Fair Routing in Mesh Networks

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Joint work with
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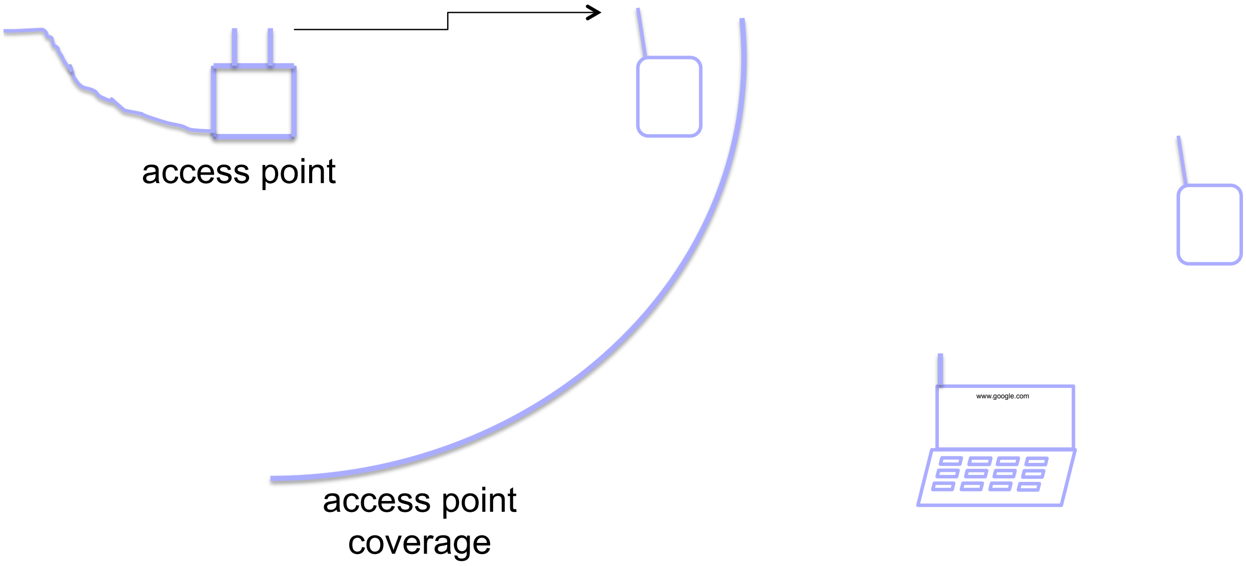
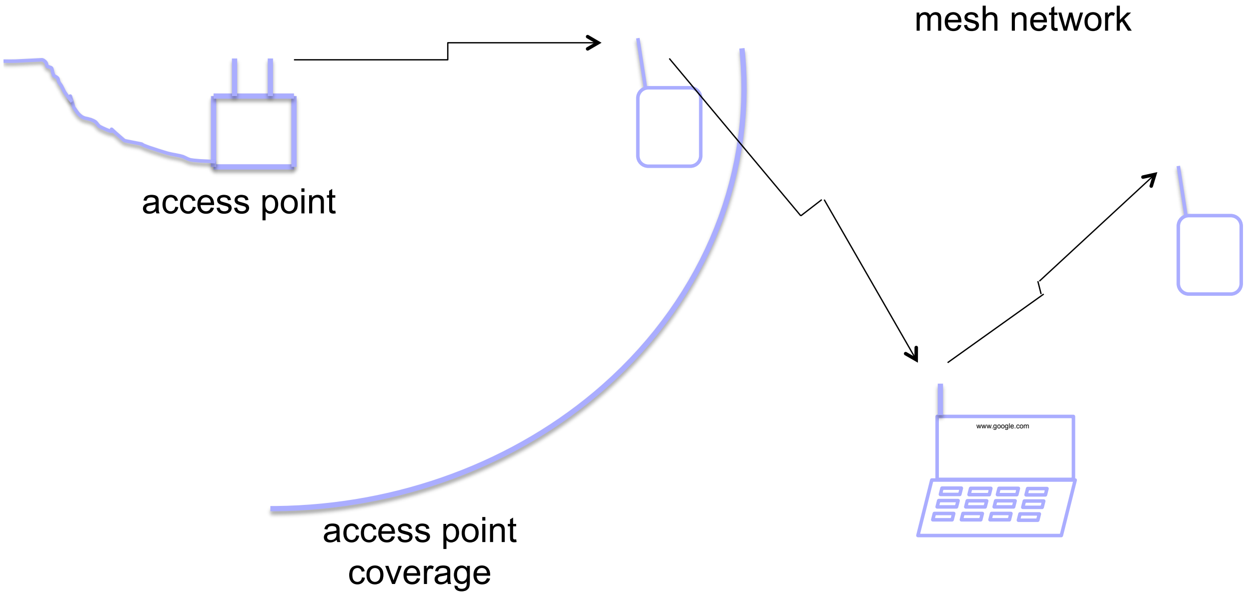
Outline

- Motivation and Problem Description
- How to measure Fairness
- A Distributed, Quick and Fair Routing Algorithm
- A Centralized Approach
- Computational Experiments
- Conclusions

Motivation



Motivation



Mesh networks

Set of nodes that communicate without using any preexisting infrastructure. Nodes participate to the network because they need to send traffic to other nodes, and direct communication is expensive or impossible. We consider nodes that use **radio links** to communicate.

Main characteristics:

• Lack of infrastructure and centralized control;

• Nodes can be owned by different entities;

• Only local information is available to nodes;

• Nodes need to exchange information, and connectivity requires intermediate nodes to forward;

• Radio links are feed by nodes' batteries which are limited.

Mesh networks

Examples of mesh networks:

• laptop and personal devices (e.g., smart phones) networks;

• sensor networks;

• disaster recovery networks;

• military battlefield networks;

• etc.

We are particularly interested in the case where the network nodes are owned by different entities

Mesh networks

Why nodes stay in the network?

Nodes get a benefit from the network, because they obtain connectivity. At the same time, being in the network is a cost because nodes have to forward other nodes' traffic, and this consumes the battery (and it may be difficult to recharge it).

How should we route the traffic in the network?

Game theory: the only Nash equilibrium is that no one forwards other nodes' traffic, thus there is no communication (unless a "payment" mechanism is designed).

Mesh networks

However, people (sometimes) cooperate if they feel that the system is fair, i.e., they may decide to use their energy for forwarding if they know that the network will forward their traffic.

In any case, a node could decide to switch-off his radio if it feels that the network is unfairly draining its battery with retransmissions of other nodes' traffic. This is highly undesirable because the network efficiency and connectivity would be jeopardized.

Thus, fairness should be ensured when routing traffic in the network.

Notation

We represent the network as a graph $G=(V,A)$, where V is the set of nodes, and A the set of available links (arcs). Let:

p_{ij} - be a positive cost associated to arc (i,j), which is the energy that node i spends when sending a unit of information to j;

p_{ij} - can be either fixed for all $(i,j) \in A$, or can be a superlinear function of the distance from i to j;

C_i - the available battery energy of node i;

E_i - the total energy that node i spends in the transmissions;

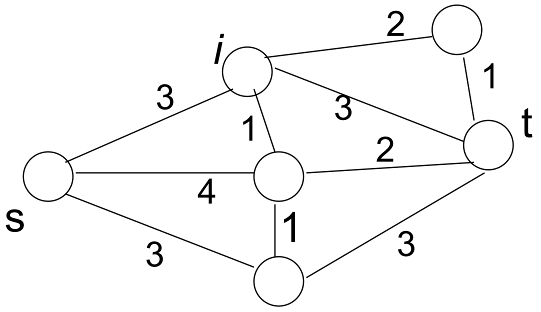
r_{st} - be the amount of traffic that node s needs to send to node t, and $K= \{(s,t): r_{st} > 0\}$.

Notation

In order to keep the analysis protocol independent, we use flows to denote how information travels across the network.

We require all the traffic to be transmitted, but traffic from one origin s to the destination t can be split on more than one path.

$r_{st}=2$



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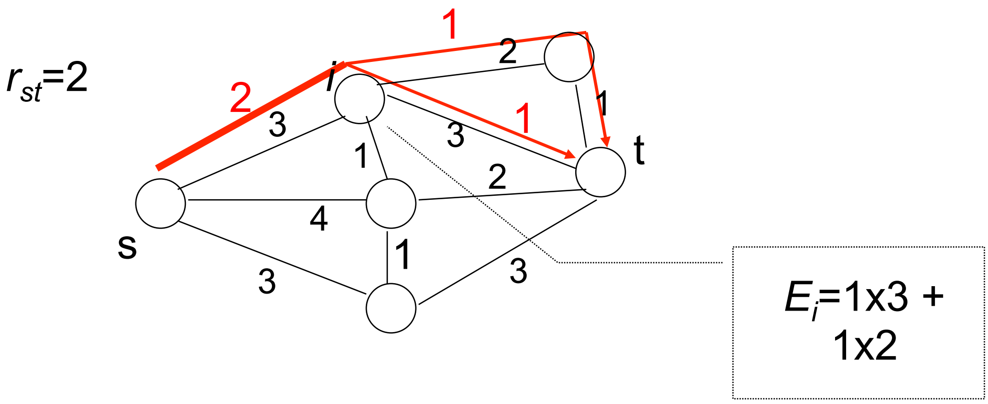

Flow model

Thus, if $x_{i,j}^{s,t}$ is the amount of traffic originated in *s*, with destination *t*, flowing on arc *(i,j)*, we are looking for feasible routings satisfying:

$$\sum_{(i,j)\in A} x_{i,j}^{s,t} - \sum_{(j,i)\in A} x_{j,i}^{s,t} = \begin{cases} r_{s,t} \text{ when} & i = s \\ -r_{s,t} \text{ when} & i = t \\ 0 \quad \text{otherwise} & \end{cases} \qquad (1)$$

$$E_i = \sum_{(i,j)\in A} p_{i,j} \left(\sum_{(s,t)\in K} x_{i,j}^{s,t} \right) \qquad i \in V \qquad (2)$$

$$E_i \le C_i \qquad i \in V \qquad (3)$$

$$x_{i,j}^{s,t} \ge 0 \qquad (i,j)\in A, (s,t)\in K \qquad (4)$$

System optimum

We denote by *System Optimum (SO)* the solution which minimizes the total energy spent by the network. *SO* can be computed by solving the following *LP*:

$$\min \sum_{i \in V} E_i$$

(1)
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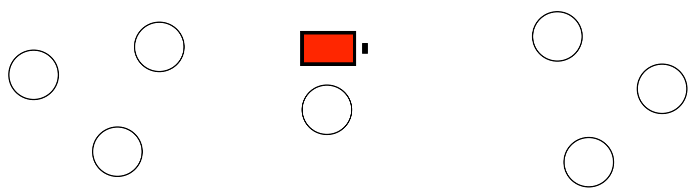
However, system optimum can be very unfair towards network nodes.



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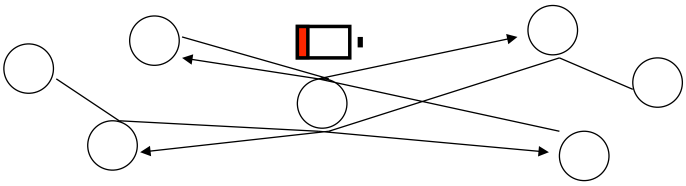


How to measure fairness?

Nodes participate to the network because they have an utility.

Def. – the utility u_i that node *i* has from being in the network is the ratio between the energy $E^{net\text{->}i}$ that the network spends for forwarding *i*'s traffic and the energy $E^{i\text{->}net}$ that node *i* spends for forwarding the traffic of other nodes in the network:

$$u_i = E^{net\text{->}i} / E^{i\text{->}net}.$$

Def. – the fairness φ of a network is defined as the minimum utility of one of the network nodes:

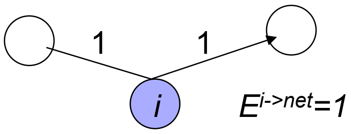
$$\varphi = \min_{i \in V} u_i.$$

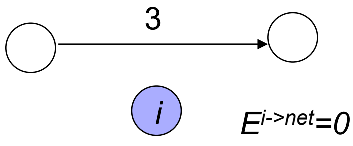
According to the above definition, $\varphi \in [0,1]$

How to measure fairness? (ctd.)

If not careful, an effort to make a routing more fair can result in solutions that are unnecessarily inefficient, or even worst, include non simple paths. The utility $u_i = E^{net\text{->}i} / E^{i\text{->}net}$ of node *i* can be increased:

by reducing the contribution $E^{i\text{->}net}$ of node *i* to the network;





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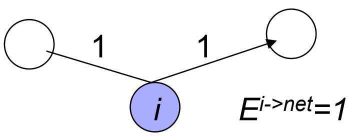

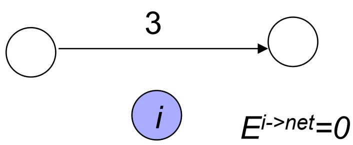

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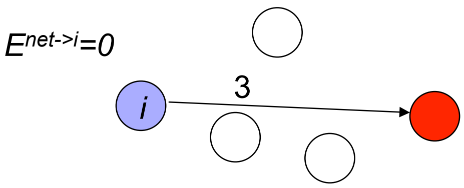

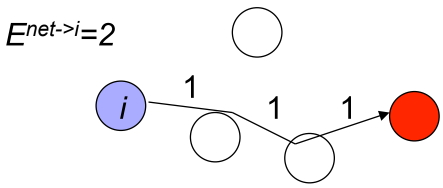

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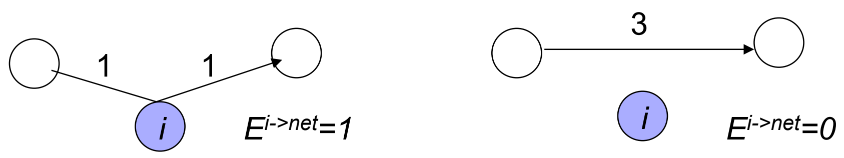

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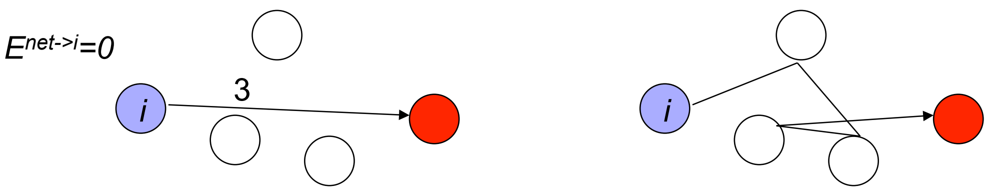
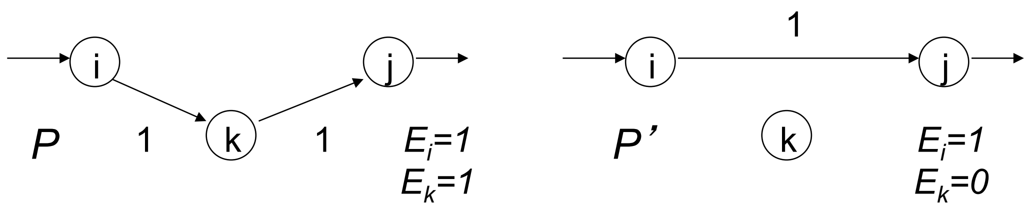


How to measure fairness? (ctd.)

Def: - We say that a path P' *dominates* a path P if P' visits a subset of the nodes visited by P, and all these nodes spend equal or less energy when P' is chosen. Obviously, if a dominating path P' exists, it makes more sense to use it instead of the path P.



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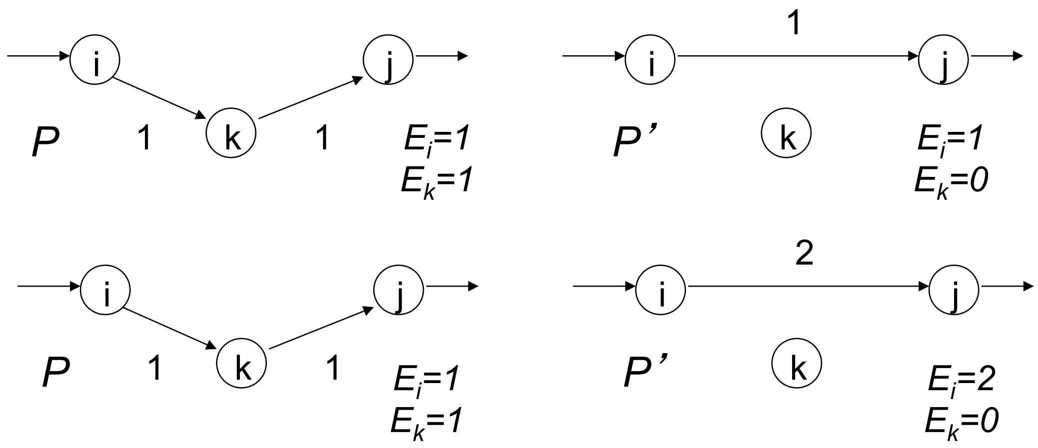


A quick and fair decentralized algorithm

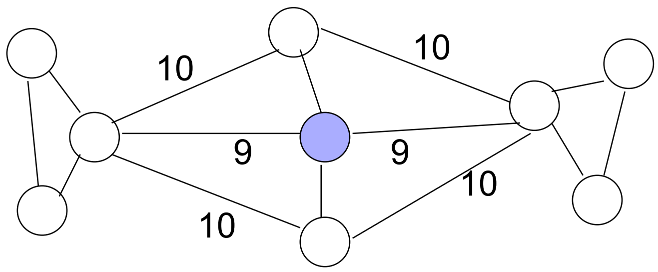
We assume that the transmitting protocol in use can compute shortest paths among nodes in the network, either exactly or approximately.

We want to design a routing algorithm which

- produces fair routings;
- does not require to know in advance the traffic demand *K* (online);
- uses only local information.

A quick and fair decentralized algorithm

The decentralized algorithm *DIST ONLINE ALG* works by periodically modifying the arcs costs according to the current utility of the nodes: the use of forwarding nodes with low (resp. high) utility is discouraged (resp. encouraged):



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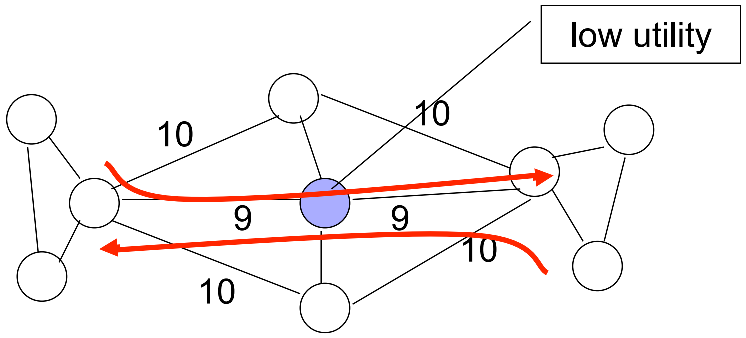

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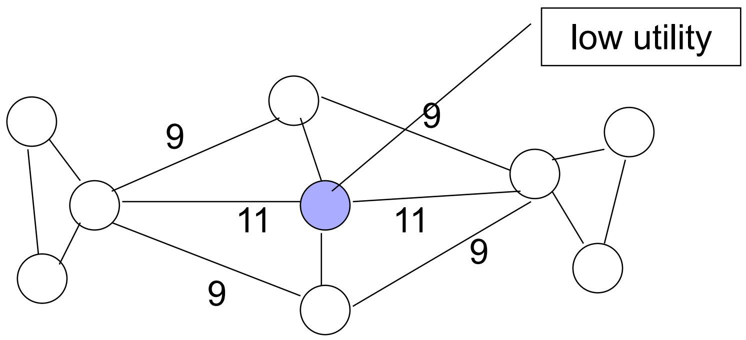

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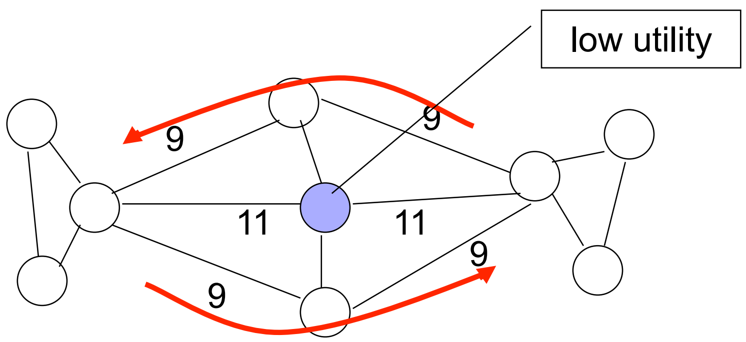

A quick and fair decentralized algorithm

DIST ONLINE ALG

with a given frequency *F*:

- $\forall (i,j) \in A$ *set* $p'_{ij} = p_{ij} / u_i^{\beta}$
- $\forall$ *st* $\in K$, route traffic from *s* to *t* through *shortest path st* computed w.r.t. costs p'_{ij}

where β is a positive parameter.

(Additional strategies are implemented to avoid using nodes with very low battery)

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Theorem: DIST ONLINE ALG always builds non dominated paths with respect to the original costs p_{ij}

A centralized approach as a benchmark

Suppose:

- We know the whole traffic demand *K* in advance;
- We have a full picture of the network (topology and batteries).

Then, we can impose the fairness *Φ* that we want to reach as a constraint and look for the most efficient routing by solving the following *LP*:

$$\min \sum_{i \in V} E_i \qquad (1)$$

$$u_i \geq \Phi \quad i \in V \qquad (2)$$

$$E_i \leq C_i \quad i \in V \qquad (3)$$

$$flow \quad conservation \qquad (4)$$

$$x_{i,j}^{s,t} \geq 0 \quad (i,j) \in A, (s,t) \in K \qquad (5)$$

A centralized approach as a benchmark

What we obtain is a *benchmark* that can be used to asses the performance of *DIST ONLINE ALG*. Actually, not only we are using information which is normally *not available*, but the *LP* solutions may contain *dominated* paths and even contain *cycles*.

Can we compute optimal routings composed by non-dominated path?

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*Theorem: Deciding if there exists a routing that only uses **simple paths** and that has a fairness value bigger than or equal to Φ is NP-hard, for 0 < Φ ≤ 1.*

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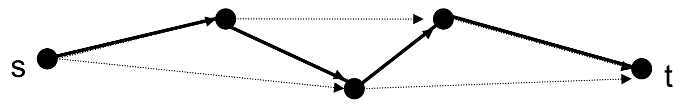
NP-hardness

(Sketch of) Proof - NP-hardness follows from reduction of the Hamiltonian Path problem: given a directed graph G, does it exist a s-t path visiting all nodes?



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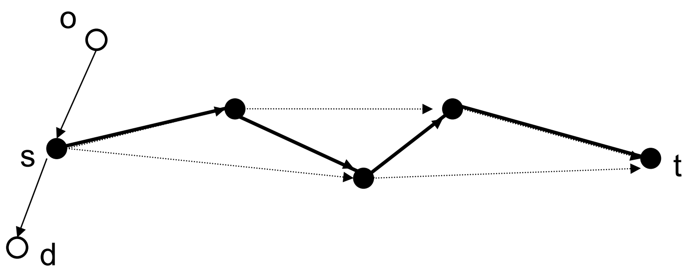


Consider an instance of the Hamiltonian Path problem on a graph *G* with *n* nodes.
Add nodes o, d, arcs o->s, s->d, and let all arcs have cost 1, $r_{od}=n-2$, $r_{st}=1$.

$E^{s\text{->}net}= n-2$. Then, $u_s=E^{net\text{->}s} / E^{s\text{->}net}= 1$ if and only if the s-t path visits $n-2$ intermediate nodes, i.e., the *s-t* path is Hamiltonian.

By adding artificial nodes and demands, we can make all nodes having $u=1$, get $\varphi = 1$, and make the *s-t* path non dominated […]

Computational experiments

We want to investigate the following issues:

• Solutions of *DIST ONLINE ALG (OL)* are significantly more fair than system optima *(SO)*. The amount of extra energy of fair *OL* solutions is limited;

• The performance of *DIST ONLINE ALG*, both with respect to energy consumption and fairness, is comparable with the benchmarks represented by the *LP* solutions;

• *DIST ONLINE ALG* is particularly beneficial to nodes in central locations that would unfairly spend too much energy under a system optimum, where all nodes would tend to use them for retransmitting their traffic.

Computational experiments

Experimental setting:

Topology - square Networks of 20 nodes with uniform random distribution.

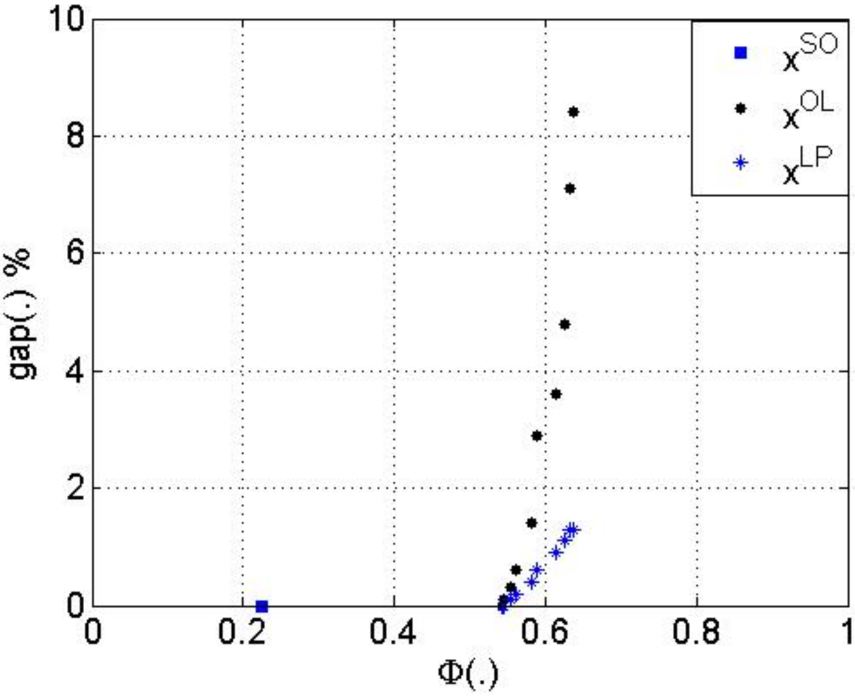
Traffic – all to all traffic pattern (many other traffic patterns considered with similar outcomes).

Arcs costs (energy needed to transmit):

- constant for all arcs;
- quadratically increasing with the distance.

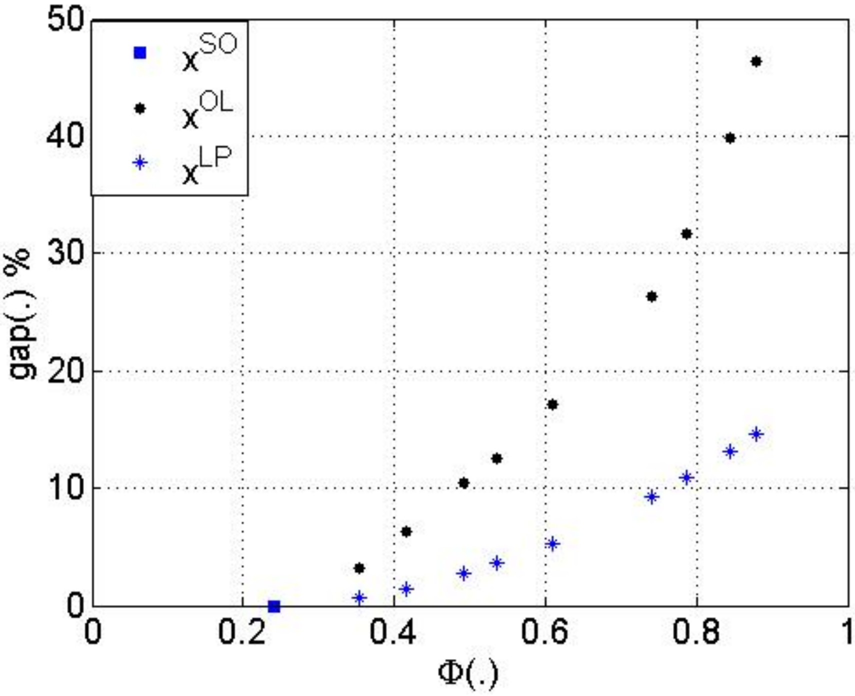
Battery constraint is not active.

Energy and Fairness: constant arc cost



$p'_{ij} = p_{ij} / u_i^{\beta}$

Energy and Fairness: quadratic arc cost



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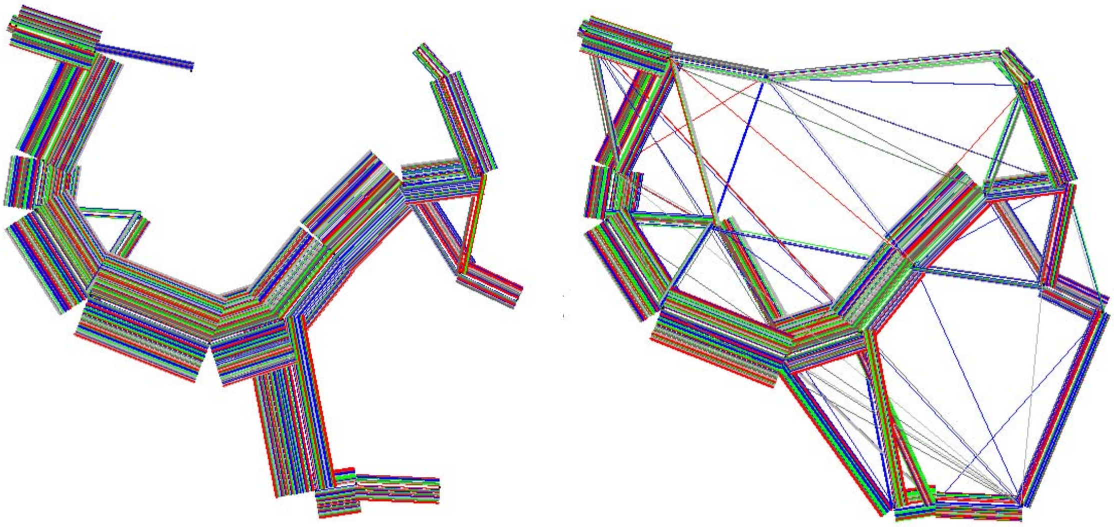
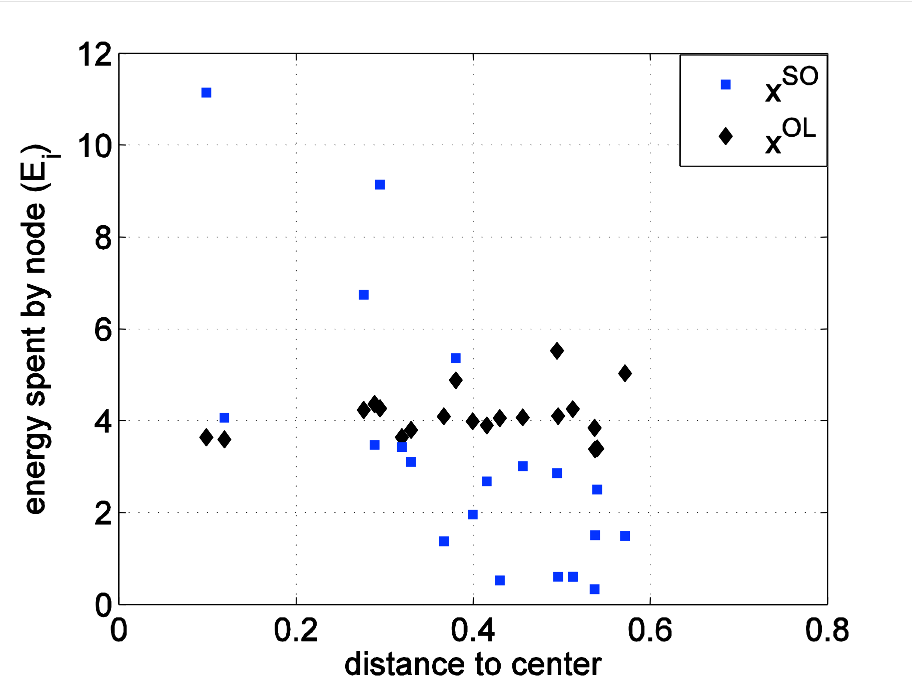
Flow distribution



Nodes energies and distance to center



Conclusions

We proposed a ***measure of fairness*** for routing in wireless mesh networks, and an ***algorithm*** for the routing of information. The algorithm is:

- ***online***

- ***fair***

- ***distributed***

Searching for fair solutions can bring to meaningless routing.
The algorithm that we propose always builds routings which are reasonable.

The algorithm performs well with respect to a centralized and off line benchmark, which is computed through a LP model.