

MacroEconomia Avanzata

Esercitazione 2

Esercizio 2.3.

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1 Question 2.3.

The maximum problem for the social planner is

$$\max H = \int_{t=0}^{\infty} e^{\rho-n} \ln c_t - \lambda [c - f(k) - nk]. \quad (1)$$

FOCs are

$$\begin{aligned} \frac{\partial H}{\partial c_t} &= c_t^{-1} - \lambda = 0 \\ c_t^{-1} &= \lambda; \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial H}{\partial k} &= [f'(k) - n]\lambda = (\rho - n)\lambda - \dot{\lambda} \\ \frac{\dot{\lambda}}{\lambda} &= \rho - f'(k). \end{aligned} \quad (3)$$

Deriving (2) w.r.t. t , dividing by λ , and substituting (3) into (2) yields

$$\begin{aligned} \frac{u''(c_t)}{\lambda} \dot{c}_t &= \frac{\dot{\lambda}}{\lambda} \\ - \frac{c_t^{-2}}{c^{-1}} \dot{c}_t &= \rho - f'(k) \\ \frac{\dot{c}_t}{c_t} &= f'(k) - \rho. \end{aligned} \quad (4)$$

This is exactly the same dynamic equation for consumption obtained in the decentralised scenario.