MacroEconomia Avanzata
Esercitazione 5
Correzione.

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26 marzo 2015

1 Question 1.

1. Let us consider an individual who lives for T periods and whose lifetime utility is given by

$$U = \sum_{t=1}^{T} u(C_t), \qquad \text{with } u'(\cdot) > 0, \quad u''(\cdot) < 0. \tag{1}$$

The individual has initial wealth of A_0 and labour incomes of Y_t, $t = 1, \ldots, T$. Assuming $r = \rho = 0$, we can set up the maximum problem as follows:

$$\max_{C_t,\lambda} L = \sum_{t=1}^{T} u(C_t) + \lambda \left(A_0 + \sum_{t=1}^{T} Y_t - \sum_{t=1}^{T} C_t \right). \tag{2}$$

FOCs are:

$$\frac{\partial L}{\partial C_t} = u'(C_t) = \lambda; \tag{3}$$

$$\frac{\partial L}{\partial \lambda} = A_0 + \sum_{t=1}^{T} Y_t - \sum_{\tau=1}^{T} C_t = 0. \tag{4}$$

Since (3) holds for every t, marginal utility of consumption is constant, and consumption is also constant, thus $C_1 = C_2 = \cdots = C_T$. Using this in (4) yields

$$C_t = \frac{1}{T} \left(A_0 + \sum_{\tau=1}^{T} Y_\tau \right) \qquad \forall\ t. \tag{5}$$

Saving is defined as

$$\begin{aligned} S_t &= Y_t - C_t = \\ &= Y_t - \frac{1}{T} \left(A_0 + \sum_{\tau=1}^{T} Y_t \right). \end{aligned} \tag{6}$$

Hence, a rise (Z) in income at t will raise consumption by Z/T (which tends to zero as $T \to \infty$), and raise savings by $1 - Z/T$ (which tends to one as $T \to \infty$).

2. Within this framework, consumption is equal to permanent income ($C = Y^P$). Current income can be decomposed in two components, permanent and transitory income ($Y = Y^P + Y^T$). Since transitory income represents departures of current income from permanent income, in most samples it has a mean near zero ($E[Y^T] = 0$) and is roughly uncorrelated with permanent income ($Cov[Y^T, Y^P] = 0$). Thus, if we set up a regression like

$$C_i = \alpha + \beta Y_i + \varepsilon_i, \tag{7}$$

we can use Friedman's hypothesis to obtain estimated coefficients:

$$\begin{aligned} \hat{\beta} &= \frac{Cov[Y, C]}{Var[Y]} \\ &= \frac{Cov[Y^P + Y^T, Y^P]}{Var[Y^P + Y^T]} \\ &= \frac{Var[Y^P]}{Var[Y^P] + Var[Y^T]}; \qquad (8) \\ \hat{\alpha} &= \bar{C} - \hat{\beta}\bar{Y} \\ &= \bar{Y}^P - \hat{\beta}(\bar{Y}^P + \bar{Y}^T) \\ &= (1 - \hat{\beta})\bar{Y}^P. \qquad (9) \end{aligned}$$

Hence, the permanent-income theory predicts that the key determinant of the slope of an estimated consumption function, $\hat{\beta}$, is the relative variation in permanent and transitory income (i.e. an increase in Y is associated with an increase in C only if such an increase reflects an increase in Y^P). On the other hand, the intercept depends upon $\bar{Y}^P$.

For a discussion about empirical evidence see the textbook, pp. 368–371.

3. Since $E[Y^T] = 0$, we can say that $E[Y] = E[Y^P]$. Thus we are told that, on average, farmers have lower permanent income than non-farmers do ($E[Y_F^P] < E[Y_{NF}^P]$). Furthermore, $Var[Y_F^T] > Var[Y_{NF}^T]$.

   From equation (8), as long as $Var[Y_F^P] = Var[Y_{NF}^P]$, we see that $\hat{\beta}_F < \hat{\beta}_{NF}$. This means that a marginal increase in current income increases farmers' consumption less than it increases non-farmers'. Intuitively, this is because the increase for farmers is more likely to be due to transitory rather than permanent income.

   As for equation (9), given the fact that $E[Y_F^P] < E[Y_{NF}^P]$ (i.e. $\bar{Y}_F^P < \bar{Y}_{NF}^P$), we would expect $\bar{\alpha}_F < \bar{\alpha}_{NF}$. On the other hand, since $\hat{\beta}_F < \hat{\beta}_{NF}$, then $\bar{\alpha}_F > \bar{\alpha}_{NF}$. The overall effect is hence ambiguous.

   We can say, however, that at the average level of permanent income for farmers, the estimated consumption functions for farmers is expected to lie below that for non-farmers. If the two estimated functions cross, they do so at a level of income less than $\bar{Y}_F^P$.

   Consider a non-farmer for whom $Y_t = \bar{Y}_F^P$. Since $E[Y_F^P] < E[Y_{NF}^P]$, non-farmers are likely to have $\bar{Y}_t^P > \bar{Y}_F^P$, hence $Y_t = \bar{Y}_F^P + Y_{NF}^T$, with the second term that is negative. This means that a non-farmer with $Y_t = \bar{Y}_F^P$ will have $C_t > Y_t$.

   Consider now a farmer for whom $Y_t = \bar{Y}_F^P$. On average, this means that $Y_t = Y_F^P = C_t$. Thus, the consumption function for farmers is expected to lie below the one for non-farmers at $\bar{Y}_F^P$.

4. (a) We need to find an expression for

$$\frac{C_{t+2}+C_{t+3}}{2}-\frac{C_t+C_{t+1}}{2} \tag{10}$$

in terms of C_t and ε_t. We can write

$$C_{t+1}=C_t+\varepsilon_{t+1}; \tag{11}$$
$$C_{t+2}=C_{t+1}+\varepsilon_{t+2}=C_t+\varepsilon_{t+1}+\varepsilon_{t+2}; \tag{12}$$
$$C_{t+3}=C_{t+2}+\varepsilon_{t+3}=C_t+\varepsilon_{t+1}+\varepsilon_{t+2}+\varepsilon_{t+3}. \tag{13}$$

Substituting (11)–(13) into (10) yields

$$\frac{C_{t+2}+C_{t+3}}{2}-\frac{C_t+C_{t+1}}{2}=\frac{C_t+\varepsilon_{t+1}+\varepsilon_{t+2}+C_t+\varepsilon_{t+1}+\varepsilon_{t+2}+\varepsilon_{t+3}}{2}-\frac{C_t+C_t+\varepsilon_{t+1}}{2}$$
$$=\frac{\varepsilon_{t+3}+2\varepsilon_{t+2}+\varepsilon_{t+1}}{2}. \tag{14}$$

(b) Through similar manipulation as in part (a), we get

$$\frac{C_t+C_{t+1}}{2}-\frac{C_{t-2}+C_{t-1}}{2}=\frac{\varepsilon_{t+1}+2\varepsilon_t+\varepsilon_{t-1}}{2}. \tag{15}$$

Covariance between successive changes is then:

$$Cov\left[\left(\frac{\varepsilon_{t+3}+2\varepsilon_{t+2}+\varepsilon_{t+1}}{2}\right);\ \left(\frac{\varepsilon_{t+1}+2\varepsilon_t+\varepsilon_{t-1}}{2}\right)\right]=\frac{\sigma_\varepsilon^2}{4}. \tag{16}$$

Equation (16) uses the fact that ε's are uncorrelated with each other and that the only term that appears in both expressions is ε_{t+1}. Since covariance is positive, measured consumption in later intervals will be greater than measured consumption in earlier intervals. For a process $\{X_t\}_{t\in\mathbb{N}}$ to be a random walk, the following condition must hold

$$Cov[X_t;X_{t+1}]=0 \qquad \forall\ t. \tag{17}$$

Hence, while actual consumption C_t is a random walk, measured consumption is not.

(c) From equation (14), the change in measured consumption from $(t,\ t{+}1)$ to $(t{+}2,\ t{+}3)$ depends upon ε_{t+1}, which is known at $t+1$. Thus the change in consumption from one two-period interval to the next is not uncorrelated with everything known as of the first two-period interval. However, it is uncorrelated with everything known as of the two-period interval immediately preceding $(t-2,\ t-1)$.

(d) Let us write C_{t+3} as a function of $C_{t+1},\ \varepsilon$'s.

$$C_{t+3}=C_{t+2}+\varepsilon_{t+3}=C_{t+1}+\varepsilon_{t+2}+\varepsilon_{t+3}. \tag{18}$$

The change in measured consumption from one two-period interval to the next is

$$C_{t+3}-C_{t+1}=\varepsilon_{t+2}+\varepsilon_{t+3}. \tag{19}$$

Covariance between successive changes in measured consumption is:

$$Cov\big[(C_{t+3}-C_{t+1});(C_{t+1}-C_{t-1})\ \big]=Cov\big[(\varepsilon_{t+2}+\varepsilon_{t+3});(\varepsilon_t+\varepsilon_{t+1})\ \big]=0. \tag{20}$$

Thus in this case measured consumption is a random walk.

2 Question 2.

1. Using an Euler equation approach and supposing that the individual has chosen first-period consumption optimally given the information available. Consider a reduction in C_1 of dC and an equal increase in consumption at some future date. If the individual is optimising, then a marginal change of this type does not affect expected utility, hence

$$\begin{aligned} & u'(C_1)\mathrm{d}C = E[u\prime(C_t)|I_1]\mathrm{d}C \qquad \forall\ t = 1, \dots, T; \\ & C_1 = E[C_t|I_1] \qquad \forall\ t = 1, \dots, T. \end{aligned} \tag{21}$$

   The budget constraint is:

$$A_0 + \sum_{t=1}^{T} E[Y_t|I_1] = \sum_{t=1}^{T} E[C_t|I_1]. \tag{22}$$

   Substituting (21) into (22) and dividing by T yields

$$C_1 = \frac{1}{T}\left(A_0 + \sum_{t=1}^{T} E[Y_t|I_1]\right). \tag{23}$$

   Equation (21) implies that (by the law of iterated expectations):

$$\begin{aligned} C_t &= E[C_t|I_{t-1}] + \varepsilon_t \\ & C_{t-1} + \varepsilon_t. \end{aligned} \tag{24}$$

   Consumption, hence, is a martingale (a random walk if we assume ε's are i.i.d.), which means that if a variation in income is predictable, the individual would be better off redistributing consumption across periods (consumption smoothing). This holds if ε is exogenous.

2. For a discussion, see the textbook, pp. 375–380. Here we briefly report a scheme for discussion.

   Campbell, Mankiw suppose that a portion λ of consumers spend their current income, so that variation in consumption is equal to the variation in income for the first group, and equal to the variation in permanent income for the second.

$$\Delta C_t = \lambda(\Delta Y_t) + (1 - \lambda)\varepsilon_t. \tag{25}$$

   Since ΔY_t and ε_t are correlated, OLS estimator is biased upward. Hence, they use the IV approach. Instruments are lagged changes in consumption. Estimates of λ are between 0.42(0.16) and 0.52(0.13). The null is strongly rejected. (Problems: aggregate data, small sample, measurement errors, individuals not observed)

   Shea obtains a coefficient of 0.89(0.46) for households whose wages are covered by long-term union contracts (high degree of predictability). Results are tested for the presence of liquidity constraints (that would imply $C = Y < Y^P$) but there is no evidence that this could be the reason for Shea's results.

3. For a discussion, see the textbook, section 8.6..

4. **(Optional)**

   (a) The government must choose T_1, T_2 so that their present value is equal to the present value of the tax on interest income:

$$T_1 + \frac{T_2}{1+r} = \frac{r}{1+r}\tau(Y_1 - C_1^0). \tag{26}$$

   (b) Suppose the new tax satisfies equation (26). This means that where the individual consumes C_1^0, he pays the same with the new lump-sum tax as he did with the old tax. Thus at C_1^0, the individual has just enough to consume C_2^0 in the second period under both tax schemes. This means that the new budget line must go through $(C_1^0,\ C_2^0)$ just as the old one did. Hence this combination is just affordable.

   (c) First-period consumption must fall. The old budget line had slope $-[1+(1-\tau)r]$; the new budget line is steeper, with slope $-(1+r)$. With savings no longer taxed, giving up one unit of period-one consumption yields more units of period-two consumption $(C_2 = [1+r]C_1)$.

   As discussed in part (b), if the government sets the new lump-sum taxes so that the present value of government revenue is unchanged, the new budget line must go through the original consumption bundle. We have seen in this point, though, that return on saving has increased, hence the old bundle $(C_1^0,\ C_2^0)$ is sub-optimal. Intuitively, the government has set the tax rate so that there is no income effect, only a substitution effect.

3 Question 3.

1. Individuals' demand for assets determine expected returns. Suppose that individuals are symmetrical and consider a marginal reduction in C_t. The resulting saving is used to buy an asset i at price P_t^i, that produces an uncertain stream of payoffs D_{t+k}^i, $k = 1, \ldots$. If the representative individual is optimising, Euler equation must hold:

$$u'(C_t)P_t^i = E_t\left[\sum_{k=1}^{\infty} \frac{1}{(1+\rho)^k} u'(C_{t+k}) D_{t+k}^i\right]. \tag{27}$$

   Solving this expression for P_t^i gives

$$P_t^i = E_t\left[\sum_{k=1}^{\infty} \frac{1}{(1+\rho)^k} \frac{u'(C_{t+k})}{u'(C_t)} D_{t+k}^i\right], \tag{28}$$

   where the term in brackets that multiplies D_{t+k}^i is called the *stochastic discount factor* and represents individual's valuation of future payoffs.

   Assuming quadratic utility and that individuals hold the asset for one period only, we define the return on the asset

$$r_{t+1}^i = \frac{D_{t+1}^i}{P_{t-1}^i}. \tag{29}$$

   Using these assumptions, we can solve equation (27) for expected returns.

$$E_t\big[1 + r_{t+1}^i\big] = \frac{1}{E_t\big[u'(C_{t+1})\big]}\Big[(1+\rho)u'(C_t) + a\mathrm{Cov}_t(1 + r_{t+1}^i;\ C_{t+1})\Big]. \tag{30}$$

This means that the higher the covariance of an asset's payoff with consumption, the higher its expected return must be.

Let us define the return on a risk-free asset as

$$E_t\big[1+\bar{r}_{t+1}]\big] = \frac{1}{E_t\big[u'(C_{t+1})\big]}\Big[(1+\rho)u'(C_t)\Big], \tag{31}$$

and subtracting (31) to (30) gives the expected return premium that an asset must offer relative to the risk-free rate, that is proportional to the covariance of its return with consumption:

$$E_t[r^i_{t+1}] - \bar{r}_{t+1} = \frac{a\mathrm{Cov}_t(1+r^i_{t+1};\ C_{t+1})}{E_t\big[u'(C_{t+1})\big]}. \tag{32}$$

This means that the coefficient from a regression of an asset's return on consumption growth (*consumption beta*) is proportional to the premium.

2. Let us consider the case in which the risky asset is a broad portfolio of stocks and assume CRRA utility. The Euler equation (27) becomes

$$1+\rho = E_t\left[(1+r^i_{t+1})\frac{C^{-\theta}_{t+1}}{C^{-\theta}_t}\right]. \tag{33}$$

Let g^c_{t+1} denote the growth rate of consumption, and omit the time subscripts. Thus we have

$$E[(1+r)(1+g)^{-\theta}] = a+\rho. \tag{34}$$

We take then a second-order Taylor approximation of the left-hand side around $r = g = 0$ and obtain

$$E[r^i] - E[r^j] = \theta\mathrm{Cov}(r^i - r^j,\ g^c). \tag{35}$$

For a discussion of the empiric evidence about this equation, see the textbook, pp. 388–389.

3. (a) If the individual is optimising, the utility cost of the reduction in consumption must be equal to the expected utility benefit:

$$u'(C_t)\mathrm{d}C = E_t\left[u'(C_{t+1})\left(\frac{\mathrm{d}C}{P_t}Y_{t+1} + \frac{\mathrm{d}C}{P_t}P_{t+1}\right)\right], \tag{36}$$

The right-hand side of the equation represents the expected marginal utility times the benefits in $t+1$, i.e. an increase in consumption due to the additional output provided by the additional trees, and an increase in consumption due to the sale of the additional trees at price P_{t+1}.

Informally cancelling out the dC's and substituting the derivatives of the (log) utility gives:

$$\begin{aligned}
C_t^{-1} &= E_t\left[\frac{C^{-1}_{t+1}}{1+\rho}\frac{1}{P_t}\left(Y_{t+1}+P_{t+1}\right)\right]\\
\frac{1}{Y_t} &= \frac{1}{P_t}E_t\left[\frac{C^{-1}_{t+1}}{1+\rho}\left(Y_{t+1}+P_{t+1}\right)\right]\\
P_t &= \frac{Y_t}{1+\rho}E_t\left[\frac{Y_{t+1}+P_{t+1}}{C_{t+1}}\right].
\end{aligned} \tag{37}$$

(b) Given these assumptions, equation (36) can be written as:

$$P_t = \frac{Y_t}{1+\rho} E_t \left[1 + \frac{P_{t+1}}{Y_{t+1}} \right] = \frac{Y_t}{1+\rho} + \frac{Y_t}{1+\rho} E_t \left[\frac{P_{t+1}}{Y_{t+1}} \right]. \tag{38}$$

This holds for all periods, so we can write

$$P_{t+1} = \frac{Y_{t+1}}{1+\rho} + \frac{Y_{t+1}}{1+\rho} E_t \left[\frac{P_{t+2}}{Y_{t+2}} \right]. \tag{39}$$

Substituting (39) into (38) gives

$$P_t = \frac{Y_t}{1+\rho} + \frac{Y_t}{1+\rho} E_t \left[\frac{1}{1+\rho} + \frac{1}{1+\rho} E_{t+1} \left(\frac{P_{t+2}}{Y_{t+2}} \right) \right]. \tag{40}$$

By the law of iterated projections (i.e. $\forall\, x,\ E_t[E_{t+1}[x_{t+2}]] = E_t[x_{t+2}]$,

$$P_t = \frac{Y_t}{1+\rho} + \frac{Y_t}{(1+\rho)^2} + \frac{Y_t}{(1+\rho)^2} E_t \left[\frac{P_{t+2}}{Y_{t+2}} \right]. \tag{41}$$

After repeated substitutions we get

$$P_t = \frac{Y_t}{1+\rho} + \frac{Y_t}{(1+\rho)^2} + \cdots + \frac{Y_t}{(1+\rho)^s} + \frac{Y_t}{(1+\rho)^s} E_t \left[\frac{P_{t+s}}{Y_{t+s}} \right]. \tag{42}$$

Imposing the no-bubbles condition, the price of a tree in period t can be written as

$$\begin{aligned} P_t &= Y_t \left[\frac{1}{1+\rho} + \frac{1}{(1+\rho)^2} + \ldots \right] \\ &= Y_t \left[\frac{1/(1+\rho)}{1 - [1/(1+\rho)]} \right] \\ &= \left[\frac{1/(1+\rho)}{\rho/(1+\rho)} \right] \\ &= \frac{Y_t}{\rho}. \end{aligned} \tag{43}$$

(c) There are two effects of an increase in the expected value of dividends at some future date. The first is the fact that at a given marginal utility of consumption, the higher expected dividends increase the attractiveness of owning trees. This tend to raise the current price of a tree. However, since consumption equals dividends, higher expected dividends in that future period mean higher consumption and thus lower marginal utility of consumption in that future period. This tends to reduce the attractiveness of owning trees, lowering the current prices. In the case of logarithmic utility, these two forces exactly offset each other, leaving the current price of a tree unchanged.

(d) The path of consumption is equivalent to the path of output, then consumption is a random walk if and only if output is. In this model output is exogenously given (by the trees), so we cannot determine its properties.