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Contents

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1 Introduction

The huge amount of work in financial time series has led to a general consensus of the scientific community about some empirical statistical features known as *stylized facts* (i.e., positive correlation among square or absolute returns, conditional heteroskedasticity, clustering effects, leptokurtosis of return distributions) which have been thoroughly investigated.

On the contrary, skewness in marginal and conditional return distributions has been quite neglected and relatively little work has been done to detect it. As a consequence, the occurrence of skewness, both unconditional and conditional, is still disputable and the empirical findings are not univocal. While some authors found, assumed or declared significant asymmetries in return distributions (e.g. Engle and Patton, 2001, Cont, 2001, Chen et al., 2001, Hueng and McDonald, 2005, and Silvennoinen et al., 2005), others (e.g. Peiró, 2004, Kim and White, 2004, Lisi, 2005, and Premaratne and Bera, 2005) are more doubtful about the pervasive presence of skewness in returns.

The existence (or lack) of both unconditional and conditional symmetry is important in a number of situations relevant to both economic and statistical contexts. From a financial perspective, skewness is crucial since it may itself be considered as a measure of risk. For example, Kim and White (2004) stressed that, if investors prefer right-skewed portfolios then, for equal variance, one should expect a "skew premium" to reward investors willing to invest in left-skewed portfolios. With respect to optimal portfolio allocation, Chunhachinda et al. (1997) showed that allocation can change considerably if higher than second moments are considered in selection. Along the same lines, Jondeau and Rockinger (2004) measured the advantages of using a strategy based on higher-order moments. With respect to option pricing problems, Corrado and Su (1997) attributed the anomaly known as "volatility skew" in option pricing to the skewness and kurtosis of the return distribution. Kalimipalli and Sivakumar (2003) studied whether one can trade profitably in the option market using time-varying skewness in the underlying asset returns, finding that strategies based on skewness forecasts are profitable in out-of-sample experiments.

In the context of hedge funds, some authors showed that the funds exhibit option-like features in their returns and have significant left-tail risk (Fung and Hsieh, 2001, Mitchell and Pulvino, 2001). Kat and Miffre (2006) found that systematic kurtosis and skewness risks are the two main drivers of hedge fund performance. The role played by skewness in risk management is also described by Rosenberg and Schuermann (2006) and in general it is reasonable to expect that, when skewness is present, accounting for it may lead to better estimation of risk measures such as Value-at-Risk or Expected Shortfall.

Several economic theories have been offered as an explanation of the the mechanism generating the asymmetry, including leverage effects (Black, 1976, Christie, 1982), the volatility feedback mechanism (Campbell and Hentschel, 1992), stochastic bubbles models (Blanchard and Watson, 1982) and investor heterogeneity (Hong and Stein, 2003).

On the other hand, the interest in possible asymmetries is motivated also by statistical reasons. For example, often estimation procedures assume conditional symmetry and thus a proper evaluation of this assumption may be advisable. In particular, Newey and Steigerwald (1997) showed that consistent estimation of the GARCH parameters can be obtained by QMLE if both the true and the assumed innovation densities are symmetric around zero and unimodal. When conditional symmetry does not hold, an additional parameter is necessary to identify the location of the innovation distribution. The assumption of conditional symmetry is also commonly used in adaptive estimation.

It should also be noted that a good modeling of the conditional distribution may be crucial in any dynamic analysis such as dynamic optimal portfolio allocation or Value-at-Risk estimation.

An important statistical issue is the relation between unconditional and conditional skewness. It is well known that asymmetry of marginal and conditional distributions do not necessarily coincide. Referring to the coefficient of skewness given by the standardized third moment, Engle and Gonzáles-Rivera (1991) show that unconditional skewness is greater or equal than the conditional one. Their proof assumes constant conditional skewness but, as we will show in this paper, this is not always the case.

Within this context, we first analyze the statistical significance of unconditional and conditional skewness in order to assess whether asymmetry is a widespread characteristic of financial returns. In our analysis we consider nine time series of stock index returns for which marginal symmetry is investigated by means of a suitable test. Then, for the same series, conditional skewness is studied using tests and a non-Gaussian GARCH-type model. In both steps, skewness is assumed to be constant. The possibility of conditional time varying skewness is introduced in a third step, through a generalization of the previous GARCH-type representation, that allows to dynamically model conditional variance, skewness and kurtosis. Although other models with dynamic conditional skewness and kurtosis have been studied in the literature (Hansen, 1994, Harvey and Siddique, 1999, Brooks et al., 2005), the specific form of the proposed model is new. A second goal of this paper is to analyze the economic significance and the financial impact of a correct modeling of skewness. With this purpose Value-at-Risk, Expected Shortfall and connected capital requirements as defined by the second Basel Accord (Basel Committee on Banking Supervision, 1995, 1996) were considered for the stock index returns. The performances of different models were compared.

The paper is organized as follows. Section 2 reviews some tests for marginal and conditional skewness. Section 3 introduces a model which allows to study both constant and time-varying conditional skewness and kurtosis. Empirical evidences and statistical and economic significance of skewness are investigated in Section 4. Some concluding remarks are presented in Section 5.

2 Testing for skewness

The first step of our study consists in testing for unconditional skewness by means of the standardized third moment $S = \mu_3/\mu_2^{3/2}$, where μ_j is the $j-$th central moment.

In this context, it should first be noted that the standard asymptotic test based on the relationship $\sqrt{n}\,\hat{S} \xrightarrow{d} N(0,6)$ does not work correctly, either for dependent Gaussian or independent non-Gaussian data (Bai and Ng, 2005, Premaratne and Bera, 2005, and Lisi, 2005). In particular, for leptokurtic distributions this test strongly overestimates the asymmetry while the opposite happens when the distribution is platykurtic.

Bai and Ng (2005) proposed a test for unconditional skewness, based on the distribution of $\hat{S}$, that works properly also for dependent and non-Gaussian data. It is thus particularly useful for time series of financial returns. Let us denote by $y_1, \ldots, y_n$ the observed series. Under the hypothesis of symmetry, assuming the existence of the sixth moment and some mixing conditions, but without any assumption of independence or Gaussianity, it is shown that

$$\sqrt{n}\,\hat{S} \xrightarrow{d} N(0, V)\ , \tag{1}$$

where $V = \alpha\,\Gamma\,\alpha'/\sigma^6$, with $\sigma^2 = \mu_2$, $\alpha = [1, -3\,\sigma^2]$ and Γ defined as the 2×2 matrix given by $\lim_{n\to\infty} n\,\mathrm{E}(\bar{Z}\,\bar{Z}')$, with $\bar{Z}$ being the sample mean of

$$Z_t = \left[\begin{array}{c} (y_t-\mu)^3 \\ (y_t-\mu) \end{array}\right]\ .$$

In this framework, the serial dependence in the observed series $\{y_t\}$ is explained through Γ, which represents the spectral density matrix of Z_t at frequency 0. It is clear that relationship (1) allows to verify the hypothesis H_0: $S=0$ by means of a normal test.

Bai and Ng (2001) proposed an asymptotically distribution free test for conditional symmetry in time series models. In particular, conditional symmetry is tested by considering the empirical distribution function of the estimated residuals $\{\hat{\varepsilon}_t\}$ from the general model

$$y_t = \mu_t(I_{t-1}, \theta) + \sigma_t(I_{t-1}, \theta)\ \varepsilon_t, \tag{2}$$

where θ represents a suitable set of parameters, I_{t-1} is the information up to time $t-1$, μ_t is the conditional mean of y_t, σ_t^2 is the conditional variance and ε_t is a zero mean and unit variance disturbance, independent of the elements of I_{t-1}. Since conditional symmetry of y_t is equivalent to symmetry of ε_t about zero, the former may be studied by comparing the empirical distribution function of ε_t to that of $-\varepsilon_t$. To this end the standardized residuals $\hat{\varepsilon}_t = (y_t - \hat{\mu_t})/\hat{\sigma_t}$ are computed. Then, the statistic

$$CS = \max_y |S_n(y)| \tag{3}$$

is introduced, where

$$\begin{aligned} S_n(y) &= \hat{W}_n(y) - \hat{W}_n(0) + \int_y^0 h_n^-(x)\ dx\ , \qquad \text{if } y \le 0\ , \\ S_n(y) &= \hat{W}_n(y) - \hat{W}_n(0) - \int_0^y h_n^+(x)\ dx\ , \qquad \text{if } y > 0\ , \end{aligned}$$

with $\hat{W}_n(y) = n^{-1/2} \sum_{t=1}^n [I(\hat{\varepsilon}_t \le y) - I(-\hat{\varepsilon}_t \le y)]$ and

$$\begin{aligned} h_n^-(x) &= g_n(x)\, f_n(x) \left[\int_{-\infty}^x g_n(z)^2 f_n(z)\ dz \right]^{-1} \int_{-\infty}^x g_n(z)\ d\hat{W}_n(z)\ , \\ h_n^+(x) &= g_n(x)\, f_n(x) \left[\int_x^{\infty} g_n(z)^2 f_n(z)\ dz \right]^{-1} \int_x^{\infty} g_n(z)\ d\hat{W}_n(z)\ . \end{aligned}$$

Here f_n represents an estimate of the density f of ε_t, and g_n is an estimate of the ratio $\dot{f}/f$, with $\dot{f}$ being the derivative of f. Bai and Ng (2001) suggest to approximate the integrals by summations and to estimate the density and its derivative by kernel methods. In particular, they propose a Gaussian kernel with a bandwidth equal to $1.06\, n^{-1/5}$ times the standard error of ε_t. This choice minimizes the approximate mean integrated squared error of the density estimate. Under some technical assumptions, when conditional symmetry holds, $CS \xrightarrow{d} \max_{0 \le s \le 1} |B(s)|$, where $B(s)$ is a standard Brownian motion on $[0,1]$. The asymptotic critical values of the test at 1%, 5% and 10% levels of significance are 2.78, 2.21 and 1.91, respectively.

Note that the Bai and Ng (2001) test is not based on the standardized third moment. A test for conditional skewness based on S can be obtained by applying the Bai and Ng (2005) test (or the standard asymptotic test) to $\{\hat{\varepsilon}_t\}$ in model (2).

Obviously, for all tests which assume model (2), results depend crucially on a correct specification and estimation of μ_t and σ_t.

3 Models for skewness

Conditional skewness can also be assessed by using suitable models for asymmetric behavior. In this study we propose to analyze the presence of conditional skewness using a GARCH-type model with innovations having a Pearson's Type IV (henceforth Pearson$_{IV}$) distribution. This model represents a generalization of the standard GARCH model because it can account for asymmetry and kurtosis in the conditional distribution. Conditional skewness and kurtosis can be time-varying, thus allowing to study possible dynamics in higher-order moments. In the following, the acronym GARCHDSK (GARCH with dynamic skewness and kurtosis) will be used to denote this model.

Time varying skewness and kurtosis were first introduced by Hansen (1994), who extended the ARCH framework by proposing the adoption of a conditional generalized Student's t distribution, and modeling its parameters as functions of the lagged errors. Approaches in which dynamics are imposed on shape parameters, thus inducing time-varying skewness and kurtosis, have also been adopted, among others, by Jondeau and Rockinger (2003) and Yan (2005). In other cases, higher order moments are modeled directly. For example, Harvey and Siddique (1999) introduce a GARCH-type expression for the conditional skewness, while Brooks et al. (2005) use a similar representation for the kurtosis. León et al. (2005) employ a GARCH specification for both conditional skewness and kurtosis.

In the spirit of Hansen (1994), here dynamics on skewness and kurtosis are introduced by modeling shape parameters, rather than directly skewness and kurtosis. As remarked by Yan (2005), this approach is less computationally intensive and allows skewness and kurtosis to explode, while the shape parameters remain stationary. This is particularly useful when modeling extremal events.

Concerning the choice of the conditional distribution, in the present paper we follow Premaratne and Bera (2001) in the use of a Pearson$_{IV}$ distribution. This distribution is flexible, in the sense that it implies a wide range of feasible skewness-kurtosis couples. For example, the range associated with the Gram-Charlier density studied in Jondeau and Rockinger (2001) and adopted by León et al. (2005) is relatively rather limited (Yan, 2005). The Pearson$_{IV}$ is also found to approximate the generalized Student's t distribution on a large area of the skewness-kurtosis plane, but is computationally less demanding (see Premaratne and Bera, 2001, and the computational techniques discussed in Heinrich, 2004).

The GARCH-type model we will use to assess skewness has the following structure:

$$y_t = \mu_t + \varepsilon_t \ , \qquad t = 1, \dots, n \ , \tag{4}$$

where $\mu_t = E(y_t | I_{t-1})$, and ε_t is such that $\varepsilon_t \mid I_{t-1} \sim \text{Pearson}_{IV}(\lambda_t, a_t, \nu_t, r_t)$. Hence, the conditional density is defined by

$$f(\varepsilon_t \mid I_{t-1}) = C_t \left[1 + \left(\frac{\varepsilon_t - \lambda_t}{a_t}\right)^2\right]^{-(r_t+2)/2} \exp\left[-\nu_t \arctan\left(\frac{\varepsilon_t - \lambda_t}{a_t}\right)\right] \ . \tag{5}$$

Jointly, parameters λ_t, a_t, ν_t and r_t control the conditional mean, variance, skewness and kurtosis. The parameter C_t is a normalizing constant depending on a_t, ν_t and

r_t. The distribution is symmetric for $\nu_t = 0$, positively skewed for $\nu_t < 0$ and negatively skewed for $\nu_t > 0$. For fixed ν_t, increasing r_t decreases the kurtosis. The Pearson$_{IV}$ distribution is essentially a skewed version of the Student's t and for $\nu_t = 0$, $r_t = g_t - 1$ and $a_t = \sqrt{g_t}$ reduces to a Student's t with g_t degrees of freedom. The normal distribution is a limit case where $\nu_t = 0$ and $r_t \to \infty$.

Setting $\lambda_t = a_t\, \nu_t / r_t$ in order to have a zero mean error term, for the conditional distribution of ε_t we have

$$
\begin{aligned}
E(\varepsilon_t | I_{t-1}) &= 0\ , && (6)\\
\sigma_t^2 &= \mathrm{Var}(\varepsilon_t | I_{t-1}) = \frac{a_t^2\,\left(r_t^2 + \nu_t^2\right)}{r_t^2\,(r_t - 1)}\ , && (7)\\
S_t &= S(\varepsilon_t | I_{t-1}) = \frac{-4\,\nu_t}{r_t - 2}\sqrt{\frac{r_t - 1}{r_t^2 + \nu_t^2}}\ , && (8)\\
K_t &= K(\varepsilon_t | I_{t-1}) = \frac{3\,(r_t - 1)\left[(r_t + 6)\,(r_t^2 + \nu_t^2) - 8r_t^2\right]}{(r_t - 2)\,(r_t - 3)\,(r_t^2 + \nu_t^2)}\ , && (9)
\end{aligned}
$$

where S_t and K_t are the conditional skewness and kurtosis coefficients, given by the standardized third and fourth moments.

In this framework, the conditional variance σ_t^2 depends jointly on a_t, ν_t and r_t, whereas conditional skewness and kurtosis depend only on ν_t and r_t. In particular, if $\nu_t = 0$ then $S_t = 0$ and this is why ν_t can be interpreted as the "skewness parameter". When $\nu_t = \nu$ and $r_t = r$, $\forall t$, conditional skewness and kurtosis are constant.

For a complete model specification a critical point is how to describe the dynamics of σ_t^2, S_t and K_t. Our proposal is to define it through the evolution of the parameters a_t, ν_t and r_t which, in turn, is induced by the following autoregressive GARCH-type structure:

$$a_t^2 = \omega_a + \alpha_a\ \bar{a}_{t-1}^2 + \beta_a\ a_{t-1}^2\ , \qquad (10)$$

$$\nu_t = \omega_v + \alpha_\nu\ \bar{\nu}_{t-1} + \beta_\nu\ \nu_{t-1}\ , \qquad (11)$$

$$r_t = \omega_r + \alpha_r\ \bar{r}_{t-1} + \beta_r\ r_{t-1}\ , \qquad (12)$$

with $\bar{a}_t$, $\bar{\nu}_t$ and $\bar{r}_t$ being moment-based estimators of a_t, ν_t and r_t (see Stuart and Ord, 1994, and Heinrich, 2004) defined by

$$
\begin{aligned}
\bar{a}_t &= \frac{\sqrt{\bar{\sigma}_t^2\left[16\,(\bar{r}_t - 1) - \bar{S}_t^2\,(\bar{r}_t - 2)^2\right]}}{4}\ , && (13)\\
\bar{\nu}_t &= -\frac{\bar{r}_t\,(\bar{r}_t - 2)\,\sqrt{\bar{S}_t}}{\sqrt{16\,(\bar{r}_t - 1) - \bar{S}_t^2\,(\bar{r}_t - 2)^2}}\ , && (14)\\
\bar{r}_t &= \frac{6\,(\bar{K}_t - \bar{S}_t^2 - 1)}{2\,\bar{K}_t - 3\,\bar{S}_t^2 - 6}\ . && (15)
\end{aligned}
$$

By $\bar{\sigma}_t^2$, $\bar{S}_t$ and $\bar{K}_t$ we have denoted suitable estimates of the variance, skewness and kurtosis coefficients. In particular, the estimates defined in (13), (14) and (15) are "local", in the sense that only the m more recent values of the series are used in the computation of $\bar{\sigma}_t^2$, $\bar{S}_t$ and $\bar{K}_t$. In the following, the choice of m will be based on goodness-of-fit criteria.

Since a_t, ν_t and r_t depend only on past information, conditional variance, skewness and kurtosis at time t can be computed at time $t-1$.

The introduction of the constraints $\alpha_\nu = \alpha_r = \beta_\nu = \beta_r = 0$ allows to estimate models with constant skewness and kurtosis. However, note that for a dynamic behavior of both conditional skewness and kurtosis, it is sufficient that at least one of these parameters is different from zero.

Modeling a_t, ν_t and r_t rather than directly variance, skewness and kurtosis turns out to be easier because the latter quantities need to satisfy nonlinear constraints which are difficult to impose at each point in time, while the constraints concerning a_t, ν_t and r_t can be implemented straightforwardly.

The issue of what constraints are necessary and sufficient to ensure the stationarity of the model requires further study. However, by simulations, we found that the following conditions, besides guaranteeing the positivity of the variance and kurtosis parameters, are sufficient for a non-explosive behavior: $\omega_a > 0$, $\omega_r > 3$, $\alpha_i, \beta_i \geq 0$, $\alpha_i + \beta_i < 1$, for $i = a, \nu, r$. In particular, the constraint $\omega_r > 3$ is needed to ensure existence of the kurtosis.

Estimates for the ω_i, α_i and β_i $(i = a, v, r)$ parameters are obtained by maximizing the log-likelihood function

$$\sum_{t=1}^{n} \left\{ \log C_t - \frac{r_t + 2}{2} \log \left[1 + \left(\frac{\hat{\varepsilon}_t - \lambda_t}{a_t} \right)^2 \right] - \nu_t \arctan \left(\frac{\hat{\varepsilon}_t - \lambda_t}{a_t} \right) \right\} , \qquad (16)$$

where $\hat{\varepsilon}_t = y_t - \hat{\mu}_t$. The estimate $\hat{\mu}_t$ is computed in a first step of the procedure, by estimating a suitable ARMA model, which in the present context usually represents a very weak correlation structure. Since parameters a_t, ν_t and r_t are functions of ω_i, α_i and β_i $(i = a, \nu, r)$, expression (16) can be maximized with respect to these latter. In principle, maximum likelihood can also be used to estimate the parameter m in the definition of $\bar{a}_t$, $\bar{\nu}_t$ and $\bar{r}_t$. However, this would imply a large computational burden. Hence, the choice of m will be based on goodness-of-fit considerations (see the next section).

4 Applications

4.1 Empirical evidences and statistical significance of skewness

We now look for empirical evidences of asymmetry by applying the previous methodologies to the daily returns, adjusted for split and dividends, of 9 major international stock indexes, namely the indexes CAC40, DAX, FTSE100, MIB30, Dow Jones, S&P500, Nasdaq, Nikkey225 and SMI. The time series refer to different periods but all end on December 13, 2005. The series are composed by a number of observations between 1547 and 4023 (Table 1).

Most of the series present some abnormal values that do not seem to belong to the standard dynamics of the phenomenon and can be, thus, classified as outliers. Identification of outliers in heteroskedastic models is a delicate and relatively unexplored issue and is particularly important in skewness analysis. We do not consider in depth this problem here. However, since we are interested in the systematic skewness in the data, in order to avoid strong dependence of results on possible outliers we removed them. Identification was based on graphical examination of both the return time series and the standardized residuals of a GARCH(1,1) model. The number of identified outliers was very small: no outliers in 3 series, 1 outlier in 4 series, 2 outliers in 1 series and 3 outliers in 1 series. Outliers were replaced with the mean of the data. All the analyzes were conducted on these outlier-adjusted time series.

Sample skewness and kurtosis coefficients are given in Table 1: all indexes have negative skewness and severe excess kurtosis. Only the Nikkey225 index has positive, but very small, skewness. These results are consistent with other findings in the literature (e.g. Cont, 2001, Belaire-Franch and Peiró, 2003, Kim and White, 2004, and Peiró, 2004).

As a starting point, we looked for unconditional skewness by applying the Bai and Ng (2005) test and, as a benchmark, the asymptotic standard test. The $p-$values for the null hypothesis of symmetry are reported in Table 1, and show that the Bai and Ng test accepts the null hypothesis, at the 5% level of significance, in 8 cases on 9, with a $p-$value of 0.0464 for the FTSE100. Therefore, at the 1% level the Bai and Ng test never rejects the null hypothesis. On the contrary, an erroneous, in the sense described in Section 2, use of the standard asymptotic test would have led to strongly reject the symmetry in all cases except for the Nikkey225.

These analyzes indicate that no clear evidence of unconditional asymmetry was found in the analyzed time series.

As a second step, we investigate conditional skewness of the series. For the moment we will assume skewness to be constant. The previous two tests and the Bai and Ng (2001) test are employed.

In order to test for constant conditional skewness we assumed the data generating process to be represented by model (2), then assessing the symmetry of ε_t. In practice, this amounts to estimating suitable ARMA-GARCH models, then applying the tests on the standardized residuals $\hat{\varepsilon}_t = (y_t - \hat{\mu}_t)/\hat{\sigma}_t$. The models estimated included possible leverage effects used to represent the asymmetric impact that returns may have on volatility.

Table 2 shows the sample skewness and kurtosis coefficients for standardized residuals and the results of the tests. We note that although kurtosis coefficients for the conditional distributions are always smaller than the marginal ones, this is not the case for skewness coefficients, that tend to be higher, in absolute value, than their unconditional counterparts. This is in contrast with the results by Engle and Gonzáles-Rivera (1991). As in the marginal case, for the Nikkey225 index skewness is very close to zero.

Concerning conditional skewness, results about the statistical significance are quite different from those on the marginal distributions and show more evidence of asymmetry. In particular, at the 5% significance level, the null hypothesis of symmetry is rejected in 6 cases on 9 by the BN05 test (in other two cases skewness is significant at the 9% level) and in 5 cases by the BN01 test. The Nikkey225 index is the only one for which all tests, even the asymptotic one, clearly agree on accepting symmetry. In the whole the three tests lead to the same conclusions in 5 cases on 9.

The presence of significant conditional skewness was further investigated by estimating GARCHDSK models, as defined in Section 3, assuming constant conditional skewness and kurtosis. This amounts to imposing $\alpha_\nu = \beta_\nu = \alpha_r = \beta_r = 0$, thus obtaining a subset of models that we will indicate with GARCHSK. The Kolmogorov-Smirnov goodness-of-fit test described below led us to the choice of $m = 10$ in the definition of $\bar{a}_t$, $\bar{\nu}_t$ and $\bar{r}_t$.

The maximum likelihood parameter estimates, with their asymptotic standard deviations and t-statistics, are given in Table 3. The t-statistics in Table 3 indicate that conditional skewness is statistically significant for all series except Nikkey225. Table 4 lists the conditional skewness an kurtosis implied by models estimated in Table 3 and shows that all indexes are negatively skewed with the Nikkey225 having the smallest coefficient. Again, the absolute conditional skewness entailed by the models is generally greater than the marginal one.

The model introduced in Section 3 also allows to investigate the presence of dynamic, rather than constant, conditional skewness.

Table 5 lists the estimated parameters and shows that for 7 indexes the parameter α_ν is significant implying that both skewness and kurtosis are time varying. For all these models, the Ljung-Box test at lag 15 on standardized squared residuals accepts the hypothesis of no residual correlation.

Some examples of time series of the estimated dynamic conditional skewness and kurtosis are given in Figures 1, 2 and 3, where the horizontal lines are the levels of skewness and kurtosis implied by the models when assumed constant. As expected, time-varying conditional skewness moves around these levels as also evidenced in Table 4.

In order to check goodness of fit, we applied the Kolmogorov-Smirnov test to assess the uniformity of the values $\hat{F}(\hat{\varepsilon}_t|I_{t-1})$, $t = 1, \ldots, n$, where $F(\cdot|I_{t-1})$ denotes the c.d.f. corresponding to the density defined in (5), and $\hat{F}(\cdot|I_{t-1})$ is obtained by substituting the ML parameter estimates in the c.d.f. definition. Table 6 lists the test p-values for each series. The p-values for other models, described in the next section, are also shown. At the standard 5% significance level, the null is accepted for all models, suggesting that the GARCHDSK models are appropriate.

In summary, the results on the nine analyzed time series indicate that there are no strong evidences of unconditional skewness, which seems to be more the exception than the rule. Conditional skewness, on the other hand, appears to be more widespread. In particular, there are clear indications of dynamic skewness that, if modeled, may allow for a more realistic description of the evolution of financial quantities of interest.

4.2 Measures of risk and economic significance

In the previous section we analyzed skewness mainly from a statistical viewpoint, by looking at its statistical significance. In this section we mean to study the economic and financial importance of skewness by analyzing its role in risk modeling, and examining the performance of GARCHDSK models in this context.

With this purpose, for the nine stock indexes the time-varying Value-at-Risk

(VaR_t) and Expected Shortfall (ES_t) were computed, with GARCHDSK and some alternative models, in order to compare them.

Market Value-at-Risk is a crucial component of most risk analyzes and management systems in financial and insurance industries. It measures how the market value of an asset, or of a portfolio of assets, of value P is likely to decrease over a certain time period under normal market conditions. It is typically used by security houses or investment banks to measure the market risk of their asset portfolios, but is actually a very general concept that has broad application.

VaR has two parameters: i) the holding (or horizon) period h, that is the length of time over which the portfolio is planned to be held; ii) the confidence level, denoted by $(1-\alpha)$, at which we plan to make the estimate. Given these, the VaR is a bound such that the loss over the holding period is less than this bound with probability $1-\alpha$. Basically, VaR is a high quantile of the profit/loss distribution. If we assume the portfolio value at time t is P_t and the profits and losses over h periods are represented by the log-returns of the portfolio, $r_{t,h} = \log(P_t/P_{t-h})$, with distribution F_h, then VaR is given by

$$\mathrm{VaR}_{h,\alpha} = -P_{t-h} \ \left[F_h^{-1}(\alpha)\right] \ . \tag{17}$$

Another measure of risk is the the Expected Shortfall – or conditional VaR – which describes the expectation of all losses exceeding a VaR. ES has recently been advocated as an alternative to VaR because it takes into account both the probability of a large loss (larger than VaR) and the expected loss given that the loss exceeds VaR. Formally, it is given by

$$\mathrm{ES}_{h,\alpha} = -P_{t-h} \ E\left[r_{t,h} | r_{t,h} < F_h^{-1}(\alpha)\right] \ . \tag{18}$$

When the distribution of $r_{t,h}$ is not constant over time, VaR and ES are also time-varying. In the following, we will use the typical holding period of 1 day, the 99% confidence level, and will assume to have a unitary position ($P = 1$) in a portfolio given by the index.

We computed time-varying VaR_t and ES_t by using the GARCHDSK models estimated in the previous subsection, the Riskmetrics approach with the usual smoothing parameter $\lambda = 0.94$ (see Alexander, 2001), a Gaussian GARCH(1,1) and a GARCH(1,1) with Student's t innovations. The means of the estimated values are given in Table 7.

Table 8 shows the observed in-sample significance level $\hat{\alpha}$ when the nominal level is 0.01. To compare nominal and observed levels a two-sided test for the null H_0: $\alpha = 0.01$ was conducted. The asterisks in Table 8 indicate that the observed level is not significantly different from the nominal one, at the 5% level. For the GARCH-t and GARCHDSK models the levels are always statistically correct, while for the Riskmetrics model the level is correct 8 times on 9. On the contrary, for the Gaussian GARCH model, observed levels are generally significantly greater than the nominal one. For this reason we will not comment further the results for this model.

In the whole, the Riskmetrics model leads to the smallest absolute mean values of VaR, followed by the GARCH-t and GARCHDSK models. However, it is interesting to note that the VaR_t^{DSK} is not always (i.e. $\forall t$) greater than the VaR_t^{R}; for example,

$\text{VaR}_t^{DSK} < \text{VaR}_t^R$ approximately 17% of the times for MIB30, 10% of the times for CAC40, 13% of the times for DAX and 15% of the times for FTSE100.

Value-at-Risk is also connected to the Market Risk Capital Requirements (MRCR) adopted in 1995 by the Basel Accord on Banking Supervision (Basel Committee on Banking Supervision, 1995, 1996). The Basel Accord sets minimum capital requirements which must be met by banks to face market risks, defined as the risk of losses in on- and off-balance sheet positions arising from movements in market prices. The Basel Committee carefully examined how banks' value-at-risk measures can be converted into a capital requirement that appropriately reflects the prudential concerns of supervisors. The Committee eventually defined the Market Risk Capital Requirements as a function of past VaRs and of their violations. In particular, the accord establishes that MRCR is expressed as the higher of: i) the previous day's Value-at-Risk; ii) an average of the Value-at-Risk measures on each of the last sixty business days, multiplied by a factor K which depends on the number of VaR violations in the last 250 business days, described in Table 9. MRCR_t can be thus formally defined as

$$\text{MRCR}_t = \max\left(\text{VaR}_{t-1};\ K\,\frac{1}{60}\sum_{i=1}^{60}\text{VaR}_{t-i}\right). \tag{19}$$

In a prudential spirit, one of the proposals of Basel Committee is to provide incentive to improve risk management systems, penalizing inaccurate risk models and rewarding the most accurate ones. With this purpose, it allows banks to use internal models for measuring exposure to market risk at the cost of proving that their models work correctly.

In this framework, another way to evaluate a model – given the correct coverage level – is to analyze the series of the number of VaR violations, since it is directly connected to MRCR.

Figures 4, 5, 6 and 7 show these series for some of the considered indexes. As expected, the Gaussian GARCH model leads to the greatest number of violations. Despite the Gaussian assumption, the Riskmetrics approach gives results that are comparable with those of a GARCH-t. However, the best results are those relative to the GARCHDSK which, considering (dynamic) skewness and kurtosis, gives the smallest number of violations greater than four. This is apparent for the CAC40 and MIB30 indexes, two examples of series with dynamic skewness and kurtosis. This is also true for the S&P500 index, for which the GARCHDSK model leads to only 9 occurrences of 6 VaR violations against the 174 of the Riskmetrics approach. Advantages can be found even for the Nasdaq index, an example of significant constant skewness. Using the Riskmetrics model, for Nasdaq we found 503 occurrences of 5 violations, 174 of 6 and 97 of 7 whereas using the GARCHSK model they are, respectively, 68, 108 and 0 (Figure 6). When there are no findings of skewness, for example in the Nikkey225 case, the Riskmetrics model and the GARCHDSK model give exactly the same small number of exceedances of four violations.

5 Conclusions

This paper has focused on the issue of empirical evidence of asymmetry for time series of financial returns. Nine series of daily stock index returns have been analyzed, in order to assess whether skewness can be considered a *stylized fact* for real data.

We studied both unconditional and conditional skewness, by means of tests and models. In particular, we proposed a new GARCH-type model, the GARCHDSK model, which allows to take into account both skewness and kurtosis. A characteristic feature of this approach is that skewness and kurtosis are allowed to evolve dynamically. This is done by assuming Pearson's Type IV errors and defining suitable dynamics for the distribution parameters. The dynamic structure depends on moment-based estimators.

Our results indicate that for the considered series there are no strong evidences of unconditional asymmetry which, therefore, does not appear to be a common feature of financial returns.

Different conclusions are drawn with respect to conditional skewness, which was found to be significantly present in eight of the nine stock index returns analyzed. In particular, in seven of the eight cases, we found significant time-varying skewness and kurtosis. These findings are consistent with those of studies by, among others, Brooks et al. (2005), León et al. (2005) and Cappuccio et al. (2006).

To investigate the economic importance of a correct modeling of skewness, different models were compared with respect to Value-at-Risk, Expected Shortfall and the Market Risk Capital Requirements adopted by the Basel Accord. All these analyzes confirm that skewness is important not only from a statistical point of view, but also from a financial perspective, particularly in risk management.

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Series	n	$\hat{S}$	$\hat{K}$	BN05	AS
CAC40	3977	-0.103	5.798	0.393	0.008
DAX	3792	-0.117	6.204	0.326	0.003
FTSE100	5483	-0.264	6.362	0.046	0.000
MIB30	1547	-0.189	6.608	0.410	0.002
SMI	3797	-0.200	6.861	0.177	0.000
Dow Jones	4023	-0.223	7.548	0.227	0.000
Nasdaq	4023	-0.174	7.638	0.278	0.000
S&P500	4023	-0.103	6.767	0.504	0.008
Nikkey225	3926	0.038	5.098	0.691	0.332

Table 1: Unconditional symmetry tests for index returns. $\hat{S}$ and $\hat{K}$ are the empirical skewness and kurtosis coefficients for the observed series; columns BN05 and AS give the $p-$values for the Bai and Ng (2005) and the standard asymptotic tests.

Series	$\hat{S}$	$\hat{K}$	BN05	BN01	AS
CAC40	-0.366	5.336	0.083	1.042	0.000
DAX	-0.124	3.974	0.130	1.874	0.002
FTSE100	-0.208	3.941	0.008	2.608	0.000
MIB30	-0.419	4.237	0.003	2.43	0.000
SMI	-0.280	3.915	0.000	2.777	0.000
Dow Jones	-0.348	4.731	0.002	1.156	0.000
Nasdaq	-0.412	4.316	0.000	4.665	0.000
S&P500	-0.345	4.759	0.002	0.719	0.000
Nikkey225	-0.053	4.578	0.616	0.956	0.179

Table 2: Conditional symmetry tests for index returns. Standardized residuals of ARMA-GARCH models have been used. $\hat{S}$ and $\hat{K}$ are the empirical skewness and kurtosis coefficients for the standardized residuals; columns BN05 and AS give the $p-$values for Bai and Ng (2005) and standard asymptotic tests; column BN01 gives the value of the test statistic for the Bai and Ng (2001) test, to be compared to critical values 2.21 and 2.78, for 5% and 1% levels of significance.

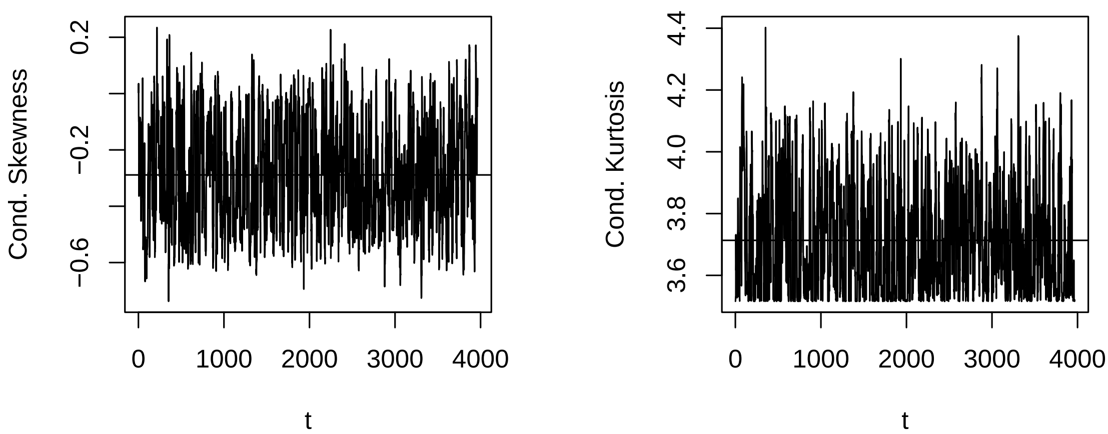

Figure 1: CAC40, conditional skewness and kurtosis.

Parameter	Estimate	Std. err.	t-stat.	Parameter	Estimate	Std. err.	t-stat.
CAC40				Dow Jones			
ω_a	0.214	0.073	2.92	ω_a	0.052	0.013	3.78
α_a	0.075	0.009	7.60	α_a	0.051	0.005	9.52
β_a	0.914	0.011	83.12	β_a	0.938	0.005	170.6
ω_ν	1.801	0.678	2.65	ω_ν	0.867	0.299	2.89
ω_r	11.46	2.109	5.43	ω_r	7.042	0.909	7.74
DAX				Nasdaq			
ω_a	0.191	0.056	3.37	ω_a	0.147	0.039	3.73
α_a	0.101	0.010	9.66	α_a	0.085	0.006	12.84
β_a	0.888	0.010	83.01	β_a	0.905	0.007	135.1
ω_ν	1.386	0.479	2.89	ω_ν	3.389	0.872	3.88
ω_r	9.602	1.587	6.05	ω_r	11.09	1.888	5.87
FTSE100				S&P500			
ω_a	0.141	0.046	3.03	ω_a	0.083	0.024	3.47
α_a	0.074	0.009	8.39	α_a	0.098	0.011	9.31
β_a	0.915	0.010	88.85	β_a	0.891	0.011	80.28
ω_ν	3.138	0.913	3.44	ω_ν	0.891	0.315	2.82
ω_r	14.39	2.512	5.73	ω_r	7.168	1.016	7.05
MIB30				Nikkey225			
ω_a	0.112	0.045	2.49	ω_a	0.195	0.057	3.44
α_a	0.088	0.013	6.73	α_a	0.081	0.009	8.58
β_a	0.899	0.013	68.67	β_a	0.906	0.010	87.12
ω_ν	1.782	0.729	2.44	ω_ν	0.307	0.252	1.21
ω_r	7.829	1.934	4.05	ω_r	6.859	0.906	7.56
SMI							
ω_a	0.163	0.049	3.32				
α_a	0.097	0.012	8.02				
β_a	0.886	0.013	64.71				
ω_ν	2.212	0.587	3.76				
ω_r	9.365	1.502	6.23				

Table 3: ML estimates of GARCHSK model parameters (constant conditional symmetry and kurtosis), asymptotic standard errors and t-statistics.

	GARCHSK		GARCHDSK	
Series	S	K	S_{av}	K_{av}
CAC40	-0.212	3.78	-0.288	3.71
DAX	-0.219	3.99	-0.264	3.99
FTSE100	-0.253	3.62	-0.269	3.60
MIB30	-0.390	4.52	-0.354	3.40
SMI	-0.365	4.17	-0.404	4.02
Dow Jones	-0.239	4.59	-0.321	4.50
Nasdaq	-0.407	4.02	-0.408	4.02
S&P500	-0.237	4.54	-0.329	4.23
Nikkey225	-0.089	4.56	0.00	4.55

Table 4: Conditional skewness and kurtosis implied by the GARCHSK and GARCHDSK models (in the latter case the average conditional skewness and kurtosis are given).

parameter	estimate	std. err.	t-stat.	parameter	estimate	std. err.	t-stat.
CAC40				Dow Jones			
ω_a	0.242	0.085	2.83	ω_a	0.058	0.017	3.41
α_a	0.062	0.007	7.97	α_a	0.054	0.006	8.80
β_a	0.925	0.009	103.9	β_a	0.935	0.007	138.4
ω_ν	4.069	1.611	2.52	ω_ν	1.658	0.551	3.00
α_ν	0.218	0.048	4.54	α_ν	0.280	0.068	4.10
ω_r	14.63	3.598	4.06	ω_r	8.064	1.334	6.05
DAX				Nasdaq			
ω_a	0.166	0.053	3.11	ω_a	0.147	0.039	3.73
α_a	0.087	0.009	9.22	α_a	0.084	0.007	12.85
β_a	0.903	0.009	93.83	β_a	0.905	0.007	135.07
ω_ν	2.151	0.705	3.04	ω_ν	3.389	0.872	3.88
α_ν	0.239	0.056	4.22	α_ν	–	–	–
ω_r	10.58	1.997	5.29	ω_r	11.090	1.888	5.87
FTSE100				S&P500			
ω_a	0.155	0.047	3.28	ω_a	0.079	0.027	2.85
α_a	0.063	0.006	9.21	α_a	0.076	0.008	9.17
β_a	0.923	0.008	113.5	β_a	0.913	0.008	106.6
ω_ν	4.656	1.20	3.85	ω_ν	2.296	1.192	1.92
α_ν	0.210	0.041	5.13	α_ν	0.275	0.061	4.47
ω_r	16.86	3.119	5.40	ω_r	9.431	2.533	3.72
MIB30				Nikkey225			
ω_a	0.410	0.253	1.61	ω_a	0.195	0.057	3.44
α_a	0.072	0.010	6.85	α_a	0.081	0.009	8.58
β_a	0.913	0.011	83.5	β_a	0.906	0.010	87.12
ω_ν	23.59	19.38	1.21	ω_ν	0.306	0.252	1.21
α_ν	0.153	0.071	2.14	α_ν	–	–	–
ω_r	35.87	21.99	1.63	ω_r	6.859	0.906	7.56
SMI							
ω_a	0.176	0.057	3.08				
α_a	0.085	0.011	7.73				
β_a	0.897	0.012	74.7				
ω_ν	3.918	1.215	3.22				
α_ν	0.224	0.052	4.33				
ω_r	11.77	2.485	4.73				

Table 5: ML estimates of GARCHDSK model parameters (time-varying conditional skewness and kurtosis), asymptotic standard errors and t-statistics.

	Riskmetrics	GARCH-N	GARCH-t	GARCHDSK
CAC40	0.427	0.044	0.450	0.090
DAX	0.183	0.003	0.339	0.182
FTSE100	0.033	0.049	0.022	0.104
MIB30	0.002	<0.001	0.006	0.415
SMI	0.056	0.002	0.040	0.117
Dow Jones	0.028	<0.001	0.364	0.243
Nasdaq	<0.001	<0.001	<0.001	0.295
S&P500	0.011	<0.001	0.297	0.090
Nikkey225	0.174	0.005	0.768	0.658

Table 6: p-values for the Kolmogorov-Smirnov goodness-of-fit test.

Series	VaR^R	VaR^N	VaR^t	VaR^{DSK}	ES^R	ES^N	ES^t	ES^{DSK}
CAC40	2.90	2.94	3.14	3.37	3.32	3.36	3.82	4.07
DAX	2.96	3.02	3.24	3.43	3.39	3.46	3.96	4.20
FTSE100	2.16	2.19	2.33	2.46	2.48	2.51	2.81	2.96
MIB30	2.72	2.77	3.04	3.18	3.11	3.17	3.80	3.79
SMI	2.38	2.41	2.58	2.86	2.72	2.75	3.15	3.51
Dow Jones	2.15	2.20	2.39	2.55	2.47	2.52	2.99	3.19
Nasdaq	3.08	3.09	3.28	3.62	3.53	3.54	3.99	4.45
S&P500	2.17	2.21	2.39	2.59	2.48	2.53	2.98	3.20
Nikkey225	3.26	3.32	3.60	3.63	3.73	3.80	4.496	4.51

Table 7: Mean Value-at-Risk (VaR) and expected shortfall (ES) for returns time series. The considered models are: Riskmetrics, Normal GARCH(1,1), Student's t GARCH(1,1) and GARCHDSK.

Series	VaR^R	VaR^N	VaR^t	VaR^{DSK}
CAC40	0.008*	0.014	0.011*	0.008*
DAX	0.008*	0.013*	0.010*	0.009*
FTSE100	0.007	0.013	0.010*	0.008*
MIB30	0.012*	0.020	0.013*	0.010*
SMI	0.009*	0.016	0.011*	0.008*
Dow Jones	0.008*	0.013*	0.009*	0.008*
Nasdaq	0.010*	0.014	0.012*	0.008*
S&P500	0.008*	0.014	0.009*	0.008*
Nikkey225	0.009*	0.015	0.009*	0.009*

Table 8: Value-at-Risk (in-sample observed levels when the nominal level is 0.01). The asterisk indicates that the observed level is not significantly different from 0.01, at the 5% level. The considered models are: Riskmetrics, Normal GARCH(1,1), Student's t GARCH(1,1) and GARCHDSK.

Zone	n. of violations	K
Green	≤ 4	3.00
	5	3.40
	6	3.50
Yellow	7	3.65
	8	3.75
	9	3.85
Red	≥ 10	4.00

Table 9: Basel accord penalty factor K. The number of violations refers to the last 250 business days.

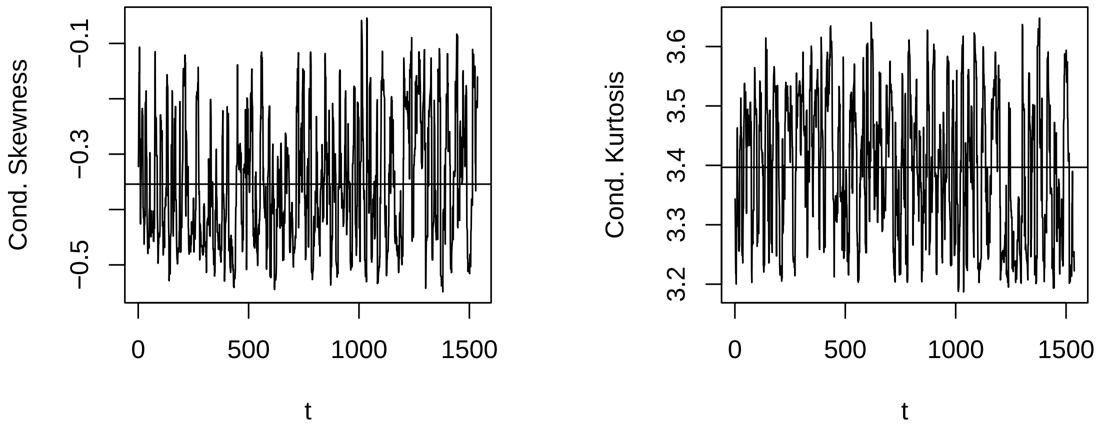

Figure 2: MIB30, conditional skewness and kurtosis.

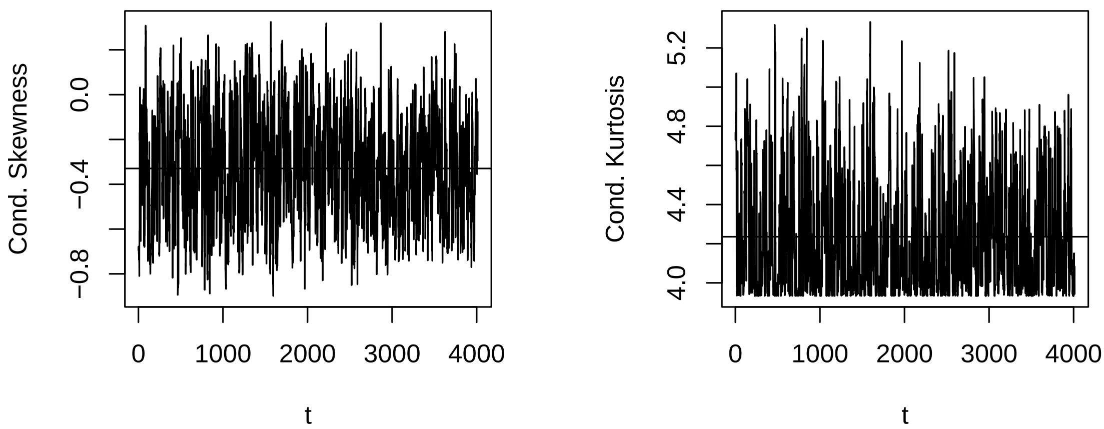

Figure 3: S&P500, conditional skewness and kurtosis.

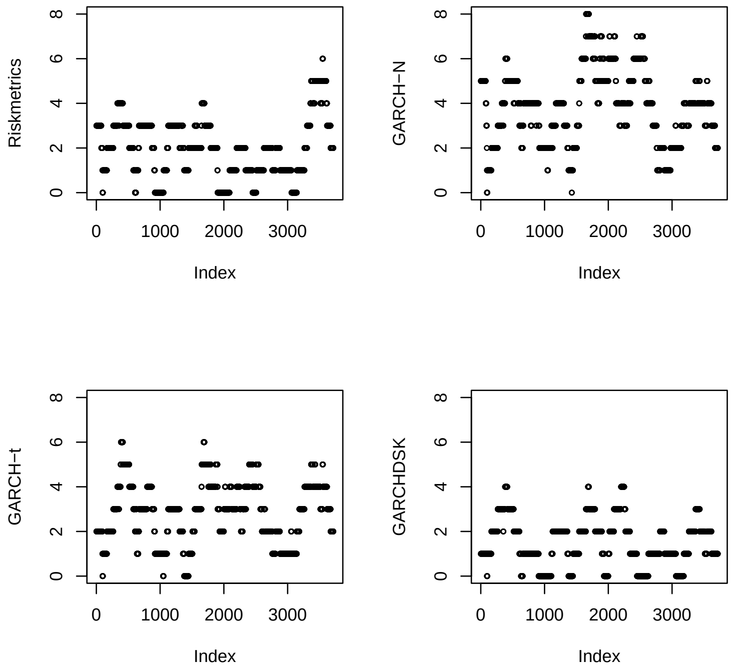

Figure 4: CAC40, number of VaR violations for the last 250 business days.

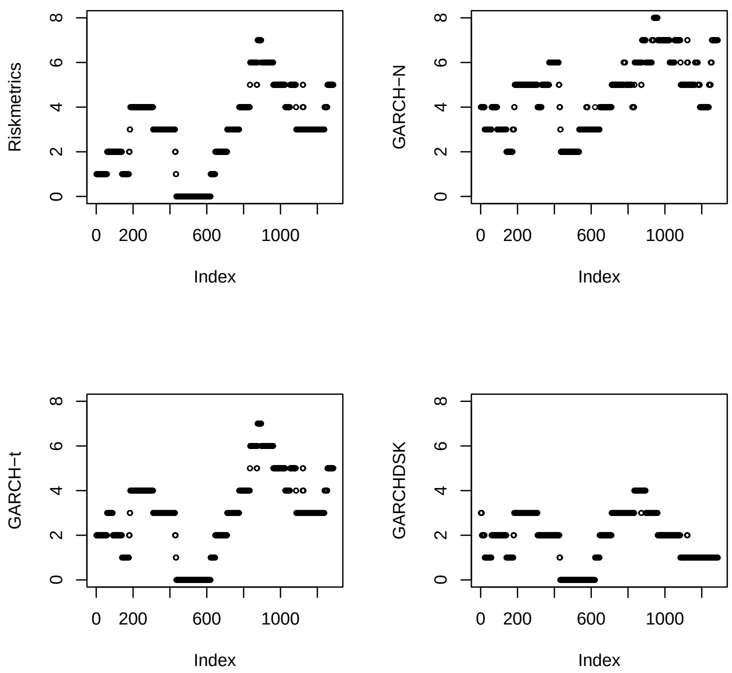

Figure 5: MIB30, number of VaR violations for the last 250 business days.

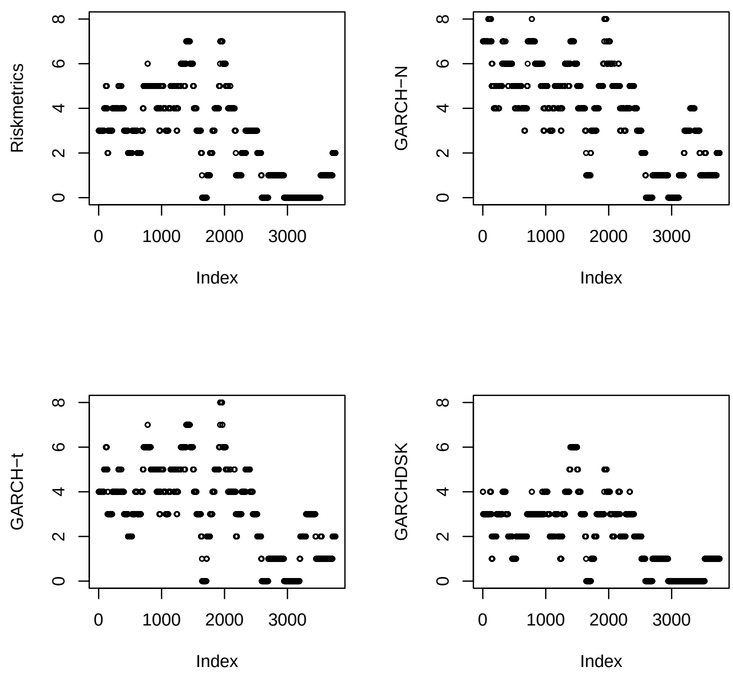

Figure 6: Nasdaq, number of VaR violations for the last 250 business days.

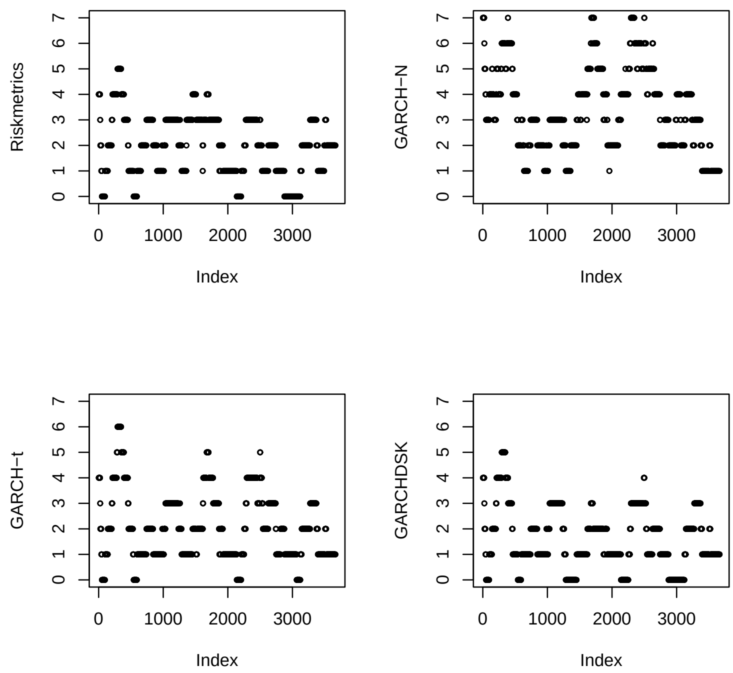

Figure 7: Nikkey225, number of VaR violations for the last 250 business days.

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