MacroEconomia Avanzata
Esercitazione 6
Correzione.

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1 Question 1.

1. Firms' profit is given by

$$\Pi = \pi(K(t))k(t) \qquad \text{where } \pi'(\cdot) = 0; \tag{1}$$

and (internal) adjustment costs are defined as a convex function of the rate of change of the firms' capital stock, $\dot{k}$. Specifically,

$$C(0) = 0; \qquad C'(0) = 0; \qquad C'' > 0. \tag{2}$$

We assume $\delta = 0$ for simplicity, and price of capital goods equal to 1, so that $\dot{k}(t) = I(t)$, where I is the firm's investment.

Hence, the firm's profits at a point in time are

$$\pi(K)k - I - C(I) = \int_{t=0}^{\infty} e^{-rt}\big[\pi(K(t))k(t) - I(t) - C(I(t))\big]\mathrm{d}t, \tag{3}$$

where we assume that the real interest rate r is constant. The firm takes the path of the aggregate capital stock K as given, and chooses its investment over time to maximise Π given this path. Let us set up the current-value Hamiltonian

$$H(k(t), I(t)) = \pi(K(t))k(t) - I(t) - C(I(t)) + q(t)I(t). \tag{4}$$

FOCs are

$$\frac{\partial H}{\partial I(t)} = 1 + C'(I(t)) = q(t); \tag{5}$$

$$\frac{\partial H}{\partial k(t)} = \pi(K(t)) = rq(t) - \dot{q}(t); \tag{6}$$

and the transversality condition is assumed to hold. Equation (5) implies that each firm invests to the point where the purchase price of capital plus the marginal adjustment cost equals the value of capital. Since q is the same for all firms, all firms choose the same value of I. Thus

$$\dot{K}(t) = NC'^{-1}(q-1) \qquad = 0 \text{ iff } q = 1. \tag{7}$$

Equation (6) implies that the marginal revenue product of capital equals its user cost. We can rewrite this as

$$\dot{q}(t) = rq(t) - \pi(K(t)) \qquad = 0 \text{ iff } q = \pi(K)/r. \tag{8}$$

The phase diagram can be found on the textbook, pp.416–419.

2. In the continuous-time case, q can be expressed as

$$q(t) = \int_{\tau=t}^{\infty} e^{-r(\tau-t)}\pi(K(\tau))\mathrm{d}\tau. \tag{9}$$

There are three possible interpretations of q:

(a) q contains all relevant information about the future that firms need for choosing optimally, i.e. it represents the present discounted value of the future marginal revenue products of a unit of capital;

(b) A unit increase in the firm's capital stock increases the present value of the firm's profits by q, thus raises the value of a firm by q, hence it represents the market value of a unit of capital;

(c) Since the purchase price of capital is fixed at 1, q is also the ratio of the market value of a unit of capital to its replacement cost. Thus equation (5) states that a firm increases its capital stock if the market value of capital exceeds the cost of acquiring it, and that it decreases its capital stock if the market value of the capital is less than the cost of acquiring it.

This last interpretation is known as the Tobin's q.

3. See the textbook, Section 9.6..

4. **(Optional — analytical passages)** Let us assume that an entrepreneur has the opportunity to undertake a project that requires 1 unit of resources, and that he possesses only $w < 1$, so that he must obtain $1 - w$ units of outside finances.

If the project is undertaken, it has an expected output $\gamma > 0$, heterogeneous across entrepreneurs and publicly observable. Actual output is given by γ, which is uniformly distributed on $[0, 2\gamma]$.

If both the entrepreneur and the investor do not undertake the project, they can invest at the risk-free interest rate r. Since they are both risk-neutral, they will pursue this strategy if in equilibrium their expected return is greater than what they would gain by undertaking the project.

The key assumption is that entrepreneurs are better informed than outside investors about their project's actual output. This is modelled by assuming that, while entrepreneurs can observe their projects' actual output costlessly, the outside investors must pay a cost $0 < c < \gamma$ (costly state verification).

(a) If also investors could observe actual output costlessly, the equilibrium would be straightforward. Entrepreneurs with $E[\gamma] > 1 + r$ would obtain financing. Expected payment for investors would be $(1 - w)(1 + r)$ times the actual output γ. This would leave the entrepreneur with $\gamma - (1-w)(1+r) = w(1+r) + \gamma - (1+r)$. Since $E[\gamma] > 1+r$, then $w(1 + r) + \gamma - (1 + r) > w(1 + r)$. The entrepreneur would be made better off by undertaking the project.

(b) Let us assume that each project has only one outside investor. Since investors are risk-neutral and competitive, the entrepreneur's expected payment to the investor must equal $(1-w)(1+r)+c$, where c is the cost of observing actual output. The optimal contract is then that if $\gamma > D$, where D is some threshold, then the entrepreneur pays D and the investor does not verify output. On the other hand, if $\gamma < D$, the investor checks output and takes it all.

If $D < 2\gamma$, actual output can be either more or less than D, hence investor's revenue will depend upon the appropriate probabilities; if $D > 2\gamma$, then output is always less than D, and investor's revenue will be actual output minus verification costs:

$$R(D) = \begin{cases} p_1 D + (1-p_1)\Big(\dfrac{D}{2} - c\Big), & \text{if } D \leq 2\gamma; \\ \gamma - c, & \text{if } D < 2\gamma; \end{cases} \tag{10}$$

where $p_1 = D/(2\gamma)$. Note that

$$\frac{\partial R(D)}{\partial D} = \begin{cases} 1 - \dfrac{c}{2\gamma} - \dfrac{D}{2\gamma}, & \text{if } D \leq 2\gamma; \\ 0, & \text{if } D < 2\gamma. \end{cases} \tag{11}$$

$R(D)$ is maximised for $D = 2\gamma - c$ and is increasing in D up to that point. After $R(D)^{\max}$, $R(D)$ decreases up to the point in which $D = \gamma - c$; thereafter further increases in D do not affect $R(D)$.

If $(1+r)(1-w) \leq \gamma - c$, then there is a unique value of D that yields the investor the required net revenues:

$$D^\star = 2\gamma - c - \sqrt{(2\gamma - c)^2 - 4\gamma(1+r)(1-w)}. \tag{12}$$

Expected verification cost is

$$A = \frac{D^\star}{2\gamma}c = \left[\frac{2\gamma - c}{2\gamma} - \sqrt{\left(\frac{2\gamma - c}{2\gamma}\right)^2 - \frac{(1+r)(1-w)}{\gamma}}\right]c. \tag{13}$$

So the project is undertaken if

$$\begin{aligned} &\gamma - (1+r)(1-w) - A(c,r,w,\gamma) > (1+r)w \\ &\gamma > 1 + r + A(c,r,w,\gamma). \end{aligned} \tag{14}$$

The implications of this model are:

(a) Agency costs arising from asymmetric information raise the cost of external finance, and therefore discourage investment.

(b) Agency costs (that are function of output and risk-free rate) alter the impact of such variables on investment. Because output movements affect firms' current profitability, they affect firms' ability to provide internal finance. For instance, a reduction in w increases agency costs, hence, holding γ fixed, the likelihood that a project will be undertaken is lower. Similarly, an increase in r raises A and discourages investment.

(c) Many variables (e.g. w) that do not affect investment when capital markets are perfect do matter when frictions arise. This can magnify the effect of shocks that occur outside the financial system.

(d) Also the financial system itself can impact investment: agency costs are increasing in c, hence in the degree of imperfection of the financial system.

2 Question 2.

1. At each point in time from t to $t+T$, the firm is allowed to deduct P_K/T from its taxable income. This allows it to save $\tau(P_K/T)$ in takes at each point in time from t to $t+T$. If i is the constant interest rate, the present value of the reduction in the firm's corporate tax liabilities is

$$\begin{aligned}\text{saving} &= \int_{s=t}^{t+T} e^{-i(s-t)}\tau(P_K/T)\mathrm{d}s \\ &= \tau(P_K/T)\int_{s=t}^{t+T} e^{-i(s-t)}\mathrm{d}s \\ &= \tau(P_K/T)\left[\frac{-1}{i}e^{-i(s-t)}\ \Bigg|_{s=t}^{s=t+T}\right] \\ &= \tau(P_K/T)\left[\frac{1-e^{-iT}}{i}\right]. \end{aligned} \tag{15}$$

Since the after-tax price of the capital good is its pre-tax price minus the present value of the tax saving, we have

$$P_K^a = P_K - \tau(P_K/T)\left[\frac{1-e^{-iT}}{i}\right] = P_K\left[1-(\tau/T)\left(\frac{1-e^{-iT}}{i}\right)\right]. \tag{16}$$

(To get a feel for the magnitude involved, if $\tau = 0.20$, $P_K = 1000$, $T = 10$ and $i = 0.05$, then $P_K \approx 843$. This means that the tax saving is approximately 16.7% of the purchase price of the capital good.)

2. Substituting $i = r + \pi$ into equation (16) gives

$$P_K^a = P_K\left[1-(\tau/T)\left(\frac{1-e^{-(r+\pi)T}}{r+\pi}\right)\right]. \tag{17}$$

A rise in π, all else equal, increases i and thus reduces the present value of the tax savings. This means that it raises the after-tax price of the capital good to the firm. (With reference to the numbers of part (a), if π rises so that $i = 0.10$, then the tax saving is only 13.6%.)

3 Question 3.

The dynamics of the market value of capital are given by

$$\dot{K}(t) = NC'^{-1}(q-1) \qquad = 0 \text{ iff } q = 1; \tag{18}$$

$$\dot{q}(t) = rq(t) - \pi(K(t)) \qquad = 0 \text{ iff } q = \pi(K)/r. \tag{19}$$

1. If capital stock falls from K to $K/2$ neither the $\dot{K} = 0$ nor the $\dot{q} = 0$ loci shift. The economy then jumps to a lower level of capital ($K_N < K$) and q adjusts by jumping to a higher level ($q_N > q$). Since this point is not an equilibrium, q falls and K rises to the saddle path, until they reach the original equilibrium.

   Intuitively, less capital implies that what is left is more valuable, so the market value of capital jumps up. The higher market value of capital attracts investment, so the capital stock begins to build back up. As it does so, profits begin to fall and thus so does the market value of capital.

2. Condition required for $\dot{q} = 0$ is now

$$q^{\star N} = (1-\tau)\pi(K)/r, \tag{20}$$

which is lower than the original condition given in (19), so the locus $\dot{q} = 0$ shifts down. Slopes are given by

$$\frac{\partial q^\star}{\partial K} = \frac{\pi'(K)}{r}; \qquad \frac{\partial q^{\star N}}{\partial K} = (1-\tau)\frac{\pi'(K)}{r}. \tag{21}$$

Hence, the new $\dot{q} = 0$ is flatter than the old one. K cannot jump at the time of the implementation of the tax, thus q must jump down so that the economy is on the new saddle path at point A, then it moves up the new saddle path with K falling and q rising. The new equilibrium has $q_N = q$ and $K_N < K$.

Intuitively, since government is taking a fraction of profits, existing capital is less valuable, so market value of capital falls. The lower market value of capital discourages investment, hence the capital stock falls. As it does so, profits begin to rise back up and thus so does the market value of capital. In the new equilibrium, the lower capital stock (and higher pre-tax profits) offset the fact that the government takes a fraction of those profits.

3. With this tax on investment, the $\dot{K} = 0$ locus is now given by

$$q = 1 + \gamma. \tag{22}$$

Thus the locus shifts up by γ. K cannot jump at the time of the announcement t_0, thus q must jump up to a point A below the new saddle path. Since now q is too high, K rises so that at implementation (t_1), the economy is on the new saddle path. The new equilibrium has $K_N < K$ and $q_N > q$ (pre-tax market value — the after tax market value is again equal to 1).

Intuitively, since the tax will reduce investment, industry profits will eventually be higher, and thus existing capital is more valuable. Anticipating, firms will expand investment until the tax is implemented and then they will reduce it to the new equilibrium.

4. Consider a market for shares in firms. For individuals to be willing to hold shares through the interval where the one-time tax on capital is imposed, the market value of capital before the levy must equal the value of capital after the levy. Otherwise, holders of shares in firms would be expecting capital losses that could be avoided. Therefore, for $\varepsilon \to 0$,

$$\begin{aligned} q(t_1-\varepsilon) &= (1-f)q(t_1+\varepsilon) \\ \frac{q(t_1-\varepsilon)}{q(t_1+\varepsilon)} &= (1-f). \end{aligned} \tag{23}$$

Thus, at the time of announcement t_0, q must jump down to a point A. Here, q is too low to maintain current level of K, hence both q and K fall to a point B south-west of A. At time t_0, q jumps up to a point C on the old saddle path, so after the levy the economy converges back to the original equilibrium.

Intuitively, at t_0, firms begin decumulating capital in anticipation of the one-time levy. Once the one-time tax is over with, since K is lower, profits are higher and so investment is attractive once again. Thus, capital stock begins rising back to its initial level.

4 Question 4.

1. The evolution of the stock of housing is given by

$$\dot{H} = I(p_H) - \delta H \qquad = 0 \text{ iff } I(p_H) = \delta H. \tag{24}$$

   That is, in order for the stock of housing to remain constant, new investment in housing must exactly offset depreciation of the existing housing stock. Differentiating w.r.t. H gives the slope of the $\dot{H}$ locus

$$\begin{aligned} I'(p_H)\frac{\partial p_H}{\partial H} &= \delta \\ \frac{\partial p_H}{\partial H} &= \frac{\delta}{I'(p_H)} > 0. \end{aligned} \tag{25}$$

   Hence, the $\dot{H} = 0$ locus is upward sloping in the (H, p_H) space.

   Rental income plus capital gains must equal the exogenous rate of return:

$$\frac{R(H) - \dot{p}_H}{p_H} = r. \tag{26}$$

   Solving for $\dot{p}_H$ yields

$$\dot{p}_H = rp_H - R(H) \qquad = 0 \text{ iff } p_H = \frac{R(H)}{r}. \tag{27}$$

   Differentiating both sides w.r.t. H gives the slope of the $\dot{p}_H = 0$ locus:

$$\frac{\partial p_H}{\partial H} = \frac{R'(H)}{r} < 0. \tag{28}$$

   Hence, the $\dot{p}_H$ locus is downward sloping in the (H, p_H) space.

2. Since $I'(p_H) > 0$, then from equation (24), $\dot{H}$ is increasing in p_H. This means that above the $\dot{H} = 0$ locus, $\dot{H} > 0$ and so H is rising. Intuitively, at a given H, if p_H is higher than the price necessary to keep the stock of housing constant, investment (which is increasing in p_H) is higher than necessary to offset depreciation; thus, the stock of housing is rising. Similarly, below $\dot{H} = 0$, $\dot{H} > 0$ and H is falling. Here prices and thus investment are too low to offset depreciation and keep the stock of housing constant.

   Since $R'(H) < 0$, then from equation (27), $\dot{p}_H$ is increasing in H. This means that to the right of the $\dot{p}_H = 0$ locus, $\dot{p}_H > 0$, and p_H is rising. Intuitively, at a given p_H, if H is higher — and thus rent lower — than the level necessary to keep the price of housing constant, this lower rent must be offset by capital gains — a rising price — if investors are to earn the required exogenous return of r. Similarly, to the left of the $\dot{p}_H = 0$ locus, $\dot{p}_H < 0$, and p_H is falling. If H is lower — and thus rent higher — than the level necessary to keep the price of housing constant, this higher rent must be offset by capital losses in order for investors to earn the rate of return r.

3. A rise in r means that the p_H that makes $\dot{p}_H = 0$ is lower. Thus the new $\dot{p}_H = 0$ locus lies below the old one. In addition, the slope is less negative and the new locus is flatter. The $\dot{H} = 0$ locus is unaffected.

   At the time of the increase, H cannot jump discontinuously. The real price of housing p_H must jump down to the point A on the new saddle path. The economy reaches the new equilibrium with $H_N < H$, $p_{HN} < p_H$, $R_N > R$, $I_N < I$.

4. At the time of the announcement, t_0, prices jump down to a point A above the new saddle path. The economy is then in the region in which both H and p_H fall to a point B on the new saddle path at the time of the increase in r. The new equilibrium has $H_N < H$, $p_{HN} < p_H$, $R_N > R$, $I_N < I$.

5. Adjustment costs are not internal. There are no direct costs of building housing in this setup. Internal costs would the actual costs of building the new capital (here, housing). This model exhibits external adjustment costs. As firms undertake more investment in housing, the real price of housing adjusts so that individuals do not wish to invest or disinvest at infinite rates.

6. The $\dot{H} = 0$ locus is not horizontal because investment depends upon the real price of housing, p_H. Depreciation is proportional to the stock of housing and thus is higher at higher levels of H. Therefore, to keep H constant at higher levels of H requires more investment. But in order to have more investment, prices must be higher. So the $\dot{H} = 0$ is upward sloping.

5 Question 5 (Optional).

1. Firm's profit in the two alternative scenarios are

$$E[\pi^{\mathrm{NO}}] = 0; \qquad E[\pi^{\mathrm{YES}}] = \pi_1 + E[\pi_2] - I. \tag{29}$$

The firm will undertake the investment when

$$\begin{aligned} &E[\pi^{\mathrm{NO}}] < E[\pi^{\mathrm{YES}}] \\ &\pi_1 + E[\pi_2] > I. \end{aligned} \tag{30}$$

2. Suppose the firm does not invest in period 1. Then, in period 2, it will invest if $\pi_2 > I$, thus the expected profit from not investing in period 1 are:

$$E[\pi^{\mathrm{P2}}] = \Pr(\pi_2 > I) E[\pi_2 - I | \pi_2 > I]. \tag{31}$$

From equation (29) we obtain the difference in the firm's expected profits between not investing and investing in period 1:

$$E[\pi^{\mathrm{P2}}] - E[\pi^{\mathrm{YES}}] = \Pr(\pi_2 > I) E[\pi_2 - I | \pi_2 > I] - \pi_1 + E[\pi_2] - I. \tag{32}$$

Even if (30) holds, as long as $E[\pi^{\mathrm{P2}}] < E[\pi^{\mathrm{YES}}]$, the firm's expected profits are higher if it does not invest in period 1 than if it does.

3. The cost of waiting is that the firm forgoes period-1 payoffs, that is, π_1. The benefit of waiting is that the firm can observe π_2, see if it is less than I and decide not to invest and avoid loss if this is the case. The expected loss that the firm avoids by waiting is equal to the probability that π_2 is less than I, multiplied by the expected loss given that $\pi_2 < I$, that is $\Pr(\pi_2 < I) E[I - \pi_2 | \pi_2 < I]$.

By the definition of conditional expected values, we can write

$$E[I - \pi_2] = \Pr(\pi_2 > I) E[\pi_2 - I | \pi_2 > I] + \Pr(\pi_2 < I) E[\pi_2 - I | \pi_2 < I]. \tag{33}$$

Substituting this into (32) yields:

$$E[\pi^{\mathrm{P2}}] - E[\pi^{\mathrm{YES}}] = \Pr(\pi_2 > I)E[\pi_2 - I|\pi_2 > I] - \pi_1 - \Pr(\pi_2 > I)E[\pi_2 - I|\pi_2 > I] - \\ - \Pr(\pi_2 < I)E[\pi_2 - I|\pi_2 < I]. \tag{34}$$

Note that we can write $\Pr(\pi_2 < I)E[\pi_2 - I|\pi_2 < I] = -\Pr(\pi_2 < I)E[I - \pi_2|\pi_2 < I]$, equation (34) becomes

$$E[\pi^{\mathrm{P2}}] - E[\pi^{\mathrm{YES}}] = -\pi_1 + \Pr(\pi_2 < I)E[I - \pi_2|\pi_2 < I]. \tag{35}$$

Since π_1 is the cost of waiting and the second term on right-hand side of equation (35), we do have

$$E[\pi^{\mathrm{P2}}] - E[\pi^{\mathrm{YES}}] = \text{benefit of waiting} \; - \; \text{cost of waiting}. \tag{36}$$