

Real-time Capable Nonlinear Model Predictive Controller Design for The Upper Stage of a Launch Vehicle

By Yunus Emre ARSLANTAS¹⁾ and Thimo OEHLSCHLÄGEL¹⁾

¹⁾The Institute of Space Systems, German Aerospace Center (DLR), Bremen, Germany

In this paper, a real-time capable Nonlinear Model Predictive Controller (NMPC) is implemented for the attitude control of an upper stage launch vehicle with liquid propellant. A mass spring model is used as an analogy to simulate the disturbance generated by the sloshing propellant. For the implementation of the NMPC, an optimal control problem (OCP) is defined with finite time horizon. The objective function is minimized while satisfying constraints on the control inputs. The resulting OCP is transcribed using single shooting method to parametrize the control inputs using uniform discretization points. The continuous control inputs are obtained by linear interpolation. A dedicated discretization algorithm in FORTRAN is coupled with a solver which used quasi-Newton algorithm to generate solutions fast. Approximation of the Hessian matrix is used to reduce computational requirements. Furthermore, the algorithm can perform parallel computation of the derivatives of the objective function with respect to optimization variables. This results in a real-time capability of generating solutions in the order of milliseconds for each iteration. The algorithm is applied for attitude maneuver and disturbance rejection for the upper stage of a launch vehicle.

Key Words: Upper stage, Nonlinear model predictive controller, quasi-Newton method

Nomenclature

x	: State vector	\tilde{f}	: State vector field
u	: Input vector	\tilde{g}_M	: Moment input map
t	: Time	\tilde{g}_F	: Force input map
X	: State space	LV	: Launch vehicle
U	: Input space	NLP	: Nonlinear programming
I	: Time interval	NMPC	: Nonlinear model predictive controller
\mathcal{J}	: Objective function	OCP	: Optimal control problem
\mathcal{U}	: Admissible input space		
Ω	: Terminal region constraint	Subscripts	
D	: Stage cost	0	: Initial
E	: Terminal cost	f	: Final
N	: Number of discretization points	p	: Prediction
B	: Hessian matrix	c	: Control
\mathbf{L}^∞	: Lebesgue space	i	: Discrete form
Q	: State performance weight	l	: Lower bound
R	: Input performance weight	u	: Upper bound
Δx	: Perturbation step	Superscripts	
α	: Continuous function	m	: Dimension of the input space
∇f	: Gradient of a function	n	: Dimension of the state space
O	: Order	∞	: Infinity
m_s	: Mass of rigid body		
m_p	: Mass of propellant		
L	: Moment arm		
l	: Length of pendulum		
J	: Moment of inertia		
F	: External force		
M	: Moment		
z	: Optimization variable		
θ	: Pitch angle of rigid body		
ψ	: Angle of fluid surface level		
w	: Penalty coefficient for terminal cost		

1. Introduction

Within the last decade, space science and technology have become an essential part of daily activities. As a result, there is an increase in demand for launching various payloads for different purposes. One of the challenges in inserting a payload into a desired orbit is the precise attitude control of the upper stage of the launch vehicle (LV). Most LVs use liquid propellant for the last stage due to various advantages of throttleable engines. Major drawback of these liquid propellant engines in terms of attitude control is the disturbance generated during the maneuver of the vehicle caused by the sloshing effect of the propellant. This paper recognizes the sloshing problem and proposes an NMPC for attitude control of an upper stage LV.

Among the available controller design schemes for systems with nonlinear dynamics, NMPC has been implemented in real world problems. For instance, it is used in chemical process control and various aerospace applications^{1) 2)}. NMPC has several advantages like its straightforward implementation, capable of handling constraints on inputs as well as states, and its ability to stabilize the system despite model mismatch or external disturbances. One drawback of NMPC is that there is no guarantee to find a solution for the associated optimization problem. The main disadvantage of this method is the demanding necessity for computational resources while for each iteration a related OCP has to be discretized and solved. Therefore, real-time application of NMPC is coupled to the advancements in computing technology. Consequently, NMPC related studies are gaining momentum in line with these advancements. Review of NMPC methods³⁾, studies about computational aspects of NMPC⁴⁾ as well as robustness and stability properties^{5) 6)} are available in the literature.

There are various controller implementations for the upper stage LV. Most of these applications are based on linear control methods^{7) 8)}. There are also adaptive based methods⁹⁾ and controllers with nonlinear dynamics¹⁰⁾.

In this paper, a real-time capable NMPC controller for the upper stage of a LV is developed. In Section 2, the NMPC is formulated considering the objective function, equality and inequality constraints for states and control inputs. The methodology for discretizing the infinite dimensional OCP into an NLP is introduced. Details of the solution for the associated NLP using BFGS method are also provided in this section. Section 2 also includes the parallel computation of derivatives of the objective function with respect to discrete control inputs using finite difference method. The dynamical model of the LV is presented in Section 3. Section 4 includes the numerical results for the simulations of disturbance rejection and attitude control of a large angle maneuver scenarios. Finally, Section 5 concludes the paper with a short summary.

2. Methodology

The NMPC problem is formulated by solving a series of finite-horizon OCPs subject to system dynamics and constraints involving states and controls. Based on the full state information at time t_0 , the controller predicts the future dynamic behaviour of the system over a prediction horizon H_p for a time interval $[t_0, t_0 + t_p]$. Then, it determines the control inputs in order to minimize the objective function for the desired performance. The control inputs are then applied over a given control horizon H_c within the time interval $[t_0, t_0 + t_c]$ where $t_c \leq t_p$. The principal model of NMPC is given in Fig. 1.

2.1. Formulation of NMPC

Consider the nonlinear dynamical system with state space $X \subset \mathbb{R}^n$ and the input space $U \subset \mathbb{R}^m$

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)), \quad t \in I = [t_0, t_f] \\ x(t_0) &= x_0 \in X \end{aligned} \quad (1)$$

The control input is contained in the space of admissible control signals; that is $u(t) \in \mathcal{U}_{ad} = L^\infty([t_0, t_f], U)$. For each point in time, the solution or state trajectory $x(\cdot) : \mathbb{R}_+ \rightarrow X$ of Eq.(1)

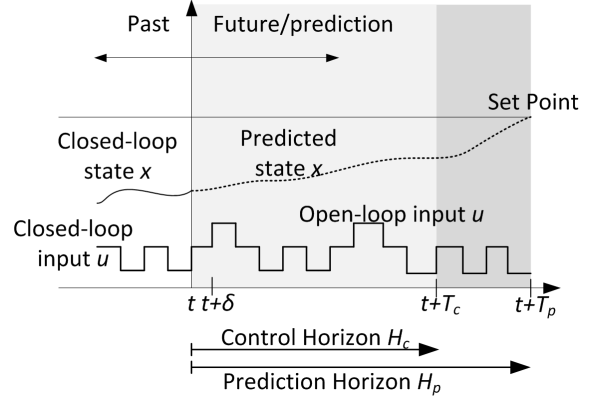


Fig. 1. The principle model of NMPC

is an element of the state space $x(\cdot) \in X$. The finite horizon open-loop OCP is formulated as

$$\begin{aligned} \min_{u \in \mathcal{U}_{ad}} \quad & \mathcal{J}(x_0, x(t_f), u(t); t_p) \\ \text{s.t.} \quad & \dot{x}(t) = f(x(t), u(t)), \quad x(t_0) = x_0 \\ & x(t) \in X \subseteq \mathbb{R}^n, \quad \forall t \in [t_0, t_0 + t_p] \\ & x(t_0 + t_p) \in \Omega \end{aligned} \quad (2)$$

with inequality constraints

$$\begin{aligned} x_L &\leq x(t) \leq x_U \\ u_L &\leq u(t) \leq u_U \end{aligned} \quad (3)$$

where $\dot{x} = f(x(t), u(t))$ is the equality constraint to be satisfied. The objective is to minimize the cost function of the following form

$$\mathcal{J}(x_0, x(t_f), u(t); t_p) = \int_{t_0}^{t_0+t_p} (\Delta u^T R \Delta u + \Delta x^T Q \Delta x) dt + E(x_f) \quad (4)$$

where $\Delta u = (u - u_{ref})$, $\Delta x = (x_f - x_{ref})$. Similarly, x_{ref} and u_{ref} are the desired constant reference values. The cost functional \mathcal{J} is defined in terms of the stage cost (the first term in Eq.(4)), which specifies the performance of the controller and terminal cost (the latter term in Eq.(4)).

A quadratic form for stage cost is used. The positive definite matrices Q and R are the weights for the deviations of states and control inputs from desired values. The state information $x(t_0)$ enters the system with initial states. In this paper, it is assumed that full state information is available. The implementation of NMPC could be summarized as follows

1. Obtain the measurements x_0 of the system.
2. Calculate the optimal inputs minimizing the cost function in Eq.(4) over prediction horizon H_p .
3. Implement the optimal inputs over the control horizon H_c and get new measurement x_0
4. Provide optimal inputs as initial guess for the next iteration.
5. Continue with (2) until the predetermined number of iterations are completed.

Fig. 3. shows the flow of the algorithm. In this study, the stopping criteria is determined as the maximum number of iterations. For each iteration, the states are propagated one second in the given time domain.

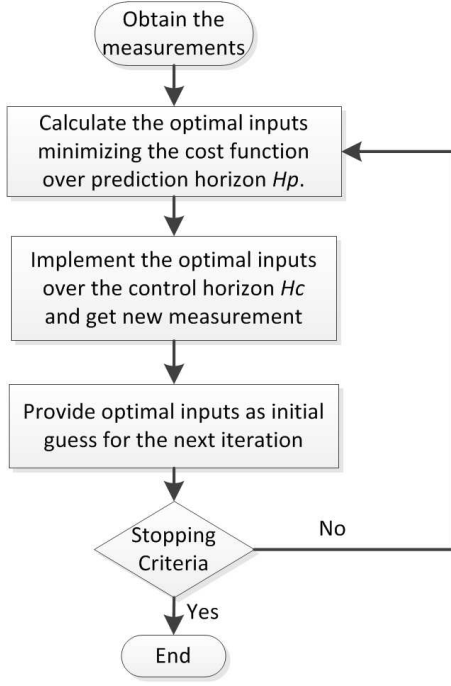


Fig. 2. NMPC flow chart

Another aspect is the stability and performance of the controller. The prediction horizon H_p plays an important role for the stability of the closed-loop system. On one side, if the H_p is short, the controller tries to steer the system to desired states with a more agile behaviour. If it is even shortened, the controller might not be able to stabilize the system around the given equilibrium point. On the other hand, if H_p is long enough, the size of the problem increases, demanding additional computational resources. The OCP defined in Eq.(2) and Eq.(3) is infinite dimensional and discretized to obtain a corresponding Nonlinear Programming (NLP). In this paper single shooting method with uniform discretization points in time is used to approximate an associated NLP of the OCP.

2.2. Discretization of the OCP to associated NLP

In order to discretize the OCP defined by Eq.(2), control inputs are parametrized using single shooting method. The control input $u(t)$ is discretized using uniform discretization points in time t_i , resulting in discrete form $u(t_i)$ of control inputs. Values of the control inputs between the discretization points are approximated by interpolating the discrete control inputs $u(t_i)$ using linear interpolation.

Uniform grid points are obtained by discretizing the time interval into equal-spaced discretization points such that

$$\begin{aligned}
 t_i &\in [t_0, t_f], & i &= 0, \dots, N \\
 x(t_i) &= x_i, & x_i &\in \mathbb{R}^n, & i &= 0, \dots, N \\
 u(t_i) &= u_i, & u_i &\in \mathbb{R}^m, & i &= 0, \dots, N
 \end{aligned} \tag{5}$$

holds. Similarly the discrete form of the objective function is

$$\mathcal{J} = w(x_f - x_d)^2 + \sum_{i=0}^N \Delta u_i^T R u_i + \Delta x_i^T Q \Delta x_i, \quad t_i \in [t_0, t_p], \tag{6}$$

$$i = 0, \dots, N.$$

The trajectories of the states are obtained by integrating the control input by using 4th order Runge-Kutta numerical integration

scheme. In order to find the values of the discrete control inputs at the discretization points, resulting NLP needs to be solved by a numerical solver.

2.3. Solution of the NLP

In this paper, the Broyden–Fletcher–Goldfarb–Shanno (BFGS) method is used to solve resulting NLP. The BFGS method is a quasi-Newton method for solving unconstrained nonlinear optimization problems. As in Newton’s method, one uses a second order approximation to find the minimum of a function $\alpha(z)$. The Taylor series of $\alpha(z)$ around an iterate is:

$$\alpha(z_k + \Delta z) \approx \alpha(z_k) + \nabla \alpha(z_k)^T \Delta z + \frac{1}{2} \Delta z^T B \Delta z, \tag{7}$$

where $\nabla \alpha$ is the gradient and B an approximation to the Hessian matrix. The gradient of this approximation (with respect to Δz) is

$$\nabla \alpha(z_k + \Delta z) \approx \nabla \alpha(z_k) + B \Delta z \tag{8}$$

and setting this gradient to zero (which is the objective of optimization) provides the Newton step:

$$\Delta z = -B^{-1} \nabla \alpha(z_k) \tag{9}$$

Different from Newton method, the BFGS method uses an approximation of the Hessian matrix of second derivatives instead of exact Hessian matrix.

Limited memory BFGS method (L-BFGS) is a modified version of BFGS method, where L-BFGS stores only a few vectors that represent the approximation implicitly. Memory requirement is linearly dependent on the number of optimization variables, which reduces the necessity of computational power.

The L-BFGS-B algorithm further extends L-BFGS to handle simple box constraints on variables. In other words, constraints of the form $u_l \leq u_i \leq u_u$ where u_l and u_u are constant lower and upper bounds of the respective optimization variable. In this paper, a FORTRAN implementation of the L-BFGS-B method is used¹¹⁾.

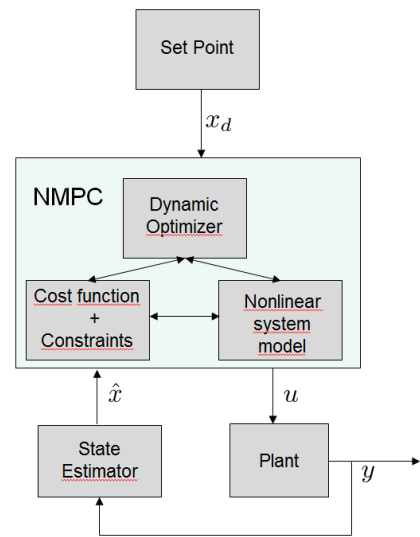


Fig. 3. Implementation of NMPC

The optimization is terminated either when the change in the objective function is sufficiently small, or infinity norm of the projected gradient becomes sufficiently small.

2.4. Numerical Derivatives

In order to obtain the solution of NLP, derivatives of the objective function with respect to optimization variables are needed. These derivatives are obtained using finite difference method by using the Taylor series expansion of the objective function.

$$\alpha(z + \Delta z) = \alpha(z) + \Delta z \alpha'(z) + \mathcal{O}(\Delta z^2). \quad (10)$$

Solving for $\alpha'(z)$ gives

$$\alpha'(z) = \frac{\alpha(z + \Delta z) - \alpha(z)}{\Delta z} + \mathcal{O}(\Delta z). \quad (11)$$

The perturbation value $\Delta z \ll 1$ is problem dependent and chosen by tuning. Obtaining the derivatives of the objective function is the most time consuming phase during solution of the associated NLP. In order to speed-up the computations, the derivatives are evaluated independently using shared memory parallelization with OpenMP. In this way, a master thread shares a specified number of slave threads and the system divides a task among them. These threads then run at the same time, with the runtime environment allocating threads to different cores of the CPU. The division of the tasks are illustrated in Fig. 4., where each slave thread works independently to carry out related commands.

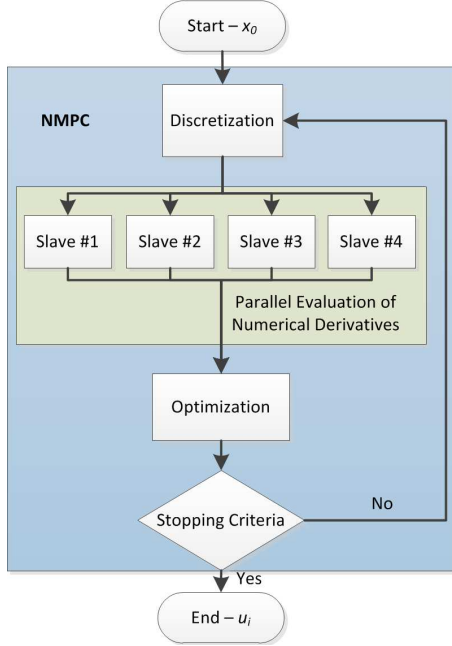


Fig. 4. Division of tasks for computation of derivative of the objective function in parallel.

3. The Nonlinear Dynamical Model of the Upper Stage

Liquid sloshing is the uncontrolled movement of the propellant inside a tank or container. During the maneuver of the upper stage of a LV, this movement acts as a disturbance to the system. There are various passive and active methods to mitigate the effects of the sloshing. For the active methods, the objective is to design feedback controllers so that the controlled LV achieves desired attitude while rejecting the disturbance caused by sloshing effect as shown in Fig. 5..

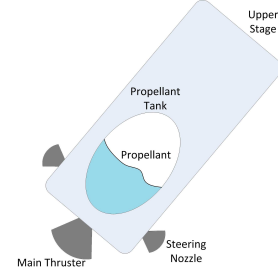


Fig. 5. Upper-stage LV with sloshing effect of propellant

The nonlinear model of the upper stage LV is taken from [12]. A mass-damper-spring system is used to model the nonlinear dynamics of the upper stage LV with the sloshing effect as illustrated in Fig. 6..

The nonlinear dynamical model consists of four states $x = (\theta, \dot{\theta}, \psi, \dot{\psi})^T$, where θ is the pitch angle of the upper stage LV and ψ is the relative angle of the fluid surface with respect to horizontal plane. The control vector includes two control inputs $u = (F, M)^T$ with F is the force provided by the main engine and M denotes the control torque. The control inputs are defined in box constraints, i.e,

$$\begin{aligned} F(t) &\in (0, 400] \text{ N} \\ M(t) &\in [-400, 400] \text{ Nm}. \end{aligned} \quad (12)$$

The state and control vectors satisfy the relation $\dot{x} = \tilde{f}(x) + [\tilde{g}_F(x) \tilde{g}_M(x)]u$ with

$$\tilde{f} = \begin{pmatrix} \dot{\theta} \\ \frac{m_f m_s \sin(2\psi - 2\theta) L^2 \dot{\theta}^2 + 2l m_f m_s \sin(\psi - \theta) L \dot{\psi}^2}{2m_f m_s L^2 \sin(\psi - \theta)^2 + 2J m_f + 2J m_s} \\ \dot{\psi} \\ -\frac{2J L \dot{\theta}^2 m_f \sin(\psi - \theta) + 2J L \dot{\theta}^2 m_s \sin(\psi - \theta) + 2L^2 \dot{\theta}^2 m_f m_s \sin(\psi - \theta) + L^2 \dot{\psi}^2 l m_f m_s \sin(2\psi - 2\theta)}{2l m_f m_s L^2 \sin(\psi - \theta)^2 + 2J l m_f + 2J l m_s} \end{pmatrix} \quad (13)$$

$$\tilde{g}_F = \begin{pmatrix} 0 \\ \frac{L m_f \sin(2\psi - 2\theta)}{2m_f m_s L^2 \sin(\psi - \theta)^2 + 2J m_f + 2J m_s} \\ 0 \\ -\frac{\sin(\psi - \theta)(J m_f + m_s(m_f L^2 + J))}{l m_f L^2 m_s^2 \sin(\psi - \theta)^2 + J l m_s^2 + J l m_f m_s} \end{pmatrix} \quad (14)$$

$$\tilde{g}_M = \begin{pmatrix} 0 \\ \frac{m_f + m_s}{m_f m_s L^2 \sin(\psi - \theta)^2 + J m_f + J m_s} \\ 0 \\ \frac{2L \cos(\psi - \theta)(m_f + m_s)}{l(2J m_f + 2J m_s + L^2 m_f m_s - L^2 m_f m_s \cos(2\psi - 2\theta))} \end{pmatrix} \quad (15)$$

The above equations represent the affine nonlinear dynamics of the system in state space. The parameters associated with the nonlinear dynamics are given in Table 1.

Table 1. Nominal values for parameters

m_s	m_f	L	l	J
10170 kg	3470 kg	1 m	3 m	105000 kgm ²

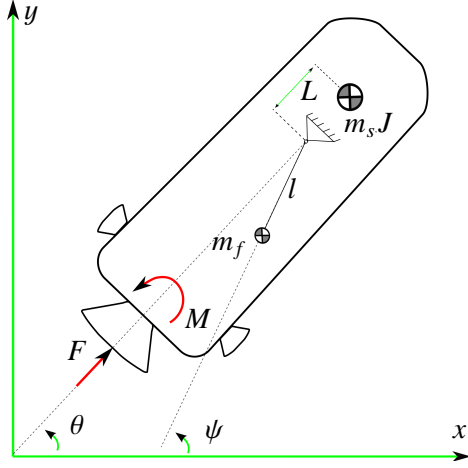


Fig. 6. Analogy of mass-spring system for modeling the sloshing effect of propellant

4. Numerical Results

The closed loop system is simulated for two different scenarios. In the first scenario, the initial angle of the fluid acts as disturbance to the system and the controller stabilizes the plant with the induced disturbance resulting from the sloshing effect of the fluid. In the second scenario, the upper stage LV performs a large angle maneuver while keeping the sloshing effect to the minimum. The computations are performed using an Intel i7 Quad Core CPU operating at 2.80 Ghz and 8 GB of RAM.

The sloshing amount is defined as the quantitative measure for sloshing and defines the relative difference of the pitch angle and the level of the fluid surface. The objective is to obtain the desired states with optimal control inputs while reducing the sloshing amount. During the simulations, the prediction horizon H_p is selected as 60 s and control horizon H_c as 1 s.

Fig. 7. shows the trajectories of the states and control inputs for the simulation of first scenario with the presence of initial disturbance with $x_0 = (60^\circ, 0, 0, 0)$. The NMPC is able to attenuate the sloshing amount, while steering the system to the desired values with $x_f = (0, 0, 0, 0)$.

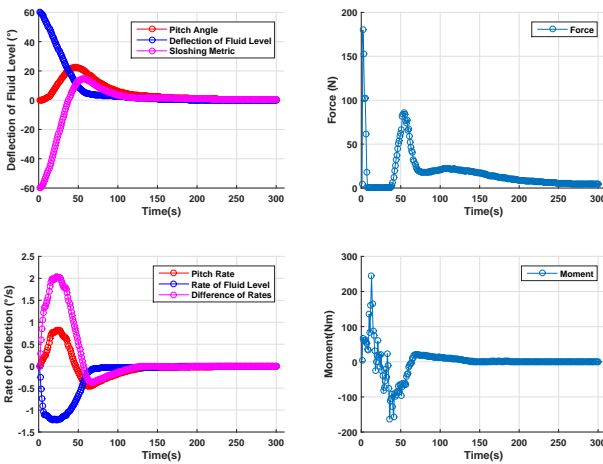


Fig. 7. State trajectories of upper-stage LV with initial disturbance

Similarly, Fig. 8. shows the trajectories of the states and control inputs for the second scenario with a large angle maneuver.

Initially, both the surface level of fluid and the pitch angle of the upper stage LV are equal with $x_0 = (0, 0, 0, 0)$. The NMPC tries to stabilize the system while the upper stage LV performs a large angle maneuver to $x_f = (90^\circ, 0, 90^\circ, 0)$. Previous comments also hold for attitude control of the upper stage LV in terms of obeying constraints and behaviour of the control inputs. In this scenario, the magnitude of the sloshing amount is bounded at around 20° .

Table 2. Comparison of Computation Time for Disturbance Rejection Scenario

	1 st Scenario (ms)	2 nd Scenario (ms)
Sequential Computing	414	583
Parallel Computing	132	188

Table 2 shows the computation time per iteration of NMPC for both cases. It is obvious that if the derivatives of the objective function with respect to discrete control inputs are computed in parallel, the computation time per iteration drops significantly. The speed-up is 3.13 for the first scenario with the Quad Core CPU. In the second case, the speed-up is likewise with the previous case with 3.10. The main reason for why the super linear speed-up is not achieved, is due to the fact that only computation of derivatives was performed in parallel. In the second scenario, the computation time for each iteration is more compared to the first scenario with initial disturbance on the fluid level. It can be deduced that, for the second scenario computing the optimal control inputs is more demanding in terms of computational resources.

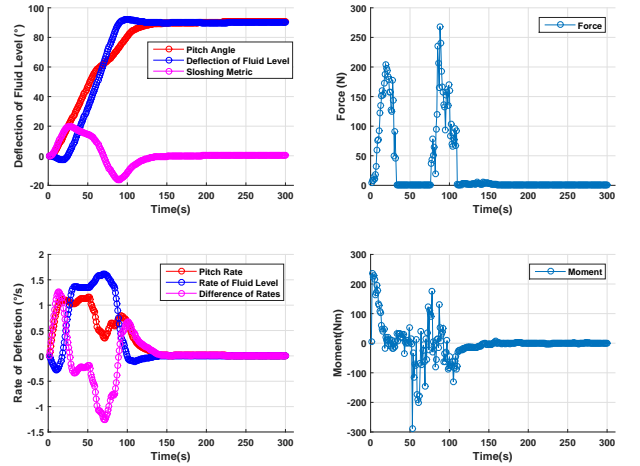


Fig. 8. State trajectories of upper-stage LV with for large angle maneuver scenario

Robustness of the NMPC is shown with Monte Carlo simulations. The pitch angle is varied with $0 \leq \theta_0 \leq 20^\circ$ and rate of fluid level is varied with $-2^\circ/s \leq \psi_0 \leq -2^\circ/s$ using uniform distribution. 250 simulations are carried out with Monte Carlo analysis for 400 seconds for each specific case.

As shown in Fig. 9., all the test cases converge to the desired value while obeying the constraints on the control inputs. Table 3. tabulate the numerical values for the Monte Carlo analysis.

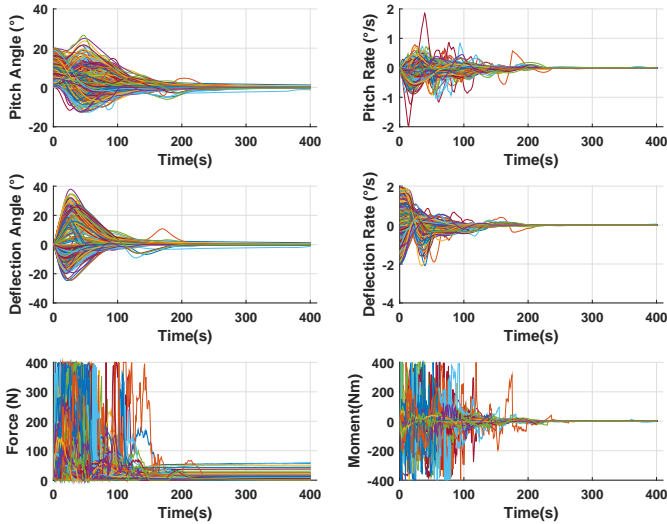


Fig. 9. Monte Carlo Analysis for the NMPC

Table 3. Monte Carlo analysis results

	Mean	Standart Deviation
θ ($^{\circ}$)	0.0164	0.1447
$\dot{\theta}$ ($^{\circ}/s$)	-0.0002	0.0017
ψ ($^{\circ}$)	0.0029	0.1239
$\dot{\psi}$ ($^{\circ}/s$)	0.0001	0.0032
F (N)	6.5562	8.4413
M (Nm)	-0.0036	0.1832

In some cases the input force doesn't tend to zero, which could be explained by the selection of stopping criteria. Although desired states are obtained, the optimizer tries to minimize the objective function, resulting in nonzero thrust values for the first second of simulations. If the simulations are terminated after achieving the desired values with sufficiently small error margins, these nonzero thrust values could be eliminated.

5. Conclusion

In this paper an algorithm for real-time capable NMPC is introduced. The NMPC is established with a given nonlinear dynamical model of the upper stage LV, equality and inequality constraints leading to an OCP. The infinite dimensional OCP is discretized with single shooting method and control inputs are parametrized with equidistant discretization points in time. All values between the discretization points are obtained by using linear interpolation.

During the two simulated scenarios, the NMPC stabilizes the upper stage LV with different initial and terminal conditions. The derivatives of the objective function with respect to optimization variables are evaluated in parallel to decrease computation time per each NMPC iteration. Computation time per each NMPC cycle is shown for these two scenarios. The solutions are obtained for discrete control inputs and state trajectories are evaluated by numerical integration of these control inputs. It was seen that for all the simulation results, the control inputs are bounded with the given upper and lower limits.

Monte Carlo analysis shows the robustness of the solution, ensuring stability with different initial conditions. Further developments include the use of nonuniform grid points in time to

parametrize the control inputs. The selection of prediction horizon H_p on stability and computation time could also be further analyzed.

Acknowledgments

This research is supported by DLR (German Aerospace Center) and DAAD (German Academic Exchange Service) Research Fellowship Programme.

References

- 1) S. J. Qin and T. A. Badgwell, "An overview of industrial model predictive control technology," in *Fifth International Conference on Chemical Process Control - CPC V*, pp. 232–256, 1996.
- 2) S. Qin and T. Badgwell, "An overview of nonlinear model predictive control applications," in *Nonlinear Predictive Control*, 2000.
- 3) F. Allgöwer, T. Badgwell, J. Qin, J. Rawlings, and S. Wright, "Nonlinear predictive control and moving horizon estimation - an introductory overview," in *Advances in Control, Highlights of ECC'99*, 1999.
- 4) L. Grüne and J. Pannek, *Nonlinear Model Predictive Controller (Theory and Algorithms)*. Springer-Verlag, 2011.
- 5) M. Alamir and G. Bornard, "Stability of a truncated infinite constrained receding horizon scheme: The general discrete nonlinear case," *Automatica*, vol. 31(9), pp. 1353–1356, 1995.
- 6) R. Blauwkamp and T. Basar, "A receding-horizon approach to robust output feedback control for nonlinear systems," in *38th IEEE Conference on Decision and Control*, (San Diego), 1999.
- 7) B. Wie, *Space Vehicle Dynamics and Control*. Reston, Va, USA: AIAA Education Series, 1998.
- 8) A. E. Bryson, *Control of Spacecraft and Aircraft*. Princeton, NJ, USA: Princeton University Press, 1994.
- 9) J. M. Adler, M. S. Lee, and J. D. Saugen, "Adaptive control of propellant slosh for launch vehicles," in *Proceedings of the Sensors and Sensor Integration*, 1991.
- 10) M. Reyhanoglu and J. R. Hervas, "Nonlinear control of space vehicles with multi-mass fuel slosh dynamics," in *Proceedings of International Conference on Recent Advances in Space Technologies*, pp. 247–252, 2011.
- 11) C. Zhu, R. H. Byrd, P. Lu, and J. Nocedal, "L-bfgs-b: Algorithm 778: L-bfgs-b, fortran routines for large scale bound constrained optimization," *ACM Transactions on Mathematical Software*, vol. 23(4), pp. 550–560, 1997.
- 12) A. Kheirkhah, "Passivitätsbasierte regelung einer wiederzündbaren kryogenen oberstufe zur unterdrückung des treibstoffschwappens während ausgesuchter manöver," Master's thesis, Leibniz Universität Hannover-Institut für Regelungstechnik, 2013.