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Phase Correlations in Turbulent and Unstable Particle Trajectories

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Introduction

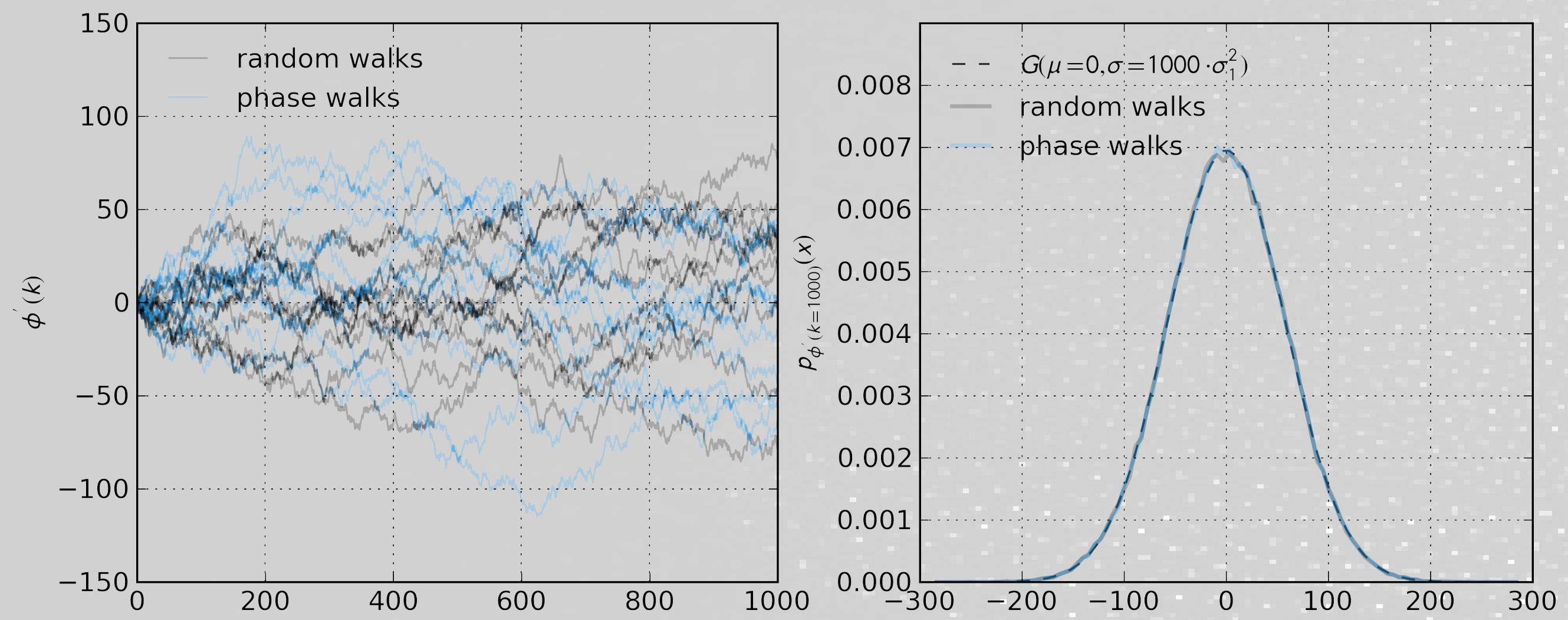
Vortex structures and shear flow instabilities often emerge in three-dimensional dusty-plasma experiments but are hard to characterize. One reason for this is that particles irregularly appear and disappear in a two-dimensional slice of a three-dimensional particle cloud - a result of common data detection setups, where only a plane of the complex plasma is illuminated by a line laser and filmed from an orthogonal angle. This often forces the data analyst to conduct her/his study in a Eulerian frame of reference, allthough, ergodicity is, stricktly taken, not provided in most setups. As an attempt to overcome this partly, we introduce a novel tool that captures nonlinearities by sensitively detecting anomalies in the Fourier phases, even in small data sets. This method allows to trace and quantify turbulent influence in single particle trajectories with outstanding significance.

Phase Walks

The discrete Fourier transform of a time series of particle accelerations $a(t)$ is given by

$FT(a(t)) = \frac{1}{N} \sum_{t=0}^{N-1} a(t) e^{i2\pi kt/N} = |A(k)| e^{i\phi(k)}$

A process, called *unwrapping* [1], can partly restore the *original* phase values $\phi'(k)$, that are not constrained onto the intervall $[-\pi, \pi]$. These series of phases show the characteristics of random walks. E.g., Gaussian noise shows uniformly distributed Fourier phases between $-\pi$ and π, which results in ideal random walks of the unwrapped phases:



SR Test

To test phase walks for anomalies, we apply common measures from random walk analysis. I.e., an adjusted *variance ratio test* can detect and quantify deviations from ideal random walks:

$$SR(\kappa) = \frac{\sigma_\kappa^{TEST}}{\sigma_\kappa^{RW}} = \sqrt{\frac{3 \cdot \sum_{k=0}^{K-\kappa-1} (\phi'(k+\kappa) - \phi'(k))^2}{\pi^2 \kappa (K - \kappa)}}$$

Values of $SR(\kappa)$:	**Behavior:**
$SR(\kappa) \approx 1$ and slowly decreasing	normal
$SR(\kappa) >\sim 3$	trendy
rapid decrease of $SR(\kappa)$	centered

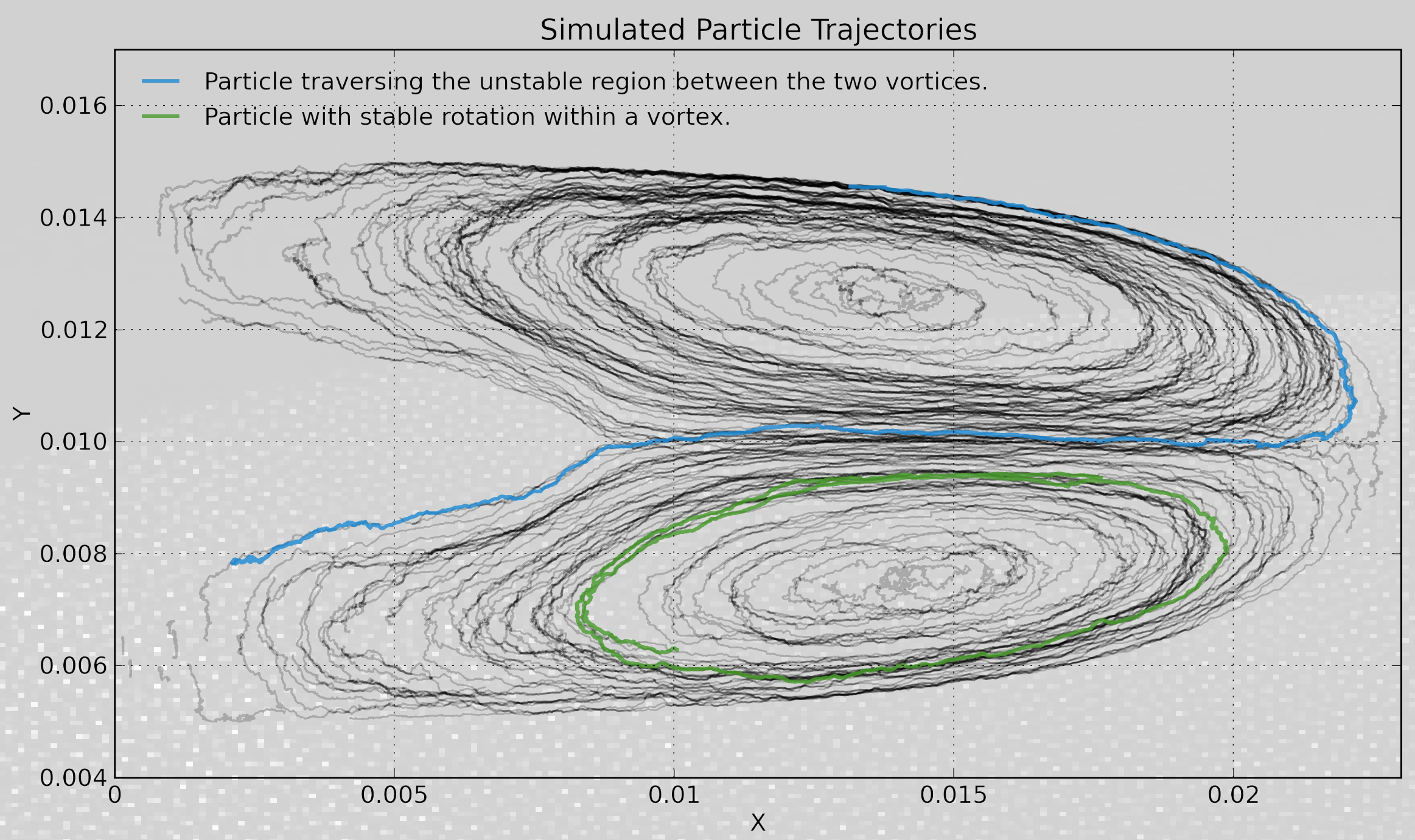
SR Maps

Comparing just the differences $\Delta\phi'(\kappa, k) = \phi'(k+\kappa) - \phi'(k)$ of a measured phase walk to the reference standard deviation yields a two-dimensional analysis:

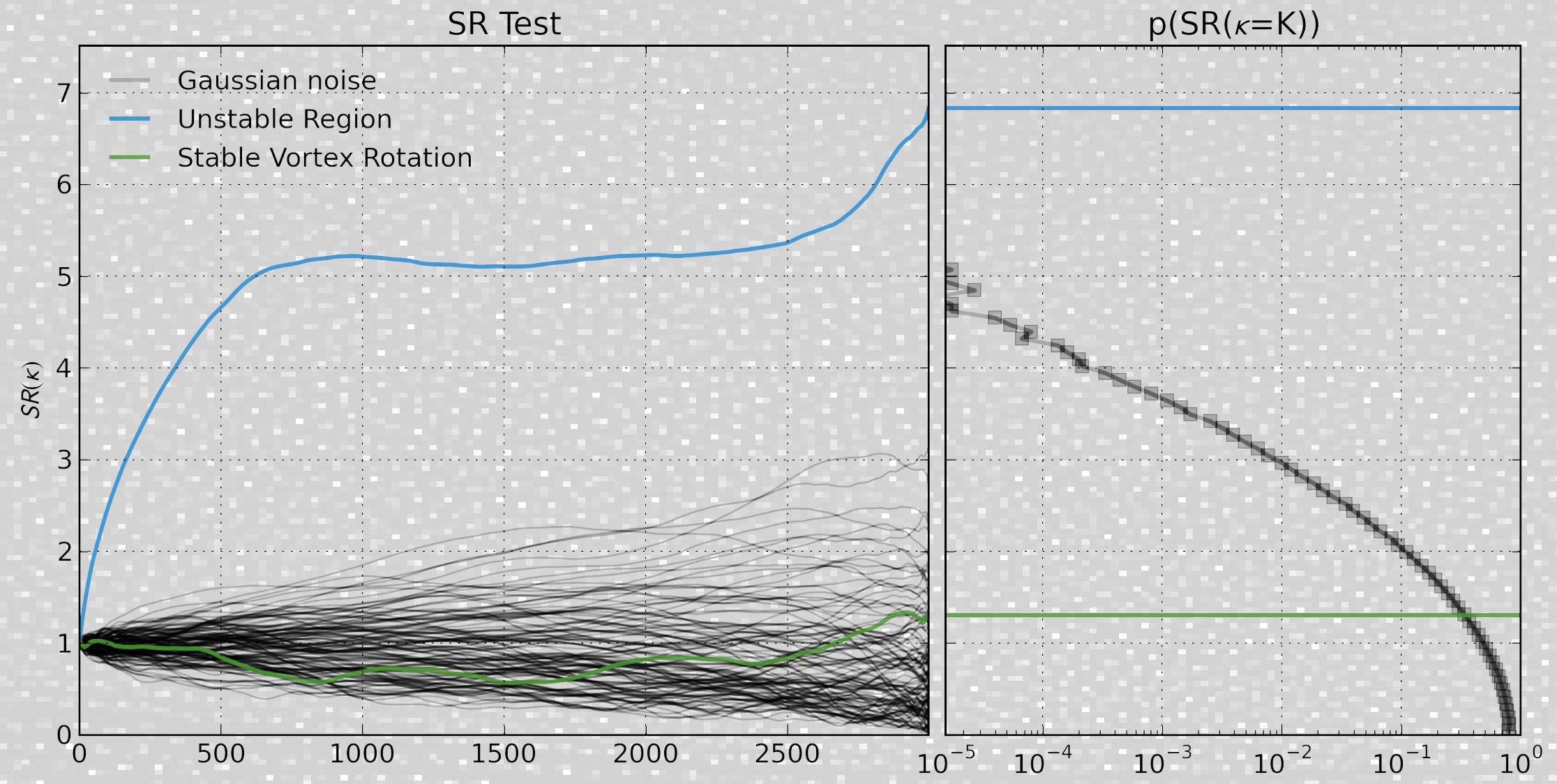
$$SR(\kappa, k) = \frac{\Delta\phi'(\kappa, k)}{\sigma_\kappa^{RW}} = \sqrt{\frac{3(\phi'(k+\kappa) - \phi'(k))^2}{\pi^2 \kappa}}$$

Nonlinearities in simulated data

Simulation of vortex behavior in the PK-3Plus chamber: [2]

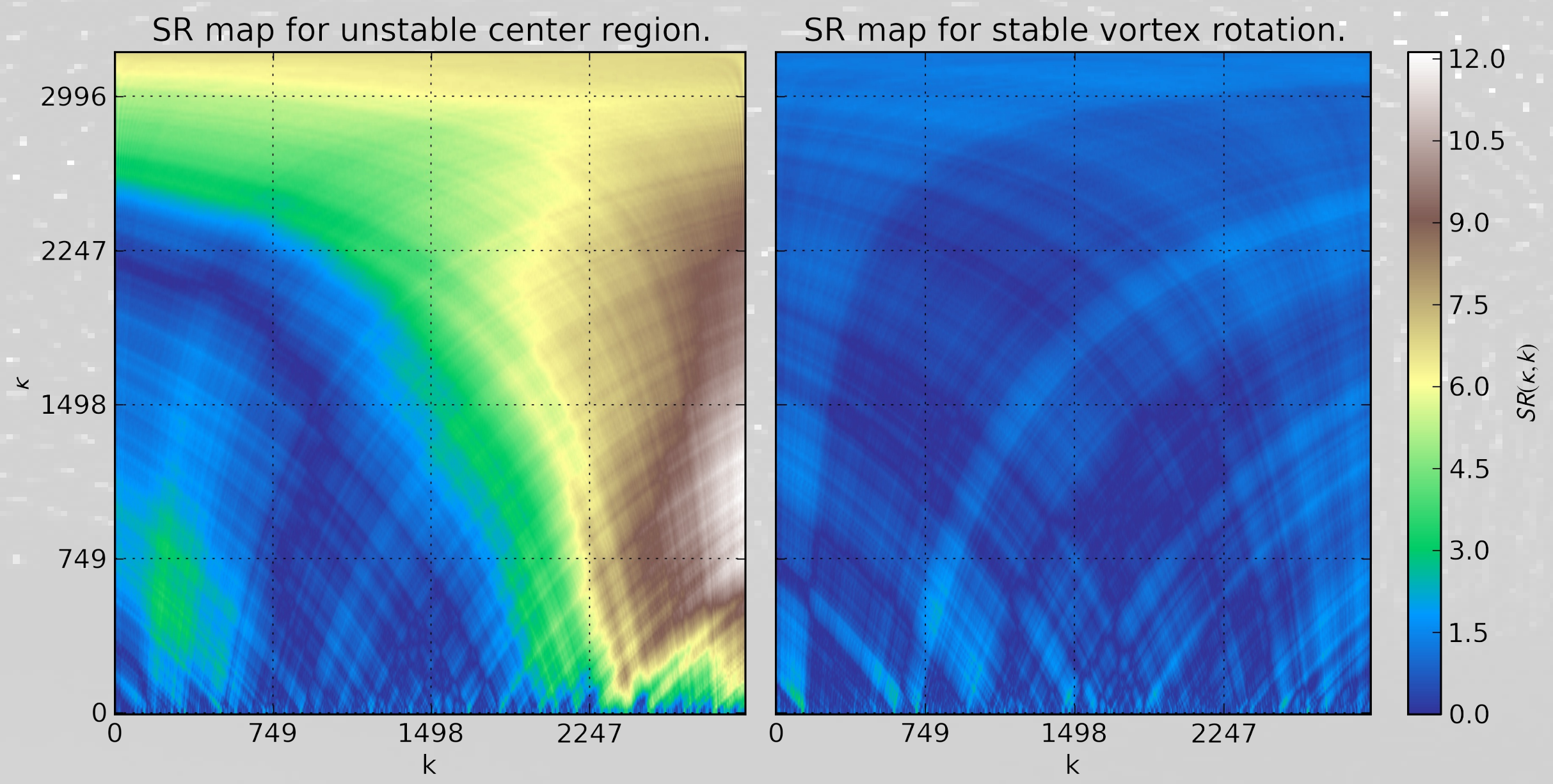


SR test for the two highlighted trajectories:



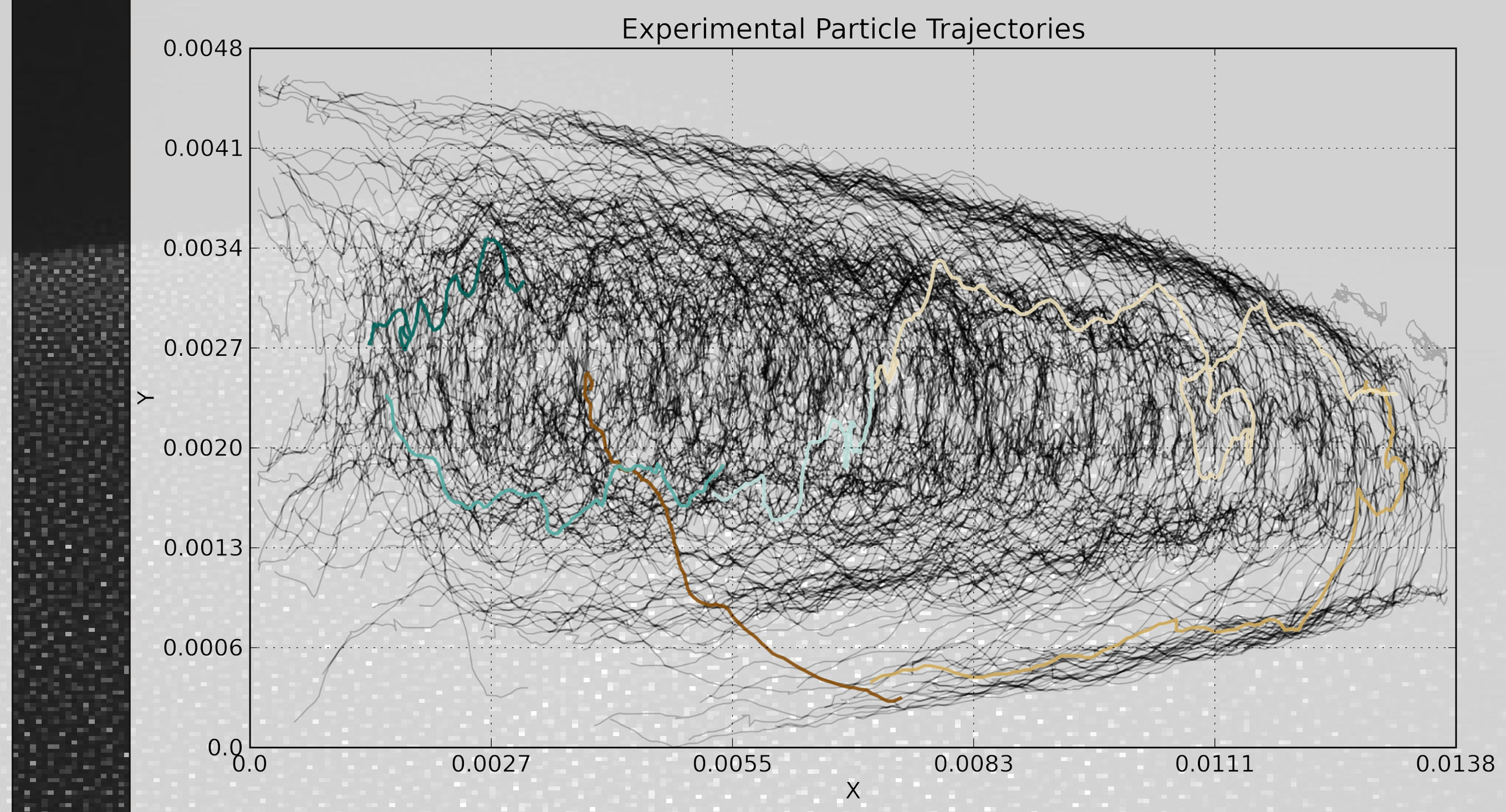
The trajectory traversing the unstable region between the two vortices causes significant diversion in the SR test. On the other hand, a stable rotation within a vortex cannot be distinguished from Gaussian noise.

SR map of the highlighted trajectories:



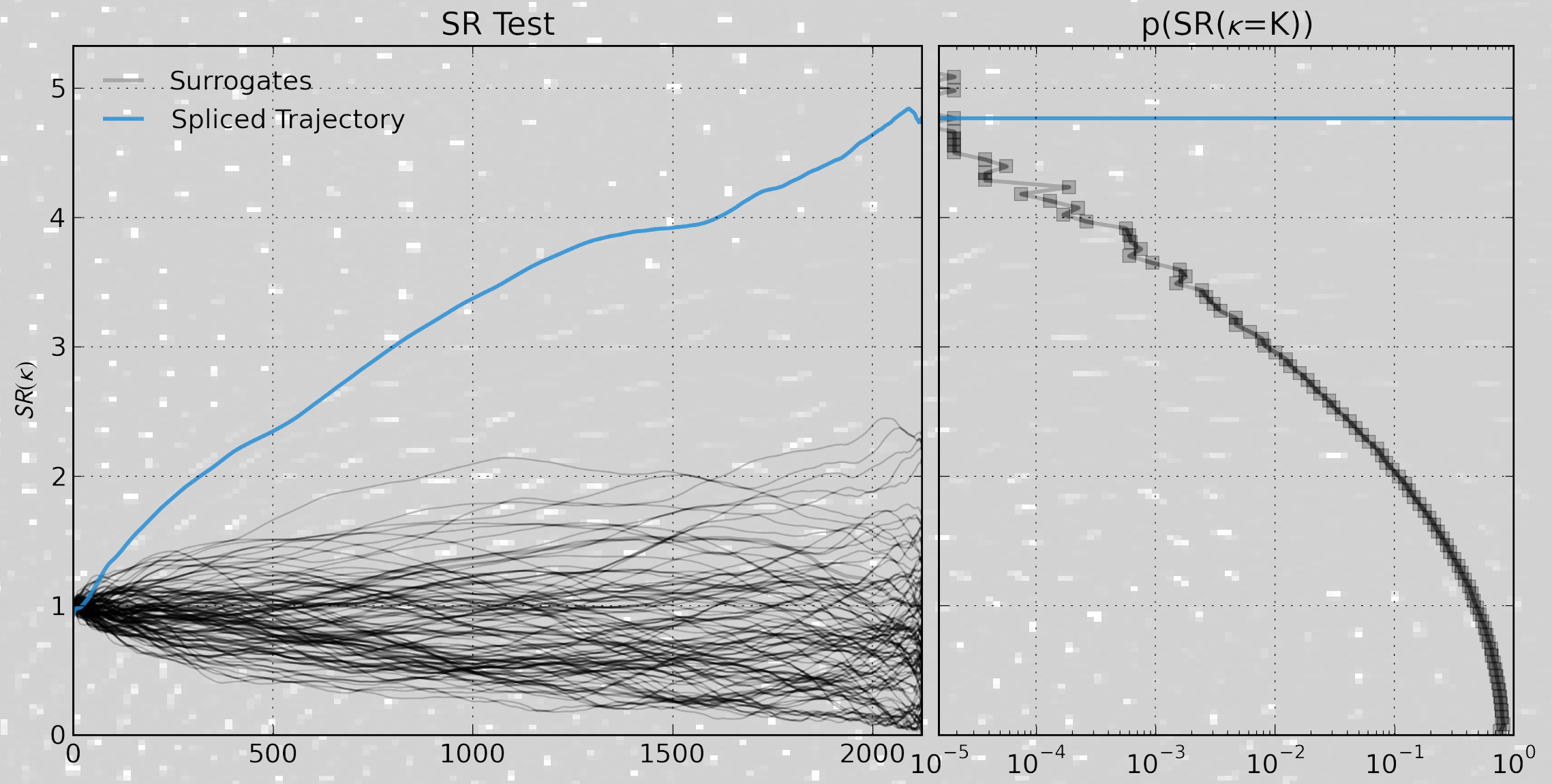
Nonlinearities in experimental data

Chaotic movement of particles in the Zyflex chamber.



Particle trajectories as they appear in the Zyflex chamber. The six highlighted trajectories were selected for data analysis.
Since the particles constantly enter and leave the illuminated slice, trajectory lenths are statistically limited to a few hundred timesteps. With time series of such short lengths no reliable phase walk analysis can be performed.
To overcome this limitation in a very vague way, the six trajectories were joined together. As the interest only lies in the accelerations of the particles, no translational correction had to be applied to obtain smooth splices between the separate sections.

Comparison of the SR Test for the spliced trajectory and 10^6 surrogates simply obtained by shuffling the original time series.



The SR Test result of the spliced trajectory appears with a probability of approximately 10^{-5} under time series with the same probability density function but with random arrangement. This clearly suggests a high degree of nonlinear behavior in the data. It is, however, not possible to decide whether this results from the movement itself or from the splicing process.

Conclusions

Phase walk analysis provides a convenient way to detect and quantify nonlinear anomalies of dynamic particle behavior in single trajectories. It can be used to identify and verify turbulent or unstable influence and regions. To reach statistical significance however, it requires comparably long trajectory lengths, which states a problem for many current data aquisition setups. To obtain good results, an approximate minimum of 5000 samples and a complete transition through the region of interest is required.

Acknowledgements

We would like to thank Christina Knapek for the acquisition of turbulent particle movement in the Zyflex chamber, Mierk Schwabe of providing us with the results of her vortex MD simulation [2], Max Weber for his awesome particle detection algorithm, and Ingo Laut for many fruitfull discussions.

References

[1] Itoh, Kazuyoshi. "Analysis of the phase unwrapping algorithm." *Applied Optics* 21.14 (1982): 2470-2470.
[2] Schwabe, Mierk, et al. "Collective effects in vortex movements in complex plasmas." *Physical review letters* 112.11 (2014): 115002.

Underlying publication:
Schreiber, K. and Räth, C. *Phys. Rev. E* (*submitted*)