Performances of O/IR telescopes

Astrophysical Lab.

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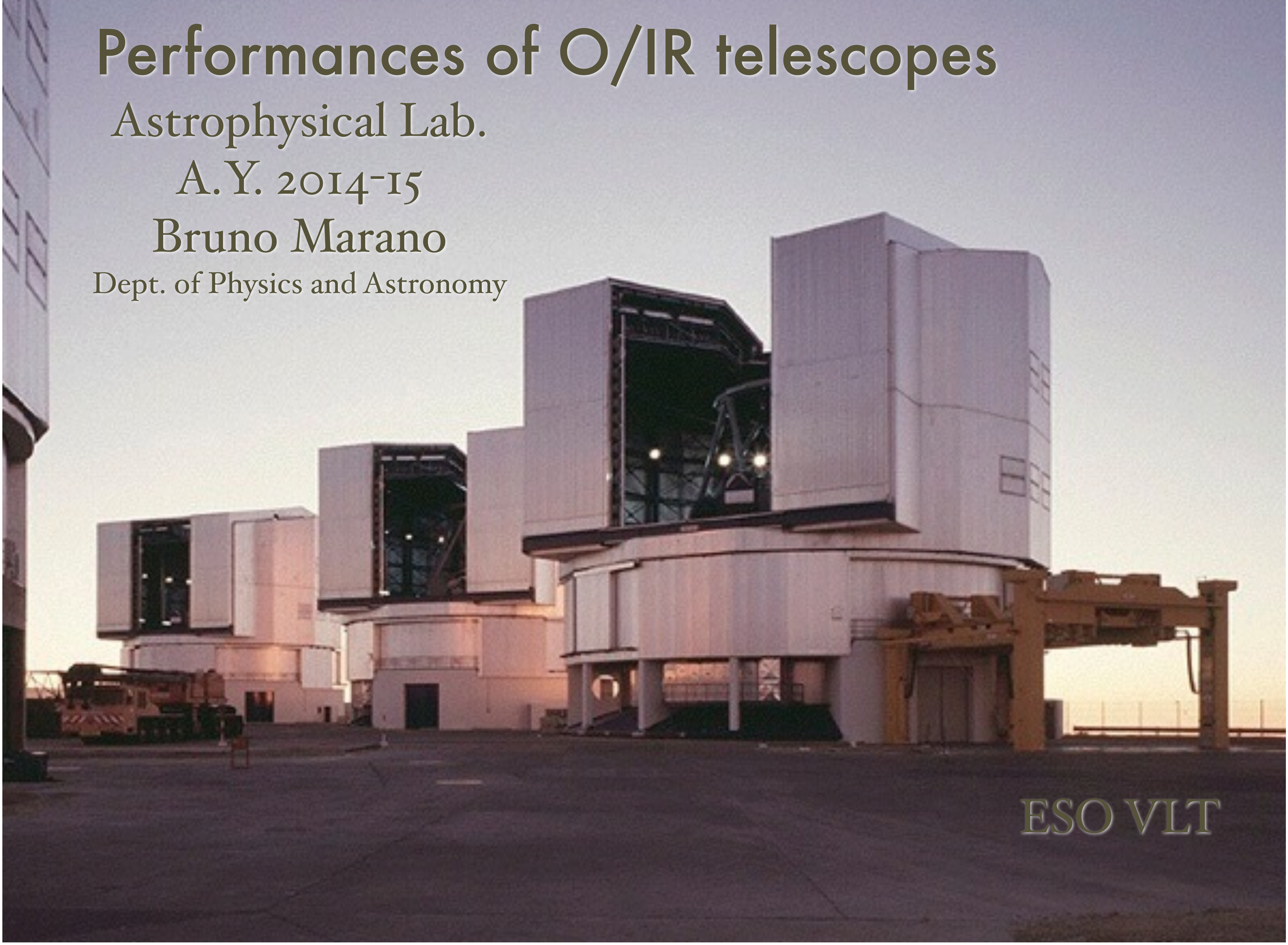


Why do we need large telescopes and/or long exposures? (How do we evaluate a photometric measure?)

- Naively: More photons give better images and more information
- How many photons? How large the telescope?
- What do "better image" and "more information" mean?
- A first (intuitive) approach:

$$\text{sensitivity} \propto N \text{ (no. of detected photons)}$$

$$N \approx n_s D^2 \tau t$$

with: n_s =photon flux, D = main mirror diameter, τ = overall efficiency, t = exp.

A more educated approach: the Signal to Noise ratio (SNR)

In photometry what actually matters is $N \pm \Delta N$, i.e.

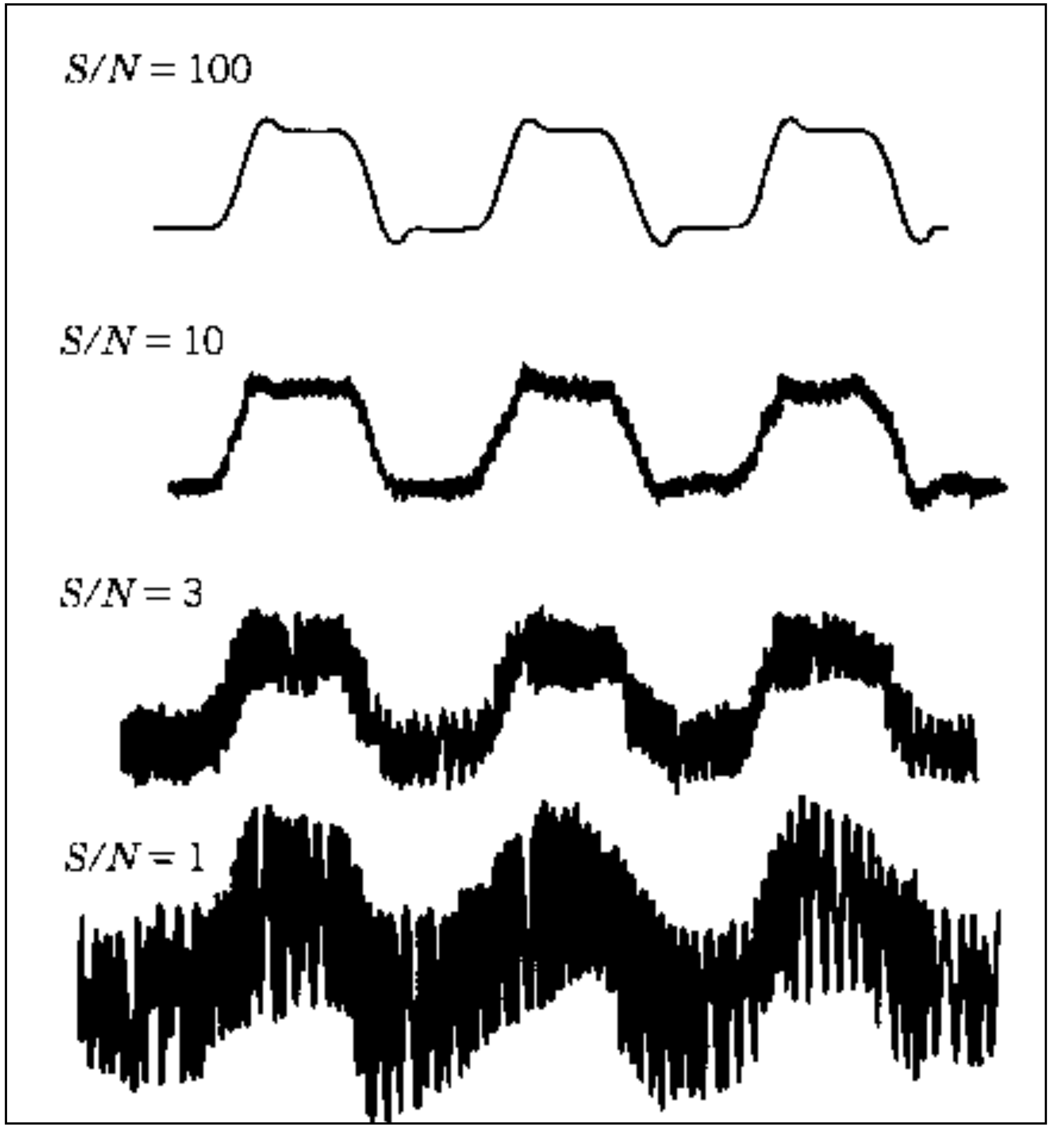
$$N/\Delta N = SNR$$

(1/SNR is the conventional "relative error" of a measurement)

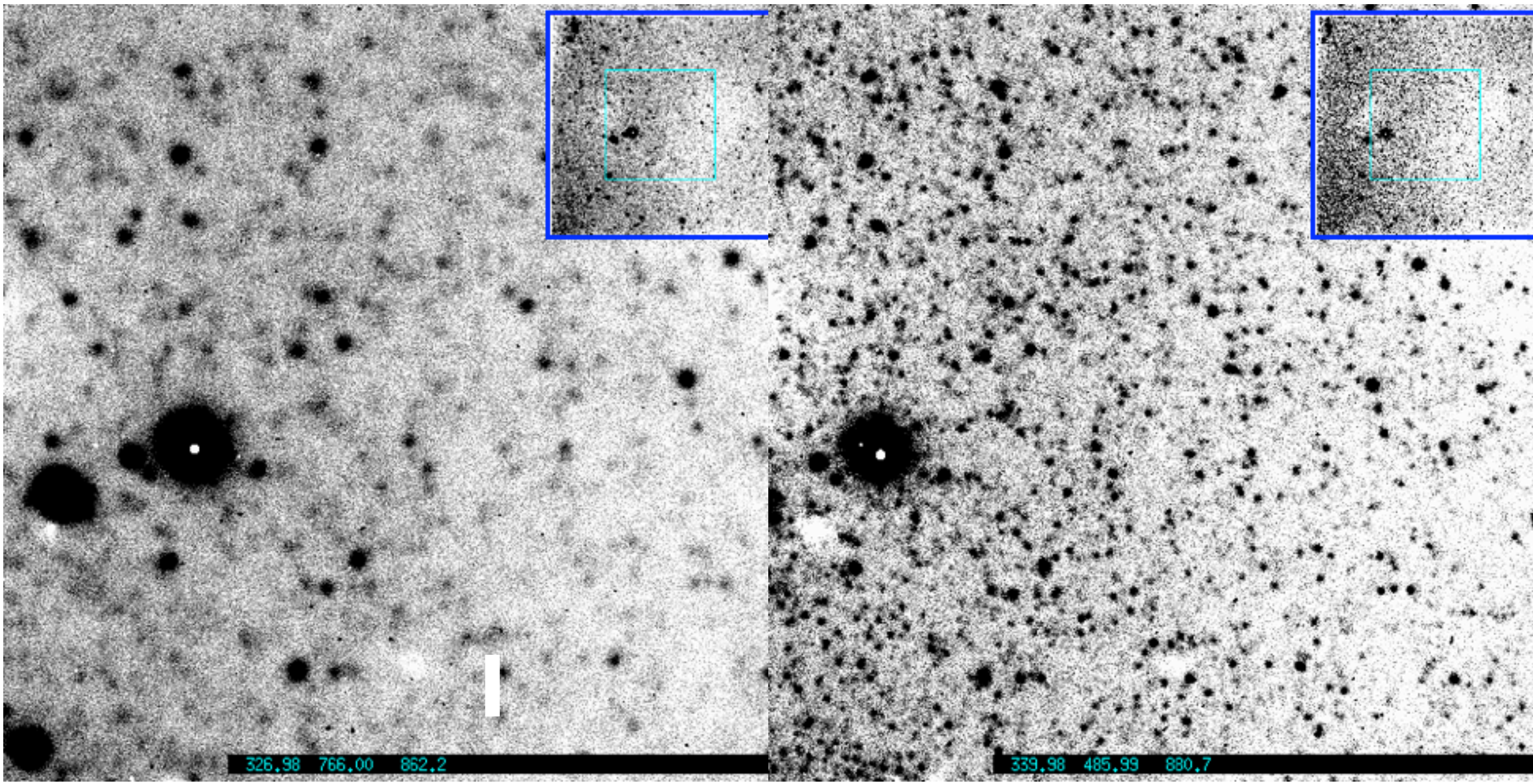
If Poisson statistics applies (best possible case), then

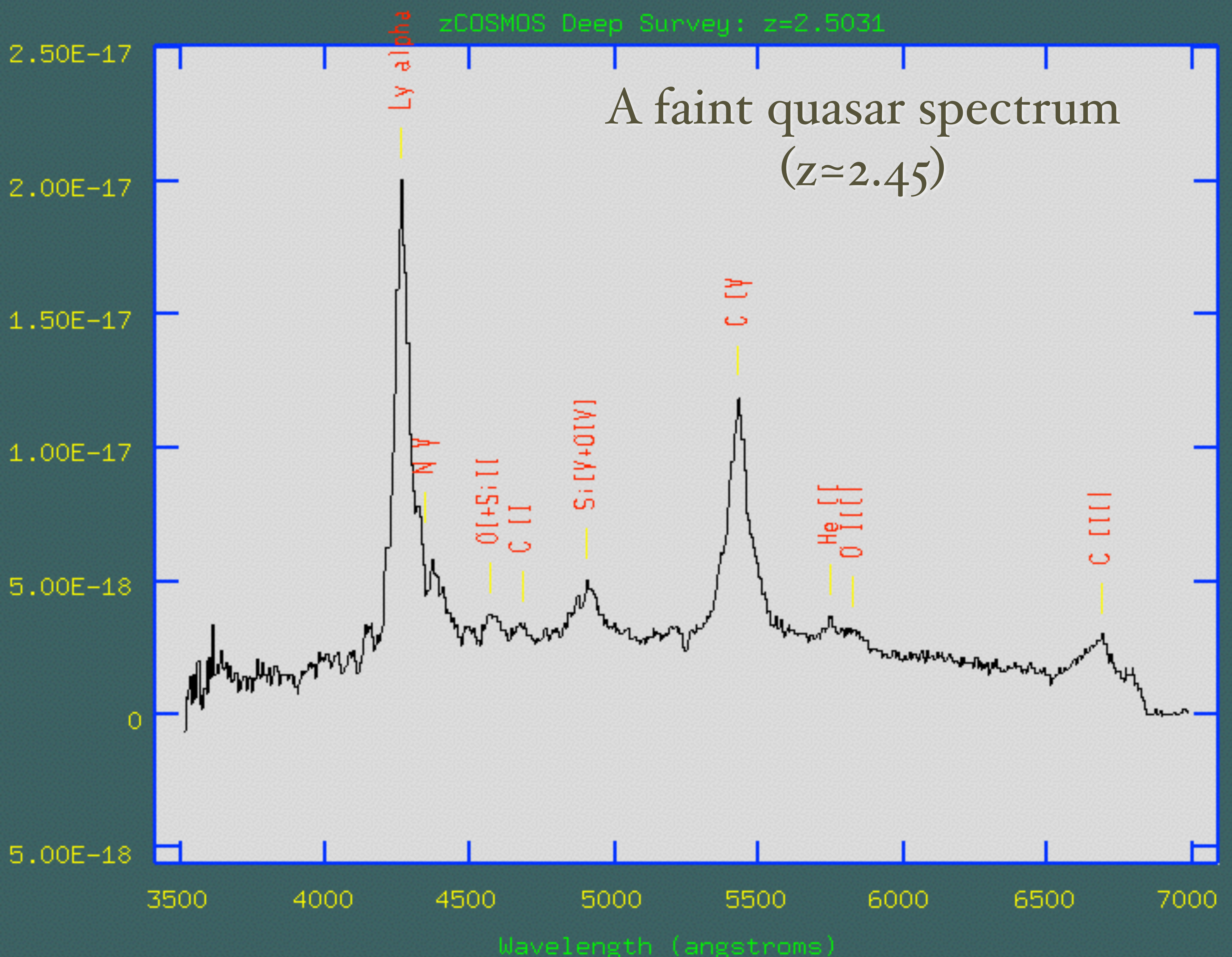
$\Delta N = (N)^{1/2}$ i.e.: $SNR = (N)^{1/2}$

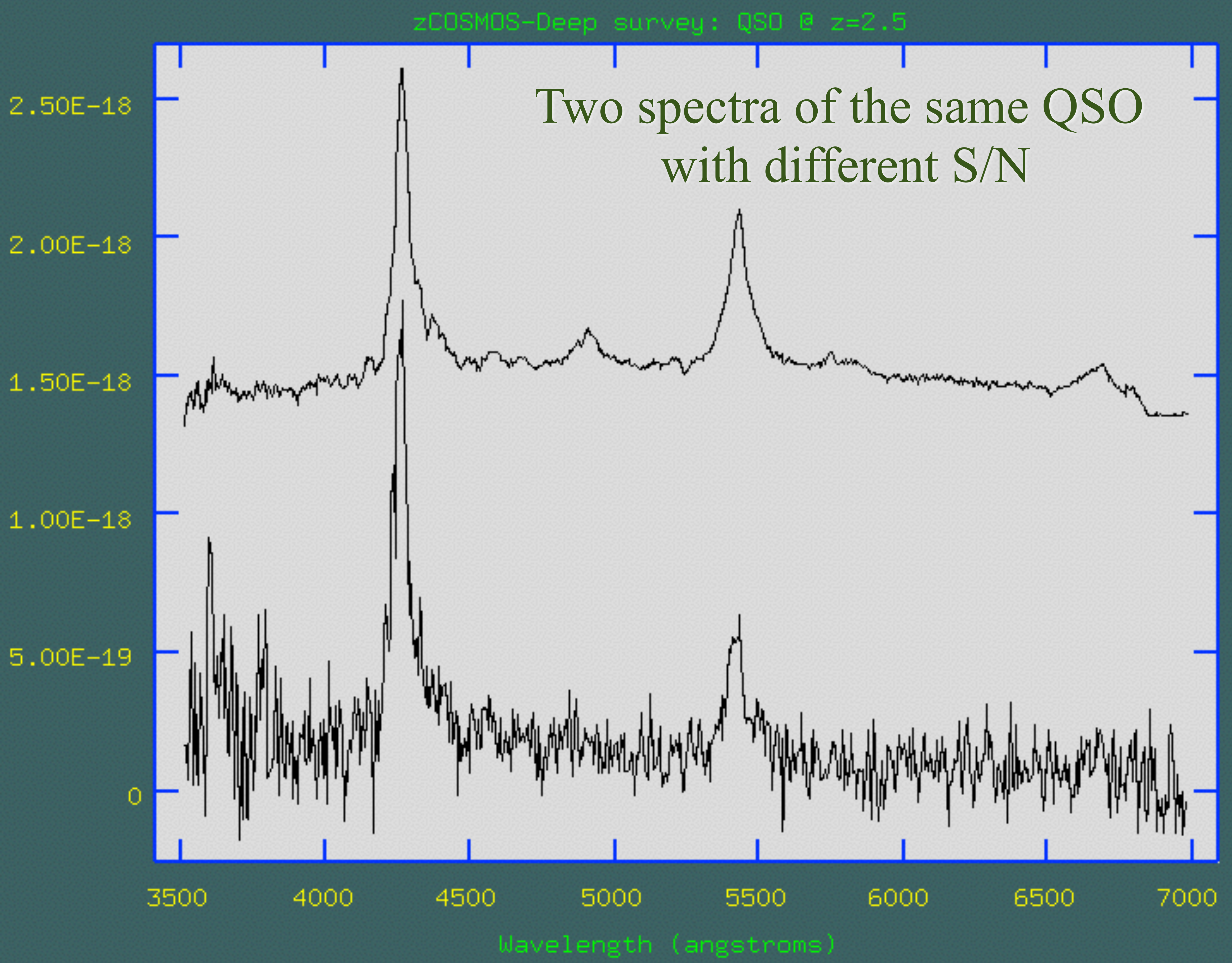
SNR (or S/N)
A non astronomical example
A square wave signal



Two exposures taken with the same telescope, same t, but different "sharpness" of the star images ("seeing")




zCOSMOS Deep Survey: z=2.5031
A faint quasar spectrum
(z≃2.45)
Ly alpha
N V
O I+Si II
C II
Si IV+OIV]
C IV
He II
O III]
C III]
2.50E-17
2.00E-17
1.50E-17
1.00E-17
5.00E-18
0
5.00E-18
3500
4000
4500
5000
5500
6000
6500
7000
Wavelength (angstroms)


zCOSMOS-Deep survey: QSO @ z=2.5
Two spectra of the same QSO
with different S/N
2.50E-18
2.00E-18
1.50E-18
1.00E-18
5.00E-19
0
3500
4000
4500
5000
5500
6000
6500
7000
Wavelength (angstroms)

SNR in astronomical observations

N = number of detected photons

- N_{S+B+D} Star+B+D
- N_B Background
- N_D Dark

From observations and calibrations we obtain:

$$N_S = N_{S+B+D} - N_B - N_D$$

If Poisson statistics applies (no systematics), at best:

$$SNR = N_S / (N_S + N_B + N_D)^{1/2}$$

SNR ESTIMATE

(See, f.i., Schroeder, Astronomical Optics)

$$N_S = n_o \cdot (1-\varepsilon) \cdot \pi/4 \cdot D^2 \cdot \tau \cdot \Delta\lambda \cdot t \approx 0.7 \cdot n_o \cdot D^2 \cdot \tau \cdot \Delta\lambda \cdot t$$

$$N_B = n_b \cdot \pi/4 \cdot \theta^2 \cdot (1-\varepsilon) \cdot \pi/4 \cdot D^2 \cdot \tau \cdot \Delta\lambda \cdot t \approx 0.7 \cdot n_b \cdot \pi/4 \cdot \theta^2 \cdot D^2 \cdot \tau \cdot \Delta\lambda \cdot t$$

$$N_D = C \cdot t + (RON)^2$$

Exposure Time Calculator

With: $n_o = N_o \cdot 10^{-0.4m}$

$(1-\varepsilon) \cdot \pi/4 \approx 0.7$ telescope geometry

τ = global tel+instr. efficiency

n_b= phot/sec/cm²/nm/□”

t =exposure time

C = dark current

$N_o = 10^4$ phot./sec/cm²/nm for a $m_V=0$ star

D = mirror diameter

$\Delta\lambda$ = Band (B, V $\Delta\lambda \approx 75$ nm)

θ = image angular size

RON = read out noise

"Photon limited" case ("bright star"):

$$SNR = N_S / (N_S + \cancel{N_B} + \cancel{N_D})^{1/2} = N_S^{\ 1/2}$$

From $N_S \propto n_o \cdot D^2 \cdot \tau \cdot \Delta\lambda \cdot t$

$$SNR = N_S^{\ 1/2} = [\ N_o \cdot 10^{-0.4\,m} \cdot \tau \cdot \Delta\lambda]^{1/2} \cdot t^{1/2} \cdot D$$

$$t \propto D^{-2}$$

Exposure time needed to reach a given SNR goes as the **inverse square of D**

"Background limited" case

$N_B >> N_S$, $N_D = 0$

$$SNR \cong N_S / (N_B)^{1/2}$$

From $N_S \propto n_o \cdot D^2 \cdot \tau \cdot \Delta\lambda \cdot t$

and $N_B \propto b \cdot \pi/4 \cdot \theta^2 \cdot D^2 \cdot \tau \cdot \Delta\lambda \cdot t$

$$SNR \propto n_o (\tau \cdot \Delta\lambda/b)^{1/2} \cdot D/\theta \cdot t^{1/2}$$

Then $$t \propto (D/\theta)^{-2}$$

The image size θ matters as much as D^{-1}!

"Background and Diffraction limited" case

Diameter of the image of a point source

$\theta \approx 1.22\ \lambda/D \propto D^{-1}$ (Airy disc),

and

$S/N \propto (D/\ \theta) \bullet t^{1/2}$

$$\mathbf{t \propto (D/\theta)^{-2} \propto D^{-4}}$$

Angular resolution is a key factor

(I am a bit overselling, real life is harder.. take $t \propto D^{-4}$ as an ideal limit !)

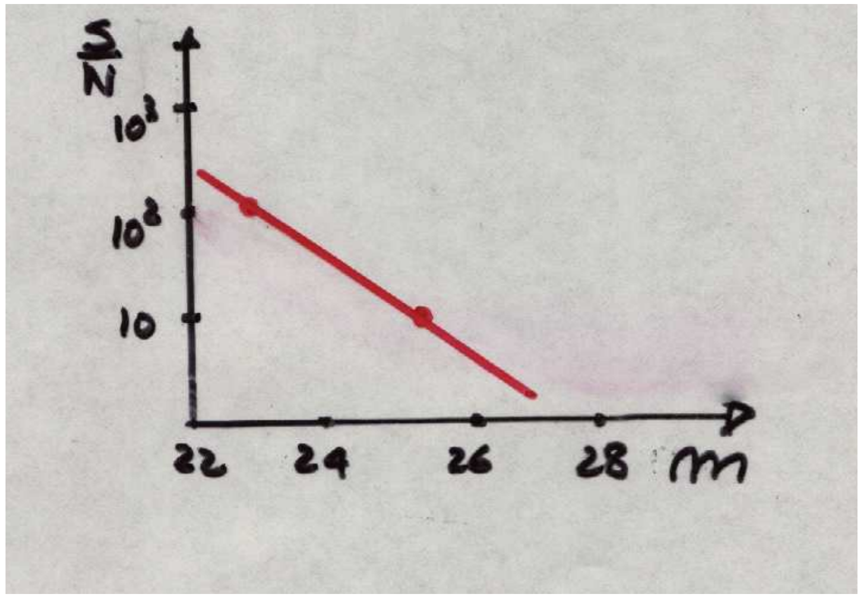
Costruiamo un “simulatore di posa”

(caso “Background limited”)

$$S/N = [\,N_o \cdot \tau \cdot \Delta\lambda\,]^{1/2} \cdot D/\theta \cdot 10^{-0.4\,m + 0.2\,b} \cdot t^{1/2}$$

$$\log(S/N) = 2.74 + \log D - \log\theta + 0.5 \log t - 0.4 m + 0.2 b$$

Se, per esempio, $D=400$ cm., $\theta=1''$, $b=22$ mag/ $''$, $t=10^3$



$$\log(S/N) = 11.24 - 0.4\,m$$

Esempi di Exposure time calculators (ETC) in rete:
1) Telescopio Nazionale Galileo:
http://www.tng.iac.es/observing/
2) European Southern Observatory:
http://www.eso.org/observing/**etc**/

Limiti di detezione per due telescopi a terra di 4m e 10m di diametro e HST.
Posa: 2400 sec. (da Schroeder: Astronomical Optics - Academic Press)

17.4. Detection Limits: Stellar Photometry