices $\mathbf{\Omega}_{AFD} = \mathbf{I}_M \otimes (\mathbf{\Delta}'\mathbf{\Delta})$ and $\mathbf{\Omega}_{PFD} = \hat{\mathbf{y}}^{-1}\left[\mathbf{I}_M \otimes (\mathbf{\Delta}'\mathbf{\Delta})\right]\hat{\mathbf{y}}^{-1} = \hat{\mathbf{y}}^{-1}\mathbf{\Omega}_{AFD}\hat{\mathbf{y}}^{-1}$, where $\hat{\mathbf{y}} = diag(\mathbf{y})$ and $\mathbf{\Delta}$ is the $((n-1) \times n)$ matrix performing first differences:

$$\mathbf{\Delta} = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}.$$

The simultaneously benchmarked series can be obtained by solving the following minimization problem:

$$\min\,(\mathbf{y}^* - \mathbf{y})'\,\mathbf{\Omega}^{-1}\,(\mathbf{y}^* - \mathbf{y}) \qquad \text{subject to } \mathbf{H}_w\mathbf{y}^* = \mathbf{w},$$

where $\mathbf{\Omega} = \mathbf{\Omega}_{AFD}$ for the additive variant, or $\mathbf{\Omega} = \mathbf{\Omega}_{PFD}$ for the proportional formulation, respectively.

In both cases matrix $\mathbf{\Omega}$ has not full rank, so that the analogy with the least squares solution (13) - with $\mathbf{V} = \mathbf{\Omega}^{-1}$ - cannot be immediately established[24]. For computational convenience, in line with the original proposal of Denton (1971), we can consider a $(n \times n)$ 'approximate' first differences matrix, $\mathbf{D}$, given by

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}.$$

In this case both matrices $\tilde{\mathbf{\Omega}}_{AFD} = \mathbf{I}_M \otimes (\mathbf{D}'\mathbf{D})$ and $\tilde{\mathbf{\Omega}}_{PFD} = \hat{\mathbf{y}}^{-1}\left[\mathbf{I}_M \otimes (\mathbf{D}'\mathbf{D})\right]\hat{\mathbf{y}}^{-1}$ have full rank, and the least-squares reconciliation can be performed by assuming

[22] In this case, it must be $y_{j,t} \neq 0$, $j = 1, \dots, M$, $t = 1, \dots, n$.

[23] Benchmarking procedures which satisfy a movement preservation principle explicitly referred to the rates of change have been derived, for either one series or a table of series, by Bozik and Otto (1988) and Di Fonzo and Marini (2003), respectively.

[24] The exact solution is derived in Di Fonzo and Marini (2006).

$\mathbf{V} = \tilde{\mathbf{\Omega}}^{-1}$. The 'approximate' multivariate Denton's benchmarking formula can thus be expressed as

$$\mathbf{y}^* = \mathbf{y} + \tilde{\mathbf{\Omega}}^{-1}\mathbf{H}_w(\mathbf{w} - \mathbf{H}_w\mathbf{y}). \tag{14}$$

Even by using this approximation, the dimensions of the matrices involved in the calculations can be considerable in practical situations, possibly giving rise to computational burdens. However, a valuable saving of computation time and of storage area can be obtained by exploiting the partitioned form of the matrices involved (Di Fonzo and Marini, 2003).

5 A data based benchmarking procedure

Guerrero and Nieto (1999) developed a benchmarking method which exploits the autoregressive features of the preliminary series to determine the unobserved values of multiple time series whose temporal and contemporaneous aggregates are known.

The notation used by Guerrero and Nieto is different from that we adopted so far, in the sense that the vectorized data-matrix is stacked by time[25]. So, let $\mathbf{y}_{0T,g} = (y_{1,0T} \dots y_{M,0T})'$ be the $(M \times 1)$ vector of temporally aggregated data for M variables at time T, $T = 1, \dots, N$. Such vectors are stacked into the $(MN \times 1)$ vector $\mathbf{y}_{0,g} = (\mathbf{y}'_{01,g} \dots \mathbf{y}'_{0N,g})'$.

The $(n \times 1)$ vector of contemporaneously aggregated data is still given by $\mathbf{z} = (z_1 \dots z_n)'$, while we denote by $\tilde{\mathbf{y}}_g = (\tilde{\mathbf{y}}'_{1,g} \dots \tilde{\mathbf{y}}'_{n,g})'$ the $(Mn \times 1)$ stacked vector of unobserved variables at higher-frequency.

Now, to deal with the temporal and contemporaneous constraints, let us denote by $(c_{j1}, \dots, c_{js})$ the coefficients defining the temporal aggregation constraint valid for variable j, and define the $(M \times M)$ matrices $\mathbf{C}_h$, $h = 1, \dots, s$, as $diag(c_{1h}, \dots, c_{Mh})$. The following relationships hold:

Temporal aggregation of flows variables: $\mathbf{C}_h = \mathbf{I}_M \quad h = 1, \dots, s$;

Temporal aggregation of index variables: $\mathbf{C}_h = \frac{1}{s}\mathbf{I}_M \quad h = 1, \dots, s$;

Temporal aggregation of end-of-period stock variables: $\mathbf{C}_h = \begin{cases} \mathbf{0} & h = 1, .., s-1 \\ \mathbf{I}_M & h = s \end{cases}$;

Temporal aggregation of beginning-of-period stock variables: $\mathbf{C}_h = \begin{cases} \mathbf{I}_M & h = 1 \\ \mathbf{0} & h = 2, \dots, s \end{cases}$.

Vector $\tilde{\mathbf{y}}_g$ satisfies both temporal and contemporaneous constraints through the expression

$$\mathbf{y}_{a,g} = \begin{pmatrix} \mathbf{y}_{0,g} \\ \mathbf{z} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_N \otimes \mathbf{C}_0 \\ \mathbf{I}_n \otimes \mathbf{b}' \end{pmatrix} \tilde{\mathbf{y}}_g = \mathbf{C}\tilde{\mathbf{y}}_g,$$

where $\mathbf{C}_0 = (\mathbf{C}_1 \ \dots \ \mathbf{C}_s)$ is a $(M \times Ms)$ matrix, $\mathbf{b}$ is the known constant vector which defines the contemporaneous aggregation constraint (in case of simple summation, $\mathbf{b} = \mathbf{1}_M$), and $\mathbf{C} = \begin{pmatrix} \mathbf{I}_N \otimes \mathbf{C}_0 \\ \mathbf{I}_n \otimes \mathbf{b}' \end{pmatrix}$ is the complete $((MN + n) \times Mn)$ aggregation matrix.

[25] In the following, where possible, we try to use a similar notation with the subscript g. A formulation where the vectorised data-matrix is stacked by variable is shown in Di Fonzo and Marini (2006).

According to Di Fonzo (1990), it is also possible to deal with both constrained and pure extrapolation problems: define the $(MR \times 1)$ vector $\tilde{\mathbf{y}}_{e,g} = (\tilde{\mathbf{y}}'_{n+1,g} \cdots \tilde{\mathbf{y}}'_{n+R,g})'$, and consider the new aggregation matrices operating on the enlarged $(M(n+R) \times 1)$ unobserved high-frequency vector $\tilde{\mathbf{y}}^e_g = (\tilde{\mathbf{y}}'_g \; \tilde{\mathbf{y}}'_{e,g})'$:

$$\mathbf{C}^{e,c} = \begin{bmatrix} \mathbf{I}_N \otimes \mathbf{C}_0 & \mathbf{0} \\ \mathbf{I}_n \otimes \mathbf{b}' & \mathbf{I}_R \otimes \mathbf{b}' \end{bmatrix} \qquad \text{constrained extrapolation}$$

$$\mathbf{C}^{e,p} = \begin{bmatrix} \mathbf{I}_N \otimes \mathbf{C}_0 & \mathbf{0} \\ \mathbf{I}_n \otimes \mathbf{b}' & \mathbf{0} \end{bmatrix} \qquad \text{pure extrapolation}$$

The proposal by Guerrero and Nieto grounds on some assumptions defining the relationship between preliminary and true (unobservable) values, that is, in our notation, $\tilde{\mathbf{y}}_{t,g}$ and $\mathbf{y}_{t,g}$, respectively:

1. Before observing $\mathbf{y}_g$, it is assumed that $\tilde{\mathbf{y}}_{t,g} - \mathbf{y}_{t,g}$ admits a stationary VAR representation of order $p \geq 1$:
$$\mathbf{\Pi}(L)(\tilde{\mathbf{y}}_{t,g} - \mathbf{y}_{t,g}) = \varepsilon_{t,g}, \qquad t = 1, \ldots, n, \tag{15}$$
where $\mathbf{\Pi}(L)$ is a matrix polynomial in the backshift operator L and $\varepsilon_{t,g}$ is a zero-mean vector white noise process. Model (15) can be expressed in matrix notation as[26]
$$\mathbf{\Pi}(\tilde{\mathbf{y}}_g - \mathbf{y}_g) = \varepsilon_g, \tag{16}$$
where $\mathbf{\Pi}$ is an $(Mn \times Mn)$ matrix formed by the coefficients' matrices $\pi_1, \ldots, \pi_p$ (for details see Di Fonzo and Marini, 2006).

2. Once $\mathbf{y}_g$ is given, (16) still holds true with $E(\varepsilon_g \varepsilon'_g | \mathbf{y}_g) = \mathbf{P} \otimes \mathbf{\Sigma}$, where $\mathbf{P}$ is a $(n \times n)$ positive definite matrix to be derived from the data (Guerrero and Nieto, 1999, p. 462).

3. $\tilde{\mathbf{y}}_g$ and $\mathbf{y}_g$ admit the same VAR representation, in the sense that $\mathbf{\Pi}(L)\tilde{\mathbf{y}}_{t,g} = \mathbf{d}_t + \tilde{\mathbf{a}}_{t,g}$, where $\tilde{\mathbf{a}}_{t,g}$ and $\mathbf{a}_{t,g}$ are white noise processes with different covariance matrices, and $\mathbf{d}_t$ is a vector of deterministic components that includes the constant term.

Now, denoting $\mathbf{V}_g$ the $(Mn \times Mn)$ matrix

$$\mathbf{V}_g = \left(\mathbf{\Pi}^{-1}\right)(\mathbf{\Pi} \otimes \mathbf{\Sigma})\left(\mathbf{\Pi}^{-1}\right)',$$

Guerrero and Nieto (1999) show that the MMSE[27] for $\tilde{\mathbf{y}}_g$ given $\mathbf{y}_g$ and $\mathbf{y}_{a,g}$ is given by

$$\mathbf{y}^*_g = \mathbf{y}_g + \mathbf{F}(\mathbf{y}_{a,g} - \mathbf{C}\mathbf{y}_g), \tag{17}$$

with $\mathbf{F} = \mathbf{V}_g\mathbf{C}'(\mathbf{C}\mathbf{V}_g\mathbf{C}')^-$, where the superscript $^-$ denotes Moore-Penrose inverse[28], and

$$Cov(\mathbf{y}^*_g - \tilde{\mathbf{y}}_g | \mathbf{y}_g) = (\mathbf{I}_{Mn} - \mathbf{F}\mathbf{C})\mathbf{V}_g.$$

[26] By assuming $\varepsilon_{1-p,g} = \ldots = \varepsilon_{0,g} = \mathbf{0}$.

[27] Guerrero and Nieto (1999, p. 464) remark that the estimator (17) is also BLUE ($\mathbf{y}^*_g$ is unbiased given $\mathbf{y}_g$), so that $\tilde{\mathbf{y}}_g$ can be estimated once $\mathbf{\Pi}$, $\mathbf{P}$ and $\mathbf{\Sigma}$ are known.

[28] As shown in section 3, it is possible to express the benchmarking formula in terms of the r linear independent constraints, which prevents from using the generalized inverse (see Di Fonzo and Marini, 2006).

5.1 Feasible benchmarked estimates

A two-stage operational procedure is proposed to validate the model (16) through the available data and to obtain a feasible expression for $\mathbf{y}_g^*$.

To begin with, a vector $\mathbf{y}_g$ of observed preliminary series corresponding to $\tilde{\mathbf{y}}_g$ is needed[29]. In the first step, a VAR(p) model is estimated for the preliminary series $\mathbf{y}_g$, with p chosen according to some reasonable statistical criterion (e.g., AIC, SC, HQ). An estimate of $\mathbf{\Pi}$, say $\hat{\mathbf{\Pi}}$, can thus be obtained through OLS equation by equation. Furthermore, through relationship (17), while assuming $\mathbf{P} = \mathbf{I}_n$, we obtain a tentative benchmarked series $\hat{\mathbf{y}}_g^*$ in line with all constraints.

A test on whiteness of the residual series $\hat{\mathbf{\Pi}}(\hat{\mathbf{y}}_g^* - \mathbf{y}_g) = \hat{\varepsilon}_g$ is then required to verify whether the assumption $\mathbf{P} = \mathbf{I}_n$ is supported by the data. If it is not, then another VAR model is considered for the residuals $\hat{\varepsilon}_g$. This model can be expressed as

$$\mathbf{\Lambda}(\hat{\mathbf{y}}_g^* - \mathbf{y}_g) = (\mathbf{Q} \otimes \mathbf{I}_M)\mathbf{\Pi}(\hat{\mathbf{y}}_g^* - \mathbf{y}_g) = \mathbf{e}_g, \tag{18}$$

where $\mathbf{Q}$ is a non-singular matrix such that $\mathbf{QPQ}' = \mathbf{I}_n$. After this transformation we have

$$E(\mathbf{e}_g|\mathbf{y}_g) = \mathbf{0} \quad \text{and} \quad E(\mathbf{e}_g\mathbf{e}_g'|\mathbf{y}_g) = (\mathbf{Q} \otimes \mathbf{I}_M)(\mathbf{P} \otimes \mathbf{\Sigma})(\mathbf{Q}' \otimes \mathbf{I}_M) = \mathbf{I}_n \otimes \mathbf{\Sigma}.$$

The coefficients in $\mathbf{\Lambda}$ can be estimated through OLS, and matrix $\mathbf{\Sigma}$ can be consistently estimated by the residuals of model (18) as $\hat{\mathbf{\Sigma}} = \frac{1}{n-p'}\sum_{t=1}^{n}\hat{\mathbf{e}}_t\hat{\mathbf{e}}_t'$, where p' denotes the number of parameters in each equation of the VAR model (18).

Now, we can calculate a new estimate of $\mathbf{V}_g$, a new smoothing matrix $\mathbf{F}$ and a new benchmarked vector $\hat{\hat{\mathbf{y}}}_g^*$ by using the equivalence relationship

$$\hat{\mathbf{V}}_g = \left(\hat{\mathbf{\Pi}}^{-1}\right)\left(\hat{\mathbf{\Pi}} \otimes \hat{\mathbf{\Sigma}}\right)\left(\hat{\mathbf{\Pi}}^{-1}\right)' = \left(\hat{\mathbf{\Lambda}}^{-1}\right)\left(\mathbf{I}_n \otimes \hat{\mathbf{\Sigma}}\right)\left(\hat{\mathbf{\Lambda}}^{-1}\right)'.$$

Guerrero and Nieto (1999) propose also a discrepancy measure to validate empirically the 'compatibility' between the benchmarked estimates $\hat{\hat{\mathbf{y}}}_g^*$ and $\mathbf{y}_g$, in other words, to test the maintained hypothesis that they share essentially the same VAR model. Assuming the normality of $\{\varepsilon_g|\mathbf{y}_g\}$, it follows that

$$\mathbf{y}_{a,g} - \mathbf{C}\mathbf{y}_g = \mathbf{C}\mathbf{\Pi}^{-1}\varepsilon_q \simeq N(\mathbf{0}, \mathbf{C}\mathbf{V}_g\mathbf{C}').$$

Thus, when $\mathbf{\Pi}$, $\mathbf{P}$ and $\mathbf{\Sigma}$ are estimated, the Wald statistic

$$DM = (\mathbf{y}_{a,g} - \mathbf{C}\mathbf{y}_g)'[\mathbf{C}\mathbf{V}_g\mathbf{C}'](\mathbf{y}_{a,g} - \mathbf{C}\mathbf{y}_g) \tag{19}$$

is asymptotically distributed as a χ^2_r, with $r = rank(\mathbf{C}\mathbf{V}_g\mathbf{C}')$. When DM rejects the compatibility assumption, other preliminary series should be found. Guerrero and Nieto (1999) suggest to compute the compatibility test also when the residuals show evidence of non-whiteness at the first stage, that is between $\hat{\mathbf{y}}_g^*$ and $\mathbf{y}_g$: the rejection of the hypothesis should be taken as an evidence that the second stage is appropriate.

[29] Guerrero and Nieto suggest to derive such series by means of related indicators using the univariate approach proposed in Guerrero (1990). In our opinion, this is not a crucial assumption, provided the disaggregation method is statistically well-founded.

6 Applications on simulated and real-life data

In this section we present some results obtained in applying the benchmarking techniques described in the paper. First, we set up a Monte Carlo experiment to assess their accuracy in reproducing a simulated set of time series fulfilling both temporal and contemporaneous constraints. Then, we consider an application on economic time series where neither temporal nor contemporaneous constraints are fulfilled by a set of preliminary series. In this latter case the evaluation is performed by comparing the amount of adjustment made to the rates of changes of the preliminary series.

6.1 A simulation exercise

Following Guerrero and Nieto (1999), we generate quarterly bivariate series according to the restricted VAR(2) model

$$\begin{array}{rcl} \tilde{y}_{1,t} & = & 0.02 + 0.5\tilde{y}_{1,t-1} + \tilde{a}_{1,t} \\ \tilde{y}_{2,t} & = & 0.03 + 0.4\tilde{y}_{1,t-1} + 0.5\tilde{y}_{2,t-1} + 0.25\tilde{y}_{1,t-2} + \tilde{a}_{2,t} \end{array} \tag{20}$$

with $\mathbf{\Sigma} = E(\tilde{\mathbf{a}}_t\tilde{\mathbf{a}}_t') = \begin{pmatrix} 0.04 & 0 \\ 0 & 0.01 \end{pmatrix}$ and $\tilde{\mathbf{a}}_t = (\tilde{a}_{1,t}, \tilde{a}_{2,t})'$.

The bivariate variable $\tilde{\mathbf{y}}_t = (\tilde{y}_{1,t}\ \tilde{y}_{2,t})'$ is the objective series of our exercise. To generate it we operate as follows. We extract a number from $U[1; 10,000]$ and fix it as the common seed in the procedure used to generate the normal random numbers[30]. This allows to reduce the distance between objective and preliminary series. Then, we generate the bivariate disturbance series $\mathbf{a}_t$ with covariance matrix $\mathbf{\Sigma}$, for $t = 1, \ldots, 88$ and calculate recursively the relationships in model (20), assuming $\tilde{y}_{1,0} = \tilde{y}_{1,-1} = \tilde{y}_{2,0} = 0$ as starting conditions of the VAR. The initial 44 observations are discarded from each series to cancel the effects of the starting conditions. Finally, we derive the annual and contemporaneous constraints $\mathbf{y}_{0t}$ and $\mathbf{z}_t$ by aggregating $\tilde{\mathbf{y}}_t$. The same steps are performed to simulate another bivariate series $\mathbf{y}_t = (y_{1,t}\ y_{2,t})'$, which differs from $\tilde{\mathbf{y}}_t$ only for the usage of the covariance matrix $\mathbf{\Psi} = E(\mathbf{a}_t\mathbf{a}_t') = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.02 \end{pmatrix}$. This introduces a larger volatility in $\mathbf{y}_t$, and in so far it can be considered as a preliminary version of $\tilde{\mathbf{y}}_t$.

The benchmarking techniques by Guerrero and Nieto and the extension of Denton's univariate PFD approach (hereafter denoted by GN and PFD, respectively) are used to reconcile the preliminary series $\mathbf{y}_t$ to the totals $\mathbf{y}_{0t}$ and $\mathbf{z}_t$. Their performances are evaluated by calculating simple descriptive statistics on relative and absolute discrepancies $\mathbf{y}_t^* - \mathbf{y}_t$, $t = 1, \ldots, 44$, where $\mathbf{y}_t^*$ is the benchmarked series.

We replicate the experiment 1,000 times and calculate average and standard deviation of the discrepancies statistics, as summarized in Table 1. Since we are dealing with a bivariate system, the amount of adjustment is symmetric in the variables: only the corrections made to $y_{1,t}$ are thus shown in the table.

[30] We use proc *rndn* of program GAUSS, version 6.

Table 1: Discrepancies statistics for series $y_{1,t}$. Simulated data with uncorrelated disturbances ($\mathbf{\Sigma}$ and $\mathbf{\Psi}$).

	relative discrepancies				*discrepancies*	
	median	min	max	range	mean	std
PFD						
average	0.0013	-0.0664	0.0713	0.1377	0.0154	0.0169
std. dev.	0.0014	0.0289	0.0386	0.0628	0.0050	0.0073
GN						
average	0.0001	-0.0412	0.0414	0.0826	0.0149	0.0112
std. dev.	0.0020	0.0096	0.0092	0.0160	0.0022	0.0018

PFD: Multivariate Proportional Denton First Differences
GN: Guerrero and Nieto

The minimum and maximum corrections are both larger for PFD: the range is 0.1377 against 0.0826 obtained with GN. Moreover, it can be noted that the results by GN are more stable than those by PFD (the standard deviations of the range are equal to 0.0160 and 0.0628, respectively). On average, the corrections are very close to each other, but the greater stability of the results by GN is confirmed.

The same exercise is performed again with non-diagonal covariance matrices. We use $\mathbf{\Sigma}^* = \begin{pmatrix} 0.04 & 0.005 \\ 0.005 & 0.01 \end{pmatrix}$ and $\mathbf{\Psi}^* = \begin{pmatrix} 0.05 & 0.005 \\ 0.005 & 0.02 \end{pmatrix}$, that is, we introduce a positive correlation in the disturbances, for both the objective and the preliminary series. In this case it is reasonable to expect an improvement of the performance by GN over PFD: for, the data-based method should be able to recognize some correlation pattern in the disturbance process and to exploit it in the benchmarking formula, whereas the Denton's solution always mechanically applies the same distribution scheme of the discrepancies without considering the properties of the data at hand.

Table 2 confirms the expectations: the range of corrections by GN is even reduced with respect to the case of a diagonal covariance matrix (0.0577), while PFD sensibly augments the level of correction in the benchmarked series (0.2059). The average correction increases to 0.0193 for PFD and decreases to 0.0107 for GN.

6.2 Benchmarking monthly estimates of Italian industrial value added

The second exercise is an application of benchmarking techniques to reconcile monthly estimates of Italian industrial value added. The official estimates are issued by ISTAT at quarterly frequency in the framework of quarterly national accounts. We obtain tentative estimates of value added at monthly level by using industrial production indices; in fact, the sum of these estimates will sum up neither to the official quarterly series nor to a "fictitious" monthly contemporaneous constraint. PFD and GN procedures are thus used, and their performances are evaluated through the amount of adjustment done to the preliminary series.

Table 2: Discrepancies statistics for series $y_{1,t}$. Simulated data with correlated disturbances ($\mathbf{\Sigma}^*$ and $\mathbf{\Psi}^*$).

	relative discrepancies				*discrepancies*	
	median	min	max	range	mean	std
PFD						
average	-0.0010	-0.0960	0.1098	0.2059	0.0193	0.0238
std. dev.	0.0018	0.0778	0.0928	0.1659	0.0071	0.0163
GN						
average	-0.0002	-0.0299	0.0278	0.0577	0.0107	0.0080
std. dev.	0.0009	0.0067	0.0048	0.0085	0.0013	0.0011

PFD: Multivariate Proportional Denton First Differences
GN: Guerrero and Nieto

We consider the quarterly value added (at constant prices and seasonally adjusted) for six sectors of the manufacturing activity[31] and we use the monthly industrial production indices to derive monthly preliminary estimates and a monthly accounting constraint in the following way.Each quarterly series of value added is regressed on a constant, a trend and its own production index. The estimated coefficients are applied at monthly level to obtain the preliminary disaggregated figures for each sector. Since the regressions do not consider any temporal constraint, the monthly sum of these estimates does not comply with the quartlerly value added (the temporal discrepancies (in %) are shown for each variable in Figure 1). The total quarterly value added for the six sectors is monthly disaggregated using as related series the corresponding production index, derived as a weighted average of the sectoral indices. The Chow and Lin (1971) procedure with AR(1) disturbances is employed to this purpose. The resulting series represents the vector of contemporaneously aggregated data, that is $\mathbf{z}$. In Figure 2 the percentage discrepancies between the sum of the preliminary series and $\mathbf{z}$ are shown.

The practical application of the two-step GN data-based procedure we described in section 5 starts by estimating at different lag lengths a VAR model (with constant term) for the preliminary series $\mathbf{y}$. The choice of the lag length is made on the basis of various information criteria (AIC, SC and HQ) [32]: an unrestricted VAR(1) model is deemed adequate for the preliminary series.

We obtain a tentative benchmarked time series $\hat{\mathbf{y}}^*$ fulfilling both temporal and contemporaneous restrictions and validate empirically the compatibility between $\hat{\mathbf{y}}^*$ and $\mathbf{y}$ by considering the DM statistic as defined in (19). While the obtained value is not significant ($DM = 0.38$), the Ljung-Box tests on the estimated residuals show evidence of autocorrelation, supporting the choice of performing the second stage of the procedure.

[31] Manufacturing of food products, textiles, chemicals, basic metals, machinery, and electrical equipment (DA, DB, DG, DJ, DK, and DL in the NACE rev. 11 classification).

[32] Detailed results on lag length selection and estimated coefficients of the VAR can be found in Di Fonzo and Marini (2006).

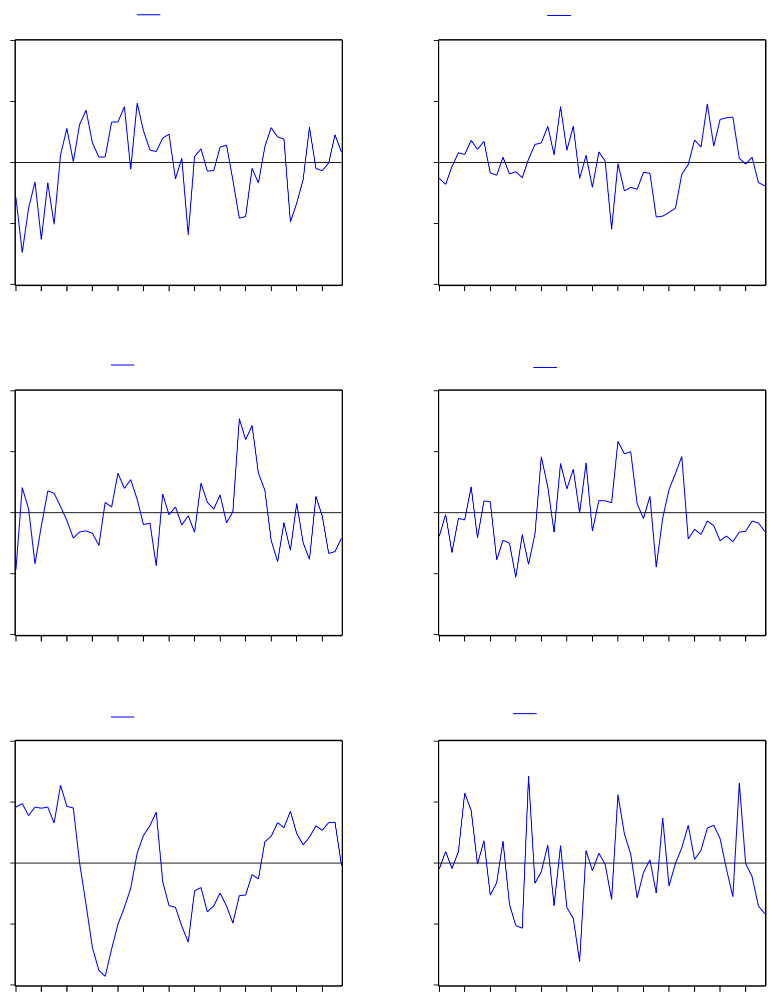

Figure 1: Benchmarking Italian monthly industrial value added. Temporal discrepancies (%) before reconciliation (quarterly).

An unrestricted VAR for the differences between the tentative disaggregated and the preliminary series, $(\hat{\mathbf{y}}^* - \mathbf{y})$, is thus estimated. In this case the selection of the lag length is not uniform among the criteria: in line with a parsimony principle, we decided to follow the indication of SC, thus a VAR(3) model has been fitted to the discrepancies $(\hat{\mathbf{y}}^* - \mathbf{y})$. The resulting estimates are used in (17) to derive the new benchmarked series $\hat{\hat{\mathbf{y}}}^*$, for which the Wald test does not reject the compatibility assumption ($DM = 0.14$), and no residual autocorrelation is detected now. According to the nature of the considered procedure, $\hat{\hat{\mathbf{y}}}^*$ is in line with all the restrictions of the problem and maintains as much as possible the autocorrelation structure of the preliminary series $\mathbf{y}$.

Finally, we use also Denton's PFD to make a comparison with a mechanical solution. In table 3 some descriptive statistics of the adjustments made to the

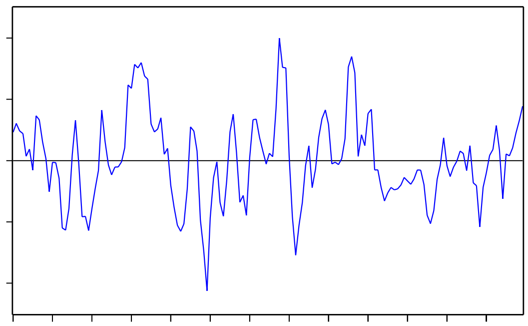

Figure 2: Benchmarking Italian monthly industrial value added. Contemporaneous discrepancies (%) before reconciliation (monthly).

preliminary monthly rates of changes are shown. The series showing the largest corrections in both cases is $\mathbf{y}_6$, which is also the most variable series: GN benchmarking produces adjustments between a maximum of 8.87% and a minimum of -6.33% (8.19% and -6.17% for PFD). Smaller adjustments by PFD are also obtained for $\mathbf{y}_5$ (5.45% against 5.55%). The results are opposite for the remaining four series: the range of the adjustment made by GN is 8.19% for $\mathbf{y}_1$ (8.59% for PFD), 6.92% for $\mathbf{y}_2$ (7.23% for PFD), 7.29% for $\mathbf{y}_3$ (7.68% for PFD), and 6.37% for $\mathbf{y}_4$ (7.76% for PFD).

Table 3: Benchmarking Italian monthly industrial value added. Performance indicators (corrections to monthly rates of change).

	PFD					GN				
sect.	*med*	*min*	*max*	*range*	*std*	*med*	*min*	*max*	*range*	*std*
1	0.00	-4.60	3.99	8.59	1.51	-0.11	-4.21	3.98	8.19	1.56
2	0.01	-3.61	3.62	7.23	1.16	0.04	-3.53	3.39	6.92	1.16
3	-0.07	-2.68	5.00	7.68	1.30	-0.16	-3.07	4.22	7.29	1.36
4	0.11	-4.10	3.66	7.76	1.53	0.03	-3.36	3.01	6.37	1.45
5	-0.03	-3.09	2.36	5.45	0.98	0.06	-2.58	2.97	5.55	0.86
6	-0.04	-6.17	8.19	14.36	1.99	-0.17	-6.33	8.87	15.19	2.72

PFD: Multivariate Proportional Denton First Differences

GN: Guerrero and Nieto

7 Conclusion

In this paper we have discussed the reconciliation of high-frequency time series subject to contemporaneous and temporal constraints in the framework of a constrained matrix problem. We have presented some general features of the problem and discussed the solutions given by (i) a mathematical/mechanical optimization procedure based on a movement preservation principle, and (ii) a data-based benchmarking procedure which exploits the autoregressive features of the time series to be reconciled.

The attention has been focused on the problem of reconciliating a table of one-way classified time series (e.g., quarterly regional time series) with both temporal (e.g., annual regional time series) and contemporaneous (e.g, the quarterly national grand-total) exogenous constraints. In order to avoid cumbersome expressions, we did not present some other cases which are of interest in practical applications: (i) benchmarking a table of one-way classified time series when the contemporaneous constraint is endogenous instead of externally given (e.g., the 'true' quarterly national grand-total is not available, while possibly a preliminary estimate has to be adjusted in line with the accounting constraints); (ii) benchmarking a table of two-way classified time series when the contemporaneous constraints are either endogenous or exogenous. First results on these issues can be found in Di Fonzo and Marini (2003).

The extension of Denton's univariate benchmarking procedure is grounded on a rather simple and appreciable principle, according to which the reconciled dynamic profiles have to be as close as possible to those shown by the preliminary series. The proportional variant of the method is generally preferred to the additive variant, particularly when the series to be reconciled have different magnitude, in order to avoid relatively large corrections to small aggregates, and the consequent risk of having unpleasant negative figures. However, the proportional adjustment of a system of time series mostly alters those component series having greater magnitude (Di Fonzo and Marini, 2003), a result which sometimes could be counter-intuitive (e.g., if one thinks that the most reliable series of a survey are generally the greater ones, and viceversa).

An attempt to overcome this kind of problem could be made by considering benchmarking according to a movement preservation principle explicitly referred to the growth rates. In this case, the criterion to be minimised would be

$$\sum_{j=1}^{M}\left[\sum_{t=2}^{n}\left(\frac{y^*_{j,t}-y^*_{j,t-1}}{y^*_{j,t-1}}-\frac{y_{j,t}-y_{j,t-1}}{y_{j,t-1}}\right)^2\right] \equiv \sum_{j=1}^{M}\left[\sum_{t=2}^{n}\left(\frac{y^*_{j,t}}{y^*_{j,t-1}}-\frac{y_{j,t}}{y_{j,t-1}}\right)^2\right].$$

Some authors (Helfand *et al.*, 1977; Bozik and Otto, 1988; Bloem *et al.*, 2001) consider this criterion as 'the ideal objective formulation' (Bloem *et al.*, 2001, p. 100), but it is not generally pursued[33] because of the inherent nonlinearity of the problem and because the proportional variant of Denton's procedure has been generally considered a good approximation (Helfand *et al.*, 1977). Di Fonzo and Marini

[33] An exception is Bozik and Otto (1988).

(2003) present a solution to benchmarking a table of time series based on a log-transformation which extends a result used in temporal disaggregation of a single time series (Salazar *et al.*, 2004; Aadland, 2000; Di Fonzo, 2003).

However expressed, though reasonable, Denton's 'closeness condition' is let down upon all the series, whatever characteristics each variable possesses, without any link with the relative accuracy of the data to be reconciled. Nevertheless, the issue of recovering measures of reliability from time series variables observed with noise is not straightforward. In line with the contributions by Weale (1992), Solomou and Weale (1993) and Smith *et al.* (1998), the data-based procedure by Guerrero and Nieto is intended to extract information about the variability of the series making use of the observed data, in order to get an estimate of the covariance matrix to be used in the least-squares adjustment according to Stone *et al.* (1942). Another interesting feature of this procedure lies on the opportunity of evaluating the compatibility of the preliminary series by means of relatively simple statistical tools, which could help in stressing possible deficiencies in (part of) the information basis.

We finally presented some results obtained in applying the benchmarking techniques described in the paper to simulated and real-world data. First, we set up a Monte Carlo experiment to assess their accuracy in reproducing a simulated set of time series fulfilling both temporal and contemporaneous constraints. Then, we considered real-life economic time series where neither temporal nor contemporaneous constraints were fulfilled by a set of preliminary series. From the simulation exercise it resulted that the data-based benchmarking technique can outperform mechanical solutions which do not take into account the properties of the data, particularly when there is contemporaneous correlation between the series. The evidence coming from the application on real-life data is less uniform. Nevertheless, it seems to confirm the ability of the data-based benchmarking procedure to induce smaller alterations to the dynamic profile of the preliminary estimates than those generated by the PFD procedure (this holds true for 4 of 6 series forming the system). In so far a larger volatility of the preliminary series is a signal of lower quality, the data-based benchmarked procedure seems to take into account this characteristics, producing relatively larger corrections to such series than those registered by the other, relatively less variable, series. However, our implementation of the procedure is still rather raw, given that modelization in levels of an unconstrained VAR has been kept into consideration. Further research will necessarily provide insights on better specified models, both in terms of restricting the VAR specification and possibly by working with log and/or differenced time series.

References

Aadland, D. (2000): "Distribution and interpolation using transformed data," *Journal of Applied Statistics*, 27, 141–156.

Almon, C. (1968): "Recent methodological advances in input-output in the United States and Canada," paper presented at the Fourth International Conference on Input-Output Techniques, Geneva.

Antonello, P. (1990): "Simultaneous balancing of input-output tables at current and constant prices with first order vector autocorrelated errors," *Economic Systems Research*, 2, 157–171.

Arkhipoff, O. (1995): "A "neglected problem" revisited. The consistency, balancing and reliability of National Accounts," in *Annali di Statistica, 124, serie X*. Istat, Rome.

Bacharach, M. (1970): *Biproportional matrices and input-output change*. Cambridge University Press, Cambridge.

Barker, T., F. van der Ploeg, and M. Weale (1984): "A balanced system of national accounts for the United Kingdom," *The Review of Income and Wealth*, 30, 461–485.

Bloem, A., R. Dippelsman, and N. Mæhle (2001): *Quarterly National Accounts Manual. Concepts, Data Sources, and Compilation*. International Monetary Fund, Washington DC.

Bozik, J., and M. Otto (1988): "Benchmarking: Evaluating methods that preserve month-to-month changes," Bureau of the Census - Statistical Research Division, CENSUS/SRD/RR-88/07.

Bregman, L. (1967): "Proof of convergence of Sheleikhovskii's method for a problem with transportation constraints," *USSR Computational Mathematics and Mathematical Physics*, 1, 191–204.

Byron, R. (1978): "The estimation of large social accounts matrices," *Journal of the Royal Statistical Society A*, 141, 359–367.

Cabrer, B., and J. Pavía (1999): "Estimating M (¿1) quarterly time series in fulfilling annual and quarterly constraints," *International Advances in Economic Research*, 5, 339–349.

Chen, Z., and E. Dagum (1997): "A recursive method for predicting variables with temporal and contemporaneous constraints," American Statistical Association, Proceedings of the Business and Economic Statistics Section: 229-233.

Cholette, P. (1987): "Concepts, definitions and principles of benchmarking and interpolation of time series," Statistics Canada, Time Series Research and Analysis Division, Working Paper No TSRA-87-014E.

——— (1988): "Benchmarking systems of socio-economic time series," Statistics Canada, Time Series Research and Analysis Division, Working Paper No TSRA-88-017E.

de Mesnard, L. (1994): "Unicity of biproportion," *SIAM Journal on Matrix Analysis and Applications*, 15, 490–495.

Deming, W., and F. Stephan (1940): "On a least-squares adjustment of a sampled frequency table when the expected marginal totals are known," *Annals of Mathematical Statistics*, 11, 427–444.

Denton, F. (1971): "Adjustment of monthly or quarterly series to annual totals: An approach based on quadratic minimization," *Journal of the American Statistical Association*, 66, 99–102.

Di Fonzo, T. (1990): "The estimation of M disaggregated time series when contemporaneous and temporal aggregates are known," *The Review of Economics and Statistics*, 72, 178–182.

——— (2002): "Temporal disaggregation of a system of time series when the aggregate is known. Optimal vs. adjustment methods," Workshop on Quarterly National Accounts, Eurostat, Theme 2 Economy and finance: 63-77.

——— (2003): "Temporal disaggregation using related series: log-transformation and dynamic extensions," *Rivista Internazionale di Scienze Economiche e Commerciali*, 50, 371–400.

Di Fonzo, T., and M. Marini (2003): "Benchmarking systems of seasonally adjusted time series according to Denton's movement preservation principle," Dipartimento di Scienze Statistiche, Università di Padova, working paper 2003.9.

——— (2006): "Benchmarking a system of time series: Denton's movement preservation principle vs. a data based procedure," Eurostat, Working Paper 2006 No KS-DT-05-008-EN.

Eurostat (1999): *Handbook of quarterly national accounts.* European Commission, Luxembourg.

Friedlander, D. (1961): "A technique for estimating a contingency table, given the marginal totals and some supplementary data," *Journal of the Royal Statistical Society A*, 124, 412–420.

Gilchrist, D., and L. St. Louis (1999): "Completing input-output tables using partial information, with an application to Canadian data," *Economic Systems Research*, 11, 185–193.

Guerrero, V. (1990): "Temporal disaggregation of time series: an ARIMA-based approach," *International Statistical Review*, 58, 29–46.

Guerrero, V., and F. Nieto (1999): "Temporal and contemporaneous disaggregation of multiple economic time series," *TEST*, 8, 459–489.

Guerrero, V., and D. Peña (2000): "Linear combination of restrictions and forecasts in time series analysis," *Journal of Forecasting*, 19, 103–122.

Harrigan, F. (1990): "The reconciliation of inconsistent economic data: the information gain," *Economic Systems Research*, 2, 17–25.

Harrigan, F., and I. McNicoll (1986): "Data use and the simulation of regional input-output matrices," *Environment and Planning A*, 18, 1061–1076.

Helfand, S., N. Monsour, and M. Trager (1977): "Historical revision of current business survey estimates," American Statistical Association, Proceedings of the Business and Economic Statistics Section: 246-250.

Jackson, R., and A. Murray (2004): "Alternative input-output matrix updating formulations," *Economic Systems Research*, 16, 135–148.

Kruithof, R. (1937): "Telefoonverkeersrekening," *De Ingenieur*, 52, E15–E25.

Lahr, M., and L. de Mesnard (2004): "Biproportional techniques in input-output analysis: table updating and structural analysis," *Economic Systems Research*, 16, 115–134.

Pedullà, G. (1995): "Recent developments in Italian national accounts: the influence of Richard Stone in social statistics, national accounts and economic analysis," in *Annali di Statistica, 124, serie X*. Istat, Rome.

Robinson, S., A. Cattaneo, and A. El-Said (2001): "Updating and estimating a social accounting matrix using cross entropy methods," *Economic Systems Research*, 13, 47–64.

Rossi, N. (1982): "A note on the estimation of disaggregate time series when the aggregate is known," *The Review of Economics and Statistics*, 64, 695–696.

Salazar, E., R. Smith, and M. Weale (2004): "Interpolation using a Dynamic Regression Model: Specification and Monte Carlo Properties," NIESR Discussion Paper n. 126.

Schneider, M., and S. Zenios (1990): "A comparative study of algorithms for matrix balancing," *Operations Research*, 38, 439–455.

Sefton, J., and M. Weale (1995): *Reconciliation of national income and expenditure. Balanced estimates of national income for the United Kingdom, 1920-1990.* Cambridge University Press, Cambridge.

Smith, R., M. Weale, and S. Satchell (1998): "Measurement error with accounting constraints: point and interval estimation for latent data with an application to U.K. gross domestic product," *Review of Economic Studies*, 65, 109–134.

Solomou, S., and M. Weale (1993): "Balanced estimates of National Accounts when measurement errors are autocorrelated: the UK, 1920-38," *Journal of the Royal Statistical Society A*, 156, 89–105.

Stone, R. (1961): *Input-Output and National Accounts*. OECD, Paris.

——— (1990): "Adjusting the national accounts," in *Nuova Contabilità Nazionale, Annali di Statistica, serie IX*, ed. by Istat, pp. 311–325. Istat, Rome.

Stone, R., D. Champernowne, and J. Meade (1942): "The precision of national income estimates," *Review of Economic Studies*, 9, 111–125.

Theil, H. (1967): *Economics and information theory*. North-Holland, Amsterdam.

Toh, M.-H. (1998): "The RAS approach to updating input-output matrices: an instrumental variables interpretation and analysis of structural change," *Economic Systems Research*, 10, 63–78.

Uribe, P., C. de Leeuw, and H. Theil (1965): "The information theoretic approach to the prediction of interregional trade flows," Rotterdam, Econometric Institute of the Netherlands School of Economics, Report 6507.

van der Ploeg, F. (1982): "Reliability and the adjustment of sequences of large economic accounting matrices," *Journal of the Royal Statistical Society A*, 145, 169–194.

——— (1984): "Generalized least squares methods for balancing large systems and tables of national accounts," *Review of Public Data Use*, 12, 17–33.

——— (1985): "Econometrics and inconsistencies in the national accounts," *Economic Modelling*, 2, 8–16.

Weale, M. (1985): "Testing linear hypotheses on national accounts data," *The Review of Economics and Statistics*, 67, 685–689.

——— (1988): "The reconciliation of values, volumes and prices in the National Accounts," *Journal of the Royal Statistical Society A*, 151, 211–221.

——— (1992): "Estimation of data measured with error and subject to linear restrictions," *Journal of Applied Econometrics*, 7, 167–174.

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